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Projection of two-dimensional diffusion in a curved midline and narrow varying width channel on a curved surface

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This study focuses on the derivation of a general effective diffusion coefficient to describe the two dimensional (2D) diffusion in a narrow and smoothly asymmetric channel of varying width that lies on a curved surface, in the simple diffusional motion of noninteracting pointlike particles under no external field. To this end we extend the generalization of the Kalinay-Percus' projection method [1, 2] for the asymmetric channels introduced in [3], to project the anisotropic two dimensional diffusion equation on a smooth curved manifold, into an effective one dimensional generalized Fick-Jacobs equation that is modified due to the curvature of the surface.

The lowest order in the perturbation parameter, corresponding to the Fick-Jacobs equation contains an extra term that accounts for the curvature of the surface by the Christoffel symbols. We found explicitly the first order correction for the invariant effective concentration, which is defined as the correct marginal concentration in one variable, and we obtain the first approximation to the effective diffusion coefficient analogous to Bradley's coefficient [4] as a function of metric elements of the manifold.

Straightforwardly we study the perturbation series up to the *n*-th order, and we derive the full effective diffusion coefficient for the two dimensional diffusion in a narrow asymmetric channel, that have modifications due to the curved metric.

Finally, we present some examples of symmetric surfaces and we study some specific channel configurations on them.

References

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