DEVELOPING LANGUAGE THROUGH COMMUNICATION AND CONVERSION OF SEMIOTIC SYSTEMS

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This paper investigates the use of language by 2 classes of 7th graders in a context involving linguistic exchanges and conversions between representation systems in mathematical setting. The pupils were given tasks requiring the verbal description of a figure by means of a written text with no drawings for one class, and the interpretation of the text for the other. Both the tasks have been dealt with through discussion. The constraints involved in the experiment (communication with people not sharing the same context of situation; conversions between figures, written and spoken texts) have been designed in order to promote the acquisition of linguistic skills suitable for the learning of mathematics.

INTRODUCTION

This study is concerned with the role of language in mathematics learning. Recently various frameworks have been proposed that underline the role of languages in the learning of mathematics. For example, Sfard (2000) interprets thinking as communication and regards languages not just as carriers of pre-existing meanings, but as builders of the meanings themselves. So, under this perspective, language heavily influences thinking. On the other hand, there is evidence that a good share of students' troubles in mathematics, at any school level, including undegraduates, can be ascribed to the improper use of verbal language. More precisely, students often produce or interpret mathematical texts[1] according to linguistic patterns appropriate to everyday-life contexts rather than to mathematical ones (for examples see Ferrari, 1999). The difference is not just a matter of vocabulary, grammar or symbols, but heavily involves the organization of verbal texts, their functions and relationships with the context they are produced within. In other words, as appropriately claimed by Morgan (1998), the distinctive features of mathematical language are not confined to the symbolic component but involve the verbal one as well.

Through the paper I am interpreting mathematical language from the functional linguistic perspective. This means that I am focusing on the functions that language plays in the various contexts. I will use the idea of *register* (defined as a linguistic variety based on use, i.e. the linguistic means developed to express meanings related to some context and goals) to link contexts, texts and the functions that the latter are expected to play. For examples see Morgan (1998), and Ferrari *et al.* (2001). Even if, as suggested by Morgan (1998, p.3) the "unity across mathematical texts that makes it

[1] Through the paper by 'text' I mean any written or spoken instantiation of language, possibly longer than a sentence, not necessarily a textbook.

possible to talk of 'the language of mathematics' … is an illusion", there are nevertheless some features of 'institutional' mathematics registers (e.g., those adopted in research, academic communication or mathematics textbooks, at almost any school level) that are common to a wide range of literate registers. Most of these features, at least in principle, are functionally appropriate for the needs of mathematical practice (but not necessarily for those of learning). Among them, in this paper I will focus on the relationships between a text and the context of situation[2] it is produced within; in most written mathematical registers, texts do not depend upon the context of situation, whereas in everyday-life ones they heavily do. The dependence of a written text on the context of situation may lie, for example, in the reference to space and time (use of expressions like 'this', 'here', 'now', ...) or to participants ('you', 'she', ...) also as physical subjects. The progressive construction of the ability to write and interpret texts not depending on the context of situation is a necessary step for both linguistic and mathematics education. Unfortunately, in mathematical practice and teaching, mathematical registers are often used or taught with little reference to their functions. Students at any level and often teachers are hardly aware of the reasons why certain constructions and styles are adopted and are induced to use them as formal[3] patterns detached from their functions. This is one of the factors that may induce students to deal with language in mathematics without taking care of meanings. It is worth mentioning that in mathematics education students (and teachers) are exposed to continual shift of registers, as it is necessary to adopt both conversational forms to effectively communicate and other forms taken from mathematical registers to express mathematical relationships. Thus a goal of mathematics education as concerns language is promoting flexibility in the use of registers and awareness of the relationships between linguistic forms and contexts and purposes. This is by no means spontaneous but must be carefully promoted[4]. This suggests to work within contexts that underline the functions and the meanings of texts rather than just their linguistic form. This means to design learning situations that force a more and more refined use of language as an answer to specific requirements and constraints explicitly posed by the context. The relative autonomy of the texts from the context of situation, for example, is to be regarded as an answer to explicit (and recognizable) communicative needs (such as communicating with people not sharing the same context of situation) rather than as conformity to some

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[2] The context of situation refers to specific aspects of the situation including also those related to the physical environment such as space, time, participants as physical subjects, and so on. Independence from the context of situation does not imply independence from any level of context.

[3] I am aware that 'formal' is used in literature on mathematics education with a variety of meanings often incompatible each other, such as: formal as 'symbolic'; as 'devoided of content'; as 'institutional'. According to the use prevalent in linguistics, here I mean 'related to linguistic form', in opposition to 'functional'.

[4] This is one of the reasons why I am not using the expression 'natural language' to denote any variety of verbal language in opposition to symbolic one.

linguistic pattern. Methods based on the teaching of grammar and lexicon detached from use can hardly achieve the goal proposed. Most often students exposed to these methods achieve some knowledge of linguistic structures, but can hardly apply this knowledge to scientific contexts.

THE EXPERIMENT

The experiment involved 2 middle-school classes (namely, A and B) located in different places (both in rural areas in North-Western Italy). Class A had to solve a measurement problem involving the ground floor of their school and to propose the same problem to the other class, communicating the data in verbal written form with the help of no drawings. The task of class B was to reconstruct a drawing, based on the verbal description only and with no knowledge of the building, to solve the problem and to submit their answers. Class A was then requested to evaluate the other pupils' solution and point out both ambiguities in their own verbal text and improperties in the drawing or in the solution, if any. All the tasks were dealt with by the whole classes and the answers were adopted through discussions which were guided by a teacher and a graduate prospective teacher, leaving the pupils completely free to decide the formulation of the problem or its interpretation.

A priori analysis

The situation involves 2 kinds of constraints both relevant from the semiotic perspective. First, the requirement of communicating (in written form) with unknown pupils (even if of the same age and from a similar socio-cultural environment) asks for an effort (mostly by class A) to make explicit a portion of the knowledge that usually is left implicit in spoken conversations, as writers and addressees did not share the same context of situation. Second, the requirements of converting[5] a geometrical figure into a text or a text into a figure asks for some explicitation and selection of the relationships embodied in both the representations. Both the requirements should prevent pupils from exploiting the context of situation and force them to convey all the information in explicit form and to reflect on the pieces of knowledge they could assume as shared by the other pupils. The situation essentially involves figural, verbal spoken and verbal written expressions. Written and spoken productions may play different cognitive functions (see Duval, 2000, for a convincing account); it may be expected that written texts and drawings should be used as relatively stable products to reflect upon, whereas spoken language could be used to explore provisional, unsettled ideas, to compare different opinions (exploiting interpersonal relationships and processes) and to focus on the aspects regarded important, temporarily neglecting the others.

All the sessions have been audio-recorded. To illustrate the arguments adopted by pupils I include some transcripts and try to analyse them. A deeper linguistic analysis

[5] The cognitive role of conversion of semiotic systems has been widely discussed by Duval (1995).

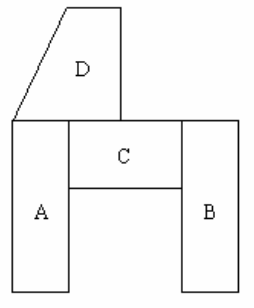
of the transcripts requires to deal with the original texts, as the english translation, although it may properly convey basic aspects of the text, may fail in preserving some features such as register and focus.

OUTCOMES

Constructing a text

Pupils of class A chose to base their text on the drawing in Fig.1. They agreed to describe the problem situation by means of the text reported below.

Fig.1



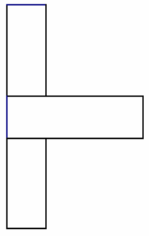
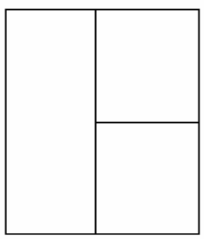
Notice that they chose to give measures referred to the drawing, rather than to the real building. I have tried to translate the text as literally as possible. To make reference easier, the sentences are numbered.

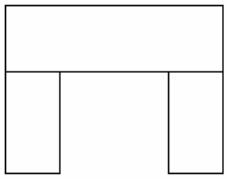
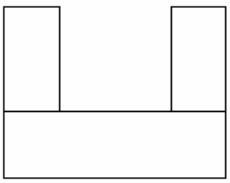
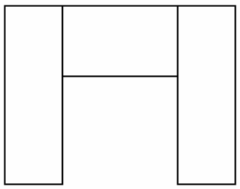
(1) Our school is much alike a cradle seen in profile.
(2) Our building consists of 3 rectangles, 2 of which are placed vertically and one horizontally, which joins them in the upper part.
(3) Let us name the 2 vertical rectangles A and B and the horizontal one C.
(4) Trapezium D (which is our gymnasium) is right-angled and is placed on rectangle A and part of rectangle C, with its oblique side adjacent to the altitude of rectangle A.
(5) The two rect.A and B are equal.
(6) Now we give you measures: the base of rect.A (thus of B too) is 11 cm long, and the altitude 21 cm.
(7) The base of rect.C is 22 cm long and its altitude is equal to the altitude of rectangle A minus a 10 cm recess.
(8) In trapezium D the greater base laid on the 2 rect.A and C is 18 cm long and the smaller one 16 cm, altitude is 19 cm long.

Interpreting the text

All the pupils of class B have been given a printed copy of the text which has been also read aloud by the teacher. Pupils immediately produced a number of drawings based on the first two sentences only. They are reproduced in figures 2.A-F, with on the right some of the arguments produced to discard some of them.

Fig.2

A 	**B** 	A: "The horizontal rectangle doesn't join both in the upper part, but one in the upper and one in the lower." B: "There are two horizontal rectangles, the text speaks of just one."

C 	**D** 	C: "It is not like a cradle seen in profile, maybe it is a front view." "Schools aren't like that." E: "The horizontal rectangle joins the other two in the lower, not in the upper part." F: "The horizontal rectangle joins them sideways, not in the upper part."
E 	**F** 	

After some discussions, all pupils agreed to discard drawings A, B, C, E. The choice between D and F elicited further discussions, focused on the interpretation of "... *which joins them in the upper part*". Here is an excerpt from an exchange between Alessandro, a boy supporting drawing F and Barbara, a girl supporting drawing D.

Barbara: "The first drawing doesn't work because it doesn't join them in the upper part, it joins them sideways"

Alessandro: "The triangle below ..."

Barbara: "It's a rectangle!"

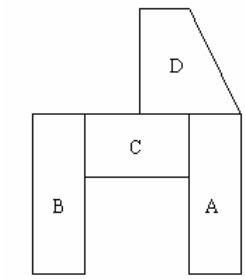
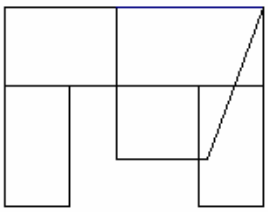
Alessandro: "Ok, the rectangle, it's the same, is joined in the upper part, but the triangle, joins them in the lower part."

Barbara: "There is no measure that tells us which is the upper part."

Through the whole discussion Alessandro often used 'triangle' in place of 'rectangle'.

When the teacher asked him "Where is your triangle?" Barbara replied for him: "Oh, he means 'rectangle', you know".

After reading the text up to sentence (4), all pupils agreed in choosing the drawing in fig.3. Immediately after 2 questions arose: the interpretation of 'on' ("...placed on rectangle A ...") and of 'adjacent'.

Fig.3	**Fig.4**
	

As regards the first, some pupils proposed drawings like that in fig.4, which were discarded at once by arguments like "It isn't like a cradle anymore" or "Where you put your trapezium there are the classrooms"

As regards the second question, pupils were puzzled by 'adjacent'. Barbara actually searched a textbook and found out that 'adjacent segments' are collinear ones. After some discussion, the contradiction between the definition of 'adjacent' and the other information was disregarded and pupils decided to adopt the drawing in fig.3 and to solve the problem accordingly.

Rearranging the text

Pupils of class A were keenly interested by the report of the interpretation of their text by class B. The discussion focused on the fact that the drawing produced by class B was not congruent to their own, even if the values of area found by both classes was the same and on the interpretation of 'adjacent'. In both cases they conceded that the interpretations produced by class B were legitimate. The text was revised by modifying sentences (3) and (4) as follows:

(3) "Let us name A the vertical rectangle on the right, B the one on the left and C the horizontal one."

(4) Trapezium D (which is our gymnasium) is right-angled and laid[6] on rectangle A and part of rectangle C, with its oblique side consecutive to the altitude of rectangle A."

Some doubts have been raised about the correctness of referring to 'left' and 'right' ("They aren't mathematical words")[7]. The whole class approved the following final remark by Barbara: "Next time we should throw four or five of us out of the classroom, then write down the text, let them in and see if they can draw the figure."

DISCUSSION

The experiment has provided a wide range of suggestions and ideas worth to be developed. Here I am focusing on some of them only.

Pupils of class A adopted various means to describe the figure, from the metaphoric reference to a cradle to the introduction of letters to name the basic figures involved. The need for communicating (along with the feedback received) forced pupils to reflect on the meaning of some of the expressions they were using (e.g.: 'upper part', 'adjacent', 'placed/laid'). The outcome has not been the adoption of a register nearer to those adopted by textbooks, but rather the exploitation of all the linguistic and cultural resources available in the context they were acting within. The use of letters to name figures, for example, is different from standard school notation, according to which capitals would represent points; the same holds for their use of 'adjacent' or the replacement of 'placed' by 'laid'.

As regards class B, at least two points of the interpretation process are worth mentioning: the interplay between the text and the drawings produced, and the spoken register adopted. When interpreting the text, pupils proposed a number of drawings immediately after reading sentences (1) and (2), and ruled most of them out after discussion or after reading sentences (3) and (4). Some of these drawings were clearly inconsistent with the first two sentences. Pupils may have paid more attention

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[6] The original italian words were 'posto' (placed) in the first version, 'appoggiato' (laid) in the revised one. 'appoggiato' suggests the idea of physical contact (including weight) much more than 'posto' (which is a more formal word common in standard geometry vocabulary).

[7] In the same context, 'top' and 'down' did not raise any doubts.

to the reading aloud of the text, rather than to their printed copies, and to some isolated words (not necessarily the same for all) rather than to the text as a whole. Anyway, this behavior may be explained by the need for an explicit representation as a common ground for reflections and discussions that may help in taking into account the meanings of the text neglected at first. Generally expert people neglect at once clearly inconsistent interpretations and rule them out through inference (or other methods) with no need for explicitly representing them. Most likely, they can deal with a text as a whole and can select the features that are relevant directly from the text; these behaviors are all typically associated to the use of literate registers. For pupils, on the contrary, reflection on explicit models and discussion and comparison of different opinions (with the activation of interpersonal processes based on their mutual relationships) seem to play a relevant function in the interpretation process. Also the final choice of drawings sometimes has proved inconsistent with the interpretation of some parts of the text. When interpreting sentence (4), no pupil objected against Barbara's interpretation of 'adjacent', which was inconsistent with the drawing adopted; nonetheless this was not rejected, maybe because the reference to a cradle or their knowledge of what schools are like were regarded (more or less consciously) as more reliable than the interpretation of a single word, taken from a textbook. Processes like those described here are quite common in everyday-life interpretation processes, whereas in mathematical registers consistency and the meaning of each word are more important.

As regards the spoken register adopted, it may appear that in some exchanges pupils have been quite inaccurate; Alessandro, for example, employs the word 'triangle' in place of 'rectangle' but the others nonetheless do understand what he is saying. Some other pupils discussing fig.2B speaks of 'two horizontal rectangles', referring to a drawing where all the rectangles have altitudes longer than their bases. These behaviors are not uncommon in spoken interactions; they might even be regarded as good features of everyday-life registers, as they allow to build sentences focusing on some aspects only (for Alessandro, the mutual positions of the figures rather than their classification) without paying too much attention to the others. So Alessandro's 'triangle' is not meant to express the whole of the meanings usually attached to it in mathematical registers but rather works as an indexical (such as 'this one' etc.), maybe associated with some gesture of hands, and is strictly related to the context of situation: his classmates understand him quite well whereas pupils of class A most likely would not understand a written transcription of his argument. Mathematically speaking, these behaviors are errors, but it is not deniable that in everyday-life registers they are standard ways of constructing and interpreting texts.

Teaching implications

The results of the study point out that 7^{th} graders may successfully use verbal language to represent mathematical ideas and communicate them to peers not sharing the same context of situation. It seems that the methods they use are quite different from those adopted in institutional mathematical registers (in particular, textbooks).

This paper has presented examples of linguistic behaviors that are inconsistent with the organization of standard mathematical registers (e.g. 'inaccurate' use of some words; lack of consideration for inconsistency) but may be regarded as effective for the purposes of communication and thinking. It seems that pupils attempt to use all the resources available at the level of context they recognize as suitable to carry out communication successfully. For example, pupils of class A when writing the text do not rely on the physical environment of their school, but do use other pieces of knowledge they expect to be shared with the other pupils (reference to a cradle, to geometry vocabulary and so on). All this means that one cannot expect that pupils (and even students at almost any level) can recognize standard mathematical registers as the answer to actual representation and communication needs, as the methods they would adopt to deal with the same needs are quite different. It is necessary from the one hand to avoid any use of mathematical language (not just lexicon but also construction and interpretation processes) that does not correspond to recognizable representative and communicative needs, from the other hand to design activities (like the one I have described) that force pupils to produce and interpret texts accurately without asking them to conform to the patterns proposed by institutional mathematical registers.

In my opinion, further research should focus on the long-term outcomes of activities like those I have described in the paper.

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