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ANALYSIS OF EMPIRICAL OBSERVATIONS ON THE SCATTERING OF SOUND BY ENCAGED AGGREGATIONS OF FISH

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ABSTRACT

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The experimental findings of RØTTINGEN (1975 and 1976) for the scattering of ultrasonic sound by encaged aggregations of saithe and sprat are analyzed. The insensitivity of the relationship of the mean time-integrated echo intensity $\bar{\varepsilon}$ and fish number density ν to both the center frequency and pulse duration of the ensonifying signal is considered qualitatively. A general theory for the scattering of sound by a collection of randomly distributed and oriented, but otherwise identical scatterers, whose individually complicated scattering behavior is described by two parameters, the backscattering and extinction cross sections, is applied to Røttingen's experiment with saithe. The empirical ε–ν relationship is reproduced successfully with respect to a unique set of parameters of a model whose main ingredients are the following: assumption of a truncated Gaussian distribution in tilt angle for the orientation distribution; expression of the scattering cross sections in terms of the mean and variance of this distribution; assumption that the mean tilt angle is independent of ν and that the variance in tilt is the sum of two variances: the intrinsic variance, which is postulated to be an exponentially decreasing function of ν, and the perspectival variance, assumed constant, which is the mean square apparent tilt of the scatterer due solely to its azimuthal orientation relative to the generally obliquely located source/receiver. By comparing predictions of the $\bar{\varepsilon}$–ν relationship with those observed for saithe, the mean extinction cross sections at the two frequencies of Røttingen's experiment are deduced.

INTRODUCTION

In an experiment performed by Røttingen in 1973 (RØTTINGEN 1975 and 1976), which was designed to help clarify the relationship between acoustic echo energy and the number density of schooling fish when ensonified by ultrasonic pulses, the echoes from encaged aggregations of fish of uniform kind and size were observed. By elementary signal processing the information contained in each echo was reduced to a single number, the time-integrated echo intensity ε. Because of the considerable pulse-to-pulse variability in this quantity, many determinations of ε were made at each observed number density ν. A large number of independent samples were then drawn from this population of measurements and their average, $\bar{\varepsilon}$, computed with respect to this ensemble, so that the true $\bar{\varepsilon}$ at a particular number density would be known with a high degree of confidence. This determination of $\bar{\varepsilon}$ was made systematically over a wide range of ν, which probably spans that observed in nature, for each of three species of fish: *Pollachius virens* (L.) or saithe, *Sprattus sprattus* (L.) or sprat, and *Scomber scombrus* L. or mackerel.

The surprising finding of the observations on the saithe and sprat was that the relationship of $\bar{\varepsilon}$ and ν is essentially non-unique, and is distinguished primarily by fish kind and size and only secondarily by pulse duration and center frequency. The basic ν-dependence of $\bar{\varepsilon}$ is the following: linear proportionality up to a certain density; thereafter, a steady decrease in the rate of increase of $\bar{\varepsilon}$ with ν until a maximum is reached; then, a decline in $\bar{\varepsilon}$ for still higher values of ν. Thus there are values of $\bar{\varepsilon}$ which obtain at each of two distinct densities.

The observations of $\bar{\varepsilon}$ for the mackerel were quite irregular. Because these fish were observed, by mean of an underwater camera, to cluster along the netting of their cage, rather than to distribute themselves more or less uniformily throughout it as both the saithe and sprat did, their observations are not considered further in this study.

It is the aim this paper, then, to explain the basic observations of Røttingen for saithe and sprat. This will be done firstly on a qualitative level: the approximate insensitivity of the $\bar{\varepsilon}$–ν relationship to the pulse duration and center frequency of the ensonifying signal will be explained and, through a dimensional analysis of the empirical findings, the requirements for a quantitative theory will be established. A general theory for the scattering of sound by collections of scatterers, which is developed in the Appendix, will then be applied to Røttingen's experiment with saithe. A similar detailed quantitative analysis for sprat will not be carried out because of the lack of ventral aspect target strength data for sprat, which will be seen to be of crucial importance in the determination of the precise form of the $\bar{\varepsilon}$–ν relationship.

METHOD

QUALITATIVE ANALYSIS OF RØTTINGEN'S OBSERVATIONS

For convenience Røttingen's results are reproduced in a condensed format in Fig. 1 and 2. In both figures, which are distinguished by fish species and size, the empirical relationship of the mean or ensemble-averaged time – integrated echo intensity $\bar{\varepsilon}$ and fish number density ν is

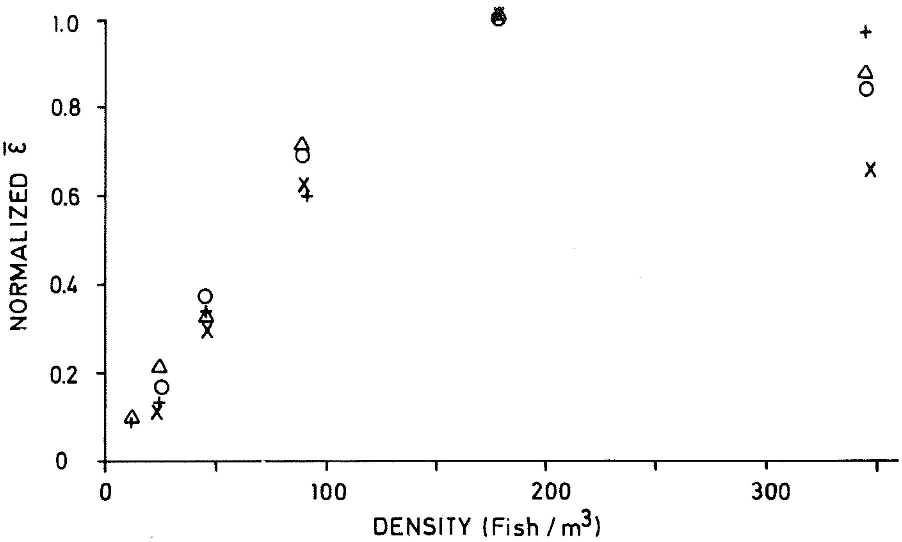

Fig. 1. Røttingen's observations of the normalized mean time-integrated echo intensity $\bar{\varepsilon}$ for saithe when ensonified by a narrowband signal for four conditions of pulse duration and center frequency: 0.3 msec at 38 kHz (o), 0.6 msec at 38 kHz (△), 0.1 msec at 120 kHz (+), and 0.6 msec at 120 kHz (x).

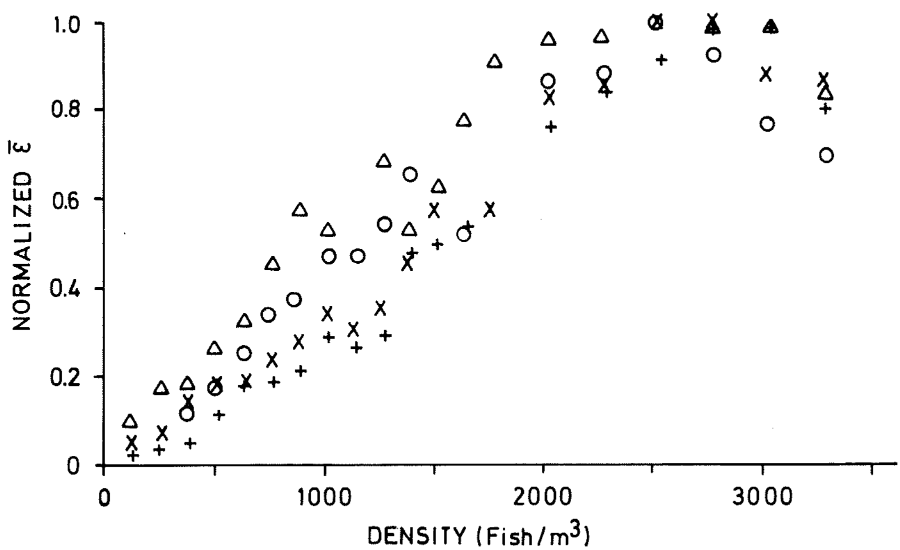

Fig. 2. Røttingen's observations of the normalized mean time-integrated echo intensity $\bar{\varepsilon}$ for sprat when ensonified by a narrowband signal for the same four conditions of pulse duration and center frequency shown in the legend of Fig. 1.

stated for the following four combinations of pulse duration T and center frequency $\omega_o/2\pi$ of the ensonifying signal: $T = 0.3$ and 0.6 msec for $\omega_o/2\pi =$ 38 kHz and $T = 0.1$ and 0.6 msec for $\omega_o/2\pi = 120$ kHz; all for a pulsed sinusoid of the form $\mathrm{rect}(t/T)\cos(\omega_o t)$, as expressed in the notation of WOODWARD (1964). The four empirical $\bar{\varepsilon}$–ν relationships are stated for saithe of uniform length distribution 35.1 ± 0.6 cm and mean mass 375 g in Fig. 1 and for sprat of uniform length distribution 12.1 ± 2.3 cm and mean mass 12 g in Fig. 2.

In view of the wide ranges in center frequency and pulse duration represented by the data, it is concluded that the $\bar{\varepsilon}$–ν relationship is generally insensitive to both center frequency and pulse duration of the ensonifying signal, at least for the narrowband pulsed sinusoidal type, and is distinguished primarily by fish kind and size.

That the $\bar{\varepsilon}$-ν relationship is relatively insensitive to frequency is not difficult to explain. This is because all estimates of a characteristic scattering size a indicate that the characteristic size-to-wavelength ratio is such that the proportional quantity $ka = 2\pi a/\lambda$, where k is the wavenumber and λ is the wavelength of the signal at its center frequency, is always in excess of unity and generally is much greater than unity. Such estimates of the characteristic scattering size include those deduced from the following three sources: (1) the gross dimensions of the fish (RØTTINGEN 1976, RASMUSSEN 1960, NAKKEN pers. com.); (2) the size of the swimbladder, the chief scattering organ of these species of fish, based on the estimate that the swimbladder occupies roughly 5 per cent of the total volume (SHIBATA 1970, MCCARTNEY and STUBBS 1971); and (3) measurements of target strength, assuming approximate equality of peak ventral and dorsal aspect target strengths (NAKKEN and OLSEN 1973). In the limit that the size is much larger than the wavelength, the phenomenon of scattering is essentially geometric, and those quantities which describe the scattering behaviour of a body, for example, a fish, adopt their respective constant high frequency values. Thus, other things remaining unchanged, a change in magnitude of the center frequency of the ensonifying signal, if sufficiently high, say, above 38 kHz, should not affect the relationship of $\bar{\varepsilon}$ and ν.

The relative insentitivity of the $\bar{\varepsilon}$–ν relationship to changes in pulse duration can also be understood rather simply. The photographic evidence of Røttingen's experiment showed the spatial distribution of fish in their net cage to be generally randomly homogeneous throughout the entire volume of the cage. The pulse-to-pulse variations in the time-integrated echo intensity were large, suggesting both the importance of coherence for particular ε and the fact that the fish did not remain stationary, but moved about. That this internal movement provides a mechanism for the randomness of ε is clear, for while the ensonifying signal was narrowband, with $\omega_o T >> 1$ for all experimental conditions of ω_o and T, the fish densities were always such that the mean nearest-neighbour fish distance was much greater than the wavelength, i.e. $v^{-1/3} \gg \lambda$. Thus the phases or relative times of the constituent echoes from individual fish, which compose the whole echo, are entirely random. The effect of this is to cause the coherent contribution to ε to vanish in the mean of large numbers of independent observations of ε at particular ν, so that $\bar{\varepsilon}$ is equal to the irreducible incoherent contribution alone. This non-vanishing component of $\bar{\varepsilon}$ is linearly proportional to T as the energy contained in each constituent echo is exactly proportional to the energy contained in the ensonifying signal, which for the pulsed sinusoids of Røttingen's experiments is proportional to T. The basic form of the $\bar{\varepsilon}$–ν relationship, when normalized by or scaled to the maximum value of $\bar{\varepsilon}$ for the particular experimental conditions of ω_o and T, should, therefore, be independent of the pulse duration T.

The implication of the insensitivity of the $\bar{\varepsilon}$–ν relationship to ω_o and T for both the saithe and sprat of Røttingen's experiments is that the phenomenon of echo formation by an encaged aggregation of fish at ultrasonic frequencies is primarily geometric and incoherent. The principal evidence for this conclusion is that a variation in frequency by a factor of three and a variation in pulse duration, thence phase, by a factor of six, with corresponding changes in the scatterer size-to-wavelength ratio and mean scatterer separation-to-pulse length ratio, have only an indiscernible effect on the ν-dependence of $\bar{\varepsilon}$. If the mean echo strength were very dependent on the coherent or physical effect of interference among the constituent echoes, then there almost certainly would be considerable variations in the $\bar{\varepsilon}$–ν relationship as T and ω_o were varied; yet, what variations are present are slight and apparently even too inconsistent to justify speculation about their origin. The fact that the $\bar{\varepsilon}$–ν relationship is stable with respect to large changes in both T and ω_o for different kinds of fish of uniform size distributions, but of widely differing mean lengths and masses, strengthens the conviction that a purely geometric theory of scattering should be entirely adequate to explain the quantitative features of the $\bar{\varepsilon}$–ν relationship.

Such a theory, which is developed in the Appendix and which has some common features with the theory of acoustic scattering by wakes (WILDT 1947), is applied to the problem of Røttingen's experiment with saithe in the next section.

QUANTITATIVE ANALYSIS OF RØTTINGEN'S OBSERVATIONS

I Scattering of sound by an aggregation of fish

The theory for acoustic scattering by a collection of randomly distributed and oriented, but otherwise identical scatterers, which is presented in the Appendix, can be applied to the problem of acoustic scattering by an aggregation of fish if the following assumptions are made:

(1) the acoustic source and receiver are essentially collocated;
(2) the ensonifying signal is narrowband and of such a center frequency that the only significant multiple scattering effect is that of extinction of the incident wave;
(3) the amplitude of the signal is sufficiently weak so that all nonlinear effects can be ignored;
(4) the fish distribute themselves randomly homogenously throughout a definite volume which is fixed relative to the source/receiver and in the farfield of the source/receiver;
(5) the fish are identical in size;
(6) the scattering parameters of a single fish can be represented by exactly two parameters, the backscattering and extinction cross sections, which generally are functions of the orientation of the fish.

When these conditions are satisfied, according to the results of the Appendix, the mean or ensemble-averaged time-integrated echo intensity $\bar{\varepsilon}$ can be written

$$\bar{\varepsilon} = \overline{\frac{I_o \varrho(0)}{4\pi} \sum_n \frac{D_{S,n}^2 D_{R,n}^2}{r_n^4} \sigma_{b,n} \exp[-4\alpha r_n - \nu \bar{\sigma}_e (r_n - r_{n,o})]}$$

where the overhead bar denotes the operation of ensemble-averaging, and where the several quantities have the following meanings

I_o is the peak instantaneous intensity of the ensonifying pressure field at unit distance from the source;
$\varrho(0)$ is the maximum value of the signal autocorrelation function;
$\sigma_{b,n}$ is the backscattering cross section of the n^{th} scatterer;
$D_{S,n}$ and $D_{R,n}$ are the respective source and receiver pressure directivity factors in the direction of the n^{th} scatterer;
r_n is the distance between source and n^{th} scatterer;
α is the attenuation rate of the medium at the center frequency of the ensonifying signal;
ν is the scatterer or fish number density;
$\bar{\sigma}_e$ is the ensemble-averaged extinction cross section;
$r_{n,o}$ is the distance from source to nearest surface of the volume in the direction of the n^{th} scatterer.

II. Approximation of $\bar{\varepsilon}$ for a particular geometry and its evaluation for constant scattering parameters.

In order to gain some insight into the above expression for $\bar{\varepsilon}$, which, under certain circumstances, could represent the mean acoustic energy backscattered by an aggregation or school of fish, $\bar{\varepsilon}$ is now examined for a particular geometry. For convenience this is chosen to be of the type used by Røttingen in his experiments.

Consider a cylindrical volume V boulding a collection of scatterers which are distributed randomly, but homogeneously throughout it. V is aligned with the common principal response axis of essentially collocated source and receiver. The position of the source/receiver defines the origin of a coordinate system whose z-axis is that of the common principal response axis. The transmit and receive beam patterns are broad with respect to V, which, further, is assumed to be sufficiently narrow so that the contribution to the echo from a single scatterer is, to a good approximation, independent of its location in any transverse plane of V. The general expression for $\bar{\varepsilon}$ can be reduced to the following:

$$\bar{\varepsilon} = \frac{I_o \varrho(0) V}{4\pi (z_2 - z_1)} \nu \bar{\sigma}_b \int_{z_1}^{z_2} \frac{\exp[-4\alpha z - \nu \bar{\sigma}_e (z - z_1)]}{z^4} dz$$

where z_1 and z_2 are the distances from source to nearer and further bounding planes of V, respectively, and where V is the measure of the cylindrical volume.

The integral in this expression for $\bar{\varepsilon}$ can be reduced to the exponential integral, so that $\bar{\varepsilon}$ can be expressed in the following alternative form:

$$\bar{\varepsilon} = \frac{I_o \varrho(0) V}{24\pi l} \nu \bar{\sigma}_b \exp(-4\alpha z_1)\, Y(4\alpha + \nu \bar{\sigma}_e)$$

where $l = z_2 - z_1$ and

$$Y(a) = \int_{z_1}^{z_2} \frac{\exp[-a(z - z_1)]}{z^4} dz = 2\left[\frac{1}{z_1^3} - \frac{\exp(-al)}{z_2^3}\right] - a\left[\frac{1}{z_1^2} - \frac{\exp(-al)}{z_2^2}\right]$$

$$+ a^2\left[\frac{1}{z_1} - \frac{\exp(-al)}{z_2}\right] - a^3 \exp(az_1)\left[E_1(az_1) - E_1(az_2)\right]$$

where $E_1(x)$ is the exponential integral

$$E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt$$

as defined in ABRAMOWITZ and STEGUN (1964).

If the ensemble-averaged scattering cross sections $\bar{\sigma}_b$ and $\bar{\sigma}_e$ are regarded as constants, independent of the scatterer number density ν, then inspection of the integral expression of $\bar{\varepsilon}$ shows that $\overline{\varepsilon(\nu)}$ is a monotonically increasing function of ν, which increases linearly with ν for small ν and gradually becomes asymptotic to a constant for large ν. These limiting forms of $\bar{\varepsilon}$ are the following:

$$\overline{\varepsilon(\nu)} \underset{\nu \to 0}{\sim} c(c_1 \nu - c_2 \nu^2) + 0(\upsilon^3)$$

$$\overline{\varepsilon(\nu)} \underset{\nu \to \infty}{\sim} c_3 - \frac{c_4}{\nu} + 0\left(\frac{1}{\nu^2}\right)$$

where

$$c = \frac{I_o \varrho(0) V \bar{\sigma}_b}{4 \pi l}$$

$$c_1 = \int_{z_1}^{z_2} \frac{\exp(-4 \alpha z)}{z^4} dz$$

$$c_2 = \bar{\sigma}_e \left[\int_{z_1}^{z_2} \frac{\exp(-4 \alpha z)}{z^3} dz - z_1 c_1 \right]$$

$$c_3 = \frac{c \exp(-4 \alpha z_1)}{\bar{\sigma}_e z^4}$$

$$c_4 = \frac{4 c_3}{\bar{\sigma}_e} (\alpha + \frac{1}{z_1})$$

These limiting forms of $\overline{\varepsilon(\nu)}$ confirm what is expected from simple physical considerations. In the limit of very low densities the several echoes from individual scatterers are generally distinct, i.e., separated in time, since a density can be found for which the signal transmission time T is relatively short compared to the mean time difference between the echoes of nearest-neighbour scatterers, so that the total echo energy is merely the sum of the individual echo energies; thence, $\bar{\varepsilon}$ is proportional to the number of scatterers in V, and thence to ν. At high densities the effect of extinction is strong; in fact, the diminution of the incident acoustic field as it penetrates V exactly counterbalances the increased number of scatterers, whose mean

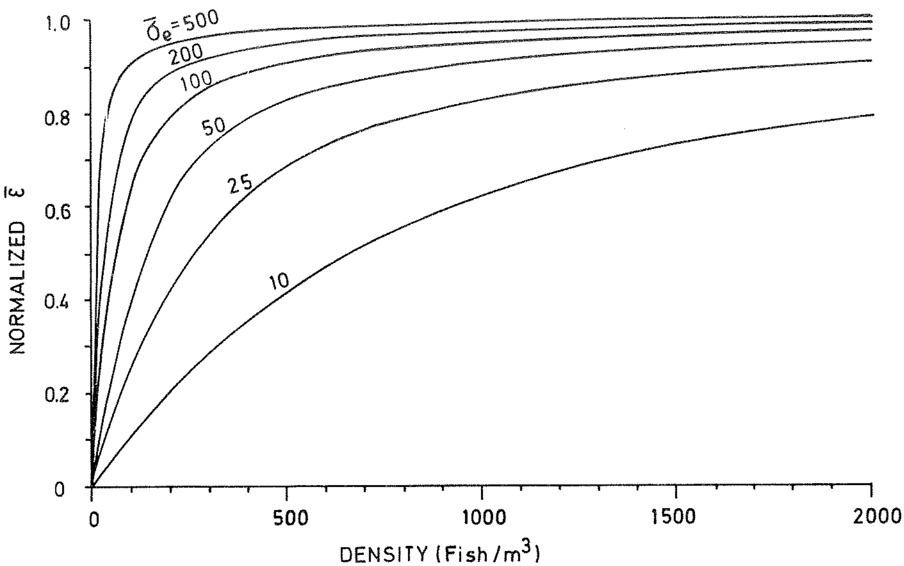

Fig. 3. Normalized mean time-integrated echo intensity $\bar{\varepsilon}$ for a collection of scatterers with constant $\bar{\sigma}_b$ and $\bar{\sigma}_e$ when ensonified by a narrowband signal of center frequency 38 kHz. $\bar{\sigma}_e$ is expressed in units of sqvare centimeters.

contribution to the irreducible incoherent part of $\bar{\varepsilon}$ is inversely proportional to the total number νV of scatterers, and thence to ν, so that $\bar{\varepsilon}$ steadily approaches a constant as ν increases. This asymptotic behaviour is a manifestation of conservation of energy: only a finite amount of energy is contained in $s(t)$ and only a finite amount of energy – at most, that which is intercepted by V – can be backscattered. If, moreover, at all values of ν the scatterers are distributed randomly homogeneously in V, so that $\bar{\varepsilon}$ is composed only of its incoherent part, then the transition from linear growth at low densities to asymptotic constancy at high densities will be completely smooth and gradual, i.e., monotonic.

If a scatterer is of finite size and simple shape, then in the limit of the highest densities the distribution of scatterers in V may become regular, in which case the interference of the constituent echoes will make a non-negligible contribution to the total echo. The amount of acoustic energy intercepted and backscattered by V is still finite in this situation, but the dependence of $\bar{\varepsilon}$ on the now significant coherent part of the echo, in addition to the irreducible incoherent part, renders the asymptotic approach to a constant at the highest density non-monotonic, or oscillatory, in the general case. The precise approach to the high density asymptote depends on the precise narrowband signal, scatterer size, geometry of V and magnitude of $\bar{\sigma}_e$ for the case of constant, density-independent $\bar{\sigma}_b$ and $\bar{\sigma}_e$.

As an illustration of the dependence of $\bar{\varepsilon}$ on ν for constant $\bar{\sigma}_b$ and $\bar{\sigma}_e$, the results of a calculation of $\bar{\varepsilon}$ for the case of a 38 kHz narrowband signal and cylindrical volume with $z_1 = 7.7$ m and $z_2 = 9.5$ are presented in Fig. 3. These results are shown for a range of values of the constant extinction cross section $\bar{\sigma}_e$ and are normalized to their respective high density asympotic values. Comparable results for the case of a 120 kHz narrowband signal are nearly indistinguishable from these and are, therefore, omitted.

III. *Application of theory to Røttingen's experiment with saithe.*

The particular geometry that was chosen for the approximation of the general expression for the ensemble-averaged time- integrated echo intensity in the preceding section is similar to that used by Røttingen in his experiments. For the measurements on saithe, the fish were contained in an approximately cylindrical net cage of height 180 cm and radius which varied smoothly between 70 cm and 72.5 cm. The cage was oriented vertically with its longitudinal axis nearly coincident with the common transmitting/receiving axes of acoustic transducers at 38 and 120 kHz, each of which was used in its dual transmitting/receiving capacity. The transducers were placed below the cage, which was located near the surface to facilitate the transfer of fish, so that the saithe were ensonified ventrally, instead of dorsally as is customany in such work. The distances from the transducers to the nearer and further bounding planes of the cylindriform net cage were 7.7 m and 9.5 m, respectively. The configuration of transducers and cage were such that the maximum degradation in sound source level over the cage, because of transducer directivity, in the worst case was less than 1.5 dB. All photographic observations of the saithe showed that they were distributed more or less uniformily throughout the volume and adopted no particular orientation or attitude as they might be expected to do when schooling, for example.

The circumstances of Røttingen's experiment evidently fulfill the conditions under which the general expression for $\bar{\varepsilon}$ was reduced to the simple one-dimensional integral approximation earlier, if the slight variation in sound level due to transducer directivity over the net cage is ignored, with a single difference: that the scattering properties of the saithe must be evaluated for the ventral aspect rather than the dorsal aspect. The applicability of the approximation to $\bar{\varepsilon}$ for Røttingen's experiment is thus established; evaluation of it, if satisfactory, should disclose the mechanisms underlying the observations shown in Fig. 1 and, presumably, by analogy, those for the sprat which are shown in Fig. 2.

That Røttingen's observations cannot be explained by a model in which the scattering parameters $\bar{\sigma}_b$ and $\bar{\sigma}_e$ are both independent of the fish number density ν is clear. Such a model can explain only a monotonic increase in $\bar{\varepsilon}$ with ν which becomes asymptotic to a constant at very high densities, as is illustrated in Fig. 3. The scattering parameters $\bar{\sigma}_b$ and $\bar{\sigma}_e$ must, therefore, depend on ν.

It is not surprising that $\bar{\sigma}_b$ and $\bar{\sigma}_e$ should depend on ν when it is considered that as the number, thence density, of fish in the net cage increases, an individual specimen has less space in which to move and, consequently, must curtail its orientation range to avoid interfering with its neighbours, which would be individually and socially harmful, thence prohibited. Thus, the ensemble-averages of σ_b and σ_e, which generally vary with the orientation of the scatterer, or fish, relative to the direction of ensonification, will depend on ν. In the absence of any quantitative information about the aggregating properties of the encaged saithe of Røttingen's experiment, or, in fact, of the encaged fish of any other study, a rather simple, but plausible model for the orientation distribution of the encaged saithe is proposed. A model which connects the spread of this distribution with the fish number density ν is also proposed so that $\bar{\sigma}_b$ and $\bar{\sigma}_e$, which are expressed in terms of the characteristic measures of the orientation distribution, can be computed directly in terms of ν.

In the course of determining the density-dependence of $\bar{\varepsilon}$ for the particular conditions that obtained during Røttingen's experiments with saithe, which is contained in the factor

$$\nu\bar{\sigma}_b \int_{z_1}^{z_2} \frac{exp\ [-4\alpha z - \nu\bar{\sigma}_e(z-z_1)]}{z^4}\, dz$$

where $z_1 = 7.7$ m, $z_2 = 9.5$ m and a is the medium attenuation rate at either 38 kHz or 120 kHz; an orientation distribution for the saithe is proposed; $\bar{\sigma}_b$ and $\bar{\sigma}_e$ are evaluated in terms of the variance and mean of this distribution; and the variance of this distribution is expressed as a function of the fish number density ν, so that the expressions for $\bar{\sigma}_b$ and $\bar{\sigma}_e$ can be evaluated directly in terms of ν.

IIIa. *Orientation distribution.*

The form of the orientation distribution that obtained during Røttingen's experiment with the saithe is unknown; it was not measured, and there is no formal theory which specifies the orientation distribution of encaged aggregations of saithe or of any other kind of fish. But insofar as the fish were observed to adopt a more or less random distribution in inclination about a horizontal or near-horizontal inclination, and insofar as saithe has a dorsal aspect target strength which is rater insensitive to roll over a 60 degree range (NAKKEN and OLSEN 1973), there appears to be plausibility to the selection of a very simple form for this distribution, as is explained below.

The swimbladder, when present in fish as it is in a well-developed state in saithe, is recognized – see, for example, SHIBATA (1970), HARDEN JONES and PEARCE (1958) and CUSHING and RICHARDSON (1955) – to contribute significantly to the echo, even at ultrasonic frequencies. The fact that this organ is approximately cylindriform and generally is more or less aligned with the fish center line, which is defined here as the line running from the front of the upper jaw to the root of the tail, suggests that the principal scattering properties of the fish, and not merely its echo, are rather insentitive to roll, at least over that angular range where the swimbladder presents a similar surface. This has been confirmed by measurements of the dorsal aspect target strength of saithe, cod and herring (NAKKEN and OLSEN 1973). Inspection of the morphology of saithe (ROLLEFSEN 1960–62 and MIDTTUN and HOFF 1962) suggests that the ventral aspect target strength and, presumably, other similar primary scattering characteristics are insensitive to roll. It may be inferred, therefore, that the dependence of the orientation distribution of saithe on roll may be ignored since the scattering properties of interest, σ_b and σ_e, and their ensemble averages will be similarly insensitive with respect to roll.

Since the saithe of Røttingen's experiment were observed to adopt a more or less random distribution in inclination about some approximately horizontal mean inclination, the orientation distribution may plausibly be described by a truncated Gaussian distribution in inclination or tilt angle θ, which is defined as the angle that the center line of the fish makes with the horizontal plane. This distribution is described by the probability density function

$$f(\theta) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\,\sigma_\theta} \; exp\left[-\dfrac{(\theta-\bar{\theta})^2}{2\sigma_\theta^2}\right] & \text{for } |\theta-\bar{\theta}| \leqq \lambda'\sigma_\theta \\ 0 & \text{for } |\theta-\bar{\theta}| > \lambda'\sigma_\theta \end{cases}$$

were λ' is a factor of the order of three if the results of OLSEN (1971) for cod in the open sea are applicable. The normalizing factor $1/\sqrt{2\pi}\sigma_\theta$ is rigorously true only for infinite λ', but is underestimated by .3% for $\lambda'=3$, 1.3% for $\lambda'= 2.5$ and 4.6% for $\lambda'=2$. The mean angle of inclination $\bar{\theta}$ may be assumed to be small and constant with respect to the density ν of fish, and the standard deviation σ_θ of the distribution is presumed to be a monotonically decreasing function of ν. The precise dependence of σ_θ on ν is discussed below, after the ensemble averages of σ_b and σ_e have been expressed as functions of σ_θ and $\bar{\theta}$, the characterizing parameters of the orientation distribution.

IIIb *Ensemble-averaged backscattering cross section.*

As the spatial distribution of the saithe in Røttingen's experiment was observed to be approximately randomly homogeneous throughout the net cage, the ensemble average of a scattering parameter such as σ_b or σ_e is just the average of the same quantity with respect to the orientation distribution, which, by the discussion of the previous section, is assumed to be described adequately by the probability density function $f(\theta)$. Thus, the average of the backscattering cross section σ_b with respect to the ensemble of naturally occuring configurations at a given fish number density ν, and thence at a fixed distribution spread σ_θ, is

$$\bar{\sigma}_b = \int \sigma_b(\theta) f(\theta) d\theta$$

where the range of integration is $[\bar{\theta} - \lambda'\sigma_\theta, \bar{\theta} + \lambda'\sigma_\theta]$.

The backscattering cross section is known through the more commonly measured target strength function $TS(\theta)$, which is related to $\sigma_b(\theta)$ by the usual definition (URICK 1975):

$$TS(\theta) = 10 \log_{10} \frac{\sigma_b(\theta)}{4\pi}$$

so that $\bar{\sigma}_b$ may be computed directly in terms of $TS(\theta)$ by the formula

$$\bar{\sigma}_b = 4\pi \int 10^{TS(\theta)/10} f(\theta) d\theta$$

It was this expression of $\bar{\sigma}_b$ that was evaluated for the particular conditions of Røttingen's experiment with saithe, which are expressed here through the two target strength functions shown in Fig. 4. These functions are distinguished by the center frequency of the ensonifying pulse, which was 38 kHz in one instance and 120 kHz in the other. Both curves represent averages of measurements of the ventral aspect target strength function with respect to a number of specimens of saithe actually used in the experiment. In particular, the target strength function at 38 kHz is the result of averaging the target strength functions of 16 specimens, while that at 120 kHz is the result of averaging the target strength functions of 17 specimens.

The result of evaluating $\bar{\sigma}_b$ numberically for the case that $\lambda' = 3$ is shown

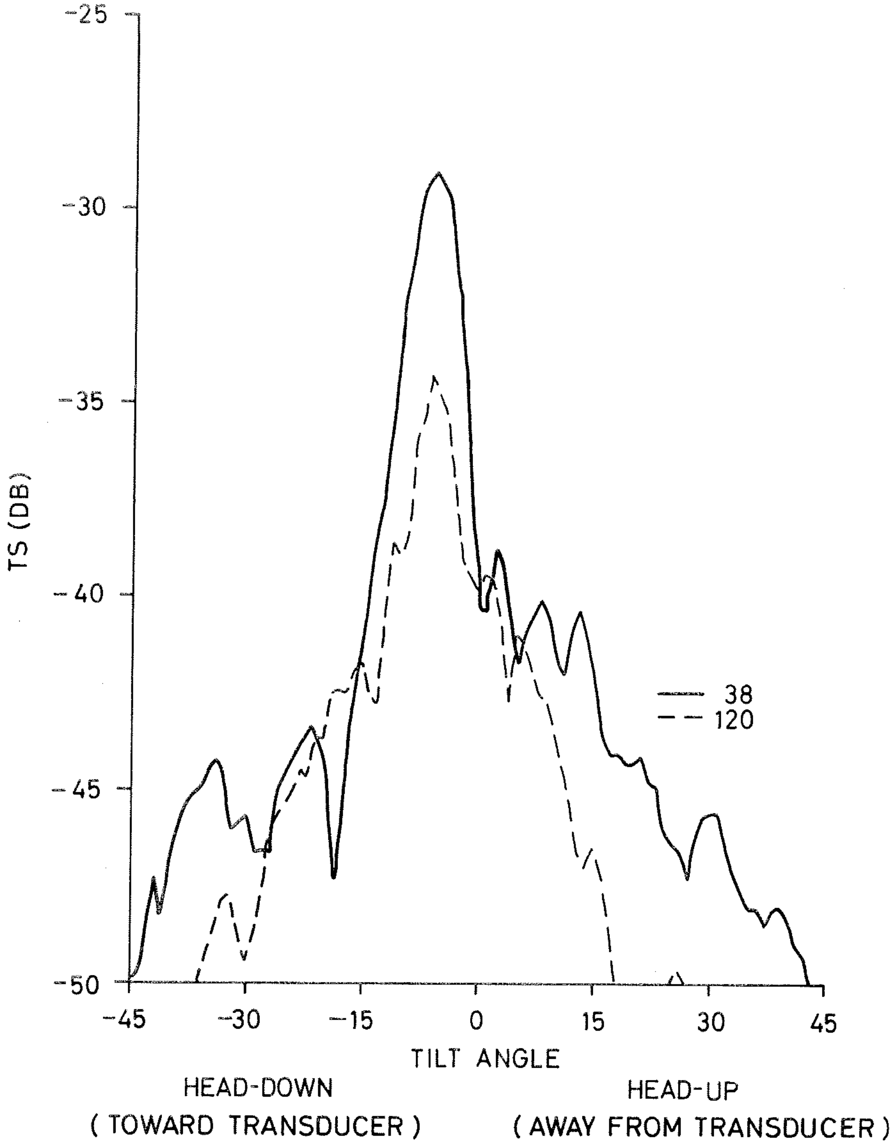

Fig. 4. Ventral aspect target strength of saithe of mean length 35.1 ± 0.6 cm at 38 kHz when averaged with respect to 16 specimens, and at 120 kHz when averaged with respect to 17 specimens.

in Fig. 5 and 6, which apply to the respective narrowband ensonifying signals with center frequencies of 38 kHz and 120 kHz. Because the mean and unknown tilt angle $\bar{\theta}$ is presumed constant with respect to the fish number density ν, while σ_θ is presumed to vary systematically with ν, $\bar{\sigma}_b$ is shown as a function of σ_θ, with $\bar{\theta}$ as a parameter which is varied over the

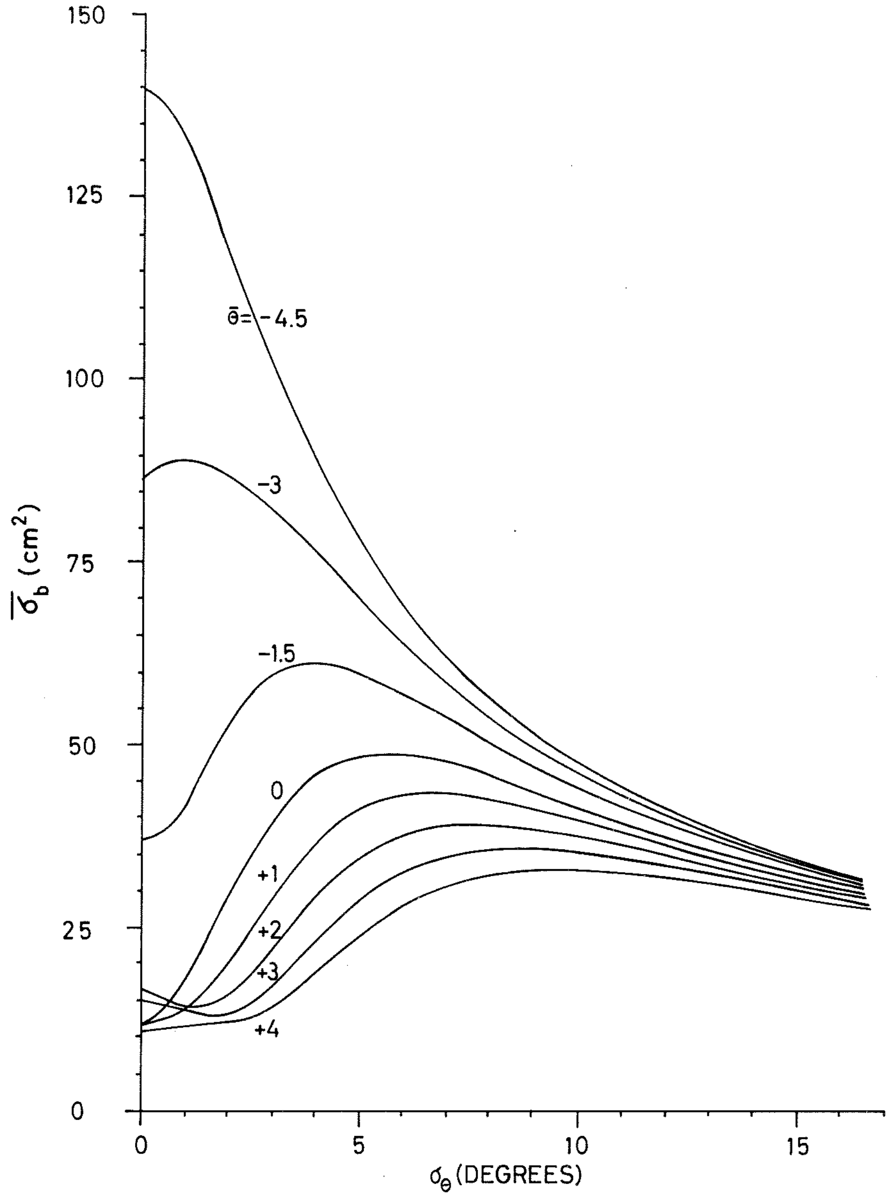

Fig. 5. Mean backscattering cross section $\bar{\sigma}_b$ of saithe in ventral aspect at 38 kHz as a function of the spread σ_θ in tilt angle distribution with mean tilt angle $\bar{\theta}$ as a parameter.

range [–4.5, 4] degrees. The sign convention used consistently throughout this paper is that negative tilt angles denote the head-down position with respect to the true horizontal, while head-up positions are denoted by positive tilt angles.

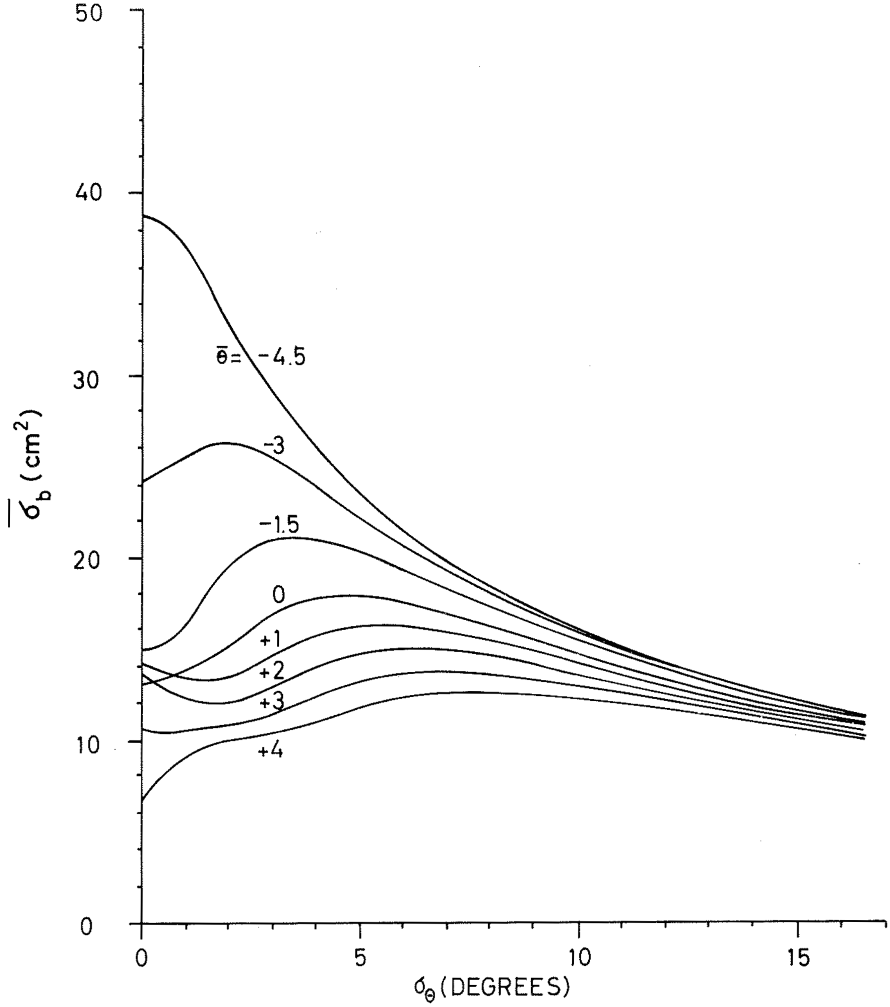

Fig. 6. Mean backscattering cross section $\bar{\sigma}_b$ of saithe in ventral aspect at 120 kHz as a function of the spread σ_θ in tilt angle distribution with mean tilt angle $\bar{\theta}$ as a parameter.

IIIc *Ensemble-averaged extinction cross section*

There is some mention of an extinction cross section for fish in the literature, but mainly in connection with the transmission of sound through fish at or near the frequency of swimbladder resonance (Davies 1973, Ching and Weston 1971, Weston *et al.* 1969). The concept of the extinction cross section is well known in other applications, as in the scattering of light by metallic spheres (Born and Wolf 1970), acoustic scattering by spherical bubbles in water (Wildt 1947), and the scattering of plane scalar fields by spheres (MORSE and FESHBACH 1953). According to the common definition of these references, the extinction cross section σ_e is the equivalent area normal to the direction of the incident ensonifying signal out of which energy is scattered and absorbed.

For the rather high frequencies used in Røttingen's experiment and for which the two-parameter description of fish scattering behaviour introduced in this study is probably most useful, σ_e may be assumed to be proportional to the surface area of the fish, or of its swimbladder, when projected onto the horizontal plane. If the principal scattering body is represented by an ellipsoid whose length to height to width ratio is designated by the respective dimensions a:b:c, then σ_e as a function of the inclination or tilt angle θ of the body with respect to the horizontal plane is

$$\sigma_e(\theta) = \sigma_{e,o}\sqrt{\cos^2\theta + \varrho^2\sin^2\theta}$$

where $\sigma_{e,o}$ is the value of σ_e at $\theta = 0$ and $\varrho = b/a$ is the ratio of height to length of the scatterer. The effect of roll on σ_e is ignored, which incurs no significant error for cylindriform bodies or quasi-cylindriform bodies which have no extreme roll, which is the plausible assumption for the saithe of Røttingen's experiment.

Given this model for $\sigma_e(\theta)$, the ensemble average $\bar{\sigma}_e$ may now be computed; it is simply the average of $\sigma_e(\theta)$ with respect to the orientation distribution $f(\theta)$, that is,

$$\bar{\sigma}_e = \int \sigma_e(\theta) f(\theta) d\theta$$

where the range of integration is $[\bar{\theta}-\lambda'\sigma_\theta, \bar{\theta}+\lambda'\sigma_\theta]$.

This expression was evaluated numerically for the case $\lambda' = 3$ for saithe having the relative dimensions 17.3:3.2:2.1, which were obtained from RASMUSSEN (1960) and by assuming that the width is two-thirds of the height (NAKKEN pers. com.). It was found that $\bar{\sigma}_e$ could be represented with an accuracy better than 0.1 percent for all $\sigma_\theta \leqq 16.5$ degrees and for all $\bar{\theta}$ in [–5, +5] degrees by the simple quadratic expression

$$\bar{\sigma}_e \doteq \sigma_{e,o}\left[1 - .038\left(\frac{\sigma_\theta}{16.5}\right)^2\right]$$

In the actual evaluation of $\bar{\varepsilon}$ this expression was used, but the variation of $\bar{\sigma}_e$ with σ_θ was found to be entirely negligible, so that $\bar{\sigma}_e$ could have been reduced to the constant $\sigma_{e,o}$, which, however, except in the true ray theory limit, does depend on the center frequency of the ensonifying signal.

IIId. *Connection of spread in orientation distribution and density of fish aggregation.*

It is reasonable to expect σ_θ, or the characteristic measure of spread in the tilt angle distribution of the encaged saithe, to be a monotonically decreasing function of the fish number density ν, for individual fish have less room in which to move, in the mean, as ν increases. It is also reasonable to assume that whatever preference there is for a mean inclination $\bar{\theta}$ should not be affected by the conditions of density, although this is purely speculative. Because of the extreme artificiality of the net cage environment and lack of any preferred direction, as might be established naturally by the fish when schooling (BREDER 1959), it is expected that $\bar{\theta}$ would be the neutral inclination, which would be horizontal or nearly horizontal. The entire dependence of the orientation distribution on ν is thus contained in the dependence of σ_θ on ν, which is described below.

Physically σ_θ may be separated into two independent components: that which expresses the biological expectation that σ_θ is a monotonically decreasing function of ν, i.e., that the intrinsic spread in tilt angle decreases with increasing ν, and that which expresses the perspectival effect that the intrinsic, or true, tilt angle generally appears different when viewed from the acoustic source/receiver, which generally is located obliquely, or transversely, with respect to the fish. These two components of σ_θ, which are denoted $\sigma_{\theta,1}$ and $\sigma_{\theta,2}$, respectively, are independent random variables and, as such, are summed in this manner:

$$\sigma_\theta = \sqrt{\sigma_{\theta,1}^2 + \sigma_{\theta,2}^2}$$

As the intrinsic tilt angle distribution of the encaged saithe is unknown, but is expected to be a monotonically decreasing function of ν, the connection between $\sigma_{\theta,1}$ and ν is specified very simply by the model

$$\sigma_{\theta,1} = \sigma_{\theta,1,0} \, exp\,(-\nu/\nu_{cr})$$

where $\sigma_{\theta,1,0}$ denotes the low density limit of the intrinsic spread in tilt angle and ν_{cr}, which is termed the critical density, is the density characteristic of the change in $\sigma_{\theta,1,0}$. Both $\sigma_{\theta,1,0}$ and ν_{cr} are regarded as parameters to be determined or learned by comparison of the results of evaluation of the theoretical $\bar{\varepsilon}$ with Røttingen's observations.

It is expected, however, that $\sigma_{\theta,1,0}$ will be of the order of 16 degrees, which is the figure determined by photographic measurements of cod in the open sea (OLSEN 1971), and which could be be thought of as a natural characteristic of cod, and thence of related fish such as saithe. The fact that the saithe are encaged should not lessen confidence in this rough estimate of $\sigma_{\theta,1,0}$ as it applies only in the low density limit where the net cage appears large.

The effect of perspective on altering the intrinsic spread when observed obliquely may be defined through the perspectival spread $\sigma_{\theta,2}$. This is approximately equal to the square root of the average of the mean square tilt angle due to the azimuthal variation in apparent tilt angle relative to an obliquely situated reference point, both with respect to the volume of the net cage, in which the fish are assumed to be randomly homogeneously distributed, and with respect to the transmit and receive beam patterns. In units of degrees

$$\sigma_{\theta,2} = \frac{180}{\pi}\left(\frac{1}{V}\int D_R^2\, D_S^2\, \Delta\chi^2\, dV\right)^{1/2}$$

where $D_R = D_R(\Psi)$ and $D_S = D_S(\Psi)$ are the respective receive and transmit beam patterns in relative units of pressure ratios, which are assumed to be functions only of the polar angle Ψ, and $\Delta\chi^2$ is the variance in apparent tilt angle distribution due to azimuthal variations in orientation. This latter quantity, expressed in units of square radians, is approximated for small mean inclination $\bar{\theta}$ by the expression

$$\Delta\chi^2 = \frac{1}{2\pi}\int_0^{2\pi}\left[\cos^{-1}\left(\sin\Psi\cos\varphi + \frac{\pi}{180}\,\bar{\theta}\,\cos\Psi\right) - \frac{\pi}{2}\right]^2 d\varphi$$

where $\bar{\theta}$ is expressed in degrees. This expression applies properly only in the limit of the highest density, where $\sigma_{\theta,1}$ vanishes, but is adopted here for convenience. D_R and D_S are not equated to unity here as they were earlier in justifiable approximations. They are equated to that beam pattern, expressed in units of relative pressure, which was observed by Røttingen for the transmitter at both 38 kHz and 120 kHz; namely, one which suffers an approximately uniform off-axis degradation which is –1 dB at the outer walls of the net cage. To the first and dominant term, then

$$\sigma_{\theta,2}^2 \doteq .75\left(\frac{180}{\pi}\right)^2 \frac{R^2}{(z_1 + z_2)^2} + .81\,\bar{\theta}^2$$

where R is the radius of the cage and z_1 and z_2, the respective distances from acoustic source to nearer and further surfaces of the cage. If the precise values of these parameters that applied during Røttingen's experiment with saithe are substituted,

$$\sigma_{0,2} = \sqrt{4.2 + .81\,\bar{\theta}^2}$$

where both $\bar{\theta}$ and $\sigma_{\theta,2}$ are expressed in units of degrees.

IIIe. *Evaluation of $\bar{\varepsilon}$*

The ensemble-averaged scattering cross sections, $\bar{\sigma}_b$ and $\bar{\sigma}_e$, can now be written, with respect to specific models which connect the parameters of the orientation distribution with ν, the fish number density, as explicit functions of ν. With respect to these same models the ν–dependence of the ensemble-averaged time-integrated echo intensity $\bar{\varepsilon}$ can be determined directly. This was done numerically by means of a digital computer. Some characteristic results of this evaluation, in addition to the principal ones which constitute a quantitative explanation of Røttingen's observations for saithe, are presented in the next section.

RESULTS

The principal results of the quantitative analysis of Røttingen's experiments with saithe are shown in Fig. 7 and 8. In Fig. 7 the result of the evaluation of the theoretical mean or ensemble-averaged time-integrated echo intensity $\bar{\varepsilon}$ for saithe at 38 kHz, according to the approximations described above, and after normalization by the peak value, is presented for the following model parameters: a mean inclination or tilt angle $\bar{\theta} = 0$

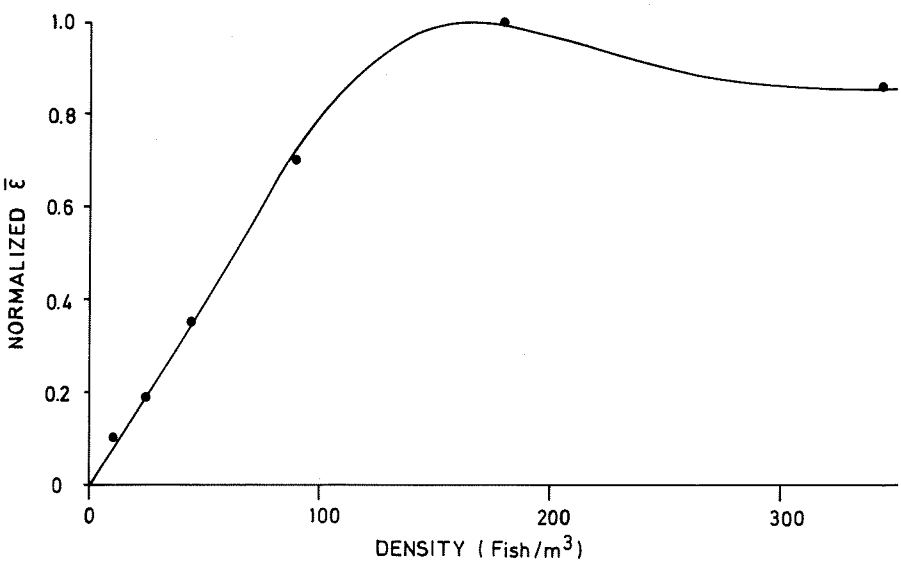

Fig. 7. Comparison of the computed normalized, mean time-integrated echo intensity $\bar{\varepsilon}$ with the mean of Røttingen's observations for saithe at 38 kHz for the following model parameters: $\bar{\theta}$ = 0 degrees; $\sigma_{e,o}$ = 60 cm^2; $\sigma_{\theta,1,0}$ = 18 degrees; ν_{cr} = 100 fish/m^3.

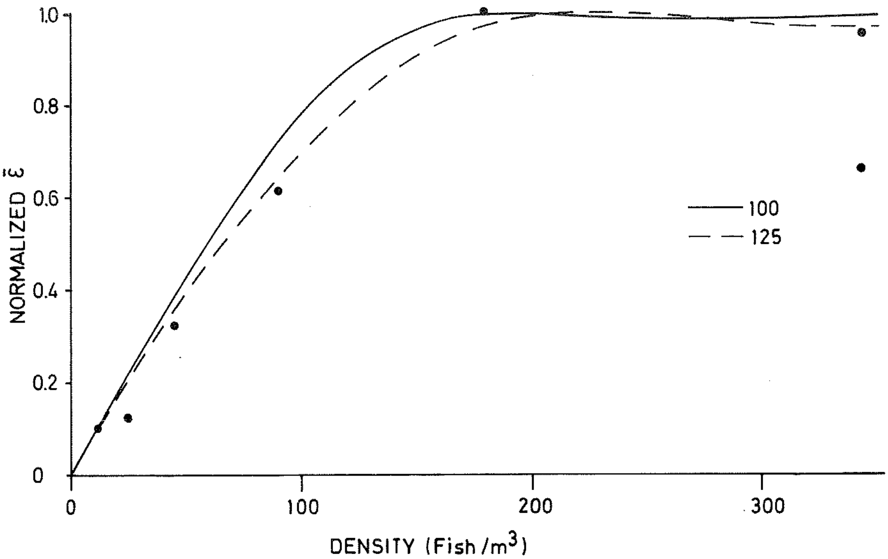

Fig. 8. Comparison of $\bar{\varepsilon}$ with the mean of Røttingen's observations for saithe at 120 kHz for the following model parameters: $\bar{\theta} = 0$ degrees; $\sigma_{e,o} = 100$ cm^2; $\sigma_{\theta,1,0} = 18$ degrees; $\nu_{cr} = 100$ and 125 fish/m^3.

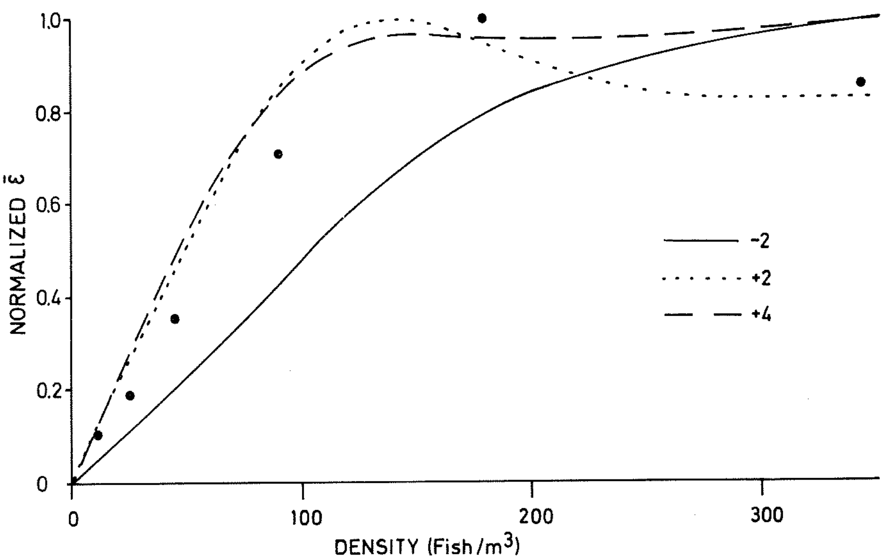

Fig. 9. Dependence of $\bar{\varepsilon}$ on $\bar{\theta}$ for 3 values of $\bar{\theta}$ for saithe at 38 kHz for the following model parameters: $\sigma_{e,o} = 60$ cm^2; $\sigma_{\theta,1,0} = 18$ degrees; $\nu_{cr} = 100$ fish/m^3.

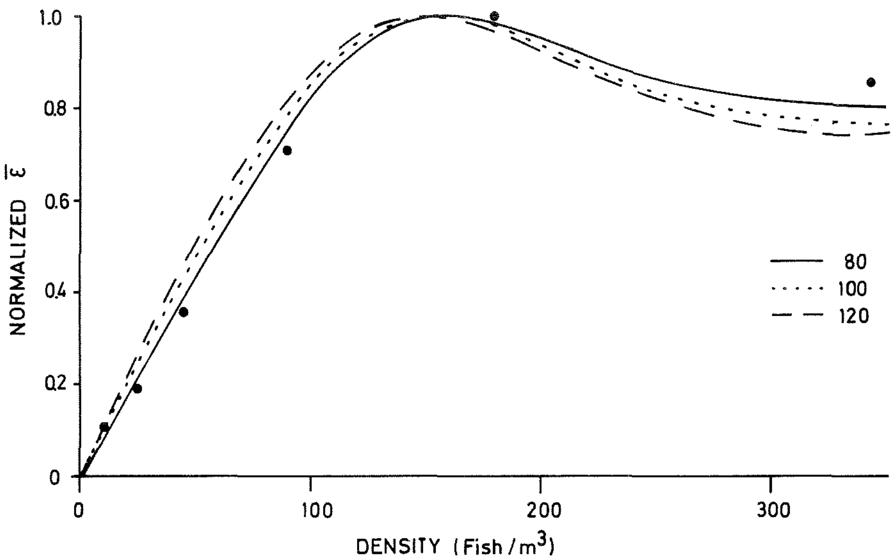

Fig. 10. Dependence of $\bar{\varepsilon}$ on $\sigma_{e,o}$ for 3 values of $\sigma_{e,o}$ for saithe at 38 kHz for the following model parameters: $\bar{\theta} = 0$ degrees; $\sigma_{\theta,1,0} = 18$ degrees; $\nu_{cr} = 100$ fish/m^3.

degrees, which describes the horizontal plane; a mean extinction cross section $\sigma_{e,0} = 60$ cm^2, which is presumed applicable in the limit of high fish number densities; an intrinsic spread in tilt angle distribution at low densities, $\sigma_{\theta,1,0} = 18$ degrees; and a decreasing exponential ν-dependence of the intrinsic spread in tilt angle distribution which is characterized by the critical density $\nu_{cr} = 100$ fish/m^3. The normalized theoretical $\bar{\varepsilon}$ is computed with respect to these several parameters at a density increment of 10 fish/m^3 over the density range [0,350] fish/m^3, as has been done consistently in Fig. 7 and 12 inclusive, although for different model parameters and thence different peak values and normalizing factors. For purposes of comparison Røttingen's averaged results at 38 kHz, i.e., the average of corresponding values of $\bar{\varepsilon}$ for the two conditions of ensonification at 38 kHz; namely, for pulse durations of 0.3 msec and 0.6 msec; are also presented in Fig. 7, as they are in Fig. 9 to 12 which likewise pertain to computations at 38 kHz.

In Fig. 8 the normalized theoretical estimate of $\bar{\varepsilon}$ at 120 kHz is computed for the same purely geometric model parameters described above for Fig. 7, but for a mean high density extinction cross section $\sigma_{e.o} = 100$ cm^2. In addition to the computation at $\nu_{cr} = 100$ fish/m^3, a computation at $\nu_{cr} = 125$ fish/m^3 is presented here. The average of Røttingen's observations of $\bar{\varepsilon}$ at 120 kHz, for pulse durations of 0.1 msec and 0.6 msec, are shown at all observed densities except at the highest density. At this density, which is 343 fish/m^3, the respective normalized observations of $\bar{\varepsilon}$ were widely different –

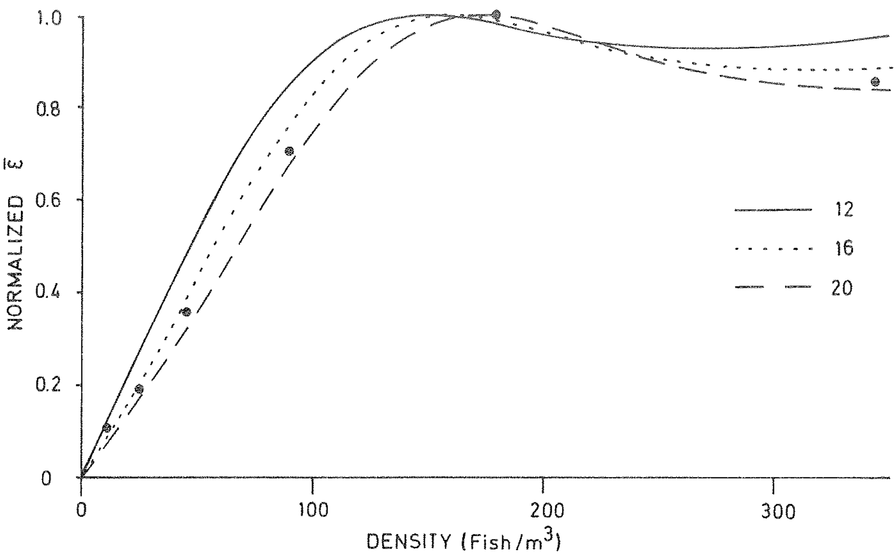

Fig. 11. Dependence of $\bar{\varepsilon}$ on $\sigma_{\theta,1,0}$ for 3 values of $\sigma_{\theta,1,0}$ for saithe at 38 kHz for the following model parameters: $\bar{\theta} = 0$ degrees; $\sigma_{e,o} = 60$ cm^2; $\nu_{cr} = 100$ fish/m^3.

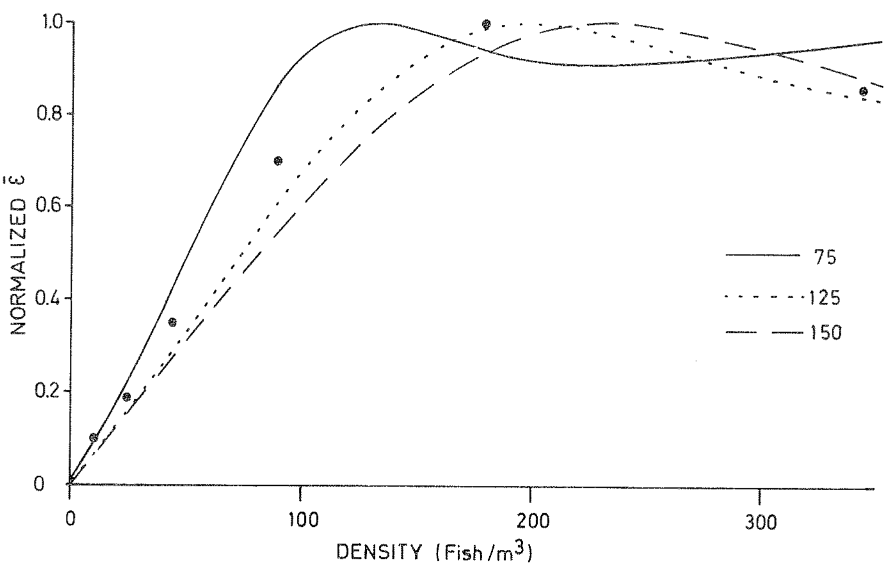

Fig. 12. Dependence of $\bar{\varepsilon}$ on ν_{cr} for 3 values of ν_{cr} for saithe at 38 kHz for the following model parameters: $\bar{\theta} = 0$ degrees; $\sigma_{e,o} = 60$ cm^2; $\sigma_{\theta,1,0} = 18$ degrees.

the only case of its kind as revealed by inspection of Fig. 1 – and are both shown in the figure.

Table 1. Values of parameters of Figures 7 to 12.

Figure number	Center Frequency (kHz)	$\bar{\theta}$ (degrees)	$\sigma_{e,o}$ (cm^2)	$\sigma_{\theta,1,0}$ (degrees)	ν_{cr} (fish /m^3)
7	38	0	60	18	100
8	120	0	100	18	100, 125
9	38	−2, +2, +4	60	18	100
10	38	0	80, 100, 120	18	100
11	38	0	60	12, 16, 20	100
12	38	0	60	18	75, 125, 150

The dependence of the normalized theoretical estimate of $\bar{\varepsilon}$ for saithe at 38 kHz on the several important model parameters; namely, $\bar{\theta}$, $\sigma_{e.o}$, $\sigma_{\theta,1,0}$ and ν_{cr}, are shown in Fig. 9 to 12, respectively. The respective values of each of these parameters in the computation of ε for these figures are presented systematically above in Table 1. Thus in Fig. 9, for example, the dependence of $\bar{\varepsilon}$ on $\bar{\theta}$ is shown for three values of $\bar{\theta}$; namely, −2, +2 and +4 degrees; while the other model parameters adopt the following values: $\sigma_{e,o} = 60$ cm^2, $\sigma_{\theta,1,0} = 18$ degrees and $\nu_{cr} = 100$ fish/m^3. Descriptions of Fig. 7 and 8 are included in the table for ease of reference.

DISCUSSION

The agreement of the theoretical prediction of the normalized mean or ensemble-averaged time-integrated echo intensity $\bar{\varepsilon}$ with Røttingen's averaged observations of $\bar{\varepsilon}$ for saithe at 38 kHz, as shown in Fig. 7, is better than can be expected, or can be justifiably stated without qualification, considering the inherent uncertainty in observations of $\bar{\varepsilon}$ and the uncertainty in representation of the data in a normalized format for such a sparsely sampled density range. Some estimates of $\bar{\varepsilon}$ were stated with confidence intervals in RØTTINGEN (1975), but these were not so significant as to alter the general finding of Røttingen as summarized in Fig. 1 and 2, for saithe and sprat, respectively. That only the averages of the respective observations at 38 kHz for the signals of pulse duration 0.3 msec and 0.6 msec are presented in Fig. 7, and in Fig. 9 to 12, is similarly insignificant, for examination of Fig. 1 shows the separate estimates of $\bar{\varepsilon}$ to correspond very closely. The fact that $\bar{\varepsilon}$ was observed only sparsely for fish number densities in excess of 90 fish/m^3; specifically, only at 179 fish/m^3 and 343 fish/m^3; could be a more serious source of error, but it was a fairly consistent finding of the theoretical work that the $\bar{\varepsilon}$–ν relationship is rather flat over the region of the peak, so the fact that $\bar{\varepsilon}$ was established at only a single density in this region is probably unimportant. It was also found that the relationship is entirely smooth and often is purely monotonic both above and below the peak, if present, so that a single estimate in the high density region, say about 350 fish/m^3, could be sufficient for determining the gross form of the $\bar{\varepsilon}$–ν relationship. It is assumed, therefore, that despite apparent shortcomings in Røttingen's observations and presentation of $\bar{\varepsilon}$ as a function of ν for saithe, it is a reasonably accurate statement for the true relationship.

If the agreement of theory with experiment at 38 kHz, cf. Fig. 7, is genuine, then evaluation of the postulated model for the same purely geometric, i.e., frequency-independent quantities at 120 kHz, should also produce good agreement with experiment. Observation of the curve with parameter ν_{cr} = 100 fish/m^3 in Fig. 8, for which all geometric quantities have the identical values as in Fig. 7, indicates approximate agreement. Evaluation of the model for the same conditions, but with ν_{cr}= 125 fish/m^3, improves this agreement. As a comparable estimate of $\bar{\varepsilon}$ at 38 kHz for this value of ν_{cr} also yields a fair agreement with experiment, cf. Fig. 12, although not so fine as the estimate obtained with ν_{cr} = 100 fish/m^3, cf. Fig. 7, it may be concluded that the true value of ν_{cr} lines in or near the range 100 to 125 fish/m^3. It is admitted that the single datum at 343 fish/m^3 for a narrowband signal of center frequency 120 kHz and pulse duration 0.6 msec is completely anomalous to a wide range of evaluations of the model, which is why the empirical estimates of $\bar{\varepsilon}$ for saithe at 120 kHz are not averaged for the presentation of Fig. 8 at this particular density, the only density at which such a discrepancy exists throughout the data for saithe, at either frequency. If the two data at 120 kHz at a density of 343 fish/m^3 are both correct, then there is, evidently, a discrimination by pulse duration. This lies beyond the scope of the model, whose development at the outset assumed a certain insentitivity to the pulse duration of the ensonifying signal. That the $\bar{\varepsilon}$–ν relationship may be significantly characterized by the pulse duration of the signal, at least for some center frequencies, is a possibility, and one that might profitably be studied with respect to the larger purposes of the use of acoustic techniques in fisheries research; the present theory simply cannot explain this supposed dependence, which, it is noted, is absent for the sprat of Røttingen's experiment.

The consequence of the consistency in predictions of theory at both 38 kHz and 120 kHz with the corresponding averaged sets of empirical observations, as shown in Fig. 7 and 8, is that the general theory for the process of echo formation by an encaged aggregation of fish, with its two-parameter representation of individual scattering behaviour, is correct. In particular, it may be concluded that the purely geometric models for the orientation distribution of the encaged saithe and for the density dependences of the characterizing parameters of this distribution, on which the $\bar{\varepsilon}$–ν relationship depends crucially, are correct. Thus, the orientation distribution of the saithe is described satisfactorily by an Olsen-type distribution (OLSEN 1971), i.e., a trucated Gaussian distribution in inclination or tilt angle θ which extends over the range $[\bar{\theta}-\lambda'\sigma_\theta, \bar{\theta}-\lambda'\sigma_\theta]$, where λ' is a constant factor of the order of three and $\bar{\theta}$ and σ_θ are, respectively, the mean and spread of the distribution. For the particular conditions of Røttingen's experiment with saithe, $\bar{\theta}$ is approximately 0 degrees, a constant independent of the fish number density ν, and

$$\sigma_\theta \doteq [\sigma^2_{\theta,1,0}\ exp(-2\nu/\nu_{cr}) + 4.2]^{1/2}$$

where $\sigma_{\theta,1,0}$ is approximately 18 degrees and ν_{cr} is approximately 100 to 125 fish/m^3. The origin of the two factors in this expression has been described fully above.

It was also determined that the maximum ensemble-averaged extinction cross section $\sigma_{e,o}$ for saithe in the ventral aspect is of the order of 60 cm^2 at 38 kHz and 100 cm^2 at 120 kHz. Evidently the pure ray theory limit of acoustics is not entirely applicable to the present problem, so that frequency-dependent variations in the scattering properties of individual fish are to be expected. This is corroborated by comparison of corresponding magnitudes of the ensemble-averaged backscattering cross section at the two frequencies, cf. Fig. 5 and 6.

The ν-dependence of the ensemble-averaged extinction cross section $\bar{\sigma}_e$, through its conjectured dependence on the spread in inclination distribution, is very slight, with only a 3.8 per cent increase in magnitude as σ_θ changes from 16.5 degrees to 0 degrees; that is, presumably, as the density of encaged saithe increases from the approximately free-space value to the highest value. If the computations of $\bar{\varepsilon}$ presented in Fig. 7 to 12 were repeated for density-independent $\bar{\sigma}_e$; specifically for $\bar{\sigma}_e = \sigma_{e,o}$, then the results would be indistinguishable from the corresponding computations with density-dependent $\bar{\sigma}_e$. In view of the influence of the magnitude of $\bar{\sigma}_e$ in determining the form of the $\bar{\varepsilon}$–ν relationship – it predicts merely a monotonic and asymptotic increase to a constant for constant $\bar{\sigma}_b$, cf. Fig. 3, and determines the rate of increase of normalized $\bar{\varepsilon}$ with ν for ν less than about 100 fish/m^3, cf. Fig. 10 – it may be concluded that the ν–dependence of $\bar{\sigma}_e$ is insignificant in determining the general form of the $\bar{\varepsilon}$–ν relationship.

Thus, because the ν-dependence of $\bar{\varepsilon}$ is completely contained in the expression shown above, in section III, which predicts only a monotonic, eventually asymptotic increase in $\bar{\varepsilon}$ with ν for constant $\bar{\sigma}_b$ and $\bar{\sigma}_e$, cf. Fig. 3, and because the ν-dependence of $\bar{\sigma}_e$ is insignificant, the ν -dependence of $\bar{\sigma}_b$ must be of crucial importance in determining the form of the $\bar{\varepsilon}$–ν relationship. This is confirmed by examination of Fig. 9, 11 and 12, which show, respectively, the sensitivities of $\bar{\varepsilon}$ to $\bar{\theta}$, $\sigma_{\theta,1,0}$ and ν_{cr}; that is, to the several purely geometric parameters of the ν -dependent orientation distribution. Variations in any one of these parameters can have a significant effect on the form of $\bar{\varepsilon}$ through $\bar{\sigma}_b$, whose dependence on $\bar{\theta}$ and σ_θ, thence $\sigma_{\theta,1,0}$ and ν_{cr}, is shown in Fig. 5 for the case of a 38 kHz signal and in Fig. 6 for the case of a 120 kHz signal.

It is noted that $\bar{\varepsilon}$, for constant $\bar{\sigma}_b$ and $\bar{\sigma}_e$, as in Fig. 3, increases linearly with ν for very small ν and begins its asymptotic approach to a constant at comparably low values of ν, which are of the order of 50 fish/m^3 for realistic values of $\bar{\sigma}_e = \sigma_{e,o}$. These latter values are of the order of 60 to 120 cm^2, which are consistent with estimates of σ_b (θ) obtained from empirical target strength data and with knowledge of the physical size of the scatterer. The fact that $\bar{\varepsilon}$ increases linearly to a rather high value of ν, of the order of 100 fish/m^3, is therefore significant. It apparently describes a σ_b which is, at least over this range of ν an increasing function of ν. This dependence of $\bar{\sigma}_b$ on ν is precisely that found at both 38 kHz and 120 kHz by computation from the measured target strength curves of saithe according to the general model in which the orientation distribution is approximately Gaussian in tilt angle and whose variance is a monotnically decreasing function of ν. The similarity in target strength curves at the two frequencies is thus seen to be responsible for the similarity in corresponding $\bar{\sigma}_b$ functions, which is also why the gross forms of the $\bar{\varepsilon}$–ν relationships are similar. If the target strength functions were very dissimilar, then the $\bar{\varepsilon}$–ν relationships also would be expected to be significantly different.

From mere inspection of the data in Fig. 1 and 2 it is difficult to assert that there are frequency-dependent differences in the $\bar{\varepsilon}$–ν relationships. In the neglect of a single datum in Røttingen's observations for saithe at 120 kHz, which was not reproduced in the observations on sprat and which remains unexplained here, what frequency-discrimination there is in the $\bar{\varepsilon}$–ν relationships is attributed to frequency-dependent differences in $\bar{\sigma}_b$ and in the conditions of ensonification of the net cage. For saithe these were found to be slight, but the uncertainty in estimates of $\bar{\sigma}_b$ at 120 kHz was considerable, so the precise form of $\bar{\sigma}_b$ is somewhat uncertain at this frequency. In the computation of the perspectival contribution to the spread in tilt angle distribution it was assumed that there was a slight, identical degradation in both transmit and receive beam patterns at the two frequencies, although this agreement was confirmed experimentally only for the two transmit beams. Admittedly the application of theory to the circumstances of Røttingen's experiment was not entirely consistent, particularly in the treatment of ensemble averages, in which the beam patterns are generally contributing factors. However, it was judged that the several ad hoc approximations used here were reasonable; at least they facilitated the computations of $\bar{\varepsilon}$ without incurring large errors and, in the case of the inferred ν-dependence of $\bar{\sigma}_b$, demonstrated the insignificant role of the beam patterns in determining the ν-dependence of $\bar{\sigma}_b$, and thence the ν-dependence of $\bar{\varepsilon}$.

In summary it is observed that the precise form of the $\bar{\varepsilon}$-ν relationship for the scattering of sound by encaged aggregations of fish, and, presumably, for the scattering of sound by schools of fish, depends on many quantities, which include the backscattering and extinction cross sections as functions of fish orientation, the spatial and orientation distributions as functions of the number density ν, the geometry of ensonification and echo reception, and the signal waveform. While little is presently known about several of these quantities, particularly the extinction cross section and the orientation distribution, much insight into their influence can be gained by the exercise of theory, by calculation of theoretical $\bar{\varepsilon}$ for different postulated models. Comparison of these predictions with observation, as in the present case with respect to Røttingen's measurements of ε for saithe at two different frequencies, can then permit selection of likely models. In particular, comparison of predicted and measured $\bar{\varepsilon}$–ν relationships for fish in a Røttingen-type experiment can permit determination of the mean extinction cross section, a quantity whose measurement is generally complicated, but whose knowledge is essential to determining the precise effect of shadowing in schools of fish.

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APPENDIX

THEORY OF ACOUSTIC SCATTERING BY A COLLECTION OF IDENTICAL SCATTERERS OF RANDOM ORIENTATION AND DISTRIBUTION

Consider a collection of randomly oriented, but otherwise identical scattering bodies which are randomly homogeneously distributed througout a definite region V of a fluid medium of homogeneous density ϱ_o, constant sound speed c_o and negligible background noise. V is ensonified by a narrowband signal $s(t)$, which is emitted by a generally directional source, or transmitting element. The corresponding echo from V is received by a generally directional receiving element, which is essentially collocated with the transmitting element with which it shares a common principal response axis is in the farfield of the source/receiver. The signal waveform $s(t)$ is sufficiently narrowband so that the pressure field modulated by it can be assumed to be attenuated by the free medium at the constant rate α, which is the attenuation constant of the medium at the center frequency of $s(t)$. The pressure due to the source, which is characterized by its value p_o at unit distance from the source, is sufficiently weak so that nonlinear effects can be strictly ignored.

Let the generally complicated scattering behavior of a single scattering body of V be represented by just two parameters: its backscattering cross section σ_b and its extinction cross section σ_e, which are defined with respect to the precise form of $s(t)$ and which generally are complicated functions of scatterer orientation $\hat{r}'$ relative to scatterer position $\underset{\sim}{r}$. The vector $\underset{\sim}{r} = \underset{\sim}{r}_n$ denoting the position of an arbitrary, or the n^{th}, scatterer, and the unit vector $\hat{r}' = \hat{r}_n'$ denoting the orientation of the same body, are defined with respect to a rectangular coordinate system whose origin is situated at the collocated source and receiver and whose z-axis is coincident with the common principal axis of source and receiver. The dependence of the two scattering cross sections on the position $\underset{\sim}{r}_n$ and orientation $\hat{r}_n$ are abbreviated thus:

$$\sigma_b(\underset{\sim}{r}_n, \hat{r}_n') = \sigma_{b,n} \text{ and } \sigma_e(\underset{\sim}{r}_n, \hat{r}_n') = \sigma_{e,n}.$$

The echo from V, in the form of the mean or ensemble-averaged time-integrated echo intensity ε, can now be developed directly and expressed as a function of the scatterer number density ν.

In the neglect of other scatterers the pressure field incident on the n^{th} scatterer is that of the free medium, time-retarded wave, viz.

$$\frac{p_o\, s\left(t-\frac{r_n}{c_o}\right)}{r_n} D_{D,n} \, exp\,(-\alpha r_n)$$

where $r_n = |\underset{\sim}{r}_n|$ is the distance from source to position $\underset{\sim}{r}_n$ of the n^{th} scatterer, and $D_{S,n} = D_S(\hat{r}_n)$ is a factor expressing the effect of source directivity on diminishing the transmitted pressure field in the generally oblique or non-axial direction $\hat{r}_n$ or the n^{th} scatterer. The effect of the presence of other scatterers on this field is its modification by multiple scattering, which might be treated deterministically by the inclusion of the sundry, calculable effects of absorption and refraction internal to the scatterers and reflection and diffraction external to the scatterers, inter alia, but which is treated here simply by the statistical concept of extinction. According to this mode of description, the composite or mutual influence of the scattering bodies in V on the pressure field within V is allowed to be precisely that of a further attenuation of the incident pressure field by scattering and absorption. Since the spatial distribution of the scatterers in V is randomly homogenous, the rate of this attenuation is the net oblique scattering area per unit area of beam. For the n^{th} scatterer this is the quantity $\nu\bar{\sigma}_e|r_n - r_{n,o}|$ where $\bar{\sigma}_e$ is the mean equivalent scattering area, or extinction cross section, which generally is a function of the location $\underline{r}_n$ of the n^{th} scatterer, and $r_{n,o}$ is the distance from source to nearest surface of V in the direction $\hat{r}_n$ of the n^{th} scatterer. Thus $|r_n - r_{n,o}|$ is the depth of penetration into V of the incident pressure field at the n^{th} scatterer, and the effect of multiple scattering or extinction is diminution of the incident energy field by the factor

$$exp\ [-\nu\bar{\sigma}_e(r_n - r_{n,o})]$$

The pressure field incident on the n^{th} scatterer, in the mean, is

$$p_{inc,\ n} = \frac{p_o\, s\left(t - \frac{r_n}{c_o}\right) D_{S,n}}{r_n}\ exp\left[-\alpha r_n - \frac{\nu\bar{\sigma}_e}{2}\,(r_n - r_{n,o})\right]$$

which is just the free-medium, time-retarded wave as modified by multiple scattering within V.

This expression for $p_{inc,n}$ has a precise meaning only in a statistical sense: as the pressure field to be expected at the n^{th} scatterer, at fixed depth of penetration $(r_n - r_{n,o})$, in the mean with respect to the ensemble of all possible, allowed configurations of scatterers in V. It is now temporarily adopted as the description of the actual field at the n^{th} scatterer for an arbitrary particular configuration of scatterers. Later, this ad hoc use will be seen to be justified when the principal quantity of interest, the mean of the integrated echo intensity with respect to the same ensemble of configurations, is computed.

In the formation of the echo from the n^{th} scatterer the incident pressure field is generally distorted. Because this distortion depends on the precise nature of the scatterer and on the conditions of its ensonification, the wave form of the backscattered pressure field is represented by $s_n(t)$. Its strength relative to the incident field is $\sigma_{b,n}/4\pi$ in the energy domain. When the additional effects of further time retardation, medium absorption and reception by a directional element at the generally non-axial direction $\hat{r}_n$ are included, the contribution to the echo from the n^{th} scatterer may be expressed by

$$p_{sc,n} = \frac{p_o\, s_n\left(t - \frac{2r_n}{c_o}\right) D_{S,n}\, D_{R,n}\, \sigma_{b,n}^{1/2}}{\sqrt{4\pi}\; r_n^2} \; exp\left[-2\alpha r_n - \frac{\nu\bar{\sigma}_e}{2}(r_n - r_{n,o})\right]$$

Here $D_{R,n} = D_R\,(\hat{r}_n)$ is the factor describing the effect of the directionality of the receiver on diminishing the received pressure field from the generally non-axial direction $\hat{r}_n$, and $\sigma_{b,n} = \sigma_b\,(\hat{r}_n)$ is the equivalent backscattering area of the n^{th} scatterer, which is a function of its orientation $\hat{r}'_n$.

The total backscattered field or echo from V after reception, in the proper weak field limit of linear acoustics, is the linear superposition of the several backscattered fields:

$$p_{sc} = \sum_n p_{sc,n}$$

where the summation extends over all νV scatterers in V. The instantaneous intensity $I = I(t)$ corresponding to p_{sc} is the product of in-phase pressure and velocity; thence, by the assumption that $s(t)$ is relatively narrowband,

$$I = \frac{1}{\varrho_o c_o} p_{sc}^2$$

If the signal is of sufficiently short duration T so that the echo from V is completely isolated in time from those echoes produced by extraneous scatterers in the larger medium, such as boundary surfaces, then the time-integrated echo intensity ε; namely,

$$\varepsilon = \int I dt$$

is precisely the energy contained in the echo field of V at the receiving element after effective «shaping» of the echo field by the generally directional receiving element. After appropriate substitution and integration,

$$\varepsilon = \frac{I_o}{4\pi} \sum_{n,m} \frac{D_{S,n}\; D_{S,m}\; D_{R,n}\; D_{R,m}\; \varrho_{nm}\; \sigma_{b,n}^{1/2}\, \sigma_{b,m}^{1/2}}{r_n^2\, r_m^2}$$

$$\times\, exp\left[-2\alpha(r_n + r_m) - \frac{\nu\bar{\sigma}_e}{2}\,(r_n - r_{n,o} + r_m - r_{m,o})\right]$$

where $I_o = p_o^2/\varrho_o c_o$ is a measure of source intensity, and ϱ_{nm} is the cross correlation function

$$\varrho_{nm}(\tau) = \int_{-\infty}^{\infty} s_n(t)\ s_m(t+\tau)dt$$

when evaluated at $\tau = 2(r_n - r_m)/c_o$, which is the difference in arrival times of the echoes from the n^{th} and m^{th} scatterers. Inasmuch as ϱ_{nm} expresses the degree of coherence in the echoes of the $(n.m)$ pair of scatterers, which always irreducibly yields the positive contribution $\varrho(0)$ when $n = m$, ε may be decomposed into two terms, viz.

$$\varepsilon = \frac{I_o}{4\pi}\varrho(0)\sum_n \frac{D_{S,n}^2\ D_{R,n}^2}{r_n^4}\ \sigma_{b,n}\ exp\ [-4\,\alpha r_n - \nu\bar{\sigma}_e(r_n - r_{n,o})]$$

$$+ \frac{I_o}{4\pi}\sum_{n \neq m} \frac{D_{S,n}\,D_{S,m}\,D_{R,n}\,D_{R,m}}{r_n^2\ r_m^2}\ \varrho_{nm}\ \sigma_{b,n}^{1/2}\ \sigma_{b,m}^{1/2}$$

$$\times\ exp\left[-2\alpha\ (r_n + r_m) - \frac{\nu\bar{\sigma}_e}{2}(r_n - r_{n,o} + r_m - r_{m,o})\right]$$

which are, respectively, the incoherent and coherent parts of ε, or those terms which are independent of and dependent on the coherence of the constituent fields of p_{sc}.

Evidently, the value of the time-integrated echo intensity ε, through its origin in p_{sc}, is strongly dependent on the precise configuration of scattering bodies in V. Only for those configurations for which $|r_n - r_m| > cT/2$ for all $n \neq m$ will particular ε be independent of coherent contributions. This condition is very strong and, for the kinds of applications that are to be addressed by this calculation, uninteresting. It is assumed, therefore, that the coherent part of ε generally makes a non-negligible contribution for particular configurations of scatterers.

It has been assumed already that the signal waveform $s(t)$ is relatively narrowband and not of very long duration, although, of course, its duration must be long compared to the inverse of its center frequency in order satisfy the condition of being narrowband. In the limit that $s(t)$ is sufficiently narrowband, the power spectra of $s_n(t)$ and $s(t)$ will be very similar and, as a consequence of the Wiener-Khintchine theorem (Horton 1969), the cross correlation function $\varrho_{nm}(\tau)$ will closely resemble the autocorrelation function $\varrho(\tau)$. If it is assumed further that $s(t)$ is unexceptional and belongs to the class of waveforms which include, for example, pulsed sinusoids and FM slides of narrow bandwidth, then $\varrho(\tau)$, and correspondingly $\varrho_{nm}(t)$, will oscillate rapidly with τ and contain nearly equal positive and negative parts, so that

$$\int_{-\infty}^{\infty} \varrho_{nm}(\tau)\, d\tau << \varrho(0)T$$

which can be shown by application of the Wiener-Khintchine theorem to the power spectrum of $s(t)$. If the collection of scatterers are randomly distributed in V, then the argument $\tau = 2(r_n - r_m)/c_o$ of ϱ_{nm} for $n \neq m$ will be a stochastic variable and the contribution of the coherent part of ε to the mean of ε with respect to the ensemble of all possible, allowed configurations of scatterers in V will be negligible. In this random-phase-type approximation, then, the mean or ensemble-averaged time-integrated echo intensity $\bar{\varepsilon}$ will be simply the ensemble-average of the incoherent contribution to ε, viz.

$$\bar{\varepsilon} = \frac{I_o \varrho(0)}{4\pi} \sum_n \frac{D_{S,n}^2 \; D_{R,n}^2}{r_n^4} \, \sigma_{b,n} \, exp \left[-4 \alpha r_n - \nu \bar{\sigma}_e (r_n - r_{n,o}) \right]$$

where all quantities written with an overhead bar are averaged with respect to the ensemble of all possible, allowed configurations of scatterers in V.