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## **Exact Solution of Bianchi Type-V Model with Variable Cosmological Term- $\Lambda$ and $G$**



A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Physics

By

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September 17, 2017

# **Dedication**

The dedication of this work goes to the beloved persons whom that I won; my mother Nusra, father Nabeel, brothers Ahmed, Mohamed and Babikir, and my lonely only sister Nidal.

# Acknowledgements

I would like to thank my supervisor Dr. Alnadhief H.A. Alfedeel for his comments, remarks, support, and encouragement to complete this work.

My special thanks goes to my family for standing behind me to complete my Master degree. Also I would like to extend my thanks and gratitude to all my friends.

Finally, I thank Shendi University for their financial support which cover all the Master degree fees, and the International University of Africa for accepting me as a M.Sc student in their postgraduate program.

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# Abstract

The homogeneous anisotropic Bianchi type- $V$  cosmological model with variable gravitational and cosmological constants is investigated. Exact solutions of the Einstein field equations are presented in terms of adjustable parameter of quantum field in a curved and expanding background. We found that the Cosmological constant decreases as time increases whereas the gravitational constants increases respectively. The universe in this model approaches isotropy state at late period of time. A clear presentation for the physical and kinematical quantities of the model are also presented.