International University of Africa

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Runge-Kutta Methods and Applications

A thesis Submitted in Partial fulfillment of the Requirements for the degree of M.Sc. in mathematics

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الإستهلال

قال تعالى

(يَرْفَعِ اللَّهُ الَّذِينَ آمَنُوا مِنكُمْ وَالَّذِينَ أُوتُوا الْعِلْمَ دَرَجَاتٍ)

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Dedication

My Parents and Family To

Who love and support me.

And

To my teachers and friends

Who enriched my knowledge.

Acknowledgment

First of all I thank Allah for all those blessings I have day by day. I would like to thanks Dr. Mohsin Hassan for his supporting and providing necessary guidance concerning project implementation. Without his superior knowledge and experience, the project would like in quality of outcomes, and thus his support has been essential.

Nevertheless, I express my gratitude toward my families and colleagues for their kind co-operation and encouragement which help me in completion of this project.

Abstract

The Initial value problems, IVPs are very important in modeling some real life problems. There are many analytical methods designed to solve IVPs. In practice these methods do not cover all types of problems. Thus, numerical methods are used.

In this study a detailed description, with implementation, is given for the class of numerical methods known as Runge-Kutta methods. Both explicit and implicit methods are discussed. Full derivations of the methods are made. Different orders are used.

Practical implementation of the methods are made using Matlab computational environment. These methods are used to solve some practical problems in Civil, Chemistry, Computer Engineering and physics.

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List of Abbreviations

Runge-Kutta method…….. ……..….RK

First order of Runge-Kutta method…….…….RK1

Second order of Runge-Kutta method……RK2

Fourth order of Runge-Kutta method…….RK4

Initial value problems ……....IVPs

Ordinary differential equations……...ODEs

Truncation error…….....Tn

Explicit Runge-Kutta……ERK

Implicit Runge-Kutta …….…… IMRK