International University of Africa

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Interior-Point Methods Applied to Linear and Semi Definite Programming.

A Thesis Submitted in Fulfilment of the Requirements for the PhD Degree in Mathematics.

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DEDICATION

*To my bride, my beloved, and my best friend*s

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ABSTRACT

Most interior-point methods (IPMs) use a modified Newton method to determine the search direction at each iteration. The system of equations corresponding to the modified Newton system can often be reduced to the so-called "normal equation" a system of equations whose matrix is positive definite, yet often ill-conditioned. Using an appropriately preconditioned iterative linear solver, the so-called "the maximum weight basis (MWB) pre-conditioner" of **J. W**. O'Neal into infeasible, long-step, primal-dual, path-following algorithms for linear programming (LP), we show that the number of iterative solver ("inner") iterations of the algorithms can be uniformly bounded by *n* (length of the decision vector) and the condition number of constrains matrix A, while the algorithmic ("outer") iterations of the IPMs can be polynomially bounded by *n* and the logarithm of the desired accuracy. We extend our study to semi definite programming in which, basing on the "Adaptive Preconditioned Steepest Descent" (APSD) algorithm of **J. W.** O'Neal, we proposed new algorithm, called "Adaptive Preconditioned Conjugate Gradient"(APCG). Numerical tests are conducted to compare our algorithm with **J.W.** O'Neal APSD algorithm basing on execution time.

مستخلص البحث

معظم طرق النقطة الداخلية تستخدم طريقة نيوتن المعدلة لتحديد متجه بحث عند أي تكرار. منظومة المعادلات الخطية لمنظومة نيوتن المعدلة يمكن اختزالها عادة لما يسمي " بالمعادلة الطبيعية " وهي منظومة معادلات مصفوفتها موجبة تحديدا وعادة معتلة الشرط. باستخدام طرق خطية تكرارية مناسبة مشروطة مسبقا ما يسمي ب "أساس الوزن الأعظم المشروط مسبقا" ل **J.W.** O'Neal في خوارزميات النقاط الخارجة منطقة الحل خطوة - طويلة بدائي – ثنائي مسار- متبع للبرمجة الخطية أثبتنا أن عدد تكرارات الطريقة التكرارية (الداخلية) للخوارزميات يمكن أن يكون محدودا بانتظام ب (n بينما الخوارزمية التكرارات (الخارجية) لطرق A طول متجه القرار) وبعدد الشرط لمصفوفة القيود ولوغاريتم الدقة المطلوبة. مددنا دراستنا إلي nالنقطة الداخلية يمكن أن تكون محدودة بكثيرة الحدود في البرمجة شبه المحددة التي فيها اعتمدنا علي خوارزمية "انحدار اشد شرطي مسبق مكيف" ل **J.W.** O'Neal أنشأنا خوارزمية جديدة تسمي "انحدار مرافق شرطي مسبق مكيف" . أجريت عدديا اختبارات اعتمادا علي زمن التنفيذ. J. W. O'Neal لمقارنة خوارزميتنا مع خوارزمية

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