UNIVERSITY OF NORDLAND

BODØ GRADUATE SCHOOL OF BUSINESS

MASTER THESIS

Master of Science in Business

Major: Finance and Capital Budgeting
Minor: Economic Analysis

Master Thesis Course Code: **BE305E**

Tail Risk and Its Implications on the Norwegian Equity Market

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Abstract

This study is a two pronged empirical study. First, it uncovers the extreme events' risk of Norwegian equity market (Oslo Børs) in the form of Hill's (1975) power law exponent. Second, it critically analyzes this time-varying tail risk (TVTR) estimator's implications on the Norwegian stock market returns. This study is significantly imperative for equity investors owing to the high persistence level of this estimator. The estimator's time series is highly persistent producing monthly AR (1) coefficient equal to **0.67**.

The empirical findings concludes that tail risk estimated using Hill's (1975) power law exponent failed in forecasting Oslo benchmark index returns over the short as well as long horizon contrary to Jiang and Kelly (2014) empirical findings using the US market data. The cross-sectional analysis yielded mixed results. Cross-sectional analysis on equally weighted tail risk sensitive portfolios also yielded insignificant results however, the results are significant cross-sectionally – when tested for value weighted tail risk sensitive quintile portfolios – controlling for Fama-French three factors, Fama-French-Carhart four-factor momentum model, as well as with respect to the Fama-French-Carhart model plus the Bernt Arne Ødegaard's Oslo stock exchange liquidity factor as a fifth control. The results suggest a long position in the highest TVTR sensitive value-weighted quintile portfolio and a short position in lowest TVTR sensitive value-weighted quintile portfolio yields annualized **26** percent Fama-French three factor alpha. This result has implications for asset pricing. This research will enrich the finance literature linked to the Norwegian equity market. This study also serves as basis for understanding the relation between continuously varying risk of Norwegian market and its repercussions on expected returns.

Preface

This thesis is developed as a final study in the Master of Science in Business with specialization of Finance and Econometrics over the period August 2014 to May 2016 at Bodø Graduate School of Business (HHB) Nord University, Bodø Campus. It was a challenging affair to write this master thesis but owing to my interest in the topic and analytical tasks it became highly interesting. It has given me valuable insight and knowledge around concepts that are not devoted much time during the finance specialized courses at the HHB.

I have gained extensive valuable analysis experience during the process of writing this thesis. I worked in Microsoft SQL server, Microsoft Excel and R to perform the desired analyses. The analyses seemed exceedingly complex in arrears of high amount of research around time-varying risk and alternative econometric models. I am truly grateful to Thomas Leirvik for suggesting this challenging as well as intriguing topic to me. I also want to thank him for continuous encouragement and feedback during the formulation of this thesis. I also like to take this opportunity to thank Svein Oskar Lauvsnes for taking me as a teaching assistant in his econometrics course which helped me a lot in revising and flourishing my learning of R software. And most importantly, I wish to express my gratitude to all my family members for their endless support throughout my master degree.

Muhammad Kashif

Nord University, Bodø – May 2016

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Chapter – 1

1.1. Background

An investment is a process of tying up money or other resources in the expectation of getting rewards in the future. These rewards correspond to the uncertainty of the future. A human being is assumed to be rational in his behavior, which means that one would avoid expected loss and inclined towards expected benefit. A rational investor invests if he anticipates that the future benefits justify both the time his money is tied up, and the exposure to future uncertainty. Investors have various investment options in the world. We can classify these options into two categories, real assets and financial assets. Real assets include developed or undeveloped real estate, crops, plants, and machinery whereas financial assets are claims on real assets. Examples of financial assets are stocks, bonds, treasury bills, certificate of deposits, and derivatives. Common stocks are the main concern in this thesis.

Stocks or shares, in particular common stock, represent ownership in a corporation. Common stock holders are entitled to receive dividends that a firm may pay and are the residual claimants on the firm's real assets. Not all companies pay dividends, and investors in such companies assumes the firm will grow more rapidly than the average firm, and thus will achieve a greater price gain than dividend paying firms. "*The greater the risk greater the reward*" is an old adage that underlines the basic elements of finance, and means that an investor should be compensated for risk. Some risks are worth taking because of the yield they carry. Stockholders or equity holders' benefit lie on two things first, the dividends they receive that the firm decides to pay and second, the increase in price of a share if that firm performs well. These benefits should be in accordance with both the time that their money is bound and the risk of the future. This benefit is the return of equity investors and the previously mentioned two factors constitute the risk of their investment. Risk of an investment usually is stated as the deviation from the expectation.

1.1.1. Risk and Return

The whole subject of finance permeates this paradigm of risk and return. Both, the risk and the return lie in the future, so a rational investor tries to balance the expected return against the expected risk and optimize his behavior. Risk is categorized into different types but can be summarized into two forms; the risk which is associated with the individual security and the risk which is associated with the market as a whole. In the case of individual risk, specific risk that affects the returns on that individual firm's stock; for example, a strike by employees can affect expected profitability of that firm. Risk as variation from expectation does not mean the possibility of a bad outcome only; it also includes variation toward better than expected outcome.

Expected return on a stock is the sum that an investor anticipates to obtain after the completion of a specific time period. Both, Risk and return of a stock lie in the future so we cannot observe them but we can observe realized returns. Realized return on a share during a specific time period is equal to the dividend received during that period in addition to the relative difference between the price of that share in the end of that period and the price of that share in the beginning of that period. It can be computed as:

$$r_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \qquad (1.1)$$

r_t = Realized return on a share

P_t = Price of the share in the end of the period

P_{t-1} = Price of the share in the beginning of the period

D_t = Dividend received during the period

Expected returns lie in the future, which is uncertain but one can estimate it using assumptions and beliefs about the future outcome of the firm and associated different probabilities for each state of the firm's performance for example the firm will perform excellent, good, below average and poorly. In this case, the expected return on a firm's stock in a specific time period, say one year, is equal to the probability-weighted average of the rates of return in each state.

$$E(r_t) = \sum_s p_t(s) \cdot r_t(s) \qquad (1.2)$$

$E(r_t)$ = Expected return on a firm's stock in the next year

$p_t(s)$ = Probability of each state of the firm's performance in the next year

$r_t(s)$= Assumed rates of return given performance of the firm in the next year

The risk is a variation from the expectation or expected mean (μ), thus standard deviation (σ) and variance (σ^2) are good measures of risk presumed normality (see Figure 1.1). Actual returns are most often not equal to μ, but either larger or lower. This is illustrated in Figure 1.1. The upside risk is the chance that the actual returns may lie after μ and the chance that actual returns may lie before μ is viewed as downside risk.

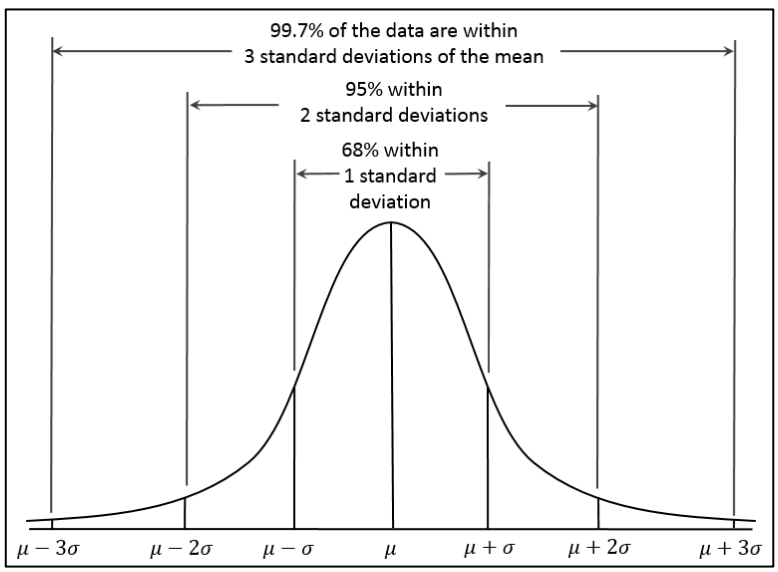

Figure 1.1 - Normal distribution showing mean and standard deviation

If returns are normally distributed, standard deviation can be a good measure of risk. Assets which are free from any kind of risk are called risk-free return, such as US treasury bills (3 months). The US T-bill is considered risk-free due to the fact that the US government has never defaulted on its debt and issues debt in its own currency. The power to tax US residents can raise money to pay the borrowed amount against T-bills, thus they are free from default or credit risk. They are short term assets so with no or insignificant interest rate risk, inflation risk, and maturity risk. A risky investment opportunity must have higher expected return to attract risk aversive investors. This excess return over risk free asset is the compensation for risk taken by investors and is called risk premium.

$$E(R_i) = E(r_i) - E(r_f) \tag{1.3}$$

$E(R_i)$ = Expected Risk Premium on Individual Asset

$E(r_i)$ = Expected Return on Individual Asset

$E(r_f)$ = Expected Risk Free Rate

The risk can be categorized into two groups, individual risk and market risk. Individual risk is the exposure to the uncertainty of returns of that individual asset. As per basic assumption of economic rationality investors are risk averse and they try to avoid risks and contemporaneously seek for higher returns. Individual risk is also characterized as avoidable risk whereas market risk is said to be unavoidable. Optimum financial behavior is pursuing an investing alternative with the least expected risk against the highest realizable expected return under given constraints. Investors use diversification as a tool to reduce individual or idiosyncratic risk. Diversification is investing in a variety of assets so that the negative performance of one individual firm cannot affect one's overall investment with same intensity. Thus in a portfolio with a variety of assets, one asset's negative shock is compensated by other asset's positive shock.

A portfolio of risky assets, and in particular risky assets portfolio optimization, is a long debated topic in finance. In his seminal paper, Nobel laureate Harry Markowitz, Markowitz (1952), characterized risk as the variance of the portfolio. Markowitz argued that investors are mean – variance optimizers. He showed that a combination of assets which are negatively correlated leads to lesser risk, variance, than the stand alone risk of both assets. In his theory, he summarized the risk-return opportunities which are available to any investors in a minimum frontier, which is widely known as efficient frontier curve. William Sharpe (1964) shows a natural relationship between expected return and variance by a model known as Capital Asset Pricing Model (CAPM) based on implications of Markowitz portfolio optimization theory.

1.1.2. Capital Asset Pricing Model (CAPM)

CAPM suggests that if all investors use the same input list, which Markowitz (1952) laid down, to find optimum risky portfolio given identical investible universe, they will end up choosing value weighted market portfolio. CAPM claims that the risk of an individual asset is determined by the contribution of that asset in the variance of market portfolio and Sharpe denoted risk as Beta (β).

$$\text{Beta } (\beta) = \frac{Cov\ (Indivial\ asset, Market\ Portfolio)}{Variance\ (Market\ Portfolio)} \tag{1.4}$$

CAPM was a breakthrough in finance as it provides estimated risk premium which an investor should require given the risk of the asset. This model served a key role in evaluating possible investments by providing benchmark expected rate of return. This model also helped professionals in pricing stocks which are not yet traded in the market. The model use individual asset contribution of risk to overall market to reach that asset's expected return. That was the time when the idea of predicting stock returns using risk factors was conceived. Risk factor calculation was done using advanced mathematics, see equation 1.4. The CAPM can be written as:

$$E(r_i) = r_f + \beta_i\ [E(r_M) - r_f] \tag{1.5}$$

$E(r_i)$ = Expected Return on Individual Asset
r_f = Risk Free Rate
$E(r_M)$ = Expected Return on Market Portfolio

William Sharpe was also awarded the Nobel Prize for his contribution in the field of economics and finance. Afterwards researchers have introduced some extensions to CAPM, for example Merton (1973) formulated the Intertemporal Capital Asset Pricing Model (ICAPM) where the investor is allowed to hedge against a shortfall in consumption, and Breeden's (1979) Consumption Based CAPM (CCAPM). The CCAPM factors in consumption as a means of understanding and calculating an expected return on investment. Merton (1973) suggested that there are a number of risk factors that affect stock returns such as uncertainty in income and price of key consumer goods (inflation risk). He relaxed the CAPM one period assumption and viewed investors as lifetime wealth optimizers; they try to optimize between current wealth and retirement wealth. However, Breeden (1979) characterized risk (β) as the covariance between asset returns and consumption growth. In his model, individual asset risk factor is attributed to the contribution of it towards consumption tracking portfolio whereas in Sharpe's CAPM its market portfolio. He defined consumption tracking portfolio as the portfolio with highest correlation with consumption growth. A more recent development is the X-CAPM, derived by Barberis et al. (2015). In the X-CAPM, some investors form beliefs about future returns by extrapolating past returns, which generate some heterogeneity in the financial market. The X-CAPM captures many features of prices and returns of stocks, as well as being consistent with survey data on investor expectations.

1.2. Introduction

CAPM's prediction of expected return was based on constant variance of historical mean and constant covariances among different assets' returns. But with the passage of time and ongoing research it is apparent that volatilities of returns keep on varying which means that there are some time periods of low volatility as well as there are some time periods of high volatility. Therefore models that encompass this time-varying nature of returns' volatility provide more realistic measure of risk than those which are based on constant volatility assumption. This time-varying behavior of returns' volatility is also termed as *heteroskedasticity*. Heteroskedasticity means that volatility of an asset or whole market tend cluster, high volatility period followed by a high volatility and low volatility period followed by low volatility.

Merton (1980) described that market return variance change over time so estimators based on time series of realized return data should be adjusted for heteroskedasticity. He also suggested of using non market data for forecasting expected return such as the surveys of investor holdings, corporate earnings and other accounting data. It is the expected volatility that is of concern, and it is time-varying. Standard deviation is a good estimator of risk implied that the distribution is normal. However, extreme values are more common in economic data than that of natural data (Mandelbrot 1963), which can affect the foundation standard asset pricing models. Shiller (2003) validated fatter tailed returns data empirically of the annual rate of return of American stock market from 1871 till 2002. All the before mentioned studies found that returns are not normally distributed. Figures 1.2a and 1.2b shows the hypothetical distributions which assets returns can take.

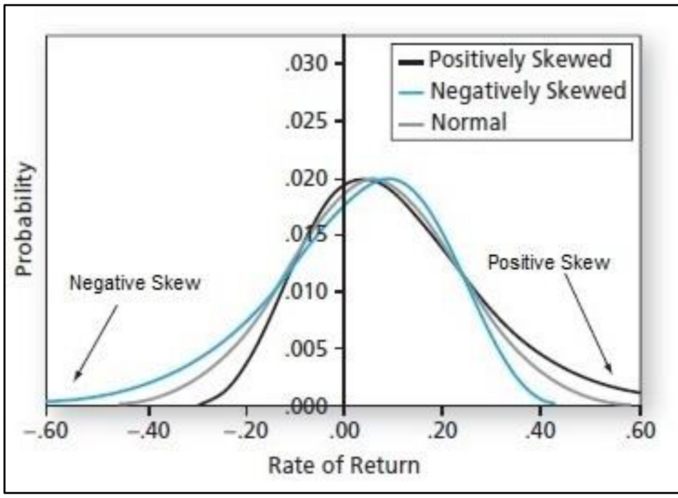

Figure-1.2a – Skewed Returns Distribution

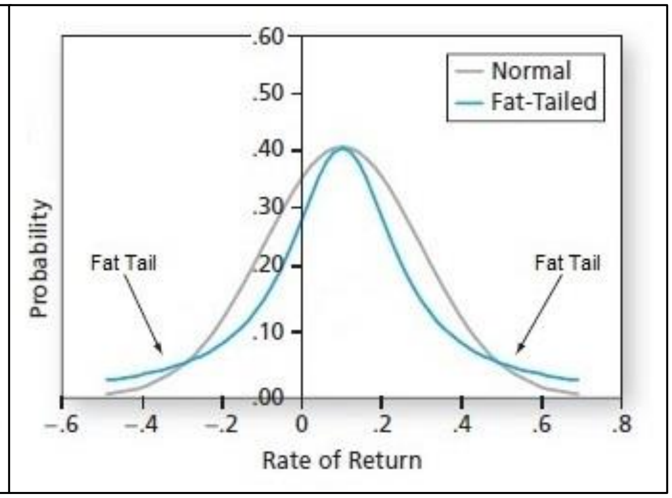

Figure-1.2b – Fat-Tailed Returns Distribution

Predicting the equity premium is of great interest for professionals and academia, thus literature provides a long list of predictors of excess returns. Examples include, but not limited to, CAPM (Sharpe 1964; Linter 1965; Black 1972), dividend-price ratio (Fama and French 1988), earnings-price ratio (Campbell and Shiller 1988), nominal interest rates (Fama and Schwert 1977; Campbell 1987), book-market value ratio (Kothari and Shanken 1997), the inflation rate (Nelson 1976; Fama and Schwert 1977), term and default spreads (Campbell 1987; Fama and French 1989), corporate issuing activity (Baker and Wurgler 2000; Boudoukh et al. 2007), consumption-wealth ratio (Lettau and Ludvigson 2001), and stock market volatility (Guo 2006). Goyal and Welch (2008) discussed various predictors of US stock market excess return and tested a long list of predictors from the literature and concluded that some predictors perform good in-sample but when it comes to out-of-sample equity premium prediction, almost all of them is found to be inconsistent with empirical data relative to historical average. In sample predictability proclaims estimating the model using data of a specific time period and then comparing the model fitted values to the actual realization during that specific time period, whereas out of sample means comparing the model fitted values to the actual observed values after that particular time period.

Almost all models in finance corroborate the relation between expected risk and expected return. There are many reasons for calculating these expectations, i.e. the estimation of expected excess return, avoiding risks which are not worth taking, for fairly pricing of assets and options etc. Investors and academia used variance, in particular square root of variance, as a measure of risk. Robert C. Merton (1980) identified many errors in estimation models of expected risk based on realized returns as well as put light on the noisiness of models which were being used for estimating expected return directly from time series of realized returns. Historically, expected volatility was estimated by a measure of standard deviation of observed returns over a specified period. Selecting a correct time period remained the question as too long horizon sample would be irrelevant for future volatility and too short sample would result in noisy estimate. This is important because volatility changes over time. The literature on time-varying volatility is large and expanding every year (see for example Franses and McAleer 2002).

Equity risk premium (ERP) prediction has evolved over the period of time whereby single variables as well as combination of variables have been used to predict returns. Goyal and Welch (2008) tested a handful of prediction variables and concluded that they all perform bad out-of-sample which triggers prompt opposing responses from econometrics scholars (Spiegel 2008). Kelly and Jiang (2014) proposed an asymptotic measure of time-varying tail risk (TVTR) for predicting aggregate market returns which is directly estimable from cross-section of returns. They argued that the measure is significantly correlated with the tail risk measures calculated from the S&P-500 index. They showed that one standard deviation increase in tail risk forecasts an increase in excess market returns of 4.5% over the following year. They showed that their variable is valid in-sample as well as out-of-sample and predicts ERP positively and this measure has a negative relation with overall economic activity. They compared it empirically with all the variables tested by Goyal and Welch (2008) and showed that their measure achieved better t-statistic and R-Square value. They also showed that, cross-sectionally, stocks with high loadings on past tail risk earn an annual three-factor (FF+Mom+Liq) alpha 5.4% higher than stocks with low tail risk loadings.

1.3. Problem Statement

There are plenty of published papers discussing categorically different variables for estimating ERP includes economic, financial, accounting and estimated variables, see, for example, Fama and French (1988), Campbell and Shiller (1988), Fama and French (1989) Kothari and Shanken (1997), Baker and Wurgler (2000), Lettau and Ludvigson (2001), Guo (2006) and Goyal and Welch (2008). Diebold and Mariano (1995) argued that if the predictor is not an asymptotically estimated variable then predictive regressions may provide a valid forecast for expected return but not otherwise. On the contrary, Torous et al. (2004) stated that the regression of stock returns on lagged financial variables – dividend yield, book to market ratio and default spread – has low power due to the noisiness of stock market returns and hence provide erroneous forecasts. Lanne (2002) also said that, on long-horizon, returns cannot be predicted with a highly persistent variable; financial ratios as they tend to correlate with the dependent variable. These opposing views create ambiguity about which view to follow as either estimated asymptotic variables such as variance, standard deviation, volatility, correlation, skewness, tail risk, value at risk, and kurtosis, predictability is more accurate than the variables extracted from actual data such as dividend payout ratio, book to market value ratio, earnings price ratio, dividend price ratio. These contradictory views of different scholars has made it difficult to choose a prediction variable that delivers superior forecasts but almost all prediction models are based on CAPM logic that premium would be higher on more risky assets and these models demand empirical standing for validity. Goyal and Welch (2008) did a comprehensive empirical study on these predictors using US market data to check how valid these variables are in predicting ERP using predictive regressions in sample and out of sample. They concluded that all these variables perform relatively worse than the historical average.

Volatility has been used as predicting returns which changes with time. After the subprime global crisis it became evident that volatilities are higher than estimated by value at risk (VaR) method. This added fuel to the fire as the European sovereign debt crisis posed a dire need to revisit the measure of tail risks, which were previously based on VaR, due to the notion that VaR is based of assumption of normality (Allen et al. 2013). Kelly and Jiang (2014) presented a new measure of time varying tail risk and showed empirically using US market data from 1963 to 2010 that their measure is correlated with other tail risk measure calculated from S&P500 and predicts returns better than the variables studied by Goyal and Welch (2008). Their study results are striking as higher t-statistic in sample and their measure showed a positive though little R-square value in out-of-sample predictability. This is a new measure and has no literature in context of Norwegian stock market. It is not necessary that all stock markets show same behavior so my contribution would be to investigate this estimator empirically in the Norwegian market context. The research is conducted based on data from the database TITLON, which contains information of the companies listed on Oslo Stock Exchange alias Oslo Børs.

The main questions to be answered by this research work[1] are:

1- What is the tail risk of Norwegian equity market given the time-varying tail risk, TVTR, measure?
2- Does the TVTR measure explain the Norwegian aggregate market returns – Return on Oslo Benchmark Index?
3- Cross-sectionally, what is the empirical standing of TVTR?

[1] All the analyses performed using Microsoft SQL Server, Microsoft Excel and R.

Chapter – 2

2.1. Literature Framework

Risk is basically the likelihood of deviation from expected return or mean. Unlike price variations, expectation is a latent variable so we cannot observe it. If the variance had been constant over time, then standard deviation of observed returns would have been the perfect estimate of risk and the volatility could have been be an observable phenomenon. But unfortunately, variance changes over time and equity return's distributions significantly deviate from normality (Mandelbrot 1963). These deviations from normality can be detected using higher moments of returns distribution. The third moment, which is the ratio of average cubed-deviations from average to cubed-standard deviation, is used to detect asymmetry in distribution called Skewness (Figure1.2a show a depiction of positive and negative Skewness). The fourth moment, which measures the degree of fat tails, is called Kurtosis. It is the ratio of deviations raised to power four to fourth power of standard deviation. A normal distribution has zero value of Skewness and the Kurtosis value of three. A negative Skewness implies that more mass of the returns probability is in the left side of the distribution. There are more extreme negative events expected to happen than positive extreme events and vice versa for positive Skewness. Kurtosis represents fewer values around the mean.

Alexander (2001) stated that volatility and correlations cannot be measured directly from market data, therefore we need econometric models to estimate them. An econometric model is an application of statistical methods to critically asses the hypothetical relationship between economic data. The improvement in these models went side by side in devising different techniques for forecasting excess return and other financial outcomes. Which parameter is a correct estimate of risk? This question is still in the exploration phase. Markowitz (1959) argued that semi-variance is a proper estimate of risk because variance penalized both upside and downside risk. Sharpe (1964) in his CAPM embodied risk as beta (β) which does not change with the high and low volatility periods. Lindenberg (1977) suggested two variations of beta, downside and upside beta, to discern the asymmetric behavior of risk (Ang et al. 2006). Volatility, the degree of variation in the price of a security, is also used as a measure of risk. There are many features of financial time series data volatility as of now; fat-tailed distributions, volatility clustering, asymmetry and volatilities comovements between assets and financial markets (Granger and Poon 2003).

Priestley (2001) argued that stock prices are very volatile and highly responsive to news about future outlook of firms. He stated that persistence exists in expected returns and the time-varying nature of expected returns is a function of time-varying volatilities. Many empirical studies found that extreme returns in lagged period entails extreme returns in the next period. Engle (1982) introduced an econometric model for dynamic volatility named Autoregressive Conditional Heteroskedasticity commonly known as ARCH. ARCH model assumes that returns volatility is clustered; high and low volatility tend to persist thus expected variance is a function of equally weighted average of the squared residuals from last n days;

$$\sigma^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ \dots + \alpha_i \varepsilon_{t-i}^2 \qquad (2.1)$$

σ^2 = Estimate of expected volatility

i = Order of ARCH process

ε^2 = Square of residuals whereas ($\alpha_0 > 0$ and $\alpha_i \geq 0$).

But as a general understanding recent events would be given more weightage than that of older events. This weightage problem of ARCH resolved as Bollerslev (1986) developed Generalized Autoregressive Conditional Heteroskedasticity Model known as GARCH. In this model the weights are assumed to decline geometrically on older residuals. Afterwards there have been many extensions in ARCH i.e. GARCH, IGARCH, Component ARCH, Asymmetric component ARCH, GED-ARCH see Eagle (2004).

Investors have to face risk and return trade-off every time when deciding about investment. The event of subprime crisis left risk observers, analysts, and managers wondering about risk measures. There comes a dire need to find adequate and realistic risk measures as insufficient risk analysis of risk results in mispricing of assets and more severely underestimation of risk (Mitra 2009). The fall of international financial institutions suddenly created an extremely tremulous situation in money markets as well as in stock markets, liquidity of the markets was evaporated, stock prices turned unstable and volatilities and risk were sky-rocked. (Degiannakis et. al 2012)

Financial behavior is fundamentally influenced by risk factors. Angelidis and Degiannakis (2009) described five risk factors in their writing which include business, credit, market, liquidity and operational risk. Business risk is attributed to a specific industry in which a firm operates. Credit risk is the likelihood of a firm being unable to meet its liabilities. Liquidity risk comes into play when an investor is unable to sell a security without a significant change in the price of the security. Operational risks are which arise due to internal systems of a firm. Market risk is the unforeseen behavior of asset prices as a whole, is also often attributed to volatility and value at risk (VaR) is a good measure of market risk.

If we consider volatility as the risk then the fundamental principle of finance that higher risk is traded off with higher return does not seem fulfilling. Baker et al. (2011) explained these inconsistencies that low volatile stocks are better off than high volatility stocks. It means that investors are not being rewarded for taking additional risk, contradicting standard financial theory such as the CAPM. They concluded that when an asset's volatility is high institutional investors, such as mutual funds, are fiercely after these assets to secure higher returns than the index S&P500 thus decreasing their expected returns. Hsu et al. (2013) claimed that financial analysts exaggerate forecasts for high volatile stocks that trigger high demand for these stocks among investors. They assumed that most of the investors over-react to analysts' high forecasts which lead to systematic overpricing of high volatility stocks thus lower returns for high volatility assets.

Ibbotson et al. (2014) answered these low volatility puzzle differently. They asked a more fundamental question: which one is rewarded – tail risk or volatility? Tails are referred to the end portions of the bell-shaped distribution curves, see figure-2.1. These curves show the statistical probabilities of possible outcomes, the left tail represent extreme negative outcomes and the right tail represent extreme positive items. They explained that volatility and beta are the risk estimators that penalized assets of non-normal behavior, either its left tail or the right tail, so these both so not seem good estimators of risk. They claimed that all investors unambiguously see left tail values as bad outcomes consequently they should be rewarded for taking high tail risks in particularly left tail risk.

The market does not behave normally as evident from various crises; stock market crash 1929, black-monday 1987, asia crisis 1997, dot com bubble crash 2000 and subprime and banking crisis 2008 (Ibbotson et al. 2014). Investors require a premium to invest in negatively skewed or left fat-tailed stocks. The definition of risk aversive investor would be "One who seeks to minimize the left tail risk of returns without inhibiting the right tail growth potential". Ibbotson et al. (2014) termed the premium as tail-risk premium and argued that tail risk premium is economically significant in US stocks. They used standard deviation for volatility and two variables – Skewness and an extension of VaR – for calculating tail risk. Figure 2.1 shows distribution with high tail risks, red color shows the probability of extreme negative returns while green color shows the probability of extreme gains.

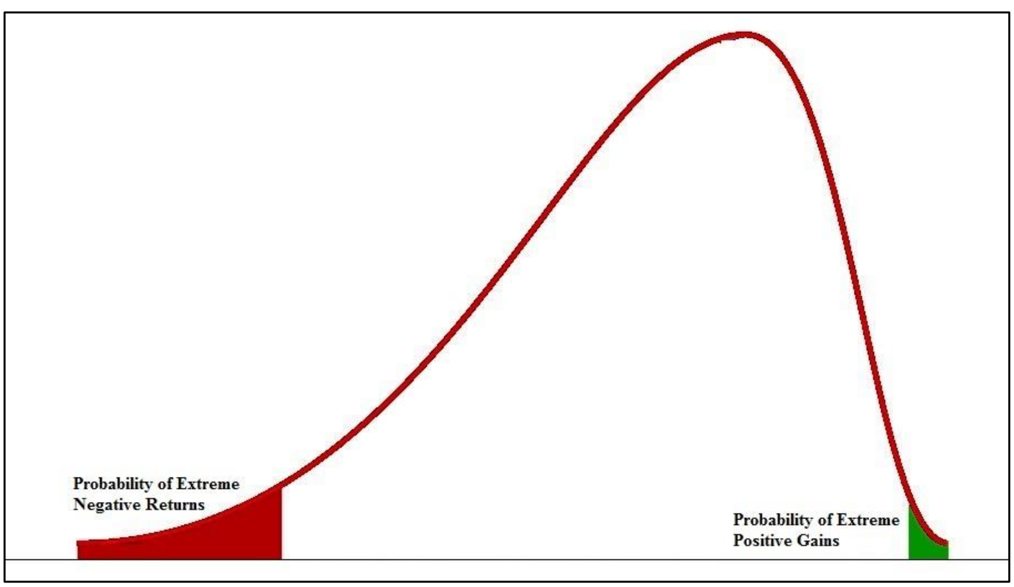

Figure-2.1 – Returns Distribution with Higher Probability of Extreme Negative Events

The global financial crisis of 2007-2009 was the main event that put many professionals and academic researchers to look for better and realistic measures of tail risk. Since then, tail risk has been a heavily discussed topic among academic researchers due to the fact that returns often violate the assumption normal distribution (Ibbotson et al. 2014). The tail risk discussion started very early when Mandelbrot (1963) and Fama (1963) studied the behavior of returns and concluded that prices in markets can take an unforeseen course which cannot be expected from Gaussian (normal) distribution. Akgiray and Booth (1988) and Jansen and de Vries (1991) further the work and deduced that the behavior of returns under the tails is fundamentally different from residual distribution. Sortino and Price (1994) argued that downside deviation should be used as a risk measure as it represents left tail risk.

Since JP Morgan introduced its matrices based risk measure VaR, VaR has been used as a benchmark by financial sector to manage their risk. Since VaR does not account for the whole distribution, it was believed to be the logical tail risk estimator (Xu 2014). Rockafellar and Uryasev (2000) proposed a new measure of tail risk, Expected Shortfall, based on VaR, though coherent. It is also termed as conditional value at risk (CVaR) and conditional tail expectation (CTE) (Rockafellar and Uryasev 2000; Bodie et al. 2004). Researchers were thriving for improved tail risk measures and resorted to using quantitative theories in parameterizing tail risk.

Bali (2003) introduced extreme value theory (EVT) in the measurement of tail risk. EVT is a statistical theory which deals with the extreme deviations from median in a probability distribution. Ze-To (2012) stated that EVT method parameterizing of extreme tail distributions entails the extension of the probability curve beyond the range of data. He proclaimed that using EVT in estimating VaR gives a better estimate of extreme values. He advanced to reaffirm the findings of Bali and Neftci (2003), they derived VaR using EVT and claimed that their model accurately forecasts the occurrence and size of extreme observations. Marimoutou et al. (2009) implemented EVT theory in estimating tail risk in energy markets and claimed that EVT models provide better estimates of tail risk than that of GARCH model. It is evident from literature that returns are dependent on tail risk and the tail risk premium is economically significant.

Agarwal and Naik (2004) stated that financial securities are associated with different kinds of risk and each risk factor provides its associated risk premium. There is no single investment strategy that entails entire risk factors. So, for investors who want to get higher returns associated with different risk factors must engage in different investment strategies. Mutual funds employ strategies that account for equity, interest rate and default risk such as buy-and-hold strategy. However, Hedge funds follow dynamic trading strategies to capture risk premia associated with dynamic factors such as bid-ask spread. Their target is to beat the passive benchmark. They offer exposure to such risk factors that other investment funds such as mutual funds cannot offer. They expose investments to higher left tail risks and the risk-return relation is nonlinear in their case. They bear significant left tail risks which are severely understated under mean-variance framework; CVaR seems to a better performing estimator for those extreme risks (Agarwal and Naik 2004).

According to Bodie et al. (2004), it is evident that the frequency of negative extreme returns is not what is represented by a normal distribution, as the distribution is skewed and often has kurtosis – fat tails. They proposed four measures to cater for the vulnerability of extreme negative events. These are value at risk, expected shortfall, lower partial standard deviation, and the frequency of extreme returns – 3-sigma. The first two are explained in this document.

2.1.1 Returns Predictability & Market Efficiency

Market efficiency refers to the concept that prices of individual stocks as well as indexes reflect all available information. The market efficiency hypothesis was formulated by Nobel Memorial Prize holder in economics, Eugene Fama, in 1970. This hypothesis explains the returns predictability phenomena. It states the equity prices fully reflect information available in the market and no one can take advantage of stock price prediction because no one has access to information which is not already available to everyone. The information does not have to be a financial news or financial analysis; it can be political, legal, law and order, social or warfare. The information does not have to be true, it may be rumored information. Thus efficient market hypothesis claims that no one can consistently beat the market meaning that one cannot gain higher returns by predicting stock prices repeatedly (Fama 1970).

According to efficient market hypothesis (EMH) stock prices follow Random Walk. Random Walk in this context refers to the idea that in a stock price series each successive price change represents a random departure from previous price. The idea asserts that if flow of information is not hampered and information is immediately reflected in equity prices, then today's stock price movement is a reflection of today's news and this movement is independent of last day price. By definition, news cannot be predicted as a consequence stock price movements are purely random and unpredictable. EMH proponents advocate the idea that strategies that aim to outperform the market consistently will fail and passive investors will be better-off. There is also transaction cost involved in deploying active beat-the-market strategy which strengthens the EMH school of thought that investment in index fund is more profitable (Heakal 2013).

Fama (1970) also explained three states of market efficiency according to their degree of intensity; weak efficiency, semi-strong efficiency and strong efficiency. Weak efficiency suggests stock prices reflect all historical market data such as past prices, sale volumes and dividends. Semi-strong efficiency refers to stock prices being a reflection of past market data as well as current publicly available information such as financial statements. In strong market efficiency state, stock prices reflect past market data, publicly available information and private information such as insider information. EMH stands for non-predictability of returns in all three states. Statisticians, however, found that returns series show persistence which made the basis for returns predictability argument. Stock returns are partially predictable and empirical studies showed statistically significant predictability evidences (Schwert 2003). He also stated that most of the prediction patterns seem to disappear after being published in finance literature.

Heakal (2013) stated that in the real world there is evidence of market inefficiency since there are investors who have outperformed the market and made billion such as Warren Buffet. Such investors' investment strategy revolves around finding undervalued stocks and making money out of it. He also explained that EMH does not mean that stock prices remain equal to their fair value all the time. EMH suggests that investors, who have beaten market, were just lucky means they did not do it out of skill. Heakal (2013) concluded that although the advancement in information technology has made markets more efficient but they cannot be absolutely efficient or fully inefficient hence a mixture of both.

Kelly and Jiang (2014) remained silent over EMH. However, they claimed that their TVTR measure, which is calculated using Hill's (1975) power law, has strong return's prediction power due to its persistence. This statement is implicitly in opposition to EMH of returns' non-predictability. They also claimed that TVTR has asset pricing implications. Assets with high loadings of left tail risk remain deterred to investors due to high vested risk. This deterrence makes these assets undervalued therefore they become good contenders for above average expected return.

The aforementioned argument relies on the basic economics' "General Equilibrium Theory" of demand and supply. However if markets are efficient TVTR predictability will disappear. According to EMH investors must sense that the markets are inefficient so investment strategies will be focused on outperforming the market. Ironically, these beat-the-market strategies serve as the impetus that keeps a market efficient.

It is my conjecture that market efficiency hypothesis is moderately in line with economics' general equilibrium theory of demand and supply, because as the prediction pattern becomes available to professionals in market. They will try to beat the market, this action will in-turn increase the demand for that particular asset with above average expected return. Therefore price of that asset will increase and the above average expected return will disappear. But it does not mean that there are no ways to beat the market. Investor, who found the pattern first and exercised it, will be better-off than other pattern followers.

2.1.2 Value at Risk

The value at risk is the loss which is associated to a left tail of the return distribution. It is actually the value linked to a very low percentile of the return distribution such as 1%, or 5%. VaR is the most commonly used estimation technique for left tail risk especially among financial sector. If p is the percentile of a distribution, p% of possible values lie below it for example 5 percentile means that 5 percent of realized value will be expected to be below that mark. Five percent VaR is commonly used among professionals; meaning that 95 percent value of return will be above VaR value and 5 percent will be below (Bodie et al. 2004). Gilli and Këllezi (2006) defined VaR as "*the capital sufficient to cover, in most instances, losses from a portfolio over a holding period of a fixed number of days*."

Xu (2014) stated that VaR serves as a benchmark in financial sector in managing risk. He further explained the concept as it is the maximum possible loss a risky portfolio holder can expect over a period of time. The common use of VaR is in banking sector as Banks use this measure to calculate capital requirements. VaR under the assumption of normality with zero mean unity standard deviation:

$$\text{VaR}\ (.05, \text{Normal}) = \text{Mean} - 1.65\ \text{SD} \tag{2.2}$$

Gregory and Reeves (2008) stated that VaR calculating methods are based on the assumptions, time-invariant distribution of portfolio returns and constant security holdings. These assumptions are not realistic due to the notion that distributions of the asset returns are time-varying and the active trading strategies results in unexpected changes in portfolio size. They put light on the two estimation methods – Historical Simulation and Variance-Covariance – and explained the latter method's underlying normality approach which is not supported by empirical returns data whereas historical simulation will always be having "Black Swan" problem. They hunch that VaR forecasts would fail as the left tail mass is not actually represented by the confidence interval.

Angelidis and Degiannakis (2009) explained various criticisms of the VaR, such as different VaR estimation techniques present different results, thus it is imprecise. The risk estimates are not sub-additive therefore no coherence in the parameter. Bodie et al. (2004) stated that it is an optimistic measure of left tail risk due to the fact that it takes into account the highest worst case scenario value. Moreover VaR estimate is very difficult to use by investors in optimizing their portfolios, see, for example, Yamai and Yoshiba (2005). VaR is usually reported in a positive number say 5%VaR of 1 million dollars means that there is a 95% chance that the portfolio return over the next specified time will be greater than 1 million dollars (Crouhy et al. 2014). As if now, there have been many improved extensions of VaR to resolve the criticized issues, see for example Pearson and Smithson (2002).

2.1.3 Expected Shortfall

Artzner et al. (1999) presented expected shortfall (ES) as a new measure of risk and criticize VaR being non-coherent. This measure has been given many names by different scholars such as Conditional Value at Risk – CVaR (Rockafellar and Uryasev 2000), Conditional Tail Expectation – CTE (Bodie et al. 2004) and Tail VaR (Gourieroux & Jasiak 2011). This measure holds the aspects of VaR methodology as well as focus more on the distribution of returns in the tail. In simple words ES is the expected value of the left tail distribution. As being the mean of the left tail, ES value is always greater than VaR in absolute sense thus it does not depict as optimistic view of risk as predicted by traditional VaR. Figure-2.2 showing values of VaR and ES.

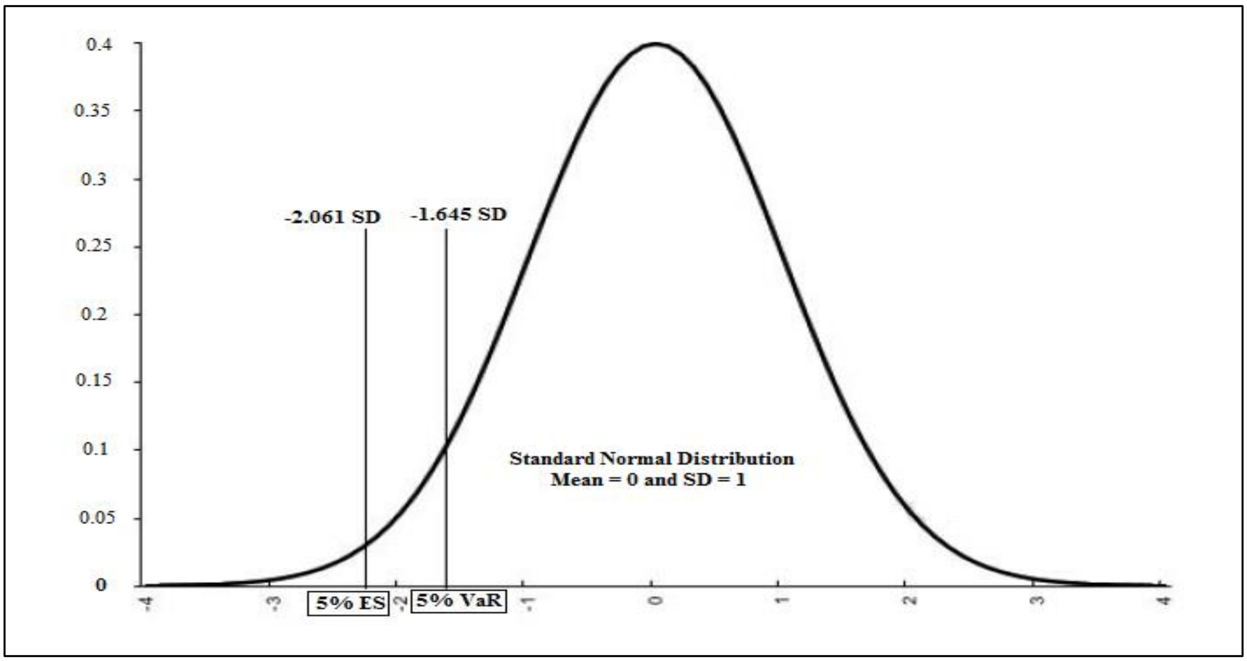

Figure-2.2: Difference between values of ES and VaR

As its clear from the figure-2.2 that ES is more reliable parameter of risk especially in market turmoil conditions and it does not discourage diversification though VaR occasionally does (Angelidis and Degiannakis 2009). ES estimates can be postulated using ARCH/GARCH volatility process or using extreme value theory (Xu 2014).

2.1.4 Time Varying Tail Risk (TVTR)

It is apparent from empirical studies that the distribution of returns is time-varying and has more probability mass in the left tail, therefore tail risk is a contender for economically significant premium. Kelly and Jiang (2014) proposed a new measure of time-varying extreme events' risk. They claimed that a dynamic univariate tail risk parameter which is calculated from a time series of market returns using GARCH model is infeasible due to the less frequent extreme events. They resorted to individual firm tail events to estimate the overall tail risk measure under the assumption that firm level tail distributions embody similar dynamics. They took refuge in advanced mathematics (Power Law) to calculate extreme return events of individual firms. Their assumption is validated due to the highly correlated TVTR estimates of different industries ranging from 57% to 87%. When calculated on individual stocks, TVTR shows highly persistent results with first degree autoregressive coefficient AR(1) equals 0.927 – AR(1) value close to one represent high persistence – which means that it has high predictive power. TVTR also have significant prediction power for aggregate stock returns, and can serve as a variable in explaining asset price behavior. The predicting power of TVTR outperforms other common predictors of equity returns such as dividend price ratio and various other variables tested by Goyal and Welch (2008). They claim that this measure can be used in any setting where large cross-section is available. The focus of my master thesis is on the TVTR measure and its implications in the Norwegian stock market. TVTR is calculated using hill's power law using equation (3.1):

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K} ln \frac{R_{k,t}}{u_t} \tag{3.1}$$

λ_t^{Hill} = Tail Risk Component

u_t = Tail Threshold

$R_{k,t}$ = The kth daily return that falls below an extreme value threshold

K_t = The total number of exceedences over threshold within a month

Chapter – 3

3.1 Dataset Creation

This section presents the sources of data used for the computation of TVTR and performing related analysis. The risk estimator TVTR depends on the cross-section of returns, so I needed a large panel of stocks to gather sufficient information about the tail events at each point in time. Norwegian stocks data is obtained from the TITLON[2] database, which contains Norwegian market (Oslo Børs) data from 1983. The database contains Norwegian data of equities, mutual funds, indices, bonds and derivatives. The database contains variety of variables such as unadjusted, fully adjusted prices, logarithmic risk free rate, logarithmic returns and many more. I used the dataset from January 1990 till December 2015 for calculating TVTR and for testing the estimator performance in explaining returns on Oslo Benchmark Index[3] (OSEBX).

The dataset consists of 1,369,396 daily records of 800 unique securities and 672 unique companies[4]. I needed only common stocks' records to compute TVTR as per the original model specified by Jiang & Kelly (2014) and to mitigate dependence among returns on categorically different stocks of the same company. The TITLON database has a variable called *Description*[5] which clarifies the type of stock issued by each company. I dwindled the dataset by 204,608 records through choosing record of stocks with *Description* title "*A-aksjer*", "*Ordinære aksjer*" and "*Konverterte A*". This elimination left me with 1,164,788 daily records of 661 unique securities. Dataset for returns on OSEBX was also gathered from the TITLON database.

[2] TITLON is a database with financial data from Oslo Stock Exchange for all universities and university colleges in Norway. It contains detailed daily financial data with fully adjusted prices. - See more at: https://uit.no/forskning/forskningsgrupper/sub?p_document_id=352767&sub_id=417205#sthash.TyM0wwCM.dpuf

[3] Linked benchmark Index of OSEBX used with the security id 2

[4] Unique companies and unique securities are identified by the company id and security id respectively

[5] A full list of stock types (Description) is available in the Table A1 in appendix (Norwegian language)

Monthly returns on OSEBX were calculated by summing up daily logarithmic returns from TITLON owing to the additive consistency property of logarithmic returns.

Descriptive Statistics					
Common Stocks Listed on OSE				**Oslo Benchmark Index**	
Year	No. of Stocks	Mean Return	Standard Deviation	Mean Return	Standard Deviation
1990	135	-1.61	58.92	-1.20	24.59
1991	122	-2.89	71.48	-0.82	25.80
1992	123	-4.84	78.62	-0.88	27.73
1993	138	5.83	66.70	4.16	19.77
1994	144	0.07	40.45	0.57	19.81
1995	159	1.68	37.65	0.91	12.55
1996	172	2.69	43.09	1.56	15.07
1997	214	1.02	43.31	2.33	13.48
1998	231	-4.77	55.19	-2.62	36.15
1999	228	2.87	58.99	3.29	17.35
2000	226	-1.02	60.14	0.26	15.65
2001	210	-3.28	69.64	-1.32	24.86
2002	196	-6.28	79.94	-3.10	26.76
2003	190	3.67	72.63	3.29	23.56
2004	183	2.25	48.06	2.71	17.02
2005	217	3.10	49.56	2.83	17.96
2006	238	2.09	38.08	2.34	15.58
2007	272	0.06	53.87	0.90	13.36
2008	266	-9.18	77.03	-6.48	47.08
2009	247	2.14	82.51	4.16	19.58
2010	238	0.29	61.46	1.40	22.81
2011	231	-3.69	60.25	-1.11	17.84
2012	221	-0.22	58.13	1.19	14.90
2013	220	1.11	62.51	1.77	9.98
2014	216	-1.70	49.72	0.40	9.50
2015	208	-1.77	61.86	0.48	12.46

Table 1 - It includes the number of common stocks listed on OSE during the year, monthly average of log returns on all common stocks and their respective annualized standard deviation. The right side of the table presents the average monthly log return on OSEBX and its respective annualized standard deviation. Return and standard deviation figures are percentages.

Table 1 shows the descriptive statistics of the data fetched from TITLON to test the hypothesis that TVTR is a sound measure of risk and it could explain positive returns in the forthcoming period. The occurrence of negative returns is visible just by a glimpse of the table above. Both losers and gainers are in the sample but losers are more intensely negative in converse to the gainers positive impact yielding the monthly mean returns of all common stocks to **-0.48** percent. The amounting figures of standard deviation show evidently that the spectrum of monthly returns is very wide. Listed number of common stocks keeps on varying due to commencement of new firms and established firms going bankrupt or private.

This research includes two types of empirical analyses to evaluate the performance of TVTR. A good performing risk estimator has to have significant effects on the returns going forward such that an increase in risk estimator should result in increase in forthcoming returns. After estimating TVTR using common stocks listed on Oslo Stock Exchange (OSE), I evaluated its performance cross-sectionally and overall. The factors data used in cross-sectional analysis downloaded from Bernt Arne Ødegaard's webpage[6] though the monthly risk free rate factor[7] fetched from the TITLON. The cross-sectional analysis conducted using data sample from January 1990 to December 2011 due to the non-availability of factors data of Norwegian market after 2011.

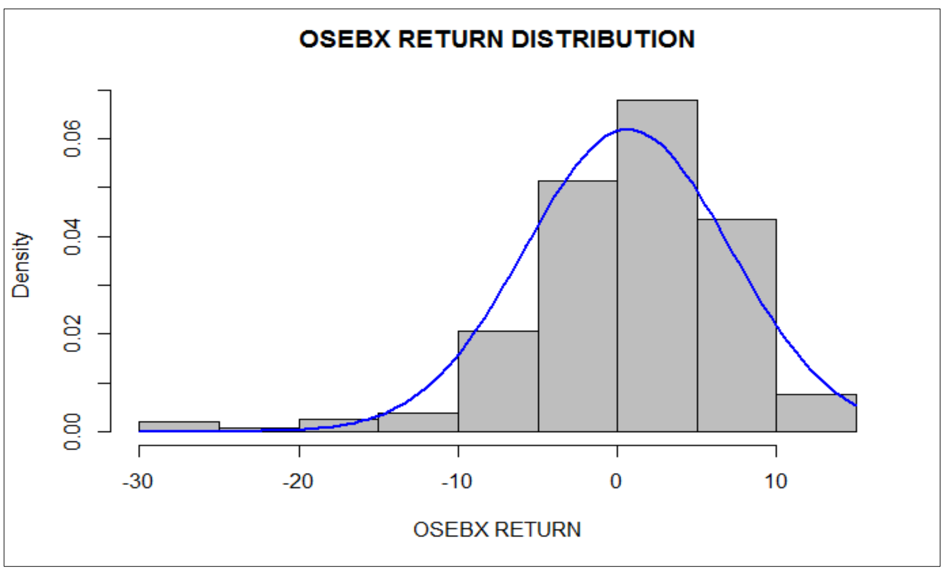

Figure-4.0: An illustration of monthly OBX log returns distribution

Figure 4.0 illustrates the realized monthly OBX log returns distribution. The left skewness is clearly visible from the graph. The occurrence of extreme negative event is more probable than that of extreme positive event.

[6] Factors data is available at https://www1.uis.no/ansatt/odegaard/financial_data/ose_asset_pricing_data/index.html

[7] The monthly risk free rate factor was calculated using the variable "bills_DayLnrate" in the TITLON macro enabled excel sheet "Stocks Advanced"

Chapter – 4

4.1 Discussion & Analyses

This part of the paper critically investigates the time varying extreme events risk of Norwegian market and its estimator TVTR. Moreover the effects of this risk estimator gauged on the returns of OSEBX. Further the performance of TVTR will be evaluated on the portfolios constructed based on the loadings of past tail risk. This section also discusses biases related to the specified analyses. This study is a two pronged empirical study. First, it uncovers the extreme events' risk of Norwegian market in the form of TVTR. Second, it critically analyzes TVTR implications on the Norwegian stock market returns. This study is significantly imperative for equity investors owing to the high persistence level of this estimator. TVTR produces significant results cross-sectionally in value-weighted quintile portfolio setting controlling for Fama-French three factors, Fama-French-Carhart four-factor momentum model, as well as with respect to the Fama-French-Carhart model plus the liquidity factor of OSE made available on Bernt Arne Ødegaard's webpage as a fifth control.

4.1.1 TVTR Estimator

There are various fundamentally different estimators to measure extreme events risk such as risk-neutral skewness and kurtosis calculated using option price data introduced by Bakshi et al (2003), Balcus et al (2011) explained disaster risk premia using index option data. The aforementioned estimators are based on options data though Jiang and Kelly (2014) time-varying tail risk estimator is based on daily return data. The potency of TVTR depends upon the key underlying assumption that the tail risks of individual stocks share similar dynamics. Under such assumption, if the sample is sufficiently large, plentiful number of stocks will experience tail events every period which will result in a precise estimate of the prevailing tail risk.

The number of common stocks listed on OSE[8] is very few to fundamentally test the underlying assumption of TVTR. Ideally, TVTR should be calculated using categorically different industries to examine the underlying assumption. The stock prices in the same industry usually co-move and have interdependence and this dependence implicitly exaggerates the commonality among TVTR estimates. Dividing the firms into different industry categories leaves very little number of companies in industries' categories that present another estimation challenge as it causes too noisy estimates. I created two sets of stocks randomly from the total listed common stocks to overcome the interdependence issue arose during evaluating the underlying assumptions of TVTR.

I estimated TVTR from these two sets of data separately to check whether the assumption hold true. I tested the assumption that tail events are governed by the same process using Spearman's Rho. There is a clear rationale behind using Spearman correlation other than the widely used Pearson correlation. Pearson correlation based on the assumption that the comparing variables are normally distributed on the other hand Spearman correlation does not presume normality of tested variables[9]. The snapshot of results from **R** is as follows:

```
        Spearman's rank correlation rho

data:  Set-1 and Set-2
S = 2640496, p-value < 2.2e-16
alternative hypothesis: true rho is not equal to 0
sample estimates:
      rho
0.4783521
```

Figure-4.1: R result of Spearman Correlation between two randomly selected set of securities

[8] The total number common stocks listed on OSE were just 194 in the last month of year 2015.

[9] First I ran the Shapiro–Wilk normality tests to check the distribution of the two TVTR series calculated from separate randomly selected securities. Both the series failed to conform normality, Snapshot of Shapiro–Wilk normality tests results of Set-1 TVTR and Set-2 TVTR are pasted in appendix – Figure A1.

It is clear from the Figure 4.1 that the two series of TVTR are significantly related. The extremely low p-value suggests that we strongly reject the null hypothesis of no relation between the two TVTR series. The two TVTR series are approximately 48% correlated which gives validation to the underlying assumption of TVTR in the Norwegian context. This result is consistent with the US data results of Jiang & Kelly (2014).

It is evident from various threads of literature that the risk fluctuates time to time of an investment and risk estimators such as standard deviation, Value-at-Risk are time invariant (Gregory and Reeves 2008). TVTR is calculated month by month using daily log returns of all common securities with no overlapping of data which leaves us with no mechanical persistence. Yet, TVTR shows high level of persistence with monthly AR(1) coefficient equals to **0.673.** Though, TVTR persistence level in Norwegian market is not as high as in the US market (AR(1) Coefficient equals to **0.927**) calculated by Jiang and Kelly (2014). This high level of persistence means TVTR can be a good predictor of individual stock returns even in Norwegian market.

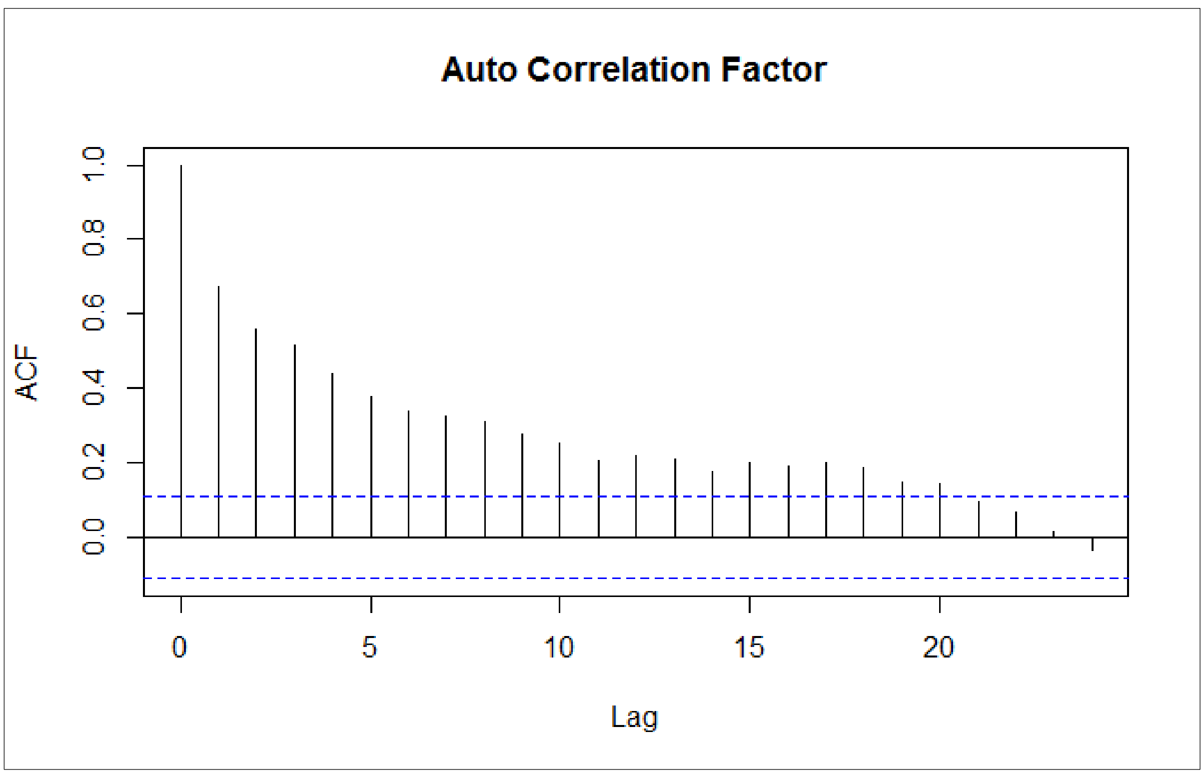

Figure-4.2: This graph shows the persistence level of TVTR series calculated each month by pooling daily log returns of all common stocks listed on OSE from 1990 to 2015.

Figure 4.2 undoubtedly demonstrates that TVTR series is weakly stationary and persistent. The effect of a shock vanishes away gradually. This means every next month estimate of TVTR is dependent on its lag so TVTR can be used as a predictor. The effect of the tail risk shock completely vanishes after 23 to 24 lags.

4.1.2 TVTR & Aggregate Market Return

The performance of TVTR in forecasting returns is based on the power law. In simple words, the events of the left tail or Pareto distribution of the assets return contain the information to describe forthcoming aggregate return distribution. An extensive research has been conducted by various renowned finance scholars on the prediction power of Pareto exponents, for example Gabaix et al (2006), Akgiray and Booth (1988), Fama (1963), Mandelbrot (1963). TVTR (λ) is also a Pareto exponent.

I estimated this Pareto exponent – TVTR (λ) using all common stocks listed on OSE. A large cross-section is crucial for this parameter to be accurate. There are two main empirical challenges arise that question the robustness of TVTR estimate; dependence among assets' returns and heterogeneity in volatility. Jiang and Kelly (2014) already answered these issues by performing Monte Carlo analysis in the presence of both issues[10].

I test the hypothesis that TVTR predicts aggregate market returns by running series of OLS regressions. All the regressions are conducted on the monthly frequency of TVTR against OBX return. These results are comparable with the results of Jiang & Kelly (2014) due to same methodology. The statistical results of TVTR predictions of aggregate market returns are significantly different in the both markets; US & Norwegian. In contrary with Jiang & Kelly (2014) results the tail risk exponent failed to predict OBX returns as a proxy for aggregate market returns. Table 2 presents the results of the monthly OLS regressions of OBX returns

[10] . The issues become more crucial in this research due to very less number of stocks in Norwegian market as compared to US market. To examine the robustness and to increase the size of cross-section I calculated TVTR series using all kind of equity securities and find extremely similar results. The correlation coefficient between both TVTR series is 96 percent.

against TVTR over the time period of one month, one year, three years and five years. The monthly observations used in these regressions are overlapping therefore Newey-West (1987) standard errors correction model used to calculate t-stats[11]. The t-stats were calculated by applying the lag length equal to the number of months over the prediction time period.

Prediction Time Period	**Norwegian Market** ($R_{OBX} = \beta_1 + \beta_2\lambda_{NR}$)			**US Market** ($R_{CRSP} = \beta_1 + \beta_2\lambda_{US}$)		
	Coeff.[12]	**t-stat**	R^2	**Coeff.**	**t-stat**	R^2
Over the period of one month	6.63	1.37	0.73	4.54	2.08	0.7
Over the period of one year	4.05	0.81	0.28	4.02	2.04	6.1
Over the period of three years	3.08	0.59	0.16	3.65	2.40	16.6
Over the period of five years	-6.20	-1.29	0.66	3.16	2.65	20.9

Table 2 – The table shows univariate TVTR prediction performance of aggregate market returns. R_{OBX} stands for excess return over risk free rate on Oslo benchmark index, λ_{NR} stands for TVTR series calculated using daily log returns of all common stocks listed on OSE. US market results are presented from the Jiang & Kelly (2014). All R-Squared values are presented in percentage form.

I scaled the series in a manner that results can be stated in a form that the percentage increase in the following OBX returns due to one standard deviation increase in tail risk. Accordingly, the first row of table 2 left section presents Norwegian market results, a one standard deviation increase in left tail risk predicts an increase in future excess return over risk

[11] Jiang and Kelly (2014) used Hodrick's (1992) standard error correction model to calculate t-stats which produces more conservative t-stats than Newey-West (1987) standard error correction model. The hypothesis, TVTR predicts OBX returns strongly rejected by Newey-West model subsequently Hodrick's model would reject the hypothesis even more strongly.

[12] All the coefficients are annualized to make them comparable with Jiang & Kelly (2014) results.

free rate of 6.63 percent per annum over the period of one month. But the low values of t-stats and R-Squared compel us to reject this claim. In the US market tail risk strongly predicts future returns and its performance becomes stronger long horizons as the t-stats and R-squared values increase with the prediction horizon. Contrarily, in the Norwegian market, the same estimator failed to forecast future excess returns and even its performance keeps on weakening with the expansion of the horizon. In the last row of Norwegian market results the prediction effect becomes negative in a sense that an increase in the tail risk parameter predicts a decrease in future returns over the horizon of five years.

The predictive regression over the horizon of one month yields the highest statistical results among all horizons forecasting results. It produces approximately the same R-Squared value in comparison of US market results. This R-squared value described as **0.7** percent variation in next month's OBX returns is explained by the TVTR estimator. However, the t-stat is very low, just **1.37**, in comparison with US **2.08**. The main statistic used to reject a hypothesis is t-stat that is very low. Therefore TVTR estimator fails to forecast OBX returns over the horizon of one month.

The same type of predictive regression is conducted however, over the horizon of next year. Strangely, the results took a swift from the US market results. TVTR prediction performance decreased when the regression is performed over the horizon of one year while it increased in the US market. The portion of OBX returns variation explained by TVTR decreased to **0.28** percent. It further decreased to **0.16** percent when predictive regression performed on OBX returns against TVTR over the horizon of three years.

The predictive regression, performed on OBX returns against TVTR over the horizon of five years, produces entirely opposing results from the hypothesis that increase in TVTR leads to increased aggregate expected returns. The coefficient becomes negative which means that an increase in tail risk forecasts a decrease in the Norwegian aggregate market returns over the next five years. This last row regression results give a feeling of cyclical behavior of Norwegian equity market alias Oslo benchmark index[13]. Data noisiness[14] can also be one of the reasons for this behavior, or the strong commonality with the oil price might be another explanation.

Actually the effect of TVTR estimator on OBX returns over the horizon of five years is in line with the auto correlation results. The monthly AR (1) coefficient of tail risk series is lower in Norwegian market than that of US market. It means that the effect of left tail risk shock dies out in a shorter time span in Norway than that of US.

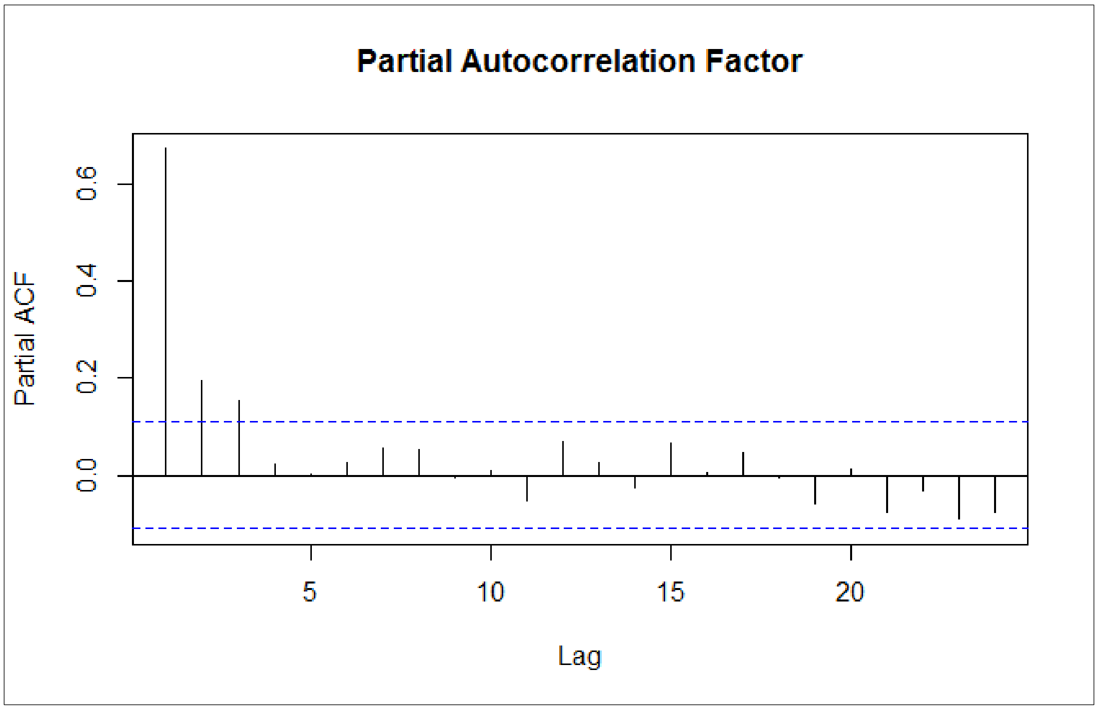

Figure-4.3: This graph shows the partial autocorrelation of monthly TVTR series calculated by pooling daily log returns of all common stocks listed on OSE from 1990 to 2015.

Figure 4.3 presents the partial auto correlation factor of monthly tail risk series. The partial auto correlation graph helps us more precisely in understanding about how the risk shock' effect vanishes. It can be said by looking at the graph that effect of a tail risk shock starts

[13] The graphs of seasonal, random and trend behavior of monthly tail risk and OBX log returns time series are presented in appendix, figures A2 and A3.

[14] The cross-section I used for the analysis is from 1990 to 2015. All the regressions are performed on the monthly observations which mean total 312 observations. The observations further decreased to 252 when predictive regressions conducted over the horizon of five years. The fewer observations lead to increase the TVTR series bias due to volatility heterogeneity among asset returns.

becoming negative in the end though not significant. The dotted line represents the significance level.

The figures 4.4a and 4.4b on the next page give a comparison of TVTR estimator performance in US and in Norway. It is clear from the figure 4.4b that TVTR[15] does not have a significant impact on future returns of Oslo benchmark index. There is no clear pattern except for some short patches. OBX constructed of 25 most liquid stocks listed at OSE, this construction approach makes it utterly different from CRSP value weighted index, which might be the reason for the insignificant performance of TVTR in the Norwegian market.

[15] The OBX returns predictive analysis by TVTR is based on log returns due to the superior properties of log returns in such kind of analysis such as log normality, time additivity and approximate equality of simple and log returns in short horizons. I also estimated TVTR series based on simple returns and the results are extremely similar for example both TVTR series are 99 percent correlated and AR (1) coefficient of log returns based series is 0.672 and 0.646.

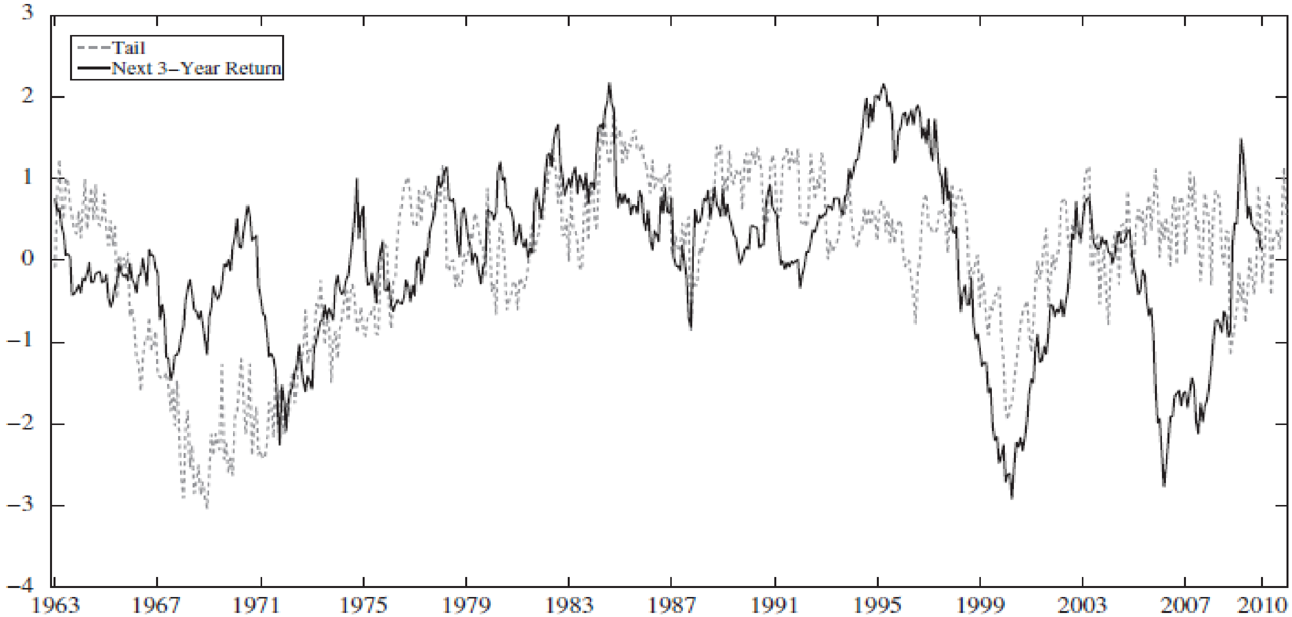

Figure-4.4a: This graph shows monthly tail risk series, calculated using returns of all common NYSE/AMEX/NASDAQ stocks, plotted against next 3 year returns of CRSP value weighted index. The graph fetched from Jiang & Kelly (2014)

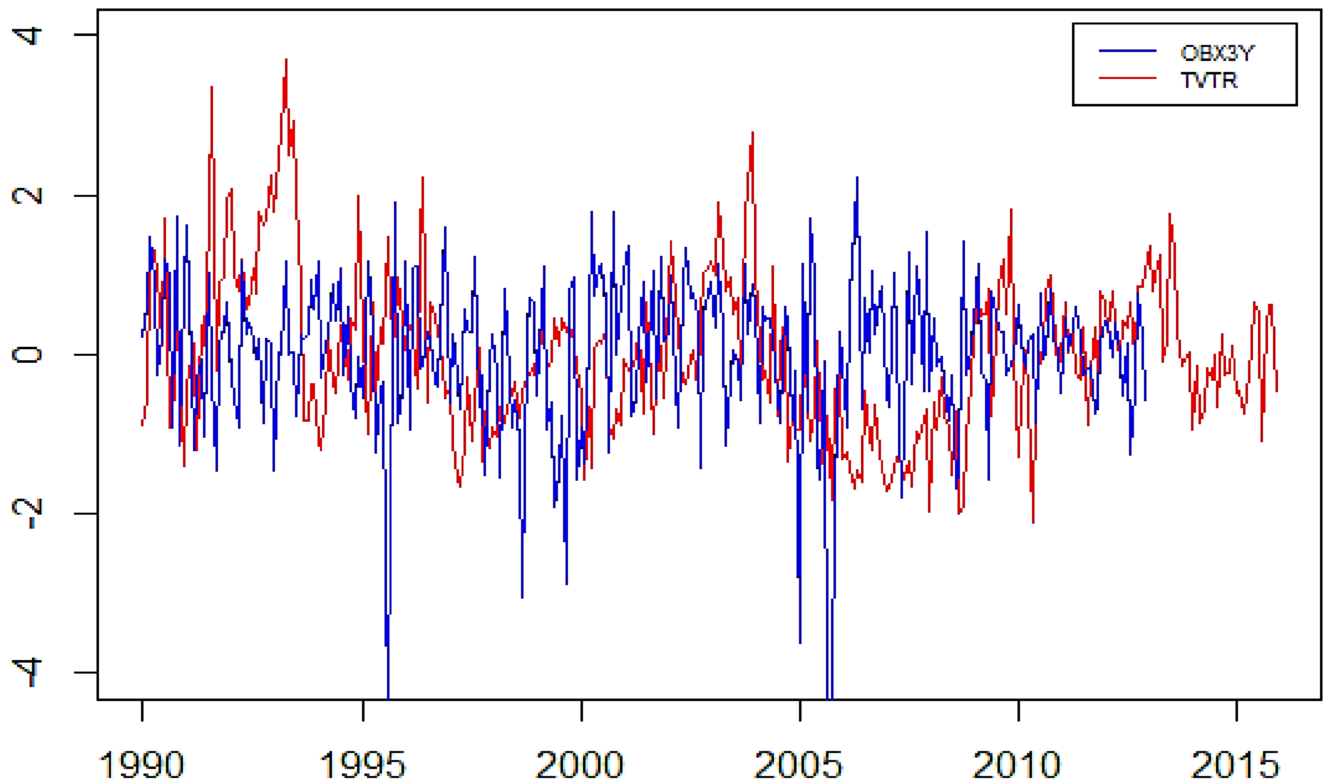

Figure-4.4b: This graph shows monthly tail risk series, calculated using returns of all common Oslo Børs stocks, plotted against next 3 year returns of Oslo Benchmark Index. Both the series scaled to zero mean and unit variance for comparison purpose.

4.1.3 TVTR Cross-Sectionally

The cross-sectional analysis conducted by following the same procedure specified in the article by Jiang & Kelly (2014) for comparison objectives. The number of stocks in the US market is plentiful in comparison with the Norwegian market, and the analysis can be affected by this. The analysis is based on the rationale that when left tail risk of a specific stock increases, investors discount the stock more that leads to low price of that particular stock and incremental expected return going forward.

The rationale is, investors are downside risk averse and shun securities with high left-tail risk. The securities that have high predictive loadings of TVTR, being a valid estimator of left tail risk, will be discounted more sharply than the securities with low TVTR predictive loadings. Consequently, securities with high TVTR loadings will have comparatively lower prices, therefore higher expected returns in the forthcoming horizon. The rationale is strengthened by the empirical results of TVTR. TVTR series yields monthly AR (1) coefficient equals to **0.673** with the **95%** confidence interval of **0.59** to **0.75**. This means that the TVTR estimator has power to forecast returns of individual stock.

The 312 monthly observations' cross-section available for this analysis therefore only monthly out-of-sample analysis is conducted. The analysis is out of sample because there if no overlapping in the data used to calculate tail risk loading and data used to calculate realized returns. Quintile portfolios constructed based on the predictive beta coefficients and results gathered for a long position in highest tail risk loadings quintile portfolio and a short position in lowest highest tail risk loadings quintile portfolio. TVTR prediction performance of aggregate market returns was relatively better in the one month horizon therefore the monthly long/short portfolio analysis is the only one that makes sense.

The tail risk loadings of all OSE common stocks calculated by using the regression equation 4.1:

$$\boldsymbol{r_{i,t+1} = \mu_i + \beta_i \lambda_t} \qquad \textbf{(4.1)}$$

$\boldsymbol{r_{i,t+1}}$= Individual stock simple returns at time **t+1**

$\boldsymbol{\mu_i}$= Error Term

$\boldsymbol{\lambda_t}$= Tail risk series estimates at time **t**

$\boldsymbol{\beta_i}$= Predictive tail loading of individual stock

Every month, the predictive regressions over the horizon of most recent **120** months conducted to estimate the tail risk loading of each stock. Stocks that have less than **36** observations in past **120** months are excluded. Quintile portfolios are constructed every month based on the predictive tail risk loadings. Stocks priced less than **35** Norwegian Kroner or **5** dollars[16] omitted in the portfolio construction phase. Monthly average value weighted and equally weighted quintile portfolio formed. Quintile portfolios' returns calculated based on one month holding period. The analysis rationale ordains that high tail risk sensitive quintile portfolio must earn significantly higher returns than that of low tail risk sensitive portfolio. Therefore, TVTR cross-section forecasting performance evaluated taking a long position in highest tail risk sensitive quintile portfolio and a short position in lowest tail risk sensitive quintile portfolio[17].

TVTR cross-sectional forecasting performance analysis produced mixed results. Equally-weighted high minus low tail risk quintile portfolio returns suggest no significant forecasting power of TVTR cross-sectionally. On the contrary, average monthly value-weighted high minus low tail risk quintile portfolio returns suggest significant forecasting power of TVTR cross-sectionally. The results are also reported controlling for Fama-French three factors, Fama-French-Carhart four-factor momentum model, as well as with respect to the Fama-French-

[16] The exclusion is made to make the results comparable with Jiang and Kelly (2014). Stock prices are available in Norwegian Krone in the TITLON database. Dollars to NOKs conversion made using conversion rates from the US federal reserve bank website http://www.federalreserve.gov/

[17] The cross-sectional analysis conducted using simple returns due to the analysis' single time period nature. Simple returns are portfolio consistent though time inconsistent, log returns are portfolio inconsistent though time consistent. The analysis conducted on the data from 1990 to 2011 due to Norwegian market factors' data non-availability after 2011.

Carhart model plus the liquidity factor of OSE made available on Bernt Arne Ødegaard's webpage as a fifth control.

Equally-Weighted Portfolio								High - Low (Annualized)
Factor	**Low**	**2**	**3**	**4**	**High**	**High-Low**	***t*-stat**	
Average Return	-0.39	0.10	0.70	0.43	0.27	0.65	1.10	8.12
CAPM Alpha	-0.81	-0.36	0.20	-0.08	-0.32	0.49	0.92	6.09
FF Alpha	-1.05	-0.45	0.21	-0.18	-0.36	0.69	1.30	8.61
FF + Mom Alpha	-1.09	-0.34	0.21	-0.19	-0.25	0.84	1.58	10.57
FF + Mom + Liq Alpha	-1.14	-0.36	0.19	-0.20	-0.30	0.84	1.56	10.53
Value-Weighted Portfolio								**High - Low (Annualized)**
Factor	**Low**	**2**	**3**	**4**	**High**	**High-Low**	***t*-stat**	
Average Return	-0.56	0.81	0.92	-0.05	1.24	1.80*	2.01*	23.89*
CAPM Alpha	-1.11	0.26	0.33	-0.62	0.60	1.71	1.92	22.59
FF Alpha	-1.42	0.24	0.44	-0.72	0.53	1.95*	2.18*	26.07*
FF + Mom Alpha	-1.46	0.25	0.59	-0.75	0.57	2.03*	2.24*	27.26*
FF + Mom + Liq Alpha	-1.55	0.28	0.63	-0.76	0.52	2.06*	2.26*	27.77*

Table 3[18] – The table exhibits monthly statistics for the quintile portfolios constructed on the basis of predictive tail risk loading – Beta coefficients, except for the right most column which presents highest minus lowest tail risk sensitive zero investment quintile portfolio annualized returns and alphas. The table also presents quintile portfolio alphas from contemporaneous OLS regressions of portfolio returns against Fama-French three factors, Fama-French plus Carhart momentum four-factors and the extended five factors with fifth factor of OSE liquidity. (*) Asterisk represents statistically significant results.

The upside and downside portions of table 3 reveals equally-weighted and average monthly value-weighted quintile portfolio results respectively. The first row shows the average

[18] The analysis performed using all common stocks' records, however, there are five companies that have inconsistent records in the TITLON database so those companies' stocks were omitted while performing the analysis. The five companies are "Simrad Optronics", "BW Gas", "Bergesen d.y ser. A", "Tsakos Energy Navigation" and "Sydvaranger".

returns of the quintile portfolios. First column presents the monthly average returns and alphas of lowest TVTR sensitive quintile portfolio. Accordingly, the fifth column presents the monthly average returns and alphas of lowest TVTR sensitive quintile portfolio. The sixth (High-Low) column displays the highest-minus-lowest tail risk portfolio returns and alphas and the seventh column presents their associated t statistic values. The last column shows the annualized results of sixth column.

One-month returns	Low	2	3	4	High	High-low	*t*-stat.
			Equal-weighted				
Average return	1.14	1.24	1.28	1.4	1.45	0.31	*2.12*
CAPM alpha	0.10	0.31	0.37	0.48	0.47	0.37	*2.52*
FF alpha	−0.11	0.02	0.09	0.19	0.19	0.30	*2.22*
FF + Mom alpha	−0.06	0.06	0.11	0.22	0.24	0.29	*2.14*
FF + Mom + Liq alpha	−0.08	0.06	0.12	0.24	0.26	0.34	*2.50*
			Value-weighted				
Average return	0.84	0.96	0.98	1.18	1.20	0.36	*2.00*
CAPM alpha	−0.19	0.03	0.08	0.25	0.18	0.37	*2.08*
FF alpha	−0.19	−0.04	0.05	0.22	0.27	0.46	*2.58*
FF + Mom alpha	−0.16	−0.03	0.03	0.18	0.30	0.45	*2.22*
FF + Mom + Liq alpha	−0.21	−0.03	0.05	0.21	0.35	0.55	*2.78*

Table 3A: The table shows the results from the cross-sectional analysis conducted using US data by Jiang and Kelly (2014)

Table 3A displays the same cross-sectional analysis results of US data by Jiang and Kelly (2014) for comparison objectives. The portfolio returns in the table 3 did not show any consistent downward trend of the form present in US data analysis in table 3A, such as returns decrease on the way from highest to lowest tail risk quintile portfolio.

Table 3 reveals that the highest tail risk equally-weighted portfolio did not report highest average return among the five portfolios. Unexpectedly, the third equally-weighted portfolio reported the highest results among the five. Though the lowest tail risk portfolio reports the least returns which is according to the rationale. All equally weighted portfolio results are statistically insignificant, produced too low t-stats.

Converse to equally weighted portfolio results, average monthly value-weighted portfolios results are aligned with underlined rationale that TVTR being a valid estimator of tail risk, high tail risk portfolio reaps more returns in the future than that of low tail risk portfolio. The first row of value weighted portfolio in table 3 shows that average historical returns are highest – 1.24 percent monthly 15.94 percent annualized – for the highest TVTR sensitive value weighted portfolio. The monotonic trend also seems to appear that portfolio returns increase as the tail risk sensitivity increases except for the fourth quintile portfolio.

The highest tail risk value-weighted portfolio earned annualized average return **23.89** percent more than the lowest tail risk value weighted portfolio, with the t-statistic of **2.01** and p-value of **0.046**. The return (**23.9%**) is much higher than that of US market (**4.4%** annualized **–** **0.36%** monthly). This result makes some intuitive sense because the volatility spread of Norwegian market is much higher than the US market.

The second row of value weighted portfolio in table 3 displays the alphas of regressions of portfolio returns against market returns. The results are also consistent with the rationale of cross-sectional analysis but are not statistically significant. The high-minus-low tail risk value weighted portfolio CAPM alpha reports a t-stat **1.92** with p-value **0.0563**. The p-value is slightly over the significance level.

The third row of value weighted portfolio in table 3 displays the alphas of regressions of portfolio returns against Fama-French three factors; market factor, small minus big market capitalization firms' stock factor and high minus low book to market value factor. The high minus low three factor annualized alpha is **26.07%** with a significant t-statistic **2.18**. Similarly in table 3, the fourth row of value weighted portfolio presents four-factor alphas calculated by multivariate regressions. The alphas are robust controlling for Fama-Frech three factors in addition to Carhart momentum factor. High minus low tail risk portfolio alpha is **27.26%** annualized (t-stat= **2.24**). The alphas of the five factor regressions – the aforementioned four factors plus a liquidity factor of OSE – demonstrate similar statistically significant results.

The three factor alpha results mean that if an investor takes a long position in highest TVTR sensitive value-weighted portfolio and a short position in lowest TVTR sensitive value-weighted portfolio, he shall earn **26** percent extra returns controlling for OBX returns, small minus big value firms' returns and high minus low book to market value firms' returns. This result has implications for assets pricing.

4.1.3.1 Data Bias

The cross-sectional analysis performed accessing data from various data sources though the main source is the TITLON database. The analysis performed using all common stocks listed on OSE. The common stocks' count is very low in Norwegian market which leads to significant changes in return results of quintile portfolios. Such as a single asset extreme event impacts the whole portfolio returns significantly. This sample noisiness increases the returns volatility spread of quintile portfolio returns. Most recent 120 months data used to calculate each stock predictive tail risk loadings which left us with 143 monthly observations to examine TVTR quintile portfolios returns controlling for the five specified returns factors because Norwegian factors' data is available until 2011.

Another potential bias arises due to non-availability of stocks' delisting information. The TITLON database does not provide any tag or description about the stocks delisted from Oslo stock exchange. TITLON does not provide information about the delisted stock that whether the issuing firm went bankrupt or the firm went private. This left us with three choices to make. First, the choice to omit stocks whose data stopped appearing in TITLON. But it creates survival bias which is very crucial in the context of tail risk. Second, the choice to consider that all stocks whose data stopped appearing in TITLON went private. But this choice results in upward biased estimates. The third choice is to consider all stocks whose data stopped appearing in TITLON went bankrupt. But this choice results in underestimation of portfolio returns.

Owing to the nature of cross-sectional analysis the third choice was made to perform the analysis. The assumption produces downward biased estimates of TVTR quintile portfolio returns. The numbers of bankruptcies, observed in each TVTR quintile portfolio, are presented in table 4 below.

Tail Risk Portfolio	**Low**	**2**	**3**	**4**	**High**
Number of Bankruptcies Observed	20	16	9	9	13

Table 4 – The table reports number of bankruptcies observed in each tail risk quintile portfolio

4.1.4 Look-Ahead Bias

The look-ahead bias can be defined as the bias arises due to the use of historical data in back-testing a strategy – investment strategy in our context – that would not have been known at the time period being analyzed. Daniel et al (2009) argued that back-testing the performance of an investment strategy seems really straightforward but it contains various ex-post conditioning biases such as survival bias, the look-ahead bias and data-snooping. The survival bias is unconcerned from this both analyses because TITLON contains data of solvent companies as well as the companies that went private later on. They stated that the look-ahead bias can lead to over estimation of expected returns up to 8 percent per annum. Data snooping bias is also negligible in the performed analyses which refers to the bias arise when statistical inference is made after analyzing the data without any pre-planned inference making arrangement.

There is valid reasoning to accept the existence of look-ahead bias in the cross-sectional analysis. Because returns are calculated using adjusted prices that contain dividends and other corporate events effects. Dividend payout and other corporate events occur with some time gaps but their effects are apportioned to daily stock prices. It is very strenuous to evade look-ahead bias in back-testing investment strategy performance analyses by simulation. But one cannot be absolutely certain that the bias has been removed.

Chapter – 5

5.1 Conclusion

Time varying tail risk estimator based on Hill’s (1975) power law was introduced by Jiang and Kelly (2014). The empirical results were very significant in their research article. It is not necessary that every country’ equity market behaves similarly. There are numerous variables that cause rise and fall of asset prices. It is not only fundamentals that cause price movements, investors’ beliefs and understanding plays a vital role in that.

The empirical finds in the table 2 concludes that tail risk estimated using Hill’s (1975) power law exponent failed in forecasting Oslo benchmark index returns over the horizon of one month, one year, three years and five years contrary to Jiang and Kelly (2014) empirical findings. Though consistent with Jiang and Kelly (2014) findings the estimator showed significant persistence level with statistically significant monthly AR (1) coefficient.

The cross-sectional analysis performed to test prediction power of TVTR for individual asset returns produced mixed results. Equally weighted TVTR sensitive quintile portfolio returns analysis results presented in the table 3 reject the notion that TVTR predicts expected returns cross-sectionally. On the contrary average monthly value-weighted TVTR sensitive quintile portfolio returns analysis results presented in the table 3 fail to reject the idea that TVTR is a contender of significant expected returns. A long high tail risk and short low tail risk net investment portfolio results are significant with significant t-statistics except for CAPM alpha.

5.2 Further Research

The dynamics of Oslo benchmark index are very different from CRSP value weighted index. Oslo benchmark index constructed of just 25 most liquid stocks listed at Oslo Stock Exchange. The low number of stocks on Oslo stock exchange implies that a different sampling approach should be considered to calculate Hill's (1975) power law exponent. Because all analyses produced results in the same direction the rationale – higher the risk greater the expected returns – ordains but statistically insignificant. The above mentioned data bias creates problems in back-testing portfolio investment strategy. Delisted stocks information will make these kinds of analyses robust.

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Appendix

Figure A1

```
> shapiro.test(`Set-1`)

        Shapiro-Wilk normality test

data:  Set-1
W = 0.9572, p-value = 6.42e-08

> shapiro.test(`Set-2`)

        Shapiro-Wilk normality test

data:  Set-2
W = 0.9736, p-value = 1.708e-05
```

Figure-A1: Snapshot of R result of Shapiro–Wilk normality tests of two TVTR series calculated from two randomly selected set of securities

Figure A2

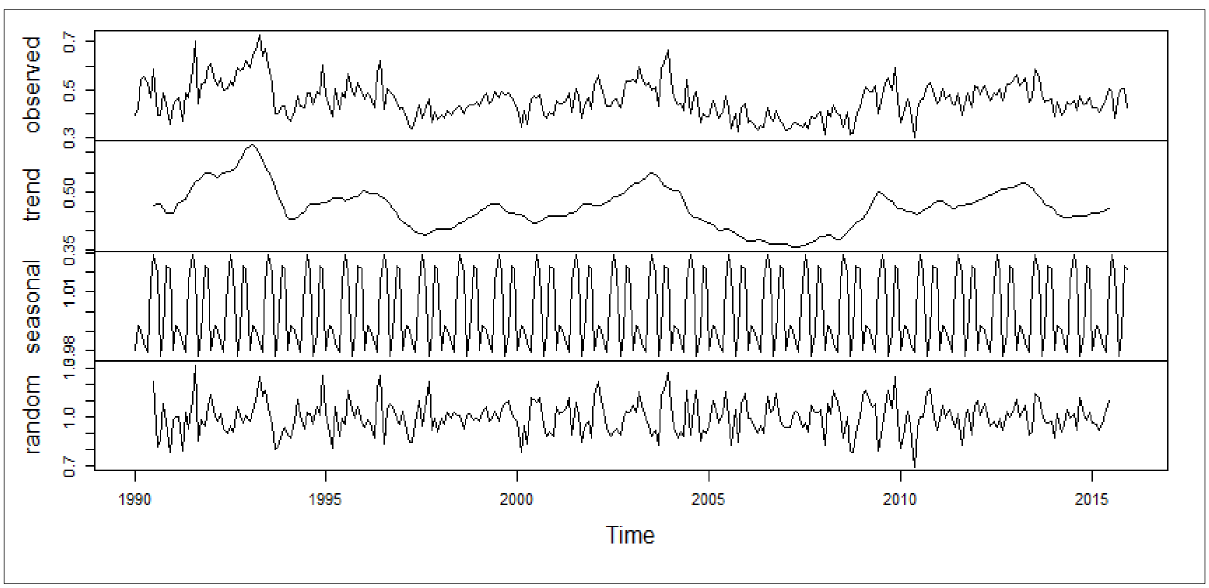

Figure-A2: The figure presents the decomposition of monthly TVTR time series

Figure A3

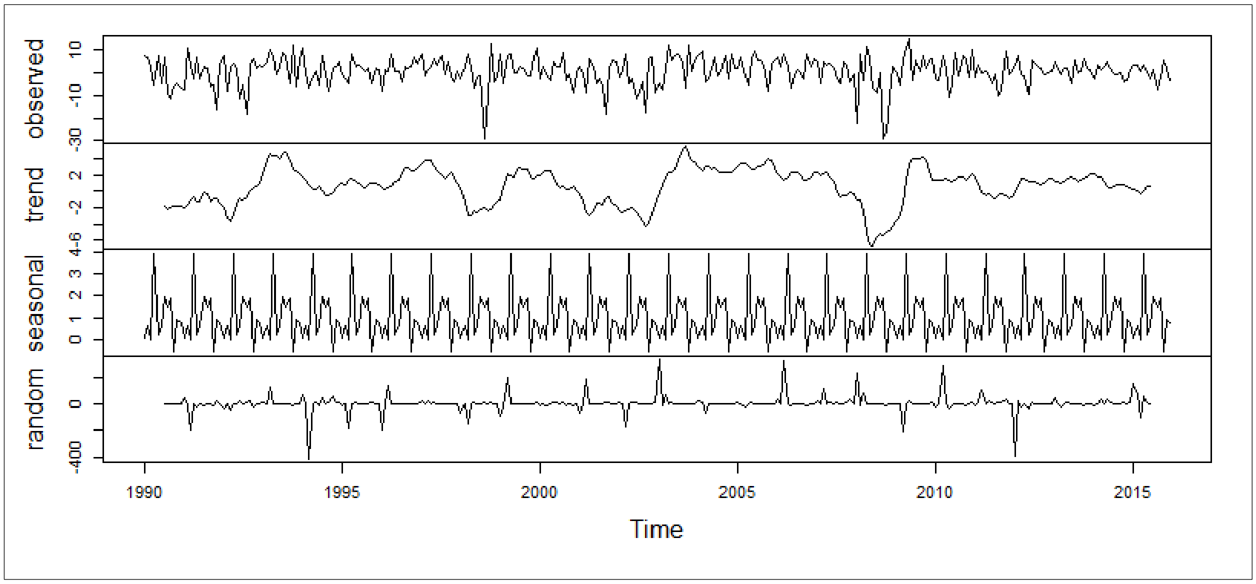

Figure-A3: The figure presents the decomposition of monthly OBX log returns time series

Table A1

Sr. No.	Description
1	A-aksjer
2	B-aksjer
3	Exchange tradeable fund
4	Frie aksjer
5	Grunnfondsbevis
6	Konverterte A
7	Konverterte aksjer
8	Konverterte B
9	Konverterte F
10	Nye aksjer
11	Nye B
12	Ordinære aksjer
13	Preferanseaksjer
14	NULL[19]

Table A1 – The table presents the full list of stock types (Description) available in the TITLON database.

[19] There are only three companies "Ugland International", "Skandia" and "Burmeister & Wain Holding B" with no description of their stocks