

MASTER THESIS

Course code: BE305E

Candidate number: 100 & 124

A Study on Risk-Adjusted Weight Construction in Portfolio Investment using Sharpe Ratio

Date: 18/05/2016

Total number of pages: 75

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Abstract

The demand for models and indices to build low-volatile portfolios for investment has been always present in the financial market. Recent blows of global financial crises around the world escalated that demand even further for developing investment strategies that can minimize the volatility associated with the financial investments. As a result, continuous efforts are being made to introduce new and better investment strategies for ensuring risk-adjusted investment opportunities. There are different investment strategies available in the financial market to adopt. Among them, use of weight models and indices are one of the most widely used both by individual and institutional investors. However, none of the conventional weight models are proved to be perfect. Therefore, continuous efforts are being made by the financial researchers and academicians to construct new models or upgrade the existing ones so that they can construct better risk-adjusted portfolios for the financial investors.

In this study, an effort has been made to construct a less complex and easy to use investment weight model that can build risk-adjusted portfolios, especially for the individual investors. To develop the new model, equity weight model was reconstructed by adding a parameter, Sharpe ratio, and including mathematical functions. The reason behind including the Sharpe ratio is, it is widely used performance indicator that measures the risk-adjusted returns to evaluate the performance of a firm or financial instruments. Therefore, use of such variable in the weight model can be effective to select risk-adjusted companies for the portfolio.

The result of the analysis done to verify the effectiveness of the Sharpe Ratio based weight model showed that portfolio built using the high-weighted companies level of risk was same as the portfolio of low-weighted companies. Interestingly, the return was much higher for high-weighted portfolio compared to its counterpart. It means the high-weighted portfolio is confirming higher return at a lower level of risk. However, this mentioned result was consistent for long time horizon and showed some discrepancies in the short time period (in this study on a yearly basis). A challenging issue like the use of standard deviation which is a simplified way of the calculating volatility of a return in Sharpe ratio was also discussed in this paper to support the findings of the study.

A conclusion was drawn to this study by suggesting possible opportunities to conduct further research on this model to quantify the effectiveness by conducting a comparison study with conventional models and also to develop the model further to increase the efficiency of the model.

1. Introduction

In today's business world, stock market investments play a crucial role to boost up the economic activities and business operations of any country. It is a significant source for firms to raise capital for both diversification and expansion of their businesses. On the other hand, these markets are good investment options for both firms and individual investors, large and small; to earn money from their savings and retained earnings outside regular banking institutions. Building portfolio for all kind of investors to increase their expected earnings and to cope with unexpected crises in the investment market is now a must for both active and passive investments. However, traditional diversification in capitalization weighted stock indices has received much criticism in the later years. This is because such a portfolio will put larger weights on companies that have already increased the price, thus increasing exposure to potentially overvalued companies. This has led to igniting demand for different experiments and research work to innovate the potential and better strategies for financial models to construct competitive investment portfolios that have been carried out by both practitioners and academic researchers. For example, there is some evidence that a portfolio weighted by the reciprocal of volatility fares better in terms of both risk and returns than a traditional portfolio constructed based on market capitalization. This way to construct a portfolio has popularly been noted as smart beta, though it is based on a classic multi-factor mode. There is intense debate over whether such portfolios are in fact adding value to an investor who wants to diversify wealth. Therefore, conducting research in this contemporary subject is a worthy effort, which can assist the interested reader to recognize possible mechanisms of competitive investment.

This chapter introduces the topic of the research and the reasons behind choosing this subject for analysis. It also explores the primary and secondary objectives of the research, identification of the problem and research question, methods of collecting data and the structure of how the research will be undertaken by analyzing the collected data.

1.1 Background

Construction of optimal investment portfolio and managing it in the financial market has turned out to be one of the most challenging tasks for both investors and investment managers in the financial market. In addition, the recent rise in the financial stability risk across the globe has made the investors more risk averse (IMF, 2015, p-93). Low volatile investments are now a preferred option for many investment managers to construct portfolios for their clients.

The weights of different asset classes in an investment portfolio play a significant role in assessing the portfolios expected returns and risks. Therefore, based on the present circumstances, the importance of calculating optimal risk-adjusted weights for asset allocation to ensure a diversifiable portfolio with improving risk-adjusted return is beyond any doubt. There are numbers of ways to calculate the weight for different stocks for a potential investment. Traditional techniques like market capitalization weighted portfolio or equally weighted portfolios are the most popular methods that are used by active investors to manage their portfolios. However, as the market is becoming more uncertain and volatile, alternative strategies, such as smart beta, have started to gain popularity. According to Bowers (2014), smart beta indices are not revolutionary rather they are a solid part in the evolution of index investing and are a part of the history of financial theory. These techniques use quantitative methodologies to calculate the weight of assets in a portfolio based on volatility, dividends, the value of the portfolio, size momentum, and preferences of the investors and so on. As the alternative strategies consider the risk and return effects of the assets before weighing itself in the total investment, it has turned out to be a more competent way of making an optimal portfolio and many investors now prefer to apply these for financial investments and moving away from traditional market capitalization based indices.

Such strategies are being developed and widely used for competitive returns at a lower cost. These alternative strategies are being continuously invented and restructured by academic and financial analysts based on changing risk and return of assets as the present financial market has become uncertain and volatile (Sullivan, n.d.).

After going through the importance of determining a portfolio based on the weights of asset classes, it is quite practical and feasible to do a study on reconstructing an investment index for building a competitive investment portfolio. Again, the reconstructed model will be tested in a real stock market to evaluate its effectiveness by comparing the results with the model that has been used to reconstruct the model, as well as with the index itself. As a result, the interested reader will get to know the reasons behind such reconstruction of an existing model and can use the new model to construct their own investment portfolios based on the suitable market conditions.

1.2 Problem Statement

In developing a portfolio, asset allocation plays the fundamental role in determining the expected return from the portfolio. According to an article written by Lummer and Riepe (1994), several studies have found that more than 90% of variations in different portfolio returns are due to the differences in asset allocations in those portfolios. Many institutional investors are still advised to maintain the 60-40 rule while investing in a portfolio, which means investing 60% in stock assets and 40% on fixed income (i.e. bonds). However, such type of asset allocation is not highly competitive anymore as the investment market has now gone global and has become rigorously uncertain. In addition, yields on bonds have been historically low the last decade, and investors are therefore searching for other ways to earn returns without adding too much risk. Throughout time, many tools were developed to calculate the asset class weights in a way that could ensure a portfolio with a higher expected return on a given level of risk.

Among all the equity weighting mechanisms some are very popular due to their simplicity and ease of use and some are widely used because of their efficiency in structuring low volatility portfolios. Among the simple tools, the most popular strategies are the Equally Weighted (EW) and Capitalization Weighted (CW) Schemes. Figure 1 illustrates a low-volatility portfolio of compared with a traditional large-cap portfolio of US market (left figure) and Developed Countries except US market (Right figure). The figures basically showed that the low volatile strategies have earned higher return than

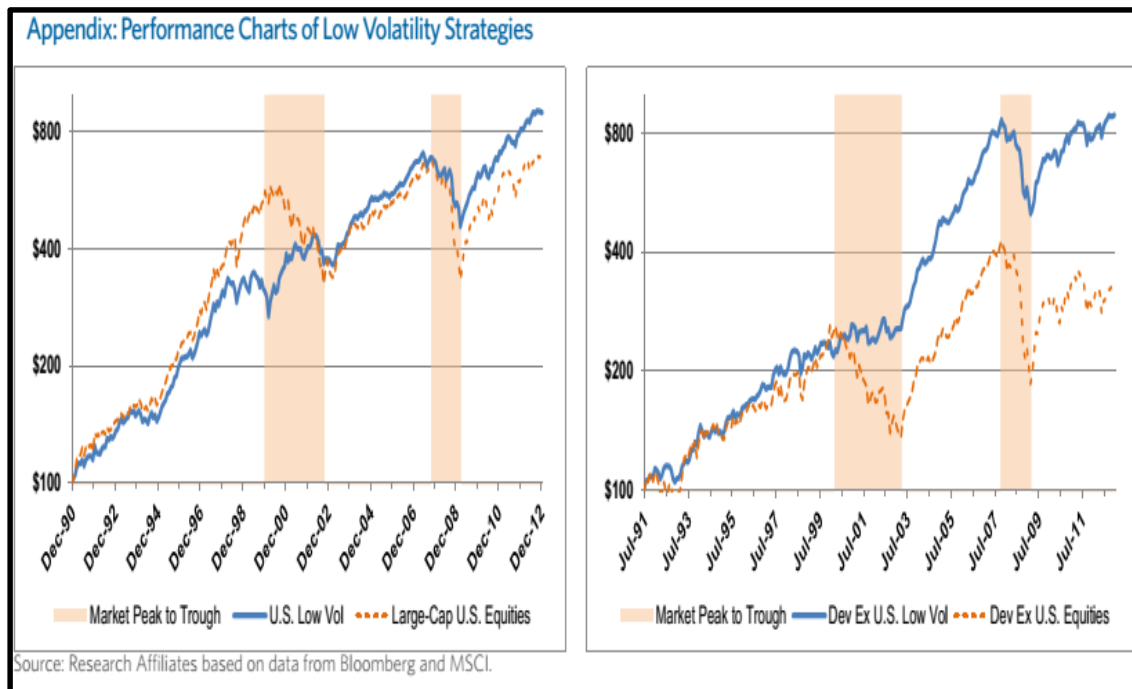


Figure 1: Performance Charts of Low Volatility Strategies (Li, 2013)

the other asset classes. In some cases the performance was better for other asset classes and that was mainly due to core fixed income associated with. However, in the long run the performance of low volatility strategies are enough to appetite the investors to invest in (Li, 2013).

The equal weight mechanism allocates equal weight to all the companies in the stock market regardless of its market capitalization, or size. Thus an EW portfolio constructed based on sample market with 100 companies will invest 1% in each company. Therefore, if investors can forecast the risk and expected return of companies stocks well then it can easily select the companies for the portfolio and can distribute the investment using equally weighted index (Arnott, et al., 2010). The CW scheme mimics the relative size of the company to the stock market as a whole. Thus a CW portfolio constructed based on a sample market with 100 companies, will invest most in the company with the largest market capitalization, and least in the company with the smallest market capitalization. Unlike the CW Scheme, the EW does not over weight overprice stocks and underweight

the underpriced stocks. Rather, the EW mechanism is considered to be highly diversified as all the stocks are equally weighted. Swenlin and Heim (2015) argued that equally weighted index can provide more return from a portfolio as it gives equal weight the smaller-cap and large-cap stocks, this is because smaller-cap stocks often performs better in the stock market that increases the expected return of the portfolio and improve the performance. Also, DeMiguel et al. (2009) found that in an out-of-sample analysis of 14 different portfolio models, none was consistently better than the EW portfolio in terms of Sharpe ratio, certainty-equivalent, and turnover.

However, the equally weighted strategy also has flaws that restrict the investors to use it in all market situations. The primary drawback of equal weighted mechanism is it does not consider its constituent stocks risks and returns and stocks are included in the portfolio only because it is a component of the target market. Giving equal weight to all kind of stocks often harms the expected risk and return. For example, sometimes this mechanism provides a significant weight to overpriced stocks which eventually increases the risk of the portfolio at the same time as lowering its expected return. Again, under-presentation and over-presentation of securities based on their presence in the target market leads to a construction of portfolios that are nowhere near to the optimal portfolio (Burton, 2013). All these facts imply that even the equal weighted strategy performs better in a bull market, but it has a high chance to underperform in a bear market. Another disadvantage of equal weighted index strategy is that it requires continuous rebalancing to maintain the equal weight of all the constituent stocks in the portfolio. As the price of the component stocks changes, the constructed portfolio does not remain equally weighted thus requires a constant rebalancing (CFA Institute Org., n.d.).

As per the discussions and evidences above, reconstructing the equally weighted index model by associating risk and return adjustments within the model can provide a better investment strategy for building optimal portfolio for investments. At the same time, it can help the investors to choose the right securities for investment to ensure better return from a portfolio at a minimum level of risk. Rather than dividing the constituent securities based on total number of assets in the target market, if the securities weights

are chosen based on their risk-adjusted returns then it will be more purposeful based on structuring an optimal portfolio. Hsu and Li (2013) show that low-volatility portfolios offer an improved risk-return profile in comparison with traditional CW investments.

To establish the discussion, a study will be conducted to restructure the equally weighted model by removing the total number of assets from the equation and replacing it with a risk adjusted return to determine the weight of the constituent stocks for a portfolio. In this study the Sharpe Ratio will be used to determine the weights.

1.3 Research Objectives

The recent global financial market suggests, the volatile nature of many investment markets is leading investors to be more risk averse. The interest in low-volatile investments strategies has increased substantially over the last few years, though such strategies comes with reduced risk, the tracking error to a benchmark with of course increase. Therefore, the significance of constructing low-volatile risk adjusted portfolios is increasing every day and financial analysts around the world continuously working on assembling proper index models; to be more specific working on developing and improving smart beta models. Hence, the primary objective of this research is to formulate a smart beta model by reconstructing equally weighted index model to produce a productive, easy to use, efficient and flexible tool for building low-volatile financial portfolios for both active and passive investors. Other secondary objectives include, identifying what type of investment time horizon is fit for the model (long-term or short-term) and for what type of market (stable or unstable) our model works more effectively. The study also has tried to analyze whether the model is indifferent to create risk-adjusted portfolios. A discussion was made in the end of the study to further to analyze the reasons behind providing different results.

1.4 Research Question

Considering the background and the objectives of the research, the following research questions were set and it is expected that the research analysis and findings will shed light to an answer that will lead the study to a worthy conclusion.

- *“Does Sharpe Ratio-based weighting methodology constructs low-volatile portfolio?”*
- *Does Sharpe Ratio-based weighting scheme confirm a difference in returns between the high and low weighted companies?”*
- *Are the differences in returns between high weight companies and low weight companies random?”*

Followings are the hypotheses for the study –

Research Hypothesis (H_1): Portfolio with High weighted are more risk-adjusted.

Research Hypothesis (H_3): A difference exists between high weighted companies and low weighted companies stock returns

Research Hypothesis (H_2): The return differences between high weighted companies and low weighted companies are systematic.

Null Hypothesis (H_0):: There is no difference between the risk and returns of high weighted and low weighted companies stocks and existing differences in few cases are random.

1.5 Research Methodology

To conduct the study successfully, a quantitative approach of the study was undertaken. A comprehensive definition of quantitative research method is given by Aliaga and Gunderson (2000), “Quantitative research is explaining phenomena by collecting numerical data that are analyzed using mathematically based methods (in particular statistics).” In this research, the numerical data or the portfolio returns were calculated using the reconstructed model. All the calculated data then analyzed using statistical data analysis and evaluated to prove the expected hypothesis mentioned above. As the investment market is usually a large area to work with sampling technique was used to collect initial data for the analysis.

1.5.1 Sources of Data

Both Primary and Secondary data were used to run the reconstructed investment weight model. Financial information both numerical and text were collected mainly using

different archival records and online database system, namely Titlon. Published documents and statistical analyses were used for precision in the research. The data collected from these sources were used to develop sample portfolios using the reconstructed investment weight model.

For the analysis, primary data was acquired as average monthly return and monthly standard deviation from the sample portfolios and was used in the statistical tools to legitimize the newly constructed model.

1.5.2 Data Analysis

Extensive analysis was done on the generated data. The generated data consist necessary financial information to evaluate the performances of the model. It is important to examine the performance details of the models because these results are the prime criteria for the hypothesis to be accepted. Therefore, statistical tests were run on all generated performance data for comparing the smart beta strategies and also for interpreting the effectiveness of the investment weight construction using Sharpe ratio. At the end of this process, the reliability and validity tests was executed to legitimize the analysis. This tests enhanced the credibility of the research and the generalized the findings for the interested people for further research.

1.6 Scope of the Study

Norway's financial market has been chosen as the scope of the study. Oslo stock exchange regulates the financial market of Norway. Therefore, necessary numerical data and information used in the study are on the Oslo stock exchange. For this study, only the stock market data was taken to test the newly constructed model.

1.6 Limitations

Certain limitations emerged during the research which limited the scope of analysis. As the research was done in the Norwegian market it was difficult to interpret the secondary data as they are mostly available in local language. Conducting a quantitative data sometimes result in lost information due to reduction of data to numbers only (InterAction.Org., n.d.). Time constraint is another problem that makes the research

method inflexible, especially when the study follows a quantitative approach. Such inflexibility occurs as the research design becomes difficult to modify once the study begins.

2. Literature Review

This chapter mainly reviews relevant literature for both fundamental and alternative weighting indices (smart beta) to show the reasoning behind conducting a study on constructing Sharpe Ratio-based weighting scheme for low-volatile portfolio. Published literatures were used to discuss the problems associated with equal weighted index and to explain the characteristics of low volatility strategies which can certainly become a replacement of EW scheme. In the study, we intended to build a low volatility index model using Sharpe Ratio as a function of investment weight calculation. An elucidation of the ratio has also been given in this chapter. We choose Norway Stock Market to test our model; hence a brief description of this market is also added in this chapter. Lastly, a theoretical framework is established based on the literature review to define the purpose of the constructed model and also to support the tests that will be run using this model. The chapter ends with the representation of the formulation process of the Sharpe ratio based weight model.

2.1 Equal-Weight Index

In general, equal weighting method allows all stocks in a portfolio to hold equal weight disregarding the price of the stocks and the size of the firms in the market (Denoiseux et al., 2014). Cap-weighted index always increases the weight of the stocks that experience a price increase in the market. This creates a ‘systematic flaw’ of increasing the weight of overpriced stocks in the portfolio compared to an equal weighted portfolio. Hence, equal-weight index is a widely used investment strategy that eliminates some errors in cap-weighted indices by exploiting the change in the stock prices over time through rebalancing (Carlisle, 2012). Plyakha et al. (2014) explained in one of their studies that, equal weighting portfolio gives higher systematic return compared to value and price weighted portfolios as it is more exposed to the value, size and market factors. They also found that higher alpha of equal weighted portfolio is a result of monthly rebalancing strategy that was used to maintain equal weights in the index. It means the rebalancing strategy plays a significant role to ensure higher returns from equal-weight indices and the choice of the method has little impact on it. Again, the equal weight index model

distorts the relative price effect of an index by investing in all stocks and contradicts the definition of a well-defined index by investing without distorting prices (Asness, 2006).

Many scholars have argued that the equal weighting scheme is most effective tool for diversification. However, Kose and Moroz (2014) showed that such diversification contains a marginal improvement in the volatility. In table -1 below, the volatility simulation done in eight countries shows that all the models have very close volatility percentage even though equal-weight model has a broader diversified portfolio. Thus the broader diversification characteristic does not ensure that the equal-weight index construct a better risk-adjusted portfolio than traditional and fundamental indices.

Equal-weight model is also considered as an expensive strategy as the implementation cost is high. Continuous rebalancing to maintain the equal weight force to replace less liquid stocks rather than moving back to the target weight thus increases the (Weinreich, 2014). A comparison of effective turnover between Cap-weight 1000, RAFI1000 and EW 1000 index in the US Market showed (Table – 2) equal-weight index has the highest effective turnover as it requires rebalancing due to additions and deletions against price movements (Aked & Moroz, 2013)¹.

¹ Effective Turnover is a linear function of additions and deletions required for reweighting of securities in portfolio

Country	Volatility		
	Cap-Weight	Fundamental-Weight	Equal-Weight
Australia	23.4%	23.2%	23.7%
Canada	18.7%	17.2%	18.4%
France	21.3%	22.3%	21.7%
Germany	22.0%	22.1%	19.8%
Italy	25.6%	26.6%	25.5%
Japan	22.1%	22.2%	21.7%
United Kingdom	18.0%	19.3%	19.2%
United States	15.0%	15.1%	16.4%

Source: Research Affiliates.

Table 1: Simulated Volatility by Country, 1985-2013 (source: Kose and Moroz, 2014)

In the EW index model, rebalancing tilts the portfolio of value stocks, but at the same time requires investors to set their minds to buy the stocks that are cheaper than usual. It is not an effective model for the investors who are not bargain-hunting minded (Burton, 2013).

TABLE 2. COMPONENTS OF EFFECTIVE TURNOVER				
Index	Component	U.S.	Dev ex U.S.	EM
Cap 1000	T _{adds & deletes}	3.9%	6.6%	11.7%
	T _{reweightings}	5.2%	5.7%	9.2%
	Effective turnover	4.2%	6.9%	12.4%
RAFI 1000	T _{adds & deletes}	3.1%	4.4%	8.4%
	T _{reweightings}	19.9%	22.3%	33.8%
	Effective turnover	9.2%	12.0%	25.2%
EW 1000	T _{adds & deletes}	17.4%	18.9%	27.2%
	T _{reweightings}	19.9%	19.6%	26.4%
	Effective turnover	22.9%	24.2%	36.2%

Source: Research Affiliates.

Table 2: Components of effective turnover (Source: Aked & Moroz (2013))

The discussion of EW model above suggests that, even though it is a simple index model that eliminates quite a few errors of market cap-weighted and fundamental index models, it is failing to serve the purpose of being a cost effective risk-adjusted index model.

2.2 Low Volatile Strategies (Smart Beta)

The demand for low volatility strategies has increased over the time as they are performing better than the benchmarks in the market from which the assets are drawn. On the other hand, the increased volatile nature of the market environment and uncertainty in receiving excess return are pushing the investors to manage the volatility of their portfolios more and more (Kuo & Li, 2013). Low-volatility strategies are known as one of the important parts of smart beta. The first generation of the smart beta was established based on the market's low-volatile inconsistencies (BNP Paribas, 2014). However, the concrete definition of smart beta is still under process as some define it simply as non-market-cap-weighted index and some believes that it is more focused on potential diversification, reducing risks and ensuring a higher return (Koenig, 2014). Tower-Watsons (2013) provided a better definition of smart beta by mentioning, "Smart beta is simply about trying to identify good investment ideas that can be structured better ... Smart beta strategies should be simple, low cost, transparent and systematic." There are multiple ways of constructing smart beta strategies, but in general, it can either be simple and sensible rules based strategy or can be optimized-based which is comparatively complex in nature and can have estimation errors (Research Affiliates, n.d.). Smart beta is constructed using both active and passive investment strategies to create potential risk-adjusted portfolios to outperform the market by earning improved returns (Shores, 2015). In recent times, due to high volatility in the global market, low-volatility strategies of the smart beta have gained much attention among the investors as it provides high risk-adjusted returns in the long run compared to the high volatility stock portfolio (Maxey, 2013). According to the study of chow et al. (2014), low volatile strategies are less exposed to the market factors that makes the portfolio less volatile and access to high Sharpe ratio factors (value, duration) helps to accumulate higher return from the market.

Both the S&P 500 low volatility index and the MSCI USA minimum volatility index showed returns of 6.95% with 10.75% standard deviation and 5.1% with 12.32% standard deviation in the US equity market. Both indices managed to acquire returns less than a market cap-weighted benchmark (i.e. S&P 500) which had a 4.12 % return with a standard deviation of 15.99% (Soe, 2012).

The construction of low volatility strategies came from the anomaly in the most common market model known as CAPM. According to CAPM stocks with high beta (risk) provides high return and vice-versa. However, the recent market has shown that this theory does not hold anymore. It has been seen in the market that less volatile stocks are generating returns that are higher than the stocks that are more volatile. The concept of low volatile investment strategy basically emerged from a low volatile anomaly. Figure 2 shows that over the long run the annualized return is higher for the least volatile stocks means the least volatile stocks are performing better than volatile stocks (Masson, 2014).

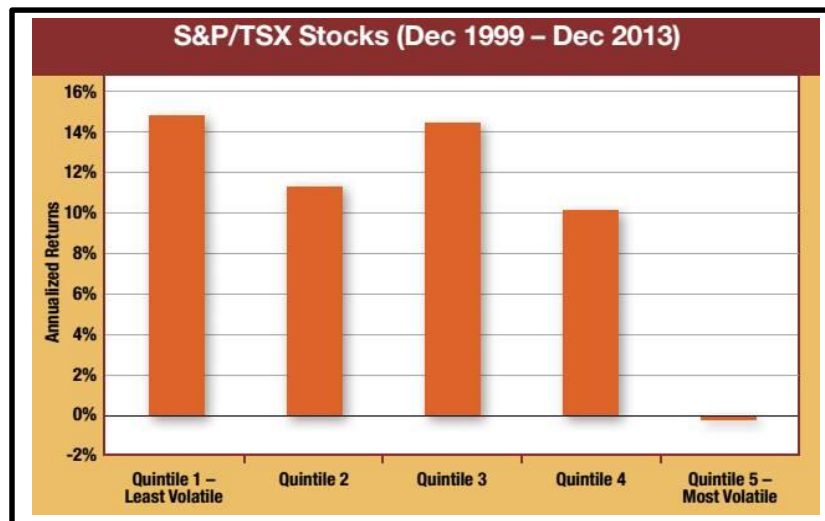


Figure 2: S&P Stocks Return Over the Long Run (Source: Masson, 2014)

Therefore, low volatility strategies of investment give the investors an opportunity to earn excess return (alpha) by exploiting an economically meaningful anomaly of volatility (Ramos & Hans, 2013).

There are numerous approaches available to construct smart beta strategies. Methods like equal weighted, economically weighted (where fundamental metrics are used for calculation), risk minimizing strategies and so on. Among them minimum volatility

strategies of the last approach have gained much attention recently. These minimization strategies, develop a framework using risk and correlation of assets to produce a low volatile portfolio, depending on the strategy (Towers Watson, 2013).

2.3 Sharpe Ratio

Sharpe Ratio is one of the most widely used measures of portfolio performance developed by William Sharpe in 1966. It generally evaluates and predicts the performance of portfolios (Goetzmann et al., 2004). The Sharpe ratio is calculated by dividing the excess return of a portfolio divided by the portfolio's standard deviation. The calculation of excess return is done by subtracting the portfolio's return and the risk-free rate of return. The interpretation of Sharpe ratio suggests that higher the ratio, more excess return can be generated from the extra volatility for holding a risky asset. The calculation does not rely on any particular market index or benchmark and uses only risk free rate of return variable which makes it more effective for comparing funds in terms of style, capitalization and market size (Landsberg, 2013). This is a versatile way to get the initial assumption of investors' reward potential by comparing all the investment vehicles. (Huy Tu Nguyen, n.d.)

Again, it uses the overall risk-adjusted excess return, thus including both beta and alpha components, thus making no discrepancy in the overall source of the risk (Christie, 2005).

The major flaw of the Sharpe Ratio is its inability to differentiate the intermittent and consecutive losses due to its use of standard deviation (R&D, 2012). Collins (2014) added to this issue that any portfolio or asset can generate weak Sharpe Ratio based even if it has a chance of better performance in the coming period.

However, the Sharpe ratio has much to do with the relative directness of the formula used to derive it. There is no need to prepare a broad financial background in the statistics or calculation to fully understand what the Sharpe ratio is theoretically trying to achieve: to pick out if the excess return gained compensations for the involved risk (Hunkar & Ozyasar, n.d.).

2.4 Norwegian Financial Market

To conduct the study successfully, it is important to have access to the necessary market information and data. The main reason for choosing Norway Stock Market is the accessibility of data through TITLON Database. This section covers a brief history of evolution of this market. Along with it, the present status of the market has also been discussed. This section is included in the literature review for showing a convincing research on the real market to make the research feasible.

2.4.1 History of Norway Stock Market

In the year of 1818, the “father” of the Oslo Stock Exchange, merchant Nicolay Andresen, firstly suggest for a commercial exchange to the Norwegian parliament. Later, four businessmen established a small committee to carry out this program. The name of the first stock market is Christiania Exchange, Christiania Børs. At that time, the exchange was only on currency and bonds.

The first exchange for stocks and shares was established in 1880 by two bank owners, N.A. Andersen and S.C. Andersen. And the exchange which stands in Oslo began to list prices for stocks and shares on 1 March 1881. At that time, the price of the stocks and shares were changed only once a month when the two brokers arrived. What is more, the process was quite simple and there was no trading happened in the exchange market. And the first list of prices was 16 bonds and 23 shares on 1 March 1881 (Oslo Bors).

At the end of the 1800s, the stock exchange committee carried out simple and basic principles of stock exchange activities in Oslo exchange. And in 1919 the Christiania stock exchange put out 578 stocks and shares.

The local exchanges were built step by step in Trondheim (1819), Bergen (1837), Kristiansand (1837), Drammen (1839), Stavanger (1878), Kristiansund (1894), Skien (1895), Ålesund (1905), Sandefjord (1912), Haugesund (1894) and Fredrikstad (1921).

Immediately after the First World War the Norway stock market had a boom. But this bad situation did not last for a long time, and the committee introduced a law which was

against over-trading. Between the two world wars the Norwegian economy experienced a crisis period.

The “black Friday” happened on 16 October 1987. On that day, New York’s share prices fell sharply. And the next two days also followed this trend. Certainly Norwegian stock market also suffered from this disaster. The Oslo market witnessed the sharpest decline in the share prices on Tuesday 20 October, with a 19% decline in all share index.

In 1989 the share prices in the Oslo market changed the growth. And in the year of 1990 the index reached at new height, 666.35. However, in the same year, all indices declined back by 46% over the remaining month of 1990.

From 1998 to 2000, due to the crisis in the banking sector, the Norwegian stock market experienced a pessimistic period. In addition, the international currency turbulence served to reinforce the prevailing mood of pessimism. After 2000, the situation changed, the price of the stocks began to increase slowly (Oslo Bors).

2.4.2 The Present Situation

Figure 1 below shows the Norway stock market trending in recent 10 years from 2005 to 2015. From 2005 to 2008, there is a great increase yearly. Because of the 2008 financial crisis, in 2009, the price of the stock dropped to a very low level. But after that, it led to a slowly, stable increase (Ola Honningdal Grytten et al., 2010).



Figure 3: Norway Stock Trading (Source: Trading Economic, n.d.)

In order to make the research more feasible, we collect the data of all the listed companies on the Oslo Stock Exchange (OSE). Until first of November 2015 there are 823 companies exist in the stock market in Norway. Necessary data will be collected from TITLON database.

A typical question is what a person could earn if he or she invested in stocks at the Oslo Stock Exchange. However, there are more than two different ways to answer this question. If someone picks a random stock, he or she wants to find the expected return of the typical stock, in which case an equally weighted average is the relevant measure. Alternatively, he or she can invest in the whole market, in which case a value weighted average is most relevant. There are two indices which are constructed to make this measurement. The OBX is a value-weighted index consisting of the thirty most liquid stocks on the stock exchange. This index was constructed to be the basis for derivatives contracts, and initiated at the beginning of 1987. Another one is also a value-weighted index of all stock on the exchange, termed TOT. The Oslo Stock Exchange has changed indices during a period; and was called the TOTX. In 1999 this index was replaced by the “All Share Index.” TOT is constructed by splicing these two indices (He Shan et al., 2015)

Until now, the OBX index is still the most commonly used index in the Oslo Stock exchange. OBX is a capitalization-weighted index, which can be described as total return and free-float as well. The index tracks the performance of the most traded securities relying on the six months turnover rating. In the Oslo Stock Exchange market of Norway, the OBX index lists the 25 most liquid companies which can be traded for futures and options on the main index of the market. And these listed companies are rotated twice a year on the third Friday of June and December (Trading Economics, n.d.).

2.5 Construction of Sharpe Ratio Based Weighting Model

Under this section, several issues were discussed to explain the motives behind choosing equal weight model for reconstruction, the reasons behind changing the equal-weight equation, the purpose behind using the Sharpe Ratio as a parameter of new model and lastly, formulation and description of Sharpe ratio based weight model.

For capital investment in the financial market, construction of a portfolio that can ensure an average return at a given level of risk is considered as the most important economic task. According to Markowitz (1952, 1959), all investors should construct portfolio in a way that they can optimize their risk- return trade-off simply by diversification. However, this optimal portfolio construction is too complicated for many investors (decision makers) as it requires both making a choice among individual alternatives and considering the correlations between the choices made (Payne, Bettman and Johnson, 1992). As a result, many investors follow simple rule-based strategies like, naïve diversification or equal-weight model to construct a portfolio (Baenartzi and Thaler, 2001). Many literatures have documented that optimal portfolio strategy do not outperform the equal weight index model (Bloomfield et al. 1977) and study conducted by Jorion (1991) further proved that performance of equal weight portfolios is similar to the mean-variance portfolios obtained with Bayesian shrinkage method. Kahn and Lehmann (1991) also suggested that investors seek diversification to avoid buying undesirable stocks from the market and this is simply because they are risk averse. They also mentioned that decision makers prefer variety when choosing from a large basket as

it takes less time and minimum effort and at the same time their risk-averse nature prefer a variety of items of same kinds. That also explains the investors' preference for the equal weight index model.

However, the equal weight index model or naïve rule has been highly criticized for its incompetent index building in large and complex market. In other words, in a volatile market, the equal weighted portfolio always lies under the efficient frontier as it does not consider the risk-return trade-off between the choice alternatives (Windcliff & Boyle, n.d.).

The simplicity of the formula and its ability to generate performance similar to the more complex index model in a small market are the most attractive characteristics of the equal weight model. Hence, reconstructing this model using risk-return adjusted parameter can generate higher returns in a large volatile market and the easiest to use characteristic will remain intact for the investors.

To associate the volatility of the present market condition, the denominator of the equal weight model replaces by a risk-adjusted parameter, Sharpe Ratio.

Given a market of N number of available assets, the Equal Weight Index model is defined as-

$$w_i^E = \frac{1}{N} \quad (1)$$

Where, w_i , represents the percentage of weight held from asset i. As the model does not consider the risks associated with each asset and simply includes equal fractions of all assets in a portfolio makes the portfolio, it completely ignores the optimization and estimation and also neglects important risk-return information related to the assets (DeMiguel et al., 2007).

As mentioned earlier in the literature review that, even though there are some flaws in Sharpe ratio, this index will be used to estimate the weight fractions of assets in the portfolio. The reason behind this is the aim of this study. In this study, we are trying to

develop a convenient model that constructs risk-adjusted portfolios and confirms an acceptable average return higher. The Sharpe ratio measures the return the investors are going to receive for the level of risk they are interested in taking on (Marte, 2012). The Sharpe ratio is in its simplicity to use. Despite the simplicity of its components, it recognizes both idiosyncratic and systematic risks of an asset to measure the performance of an asset (Sriram, 2011). The mathematical notation of *Ex-post* Sharpe Ratio is-

$$SR_i = \frac{E(R_i - R_f)}{\sigma_i} \quad (2)$$

Where, SR_i is the Sharpe ratio of asset i which is calculated dividing, $E(R_i - R_f)$, expected excess return on the difference between the realized asset return, R_i and the risk free rate of return, R_f by the standard deviation, σ_i of the asset. Here, *ex-post* Sharp ratio will be used and *ex-ante* ratio will be ignored to avoid the estimation error of predicting the expected return of an asset for the coming period.

In this study, N is replaced with ΣSR_N and 1 with SR_i to weight the risk-adjusted percentage of assets that which will be used to construct that a portfolio. Here, ΣSR_N equals the sharpe ratio of all the sample companies and SR_i refers to the sharpe ratio of specific company's of the sample. Therefore, we developed the Sharpe Ratio based Weight Model by using equation 1 and 2-

$$\frac{SR_i}{\Sigma SR_N} \quad (3)$$

However, Sharpe ratios can have both positive and negative values and to avoid the negative values in the study, Exponential function has been added in the model. The reasons behind using exponential function are, firstly it always provides positive value. Secondly, it allows exponentiation of non-zero values and shows the growth of the value over time (Ledet, 2012). Thus, the sharpe ratio weight model is constructed below by including exponential function in equation 3 –

$$\frac{\text{Exp}(SR_i)}{\Sigma \text{Exp}(SR_N)} \quad (4)$$

In this research, this model will be used to calculate the risk-adjusted weight of assets to select the worthy assets to construct a low-volatile portfolio. The risk and return performance of new portfolios will be compared with the performances of the equal weighted index and also the value-weighted indices that were already available in the Norwegian stock market. Performance measurements available in the market will be used to analyze and interpret the data.

3. Research Data and Methodology

3.1 Overview

Any research requires obtaining and assembling relevant data and use the result of those assembled data to establish support or refute to a valid conclusion (Cameron & Price, 2009). As per the literature review, a thorough analysis was done on the model's generated data to answer the research questions and to test the stated hypothesis. This chapter describes how the use of new model works effectively to determine the right companies to invest in from a large number of companies. Here right companies refer to the stocks that are less volatile and provide an acceptable average return on investment. The summary of the statistical analysis reveals that the model is operational in successfully selecting less risky stocks from the market for investment. Moreover, the model allows continuous selection of the less risky companies for any less-volatile portfolio for different periods. In the study further analysis was done based on the changing stock prices to compare the risk and returns of different periods to ensure the efficiency of the model. The result of the study entails that this Sharp Ratio weight model is efficient to allocate less volatile stocks for the portfolio over the period. A detail research methodology is explained in this chapter to clarify different stages of the analysis.

3.2 Population of the study

As mentioned in chapter 1, for this study Norway's financial market has been selected as the population of the study for the period 2005 to 2015 (ten-year period). Availability of necessary data and proximity advantages to obtaining company information is significant in conducting any study. This is why, Norway's financial market has been chosen to test the model. Norway's financial market includes trading of listed shares, unlisted shares, short-term debt securities, long-term debt securities and equity certificates (Statistics Norway, 2015).

3.3 Unit of Study

For this research paper, only the stocks of listed companies of Oslo Stock Exchange are taken as the unit of study. Adjusted closing prices of all the listed companies from period 2005 to 2015 were collected from existing database to do further calculations for the study.

3.4 Sampling Procedure

Listed Companies that are available in the database were selected and inclusion and exclusion of listed companies in the study were made based on the criteria required for the study. Hence, the convenience random sampling method was used to acquire the adjusted closing prices of the companies. Random Sampling, as Hatch and Farhady (1981) puts it, maximizes the internal and external validity of any study findings by giving equal chance to all the subjects based on the required criteria for insertion in the analysis.

As per our observed time period of 10 years (2005 – 2015), all the companies that were trading during this time horizon were selected for the study. The total number of companies are 413. However, 7 companies were excluded from the sample as data for those companies were not sufficient for the study. Therefore, the final sample size for the study is 406 companies.

Reasons behind excluding 7 companies from the analysis as they were not meeting the minimum criteria are mentioned below –

- The time period is one of the most important variables for this analysis and data of all the months and trading days are crucial to calculate the returns on the stocks. Therefore, companies that have missing months and trading days were excluded from the sample.
- Again Companies that did not have data on adjusted closing prices covering the observed period have been excluded.

3.5 Data Collection and Instrumentation

Initially, secondary data (Adjusted closing price of stock) was employed to obtain the primary data (Return on stocks) for the progression of the analysis. The data collection was done through a step-by-step process. Worksheets and macros of Microsoft Excel application were used to obtain the primary data of the sample companies. All the steps of collecting data are discussed below –

3.5.1 Calculation of Daily Return of Stocks (R_i)

The daily return of stocks is a variable of calculating the Sharpe Ratio. Hence, the return of all the sample companies was calculated to conduct a further calculation of Sharpe Ratio of all the companies. In particular,

$$R_i = (P_t - P_{(t-1)})/P_{(t-1)} \quad (5)$$

Where R_i is the daily return of stocks, P_t is the adjusted closing price of the current year and $P_{(t-1)}$ is the adjusted closing price of previous year. The daily R_i was annualized before calculating the sharpe ratio.

3.5.2 Calculation of Daily Risk-Free Rate of Return (R_f)

Primarily published risk-free rate of return on 10 years annual bonds were taken and calculated the daily rate by dividing it with 365. The new daily rate was used as the Sharpe ratio's R_f .

3.5.3 Calculation of Standard Deviation (σ)

General formula *STDEV.P* of Excel was used on the annualized average daily return to calculate the standard deviation variable. Even though this formula is to calculate the standard deviation of the population, it was used as all the data of each sample company was used for calculation.

3.5.4 Calculation of Exponential Sharpe Ratio *EXP(SR)*

The daily Sharpe ratios of all the companies for each year were calculated using the equation 2 and calculated the exponential of daily Sharpe ratios of all companies using *EXP* or exponential function in the value of SR.

As the analysis was done based on monthly data, an average of daily EXP(SR) was calculated for each year and the monthly EXP(SR) of all the companies for every period were used as the parameter on the SR weight model.

3.5.5 Sharpe Ratio Weight Model (*SR Weight Model*)

The monthly EXP(SR) of each company for every year were added to calculate the $\Sigma \text{EXP}(SR_N)$ of each month and monthly weight of each company was calculated using the SR weight model (equation 4).

To analyze the effectiveness of the SR weight model ten best and ten worst companies are selected for each year, to create a portfolio to compare the risk and return of both groups. To compare the return, Both Monthly Return of each company and average Return of both groups are calculated. For risk, the standard deviation for each month of the observed period is determined.

3.6 Procedure for Testing Hypothesis

All the research questions or Hypotheses of the study are tested as follows –

3.6.1 Hypothesis (H_1) : Portfolio with high weighted companies' stocks are more risk adjusted.

The expectation from selecting high weighted companies using SR weight model is, its volatility is low compared to the low weighted companies. In this study, the volatility is measured by observing the Standard Deviation (SD) on the monthly returns of selected portfolios. To measure the volatility, F-test was done to observe whether high weighted companies SD is lower than the low weighted companies or not.

To determine whether high weighted companies returns are less volatile than low weighted companies –

(H_0) : $SD_{Hw} = SD_{Lw}$ (Standard Deviations of high weighted companies are equal or greater than standard deviations of low weighted companies, where significance level is $\alpha = 0.05$).

$(H_1) : SD_{HW} < SD_{LW}$ (Standard Deviations of high weighted companies are less than standard deviations of low weighted companies, where, $\alpha = 0.05$).

Again, Pivot Tables and Charts of different years are examined to compare the volatility differences between high and low weighted companies. The group that has a straighter line (fewer peaks and troughs) in the histogram is considered to be less volatile.

3.6.2 Hypothesis (H_2) : A difference exists between high weighted companies and low weighted companies stock returns.

After implementation of the model to choose companies for investment, it is expected that there will be a difference between the returns of high and low weighted companies. By conducting a t-test of two samples with unequal variances the hypothesis was tested as follows –

$(H_0) : R_{HW} = R_{LW}$ (There is no difference between the returns of both high weighted low weighted companies, where, $p = 0.05$ (significance level)).

$(H_1) : R_{HW} \neq R_{LW}$ (The returns of both group exists and it is statistically significant, therefore the difference is not random. Here, $p = 0.05$ (significance level)).

3.6.3 Hypothesis (H_3) : The return differences between high weighted companies and low weighted companies are systematic.

To examine the effect of the use of SR weight model, this hypothesis is tested to prove that the differences between the mean returns are not random and it is happening due to a systematic process.

If the Hypothesized Mean Differences = 0 in the t-test of returns, the hypothesis is accepted, otherwise reject.

Apart from testing this three hypothesis, a general discussion is made to show that the use of the new model in selecting companies to invest provides a low volatility advantage in stock investment and at the same time confirms a better average return. Pivot Tables and Charts of different years are examined to compare the volatility and returns differences

between high and low weighted companies. The group that has a straighter line in the histogram is considered to be less volatile and the same group is observed to see if they consist higher bars in the bar chart of returns of both of the companies.

4. Analysis of Data

The significance of analyzing data using quantitative method has increased in business management as it is getting more complicated. The main reasons behind this increased importance are the clear and concrete results it produces that makes the decision-making process easier (Richard, 1992). The data analysis process of this study is discussed below-

4.1 Descriptive Statistics

To describe and summarize data precisely, a descriptive statistic was employed in this study. According to Zikmund (2013), Descriptive statistics transforms data in a way that makes it easier to analyze and interpret. In this study, descriptive statistics tool was run on both dependent variable Monthly Returns and Monthly Average Standard Deviation. The detail of the analyses is described below-

4.1.1 Descriptive Statistics on Monthly Returns

Table 3, shows the descriptive statistics run on both high-weighted and low-weighted companies stock monthly returns (dependent variables) on a 10 years' time period.

<i>Monthly Return (High-weighted Companies)</i>		<i>Monthly Return (Low-weighted Companies)</i>	
Mean	0.09215082	Mean	-0.027829373
Standard Error	0.038112354	Standard Error	0.006093222
Median	0.039152854	Median	-0.016608761
Mode	#N/A	Mode	#N/A
Standard Deviation	0.436215835	Standard Deviation	0.06974011
Sample Variance	0.190284255	Sample Variance	0.004863683
Kurtosis	110.3011451	Kurtosis	1.108743932
Skewness	10.14320781	Skewness	-0.739393342
Range	5.108964761	Range	0.405231499
Minimum	-0.266834688	Minimum	-0.276511873
Maximum	4.842130073	Maximum	0.128719626
Sum	12.07175737	Sum	-3.645647846
Count	131	Count	131

Table 3: Descriptive statistics of high-weighted and low weighted companies' monthly returns

The key highlights of this descriptive statistics are –

- The means of monthly returns for both high and low weighted companies showing 9.21% and (-2.78%) respectively.
- The standard deviation for the 10 years of monthly returns for high weighted companies are relatively high, 43.62% and 6.91% is for the low weighted companies.
- Again the Kurtosis for high weighted companies is 110.301 and for low weighted companies, it is (-0.739). Therefore, the variables are to have excess kurtosis and skewed than a normal distribution.

4.1.1 Descriptive Statistics on Monthly Standard Deviation

Apart from the table 4 which shows the descriptive statistics of the second dependent variable, namely monthly standard deviation; some noticeable highlights are mentioned below -

<i>Monthly Standards Deviation (High-weighted Companies)</i>		<i>Monthly Standard Deviation (Low-weighted Companies)</i>	
Mean	0.291219532	Mean	0.114404084
Standard Error	0.115769998	Standard Error	0.007756766
Median	0.115720965	Median	0.089593154
Mode	#N/A	Mode	#N/A
Standard Deviation	1.325048196	Standard Deviation	0.088780241
Sample Variance	1.755752723	Sample Variance	0.007881931
Kurtosis	117.7876253	Kurtosis	10.48874439
Skewness	10.6360435	Skewness	2.80361685
Range	14.93658818	Range	0.591642121
Minimum	0.017125613	Minimum	0.028191059
Maximum	14.95371379	Maximum	0.619833179
Sum	38.14975871	Sum	14.98693503
Count	131	Count	131

Table 4: Descriptive statistics of high-weighted and low weighted companies monthly average standard deviations

- The means of the monthly standard deviations are 29.12% for high weight group and 11.44% for low weight group.

- The median is 0.12 and 0.089 respectively for high and low-weight groups.

4.2 Research Hypothesis One: High weighted companies are more risk adjusted.

A t-test for two sample means was done on the monthly average standard deviations of both high and low weighted companies for 10 years period to test the hypothesis. Primarily the F-test: two samples for variance result showed that the variance on the standard deviations was significant at 5% level of significance. Again the test shows the p-value $6.59E-116$ is less than the alpha .05, therefore, the variance between the standard deviations for both high and low weighted companies are not equal. The mean of the standard deviations is 29.12% for high weighted companies and 11.44% for low-weighted companies.

F-Test Two-Sample for Variances

	<i>Monthly STDev (High-weighted Companies)</i>	<i>Monthly STDev (Low-weighted Companies)</i>
Mean	0.291219532	0.114404084
Variance	1.755752723	0.007881931
Observations	131	131
df	130	130
F	222.7566661	
P(F<=f) one-tail	6.59E-116	
F Critical one-tail	1.335872155	

Table 5: F-test Two Samples for variance of the monthly standard deviations of high-weighted and low weighted companies.

As the f-test proved that the variances of the standard deviations are unequal, a t-test for two samples assuming unequal variance was done and the result shows that it is not significant, at 5% level of significance where, $(p > .05)$ or $0.129949 > .05$.

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly STDev (High-weighted Companies)</i>	<i>Monthly STDev (Low-weighted Companies)</i>
Mean	0.291219532	0.114404084
Variance	1.755752723	0.007881931
Observations	131	131
Hypothesized Mean Difference	0	
df	131	
t Stat	1.52388271	
P(T<=t) one-tail	0.064974249	
t Critical one-tail	1.656568649	
P(T<=t) two-tail	0.129948499	
t Critical two-tail	1.978238539	

Table 6: t-test: Two Samples Assuming Unequal variances of monthly standard deviation of high and low-weighted companies

Again the pivot bar chart in figure 4 shows the yearly comparison of the average monthly standard deviations of both high and low weighted companies. Monthly standard deviations of both high and low weighted companies are given in Appendix – A(1). The chart explains –

- The average monthly standard deviations for high weighted companies are little higher than the low-weighted companies in 6 of the years.
- They are lower than the low weighted companies in 3 of the years.
- However, monthly average standard deviations of high-weighted companies are found to be much higher in 2009, 2014, and 2015

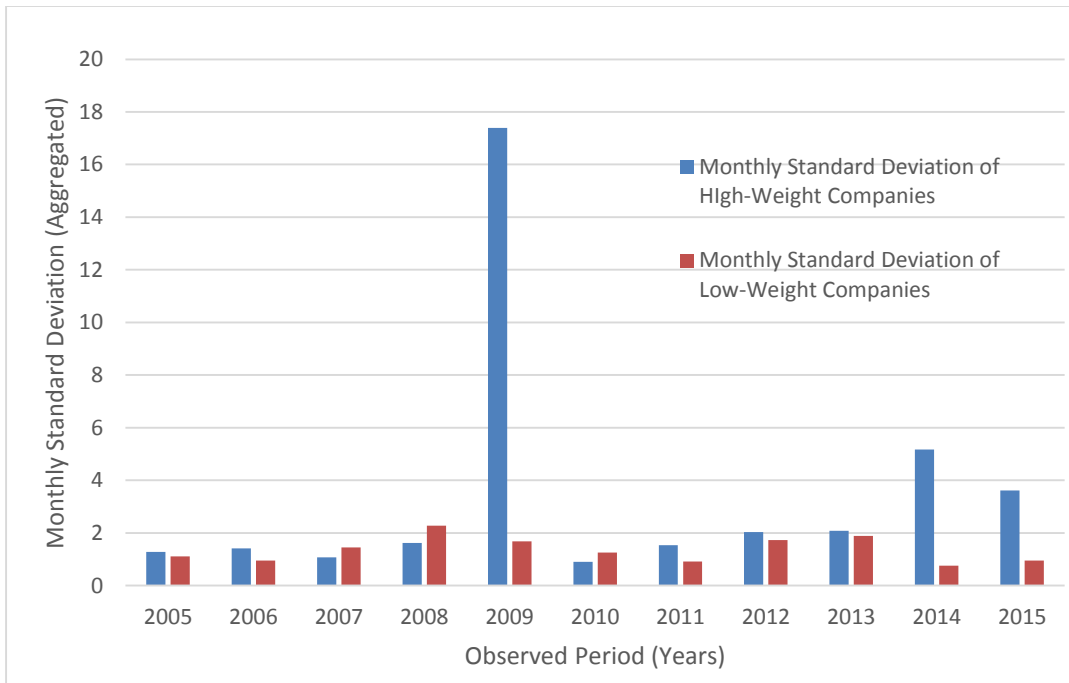


Figure 4: Yearly Comparison of Monthly Average Standard Deviation of High-Weighted and Low-Weighted Companies

4.2.1. Interpretation of the Results

According to the results to validate high weighted companies are more risk adjusted or less volatile than the low weighted companies, the null hypothesis cannot be rejected. It means there is no difference between the risk properties of high weighted and low weighted companies. Even though, the means of the average standard deviation shows different values, the risk associated with both high and low weighted companies are actually same.

The bar chart also shows that in the whole time period the monthly standard deviation of both groups are very close to each other except for the year 2009. The reason behind this was the substantial change in the stock price of Wentworth Resources company in October 2009 (Appendix A(1)). As the SR weight model selected this company among the high weighted companies, the investment in this company resulted in sizeable return due to the significant increase in stock price. Thus, the variance also increased significantly for that year.

4.3 Research Hypothesis Two: There is a difference in the returns of high weighted and low weighted companies.

4.3.1. Test Based on Overall Observed 10 Years Period

To test the null hypothesis of the returns to be same for both groups of companies, firstly an F-test was done on both groups 10 years monthly returns to find out whether the monthly returns of high and low weighted companies have equal or unequal variances or not. The result of the test shows the variance in returns is significantly different at 5% level of significance. The p-value for the test is 5.82729E-68 which is lower than the alpha .05. Again the F value > F Critical value, 39.123 > 1.34 which also proves that the null hypothesis does not hold and a difference exists in the variances between the high and low weighted groups 10 years monthly returns. Therefore, the two samples or monthly returns of the companies have unequal variances.

F-Test Two-Sample for Variances

	<i>Monthly Return (High-weighted Companies)</i>	<i>Monthly Return (Low-weighted Companies)</i>
Mean	0.09215082	-0.027829373
Variance	0.190284255	0.004863683
Observations	131	131
df	130	130
F	39.12349129	
P(F<=f) one-tail	5.82729E-68	
F Critical one-tail	1.335872155	

Table 7: F-test Two Samples for variance of the monthly returns of high-weighted and low weighted companies.

The t-test for two samples unequal variances is conducted to confirm the second research hypothesis of this study and the result shows that the 10 years monthly returns of high and low weighted companies differ significantly at 5% significance level. The p-value of this t-test is smaller than the alpha, (0.002 < .05) and the F value, 3.11 > F critical value,

1.98. Again the test shows mean monthly returns for high and low weighted companies are 9.21% and (-0.028%) and the degree of freedom is 137.

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Return (High-weighted Companies)</i>	<i>Monthly Return (Low-weighted Companies)</i>
Mean	0.09215082	-0.027829373
Variance	0.190284255	0.004863683
Observations	131	131
Hypothesized Mean Difference	0	
df	137	
t Stat	3.108588381	
P(T<=t) one-tail	0.001143321	
t Critical one-tail	1.65605208	
P(T<=t) two-tail	0.002286642	
t Critical two-tail	1.977431212	

Table 8: t-test: Two Samples Assuming Unequal variances of monthly returns of high and low-weighted companies

4.3.2. Month Based Test on Observed 10 Years Period

To confirm the research hypothesis based on monthly average stocks returns of both high and low weighted companies, per year based f-tests and t-tests were done for the observed period. Firstly the f-test for two sample variances for 10 years results showed that the p-value < alpha or (0.05) except for the year 2005 and 2013 (Appendix B). Therefore, in 80% cases, the variance between monthly average stock returns are not equal.

The t-tests were done on all the observed years monthly stock returns both for equal and unequal variances (based on the f-tests). The results showed a significant difference between the high and low-weighted companies monthly average returns at 5% significance level except for the year 2008, 2009 and 2012. In these 3 years, the p-value appeared to be higher than the alpha and which are 0.051, 0.27 and 0.10 respectively. In all other years, the p-value is less than the significant level of 5% or 0.05.

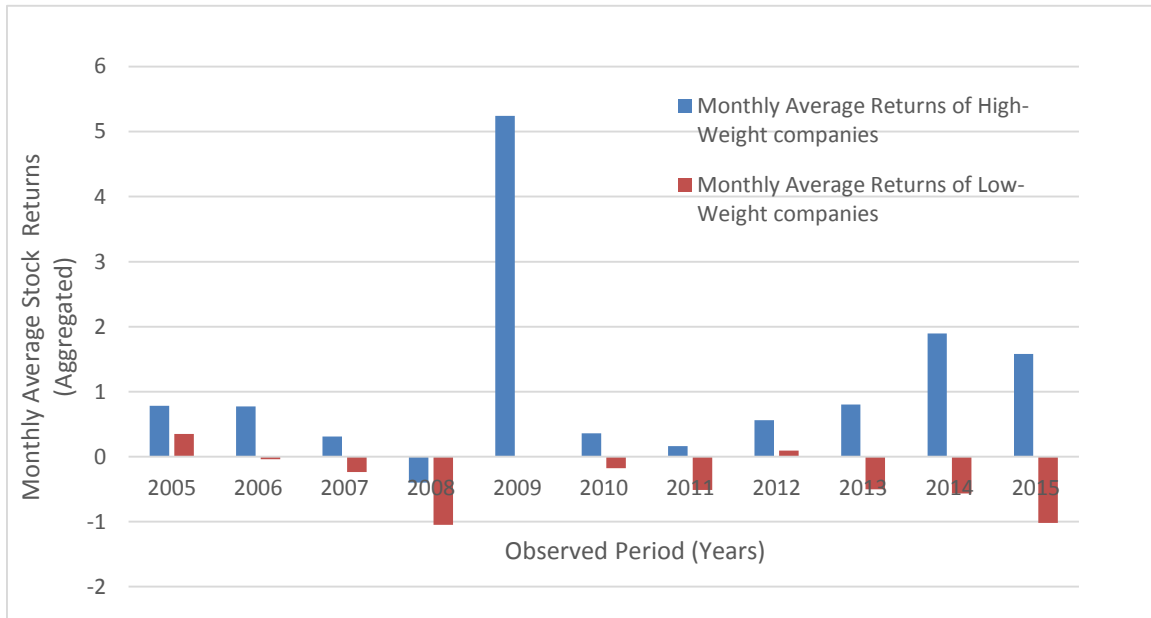


Figure 5: Yearly Comparison of Monthly total stock returns of High-Weighted and Low-Weighted Companies

A graphical representation of the monthly returns for both high and low weighted companies for the observed 10 years' time period in figure 5 depicts the structural differences between them. The bar chart shows -

- The observed periods high-weighted companies' monthly returns are always high compared to the returns of low-weighted companies.
- The monthly returns of high-weighted companies are positive even when the returns of low-weighted companies are negative except for the year 2008. However, the high-weighted companies return is still better than the low-weighted companies as it is closer to 0 whereas the low-weighted companies return is a little higher than (-1%).
- The high weighted companies return is substantially high (more than 5%) where low-weighted companies return is less than 1% for the observed periods.

4.3.1 Interpretation of the Result

The F-test results of the total monthly returns of high weighted and low weighted companies over a 10 years' period show a significant difference between them. Therefore, the variances of the monthly stock returns of both high and low-weighted companies are not same.

The t-test p-value result for unequal variances shows that it is significant and therefore the null hypothesis is rejected. It means significant difference exists between the returns of high and low weighted companies and after evaluating the mean returns of both types it can be concluded that the stock returns for high weighted companies are higher than the low-weighted companies for the observed periods.

However, when the same test was run on a yearly basis, the year 2008, 2009 and 2012 failed to reject the null hypothesis and showed that there is no difference between the monthly average returns of high and low-weighted companies. The main reason for such discrepancies is explained below –

- In 2008, the high weighted companies average monthly stock returns for October and December turned out to be lower than the low-weighted company, (-17.34%) and (-26.68%) (Appendix A(2)). This resulted in a higher p-value and the null hypothesis held.
- In 2009, the variance of mean returns of high weighted companies was extremely high compared to the variance of low-weighted companies, which are 18.74 and 0.024 respectively. Such variance of the returns from its mean return refers to the situation where an extreme gain was realized from one or more companies. In this case, it happened due to Wentworth Resources Company. As a result, the p-value was higher than the significant level and confirmed that there is no difference between the mean returns, even though the mean returns of high-weighted companies was 43.67% and 00.01% for low weighted companies (Appendix B).
- During the month of April, May, July and November 2012, the monthly average returns were lower than the low weighted companies (Appendix A(2)). This means,

during these months one or more companies realized lower returns. As a result, the test resulted in there is no difference between the Monthly returns of high and low weighted companies.

The illustration on the bar chart also shows that the yearly returns on high weighted companies are always better than the low-weighted companies even when it is negative. It can also be confirmed that the year based aggregated return from the high weighted companies is always better than the low weighted companies. During the period 2009, the high weighted companies stocks monthly returns were noticeably high due to the inclusion of Wentworth Resources company in the high-weighted companies list. In 2009, October, due to the increase in the stock price of the company, the return increased; which in turn increased the monthly average stocks return of the high weighted companies during that month (Appendix A(2)).

4.4 Research Hypothesis Three: The differences in the return are not random.

During the t-test of monthly returns for 10 years' period, the hypothesized mean difference of the test was 0 (figure 5(b)). Therefore, the null hypothesis can be rejected.

Again in the t-tests for year based monthly return of high and low-weighted companies, the hypothesized means showed 0 value (Appendix B). So again the null hypothesis can be rejected for all the years tested.

4.4.1 Interpretation of the Result:

The hypothesized mean value, 0, proved the fact that all the differences in the mean returns (both overall and yearly based means) are existing due to a systematic process and are not random figure 4(b) and (Appendix B).

Through this confirmation of the third hypothesis, it can be justified that the higher returns of the high weighted companies are happening due to the selection process of the companies.

5. Findings & Discussions

5.1 Introduction

This chapter consists the summary of findings based on the data analysis conducted in the previous chapter. A further discussion was done after summarizing the findings of the research.

As mentioned in previous chapters, this study was conducted to answer the specific research questions, *does Sharpe Ratio-based weighting methodology constructs low-volatile portfolio, does Sharpe Ratio-based weighting scheme confirm a difference in returns between the high and low weighted companies and Are the differences in returns between high weight companies and low weight companies random.* Several statistical techniques were applied to confirm the answers to the questions and graphical representations were included to support the statistical results of the tests. The tests and the graphs presented in the previous chapter addressed all the research questions successfully.

5.2 Summary of Findings

After the data analysis of the dependent variables of the study, namely, monthly stocks returns and the monthly standard deviations; numbers of findings are identified for further discussion. Followings are the summaries of the findings –

- According to the statistical results, the Sharpe Ratio based high-weight companies do not construct low-volatile portfolios compare to the low weight companies rather they are found to be same.
- The graphical and value-based comparison confirmed that the risk indicator (in this study, monthly average standard deviation) of the high-weight portfolio (in this study, monthly average standard deviation) is either equal or insignificantly high compared to low weight portfolio.
- The high-weight companies experienced extreme gains and losses in some months of the observed period which affected the portfolio's average return and standard deviation.

- Again, the statistical results ensured that among the monthly returns of both high-weight and low-weight groups, high-weight portfolio return was found to be higher for the observed periods. When the same test was run on the monthly returns of both groups (on a yearly basis), same results were confirmed for seven years from 10 years of the observed period, which comprises 70% of the result.
- 30% results that failed to hold the second hypothesis for yearly based returns actually happened due to the extreme gain and loss situations in the high weight portfolio.
- According to the value of hypothesized mean differences of both monthly average standard deviation and monthly stock returns (overall and yearly based), it is confirmed that the differences in the risk and returns between the groups are not random and is happening due to a systematic process. In this study, the systematic process is the use of Sharpe Ratio based weight model to select the companies to compare their risk and return differences.
- The excess kurtosis of the high weight companies returns reveals some interesting characteristics of the selected companies risk and return relationships.

5.3 Discussion on Findings

From the overall analysis and summarization of findings, a broad discussion can be approached to confirm that all the research questions are answered for this study.

The first research question was on identifying whether the Sharp Ratio based weight model can provide a risk-adjusted portfolio or not. Through the statistical test, it was found that the difference between the high and low-weight portfolios do not differ or carry the same level of risk in the portfolio. Therefore, to prove that the high weighted portfolio is actually risk-adjusted it became significant to confirm the second research question. The second research question was to confirm which of the portfolio provides a higher return. From the statistical analysis, it was found that high-weight portfolio provides a high return if the model is used for a long time period. However, in some cases, the portfolio can experience lower monthly return compared to its other

counterpart. In terms of yearly aggregated return, high-weight companies always surpass the low-weight companies.

Therefore, from the answers to the research question, one and two it can be concluded that Sharpe Ratio based weight model can confirm a risk-adjusted portfolio for investment as the high-weight portfolio is providing a higher return at the same level of risk.

The third research question was included to verify that the high-weight portfolio is providing a high return at the same level of risk due to the use of newly constructed model of Sharpe Ratio based weight model. From the 0 value of hypothesized mean differences, it was confirmed that such differences in the returns were due to a systematic process and in this study, the systematic process was the use of Sharpe Ratio based weight model.

The positive excess kurtosis refers to a situation where the returns are not normally distributed rather it has a chance of outlier events. Therefore, there is a chance of having extreme gain or loss in an excess kurtosis situation (Oxford Dictionary of Finance & Banking). It is also known as leptokurtic kurtosis. The kurtosis for high-weight companies is far more than +3, therefore, the high-weight portfolio can have a both extremely high return or extremely low return due to outlier events and it was experienced in some companies during some months. However, the high weight portfolio was not affected for due to the outlier events (especially in the case of negative returns) as the other companies return balanced the portfolios yearly aggregated monthly returns. The reason of the presence of excess kurtosis can be due to Sharpe Ratio's inability to distinguish between systematic and unsystematic risks as it uses standard deviation to estimate the volatility of the investment.

Before concluding the discussion on findings it can be said that this Sharpe Ratio based weight model is consistent in constructing risk-adjusted portfolio if used for a long-term.

It is also useful for making investment portfolio for a short period of time but it can be less attractive to the conservative investors as there are chances of outlier events.

6. Recommendation & Conclusion

6.1 Recommendation for Further Research

The craving for risk-adjusted portfolios that can confirm a positive return over the period is increasing among the investors. The analysis conducted and discussions made in this study, it can be said that this Sharpe Ratio based weight model has successfully constructed a risk-adjusted portfolio (high-weight portfolio). Therefore, the need of this model in the investment decisions cannot be ignored. However, further research can be done on the model to reduce its shortfalls and also to ratify the effectiveness of the model in the financial market. Followings are the recommendations for further research on the model -

- A further study to compare and contrast this model with conventional models can be conducted to determine the further efficiency of the model.
- Using this model on the other financial instruments and for more complex portfolios to analyze its effectiveness in those cases.
- confirmed its 100% validity only for long run. In the case of constructing a portfolio for short-term situations, it may have the impact of outlier events. So a Study can be done to include some effective parameters to the model that can reduce or eliminate the outlier events disadvantage.

6.2 Conclusion

According to a study conducted by EDHEC-Risk Institute, over 45% investors have changed their old weighting schemes and in Europe and around 67% realized significant problems in their current scheme (Amenc et al, 2003). Thus, the significance of constructing alternate weighting models or smart beta strategies that can reduce the risk element from the investment is beyond any argument. In such demanding state construction of risk-adjusted weighting model using simple performance indicators like Sharpe Ratio can prove to be added value for investment decision-making process for both professional and novice investors.

In the end of this study, it can be said, that the Sharpe Ratio based weight Model that was constructed and studied to legitimize its potential in creating risk-adjusted portfolios fulfilled its purpose with some shortfalls that can be eliminated with further research.

References

- Academia.edu, (n.d.). *ZEUS ASSET MANAGEMENT INC.* [Online] Available at: http://www.academia.edu/6727017/ZEUS_ASSET_MANAGEMENT_INC [Accessed 27 Oct. 2015].
- Aked, M. And Moroz, M. (2013). *The Market Impact of Index Rebalancing [White Paper]*. 1st Ed. [Ebook] Newport Beach, CA: Research Affiliates, pp. 1-4. Available at: <https://www.researchaffiliates.com/Production%20content%20library/Market-Impact-Index-Rebalancing.pdf> [Accessed 28 Oct. 2015].
- Aliaga, M. And Gunderson, B. (1999). *Interactive statistics*. Upper Saddle River, N. J.: Prentice Hall.
- Amenc, N., Goltz, F. and Martellini, L. (2013) *Smart beta 2.0*. Available at: http://faculty-research.edhec.com/_medias/fichier/edhec-position-paper-smart-beta-2-0_1378195044229-pdf (Accessed: 10 May 2016).
- Aqr.com, (2014). *AQR - Smart Beta: Not New, Not Beta, Still Awesome*. [Online] Available at: <https://www.aqr.com/cliffs-perspective/smart-beta-not-new-not-beta-still-awesome> [Accessed 26 Oct. 2015].
- Arnott, R., Kalesnik, V., Moghtader, P. And Scholl, C. (2010). *Beyond Cap Weight: The Empirical Evidence for a Diversified Beta*. [Online] www.researchaffiliates.com. Available at: https://www.researchaffiliates.com/Production%20content%20library/JOI_Jan_Feb_2010_Beyond_Cap_Weight_The_Empirical_Evidence_for_a_Diversified_Beta.pdf [Accessed 18 Nov. 2015].
- Arnott, R., And Kose, E. (2014). *What "Smart Beta" Means to Us*. [Online] [researchaffiliates.com](http://www.researchaffiliates.com). Available at: http://www.researchaffiliates.com/Our%20Ideas/Insights/Fundamentals/Pages/292_What_Smart_Beta_Means_to_Us.aspx [Accessed 27 Oct. 2015].

- Asness, C. (2006). *The Value of Fundamental Indexing*. [Online] Institutionalinvestor.com. Available at: <http://www.institutionalinvestor.com/Article/1082031/Search/The-Value-of-Fundamental-Indexing.html?ArticleId=1082031&ReservedReference=search&Keywords=The+Value+of+Fundamental+Indexing&OrderType=1&PeriodType=4&StartDay=0&StartMonth=1&StartYear=2006&EndDay=0&EndMonth=11&EndYear=2006&ScopeIndex=0&p=1#.VlcRvvlViko> [Accessed 1 Nov. 2015].
- Bacon, C. (N.d.). *How sharp is the Sharpe-ratio? - Risk-adjusted Performance Measures*. [Online] www.statpro.com. Available at: <http://pages.statpro.com/rs/statproplc/images/How%20sharp%20is%20the%20Sharpe%20ratio.pdf> [Accessed 26 Oct. 2015].
- Benartzi, S. And Thaler, R. (2001). Naive Diversification Strategies in Defined Contribution Saving Plans. *American Economic Review*, 91 (1), pp. 79-98.
- Bloomfield, T., Leftwich, R. and Long, J. (1977). Portfolio Strategies and Performance. *Journal of Financial Economics*, 5, pp.201-218.
- Bowers, M. (2014). *Exploring Smart Beta*. [Online] www.researchaffiliates.com. Available at: http://www.researchaffiliates.com/Production%20content%20library/Smart%20Beta%20PDF%20Series_WebVersion.pdf [Accessed 12 Nov. 2015].
- BNP Paribas Investment Partners, (2015). *The theory of low-volatility investing - BNP Paribas Investment Partners*. [online] Available at: <http://institutional.bnpparibas-ip.com/the-theory-of-low-volatility-investing/#acceptLicense> [Accessed 1 Nov. 2015].
- Børs, O. (2015). *The history of Oslo Børs / About Oslo Børs / Oslo Børs / Home - Oslo Børs*. [online] Oslobors.no. Available at: http://www.oslobors.no/ob_eng/Oslo-

Boers/About-Oslo-Boers/The-history-of-Oslo-Boers [Accessed 8 Nov. 2015].

- Burton, J. (2013). The Problem with All Things Being Equal. *The Wall Street Journal*, [online] p.Single. Available at: <https://www.researchaffiliates.com/Production%20content%20library/Market-Impact-Index-Rebalancing.pdf> [Accessed 27 Oct. 2015].
- Burton, J. (2013). *How equal-weight ETFs have lost their balance*. [online] MarketWatch. Available at: <http://www.marketwatch.com/story/equal-weight-etfs-arent-the-most-balanced-2013-03-04> [Accessed 16 Nov. 2015].
- Cameron, Sheila. and Price, Deborah. (2009), *Business Research Methods: A Practical Approach.*, London: Chartered Institute of Personnel Development.
- Carlisle, T. (2012). *Why Does an Equal-Weighted Portfolio Outperform Market Capitalization- and Price-Weighted Portfolios?*. [online] Greenbackd. Available at: <http://greenbackd.com/2012/05/17/why-does-an-equal-weighted-portfolio-outperform-market-capitalization-and-price-weighted-portfolios/> [Accessed 28 Oct. 2015].
- CFAinstitute.org, (n.d.). *Security Market Indices*. [online] Available at: https://www.cfainstitute.org/learning/products/publications/inv/Documents/investments_chapter2.pptx [Accessed 18 Nov. 2015].
- Chow, T., Hsu, J., Kuo, L. and Li, F. (2014). A Study of Low-Volatility Portfolio Construction Methods. *Institutional Investor Journals*, 40(4), pp.1 - 4, 6-7.
- Christie, S. (2005). Is the Sharpe Ratio Useful in Asset Allocation?. *SSRN Electronic Journal*.
- Collins, D. (2014). *Sortino Ratio: Correcting Sharpe Ratio flaws* | *Futures Magazine*. [online] Futuresmag.com. Available at: <http://www.futuresmag.com/2014/02/01/sortino-ratio-correcting-sharpe-ratio-flaws>

[Accessed 8 Nov. 2015].

- DeMiguel, V., Garlappi, L. and Uppal, R. (2009). *Optimal versus naïve diversification: How inefficient is the 1/N portfolio?* *The Review of Financial Studies*, 22,5
- Denoiseux, V., Debru, P., Warlyani, B. and Dinni, V. (2014). *Passive Insights: Equal Weighted Portfolios of ETFs*. [online] Available at: <https://etf.deutscheawm.com/GBR/ENG/Download/Passive%20Insights/287a57b1-5c15-4b37-b0a8-9be900871e7f/Passive-Insights-August-2014-Equal-Weighted-Portfolios-of-ETFs.pdf> [Accessed 28 Oct. 2015].
- Furbrot, A. (n.d.). *Avoiding the stock exchange*. [online] Bi.edu. Available at: <http://www.bi.edu/research/News/News-2011/Avoiding-the-stock-exchange/> [Accessed 26 Oct. 2015].
- Grytten, O. and Hunnes, A. (n.d.). A Chronology of Financial Crises for Norway. *SSRN Electronic Journal*.
- Hatch, E. & Farhady, H. (1981), *Research design and statistics for applied linguistics*, London: Newbury House Publishers, Inc.
- He, S. and Gu, S. (2015). *Market Anomalies in the Norwegian Stock Market: An exploration about existence of 4 Market Anomalies*. Graduate. Norges handelshøyskole.
- Hsu, J. and Li, F. (2013). Low-Volatility Investing. *Index Investing*, 4(2), pp.67-72.
- IMF.org, (2015). [Online] Available at: <https://www.imf.org/External/Pubs/FT/GFSR/2015/01/index.html> [Accessed 20 Nov. 2015].
- Interaction.org, (n.d.). *Annex 1: Strengths and weaknesses of quantitative evaluation designs InterAction*. [Online] Available at: <http://www.interaction.org/annex-1->

strengths-and-weaknesses-quant-evaluation-approaches [Accessed 18 Nov. 2015]

- Jorion, P. (1986). Bayes-Stein Estimation for Portfolio Analysis. *The Journal of Financial and Quantitative Analysis*, 21(3), p.279.
- Kose, E. and Moroz, M. (2014). *The High Cost of Equal Weighting*. 1st ed. [e-book] Newport Beach, CA: Research Affiliates, pp.1-4. Available at: https://www.researchaffiliates.com/Production%20content%20library/The_High_Cost_of_Equal_Weighting_pdf.pdf [Accessed 28 Oct. 2015].
- Kuo, L. and Li, F. (2013). An Investor's Low Volatility Strategy. *Institutional Investor Journal*, 3(4), pp.1-9 12-14.
- Landsberg, R. (2013). *Sharpe Ratio and Risk Adjusted Measurements*. [Online] <http://retirementresource.nationwide.com>. Available at: <http://retirementresource.nationwide.com/sites/default/files/Sharpe%20Ratio%20and%20Risk%20Mment.pdf> [Accessed 10 Nov. 2015].
- Lummer, S. and Riepe, M. (1994). The Role of Assets in Portfolio Management. [online] Available at: <https://corporate.morningstar.com/ib/documents/MethodologyDocuments/IBBAssociates/RoleAssetAllocation.pdf> [Accessed 15 Nov. 2015].
- Markowitz, H. (2015). Portfolio Selection. *The Journal of Finance*, 7(1), pp.77-91.
- Marte, J. (2012). Weighing Risk vs. Reward. *The Wall Street Journal*, [online] p.Single Page. Available at: <http://www.wsj.com/articles/SB10001424052702304692804577283773166995992> [Accessed 10 Nov. 2015].
- Masson, J. (2014). *Low Volatility Equity Strategies: New or Improved?*. [online] Available at: http://conferences.pionline.com/uploads/conference_admin/TD_LowVolPLusArticle1

.pdf [Accessed 6 Nov. 2015].

- Maxey, D. (2013). *Low-Volatility Strategies More Popular*. [online] WSJ. Available at:
<http://www.wsj.com/articles/SB10001424052702303722604579113244168242538>
[Accessed 20 Nov. 2015].
- Ødegaard, B. (2011). *Empirics of the Oslo Stock Exchange. Basic, descriptive, results*. Graduate. University of Stavanger.
- Oxfordstrat.com, (2012). *Sharpe Ratio: Four Problems*. *Oxford Capital Strategies Ltd. R&D*. [online] Available at: <http://www.oxfordstrat.com/ideas/sharpe-ratio/>
[Accessed 8 Nov. 2015].
- Pav, S. (2015). *Notes on the Sharpe ratio*. [online] <https://cran.r-project.org>.
Available at: <https://cran.r-project.org/web/packages/SharpeR/vignettes/SharpeRatio.pdf> [Accessed 22 Oct. 2015].
- Payne, J., Bettman, J. and Johnson, E. (1992). Behavioral decision research: A constructive processing perspective. *Annual Reviews*, 43, pp.87 - 131.
- Plyakha, Y., Uppal, R. and Vilkov, G. (2014). Why Does an Equal-Weighted Portfolio Outperform Value- and Price-Weighted Portfolios?. *SSRN Electronic Journal*, pp.1-6, 12-15.
- Ramos, Ph.D., E. and Hans CFA, J. (2013). *Finding Opportunities Through The Low Volatility Anomaly*. [Online] Available at:
<https://www.bmo.com/gam/pdf/BMOGAM-Low-VolatilityPaper.pdf> [Accessed 3 Oct. 2015].
- Researchaffiliates.com, (n.d.). *Smart Beta*. [Online] Available at:
<http://www.researchaffiliates.com/Our%20Ideas/Insights/Smart%20Beta/Pages/Home>

.aspx [Accessed 27 Oct. 2015].

- Shores, S. (2015). *SMART BETA: DEFINING THE OPPORTUNITY AND SOLUTIONS*. [Online] <https://www.blackrock.com>. Available at: <https://www.blackrock.com/institutions/en-us/literature/whitepaper/smart-beta-defining-the-opportunity-and-solutions.pdf> [Accessed 17 Nov. 2015].
- Soe, CFA, A. (2012). *The Low-Volatility Effect: A Comprehensive Look*. [Online] <http://www.spindices.com>. Available at: <http://www.spindices.com/documents/research/low-volatility-effect-comprehensive-look-201208.pdf> [Accessed 9 Nov. 2015].
- Sriram, K. (2011). Is your investment portfolio delivering good returns?. *The Economic Times*, [online] p.Single Page. Available at: http://articles.economictimes.indiatimes.com/2011-10-16/news/30286252_1_portfolio-treynor-ratio-returns [Accessed 19 Nov. 2015].
- Sullivan, R. (n.d.). *Smart Beta Definition from Financial Times Lexicon*. [Online] [Lexicon.ft.com](http://lexicon.ft.com). Available at: <http://lexicon.ft.com/Term?term=smart-beta> [Accessed 12 Nov. 2015].
- Swenlin, C. and Heim, E. (2015). *Equal-Weight Versus Cap-Weighted Indexes*. [Online] Carl Swenlin | StockCharts.com. Available at: <http://stockcharts.com/articles/decisionpoint/2015/03/equal-weight-versus-cap-weighted-indexes.html> [Accessed 18 Nov. 2015].
- Tindall, P. (2014). *Opportunities of using smart beta in asset allocation*. Understanding The Implementation Issues. [Online] London: Clear Path Analysis, pp. 25-27. Available at: <https://www.clearpathanalysis.com/product/108-sbina-2014/> [Accessed 10 Oct. 2015].
- Towers Watson, (2013). *Understanding smart beta*. [Online] Available at: <https://www.towerswatson.com/en/Insights/IC-Types/Ad-hoc-Point-of->

View/2013/08/Understanding-smart-beta [Accessed 1 Nov. 2015].

- Tradingeconomics.com, (2015). *Norway Stock Market (OBX), 1983-2015, Data, Chart, Calendar, Forecast*. [Online] Available at:
<http://www.tradingeconomics.com/norway/stock-market> [Accessed 27 Oct. 2015].
- Weinreich, G. (2014). *Which Is the Smartest Smart Beta: Equal or Fundamental Weighting?*. [Online] Thinkadvisor.com. Available at:
<http://www.thinkadvisor.com/2014/05/30/which-is-the-smartest-smart-beta-equal-or-fundamen?page=2> [Accessed 1 Nov. 2015].
- Windcliff, H. and Boyle, P. (2004). The 1/ n Pension Investment Puzzle. *North American Actuarial Journal*, 8(3), pp.32-45.
- Xi, T. (n.d.). *Advantages & Disadvantages of Using Sharpe Ratio*. [Online] People - Opposing Views. Available at: <http://people.opposingviews.com/advantages-disadvantages-using-sharpe-ratio-5979.html> [Accessed 22 Oct. 2015].
- Zikmund, William G. (2003), *Business Research Methods, 7th Edition*, Ohio: Thomson South Western.

Appendix – A

Table A(1): Monthly Stock Standard Deviation of High and Low-Weight Companies (2005 – 2015)

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2005		
February	0.052717192	0.130122996
March	0.106177991	0.076506414
April	0.064725899	0.062063622
May	0.09731966	0.059188673
June	0.186611977	0.078350694
July	0.036200787	0.076968262
August	0.117657974	0.185208798
September	0.151291176	0.092057753
October	0.075460204	0.099125412
November	0.170423902	0.039451742
December	0.228848045	0.216110117

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2006		
January	0.093872426	0.074232368
February	0.124945708	0.076035659
March	0.101718219	0.06029735
April	0.098473892	0.115886225
May	0.406671807	0.211487229
June	0.13654441	0.074526175
July	0.071588023	0.054927943
August	0.062077077	0.063931077
September	0.108498328	0.07599943
October	0.053324747	0.049794712
November	0.10968809	0.058231757
December	0.045576668	0.036276504

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2007		
January	0.073781755	0.101123137
February	0.068216855	0.126286367
March	0.049166219	0.061730252
April	0.318486259	0.06499775
May	0.042065037	0.06205641
June	0.071835276	0.061381486
July	0.048945242	0.039632895
August	0.073849535	0.068515346
September	0.079423777	0.084827256
October	0.064973845	0.044389774
November	0.100443964	0.113532231
December	0.087610596	0.619833179

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2008		
January	0.245620171	0.258443602
February	0.070829269	0.063475166
March	0.035477723	0.20993275
April	0.084239548	0.057912981
May	0.091593225	0.093184464
June	0.059655761	0.100131464
July	0.083552959	0.108569022
August	0.117251043	0.071340257
September	0.076269263	0.24410269
October	0.144454267	0.350783193
November	0.390515761	0.500537316
December	0.218194607	0.219428826

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2009		
January	0.222097298	0.242674026
February	0.273924093	0.114490591
March	0.155934485	0.101851065
April	0.165835215	0.146886822
May	0.272582683	0.202328541
June	0.248030403	0.214773285
July	0.344789611	0.091151518
August	0.271863221	0.144331834
September	0.133855257	0.125489554
October	14.95371379	0.089593154
November	0.21288527	0.149840332
December	0.133295236	0.064713367

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2010		
January	0.080649254	0.081622406
February	0.067738545	0.065788589
March	0.163924649	0.21693601
April	0.144309118	0.153573282
May	0.053615006	0.200638669
June	0.038402791	0.095299518
July	0.072287006	0.046064156
August	0.084337498	0.070678767
September	0.061750627	0.124988317
October	0.05404782	0.071706317
November	0.063177547	0.044764518
December	0.017125613	0.081475017

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2011		
January	0.122985324	0.07242223
February	0.170951598	0.105785403
March	0.207312896	0.079111807
April	0.130287396	0.045557594
May	0.080080702	0.0487235
June	0.094616047	0.103062229
July	0.147909418	0.02874144
August	0.083950441	0.128193949
September	0.115720965	0.091878227
October	0.12184197	0.092848981
November	0.114997269	0.069779084
December	0.151834793	0.051067726

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2012		
January	0.236897771	0.153380686
February	0.071693677	0.150856933
March	0.123530983	0.078751301
April	0.08304962	0.114645304
May	0.090587649	0.122002613
June	0.245357904	0.129551921
July	0.141180397	0.163304421
August	0.371999523	0.0966018
September	0.266265744	0.109457753
October	0.111771787	0.128418922
November	0.116297795	0.402889333
December	0.180115785	0.08653805

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2013		
January	0.172767914	0.311766594
February	0.113700906	0.284741745
March	0.131513026	0.107689582
April	0.194387543	0.028191059
May	0.154140227	0.26031959
June	0.259169474	0.102600059
July	0.523795816	0.134279777
August	0.042629286	0.092030983
September	0.104520716	0.316775792
October	0.090063435	0.102631427
November	0.184798816	0.112325141
December	0.11257212	0.033607872

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2014		
January	0.060922511	0.061398753
February	1.989206434	0.05874706
March	1.985825859	0.04875891
April	0.193720211	0.055156013
May	0.118375971	0.03903259
June	0.184821663	0.042053836
July	0.062633504	0.049312164
August	0.135316098	0.055558244
September	0.118908885	0.071736698
October	0.10288347	0.056337635
November	0.148106977	0.092372828
December	0.074397587	0.126388748

Row Labels	Monthly Standard Deviation of High-weighted Companies	Monthly Standard Deviation of Low-weighted Companies
2015		
January	0.065532668	0.079636227
February	0.117617969	0.102725005
March	0.08476181	0.084417469
April	2.148391591	0.038720996
May	0.349753293	0.089363925
June	0.184503644	0.124009337
July	0.052538304	0.069347115
August	0.11907307	0.049092101
September	0.113736672	0.054386036
October	0.085370682	0.062693382
November	0.165108288	0.075103464
December	0.135916624	0.125467284

Table A(2): Monthly Average Stock Returns of High and Low-Weight Companies (2005 – 2015)

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Returns of Low-weighted Companies
2005		
February	0.072713112	0.085777012
March	0.061029554	0.003670625
April	0.020590517	-0.031309702
May	0.005699401	-0.060194621
June	0.152157353	0.002144439
July	0.069410449	0.059441606
August	0.115026618	0.128719626
September	0.090652916	0.09508373
October	-0.025393126	-0.058706955
November	0.120469121	0.011482141
December	0.099199854	0.112756172

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2006		
January	0.015426342	0.053429486
February	0.085067583	-0.022585862
March	0.070523077	0.022865948
April	0.115383638	0.026562538
May	0.115114586	-0.105163871
June	0.018329882	-0.085685822
July	0.043590059	0.010534147
August	0.020523383	0.040776123
September	0.062292497	-0.008383759
October	0.043121996	0.005007031
November	0.106803415	0.021645077
December	0.078434135	0.001659829

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2007		
January	0.083153829	0.046168151
February	0.080141075	0.047319241
March	-0.006699882	-0.024939933
April	0.143466426	0.059772626
May	0.020919703	0.016450477
June	0.034439498	0.026420688
July	0.01592912	0.017807669
August	-0.03564268	-0.076891595
September	0.010317357	-0.054164188
October	0.025574554	-0.003968338
November	-0.046748427	-0.038940884
December	-0.015309034	-0.248523665

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2008		
January	0.080055089	-0.147814604
February	-0.049635051	-0.050428426
March	-0.022502141	-0.092131874
April	0.03625123	0.007648481
May	0.079513111	0.075209398
June	0.024982148	-0.006106438
July	-0.020911278	-0.045098724
August	0.019195722	-0.033204395
September	-0.06370432	-0.122050447
October	-0.173391648	-0.195392383
November	-0.042166508	-0.276511873
December	-0.266834688	-0.16170845

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2009		
January	-0.060529746	-0.092891621
February	-0.074569077	-0.09362697
March	-0.100689962	-0.084564228
April	0.129574856	0.048158002
May	0.192697672	0.060813906
June	0.153536636	0.058532118
July	-0.023801633	-0.027764891
August	0.061260952	0.040803395
September	0.094895135	0.038748417
October	4.842130073	0.00944181
November	0.043688593	0.051651494
December	-0.017752332	-0.007570127

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2010		
January	0.028001401	0.006408508
February	0.026172282	0.001714274
March	0.120145188	-0.002342631
April	0.035946627	0.009673233
May	-0.025195966	-0.112392745
June	-0.011405358	-0.055102175
July	-0.030811961	-0.05542551
August	0.041906545	0.011433319
September	0.055510249	-0.034628116
October	0.016171993	0.026596787
November	0.102333759	0.014316086
December	0.001205458	0.012370317

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2011		
January	0.09334184	0.044380362
February	0.140332755	-0.004277831
March	0.052561707	-0.032572937
April	0.068443839	0.02126764
May	-0.011374777	-0.069374766
June	-0.024165338	-0.085955411
July	0.022689041	-0.009551991
August	-0.073024891	-0.170001779
September	-0.064667456	-0.069094549
October	-0.075697553	-0.054751823
November	0.057096636	-0.04376399
December	-0.021992544	-0.03800566

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2012		
January	0.15103944	0.091058221
February	0.086235501	0.00923089
March	0.066663186	0.002357659
April	-0.045107001	-0.006100112
May	-0.033434633	-0.022165222
June	0.071166694	-0.05163626
July	0.004958134	0.012427839
August	0.156216326	0.040495507
September	0.149154308	-0.01766578
October	-0.009438928	-0.046982117
November	-0.059313149	0.064256639
December	0.022695202	0.018616028

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2013		
January	0.105090495	-0.087931907
February	0.092996743	-0.149332886
March	0.111608938	-0.013287427
April	0.081562731	-0.018454084
May	0.054099348	-0.13180979
June	0.073460392	-0.045249106
July	0.205843278	0.067212877
August	0.039152854	-9.64839E-05
September	0.032308663	-0.054816614
October	0.021603612	-0.031903043
November	-0.051269142	0.013247769
December	0.03574189	-0.047056978

Row Labels	Monthly Average Return of High-weighted Companies	Monthly Average Return of Low-weighted Companies
2014		
January	0.020858272	0.012474429
February	0.658200306	-0.039666504
March	0.767870027	-0.04129905
April	0.089322128	-0.016608761
May	0.023577068	0.00506465
June	0.150715194	0.001855994
July	0.033184058	-0.012326562
August	0.092212286	-0.015679207
September	0.035965699	-0.103215315
October	0.010268173	-0.117359757
November	0.019215196	-0.061711775
December	-0.003910012	-0.174618897

Row Labels	Monthly Return of High-weighted Companies	Monthly Return of Low-weighted Companies
2015		
January	0.03821876	-0.067667621
February	0.071156236	-0.081948977
March	0.029962101	-0.126673398
April	0.736715404	0.0020746
May	0.118832693	-0.051992085
June	0.146677061	-0.140547388
July	0.047503999	-0.064507591
August	0.113169479	-0.071199713
September	0.089382273	-0.070041437
October	0.003918413	-0.055517809
November	0.133817986	-0.129551021
December	0.051365487	-0.160495638

Appendix B

F-test and t-test Two Sample for Equal and Unequal Variances (2005 – 2015) 2005

F-Test Two-Sample for Variances

	<i>Monthly Returns 2005 (High-Weighted Companies)</i>	<i>Monthly Returns 2005 (Low-Weighted Companies)</i>
Mean	0.071050524	0.031714916
Variance	0.01809877	0.016065144
Observations	110	110
df	109	109
F	1.126586246	
P(F<=f) one-tail	0.267434007	
F Critical one-tail	1.372282589	

t-Test: Two-Sample Assuming Equal Variances

	<i>Monthly Returns 2005 (High-Weighted Companies)</i>	<i>Monthly Returns 2005 (Low-Weighted Companies)</i>
Mean	0.071050524	0.031714916
Variance	0.01809877	0.016065144
Observations	110	110
Pooled Variance	0.017081957	
Hypothesized Mean Difference	0	
df	218	
t Stat	2.232021656	
P(T<=t) one-tail	0.013315436	
t Critical one-tail	1.651873373	
P(T<=t) two-tail	0.026630872	
t Critical two-tail	1.970905601	

2006

F-Test Two-Sample for Variances

	<i>Monthly Returns 2006 (High-Weighted Companies)</i>	<i>Monthly Returns 2006 (Low-Weighted Companies)</i>
Mean	0.064550883	-0.003278261
Variance	0.021390201	0.009583231
Observations	120	120
df	119	119
F	2.23204482	
P(F<=f) one-tail	8.02824E-06	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2006 (High-Weighted Companies)</i>	<i>Monthly Returns 2006 (Low-Weighted Companies)</i>
Mean	0.064550883	-0.003278261
Variance	0.021390201	0.009583231
Observations	120	120
Hypothesized Mean Difference	0	
df	208	
t Stat	4.221942356	
P(T<=t) one-tail	1.80965E-05	
t Critical one-tail	1.652212376	
P(T<=t) two-tail	3.6193E-05	
t Critical two-tail	1.971434659	

2007

F-Test Two-Sample for Variances

	<i>Monthly Returns 2007 (High-Weighted Companies)</i>	<i>Monthly Returns 2007 (Low-Weighted Companies)</i>
Mean	0.025795128	-0.019457479
Variance	0.014693336	0.040870435
Observations	120	120
df	119	119
F	0.359510134	
P(F<=f) one-tail	2.46702E-08	
F Critical one-tail	0.738765114	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2007 (High-Weighted Companies)</i>	<i>Monthly Returns 2007 (Low-Weighted Companies)</i>
Mean	0.025795128	-0.019457479
Variance	0.014693336	0.040870435
Observations	120	120
Hypothesized Mean Difference	0	
df	195	
t Stat	2.102995657	
P(T<=t) one-tail	0.0183745	
t Critical one-tail	1.65270531	
P(T<=t) two-tail	0.036748999	
t Critical two-tail	1.972204051	

2008

F-Test Two-Sample for Variances

	<i>Monthly Returns 2008 (High-Weighted Companies)</i>	<i>Monthly Returns 2008 (Low-Weighted Companies)</i>
Mean	-0.033262361	-0.087299145
Variance	0.03463247	0.056944418
Observations	120	120
df	119	119
F	0.608180242	
P(F<=f) one-tail	0.003541177	
F Critical one-tail	0.738765114	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2008 (High-Weighted Companies)</i>	<i>Monthly Returns 2008 (Low-Weighted Companies)</i>
Mean	-0.033262361	-0.087299145
Variance	0.03463247	0.056944418
Observations	120	120
Hypothesized Mean Difference	0	
df	225	
t Stat	1.956082518	
P(T<=t) one-tail	0.025846466	
t Critical one-tail	1.651654074	
P(T<=t) two-tail	0.051692932	
t Critical two-tail	1.97056339	

2009

F-Test Two-Sample for Variances

	<i>Monthly Returns 2009 (High-Weighted Companies)</i>	<i>Monthly Returns 2009 (Low-Weighted Companies)</i>
Mean	0.43670343	0.000144275
Variance	18.74365767	0.023886676
Observations	120	120
df	119	119
F	784.6909262	
P(F<=f) one-tail	1.2176E-138	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2009 (High-Weighted Companies)</i>	<i>Monthly Returns 2009 (Low-Weighted Companies)</i>
Mean	0.43670343	0.000144275
Variance	18.74365767	0.023886676
Observations	120	120
Hypothesized Mean Difference	0	
df	119	
t Stat	1.103900672	
P(T<=t) one-tail	0.135931718	
t Critical one-tail	1.657759285	
P(T<=t) two-tail	0.271863436	
t Critical two-tail	1.980099876	

2010

F-Test Two-Sample for Variances

	<i>Monthly Returns 2010 (High-Weighted Companies)</i>	<i>Monthly Returns 2010 (Low-Weighted Companies)</i>
Mean	0.029945645	-0.015251741
Variance	0.008530371	0.014114702
Observations	120	120
df	119	119
F	0.604360709	
P(F<=f) one-tail	0.00319842	
F Critical one-tail	0.738765114	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2010 (High-Weighted Companies)</i>	<i>Monthly Returns 2010 (Low-Weighted Companies)</i>
Mean	0.029945645	-0.015251741
Variance	0.008530371	0.014114702
Observations	120	120
Hypothesized Mean Difference	0	
df	224	
t Stat	3.290160459	
P(T<=t) one-tail	0.000581494	
t Critical one-tail	1.65168456	
P(T<=t) two-tail	0.001162988	
t Critical two-tail	1.970610961	

2011

F-Test Two-Sample for Variances

	<i>Monthly Returns 2011 (High-Weighted Companies)</i>	<i>Monthly Returns 2011 (Low-Weighted Companies)</i>
Mean	0.013628605	-0.042641895
Variance	0.020641875	0.008875141
Observations	120	120
df	119	119
F	2.325808063	
P(F<=f) one-tail	2.96688E-06	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2011 (High-Weighted Companies)</i>	<i>Monthly Returns 2011 (Low-Weighted Companies)</i>
Mean	0.013628605	-0.042641895
Variance	0.020641875	0.008875141
Observations	120	120
Hypothesized Mean Difference	0	
df	205	
t Stat	3.587857372	
P(T<=t) one-tail	0.000208498	
t Critical one-tail	1.652320556	
P(T<=t) two-tail	0.000416995	
t Critical two-tail	1.971603499	

2012

F-Test Two-Sample for Variances

	<i>Monthly Returns 2012 (High-Weighted Companies)</i>	<i>Monthly Returns 2012 (Low-Weighted Companies)</i>
Mean	0.046736257	0.00809306
Variance	0.039588581	0.026683798
Observations	120	120
df	119	119
F	1.48361867	
P(F<=f) one-tail	0.016167072	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2012 (High-Weighted Companies)</i>	<i>Monthly Returns 2012 (Low-Weighted Companies)</i>
Mean	0.046736257	0.00809306
Variance	0.039588581	0.026683798
Observations	120	120
Hypothesized Mean Difference	0	
df	229	
t Stat	1.644361826	
P(T<=t) one-tail	0.050736771	
t Critical one-tail	1.651534805	
P(T<=t) two-tail	0.101473541	
t Critical two-tail	1.970377283	

2013

F-Test Two-Sample for Variances

	<i>Monthly Returns 2013 (High-Weighted Companies)</i>	<i>Monthly Returns 2013 (Low-Weighted Companies)</i>
Mean	0.066849983	-0.04162314
Variance	0.043744645	0.035142067
Observations	120	120
df	119	119
F	1.244794334	
P(F<=f) one-tail	0.116895348	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2013 (High-Weighted Companies)</i>	<i>Monthly Returns 2013 (Low-Weighted Companies)</i>
Mean	0.066849983	-0.04162314
Variance	0.043744645	0.035142067
Observations	120	120
Hypothesized Mean Difference	0	
df	235	
t Stat	4.230686444	
P(T<=t) one-tail	1.67037E-05	
t Critical one-tail	1.651363544	
P(T<=t) two-tail	3.34073E-05	
t Critical two-tail	1.970110062	

2014

F-Test Two-Sample for Variances

	<i>Monthly Returns 2014 (High-Weighted Companies)</i>	<i>Monthly Returns 2014 (Low-Weighted Companies)</i>
Mean	0.158123199	-0.046924229
Variance	0.674264393	0.007167382
Observations	120	120
df	119	119
F	94.07401074	
P(F<=f) one-tail	2.66661E-84	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2014 (High-Weighted Companies)</i>	<i>Monthly Returns 2014 (Low-Weighted Companies)</i>
Mean	0.158123199	-0.046924229
Variance	0.674264393	0.007167382
Observations	120	120
Hypothesized Mean Difference	0	
df	122	
t Stat	2.721032699	
P(T<=t) one-tail	0.003729875	
t Critical one-tail	1.657439499	
P(T<=t) two-tail	0.007459749	
t Critical two-tail	1.979599878	

2015

F-Test Two-Sample for Variances

	<i>Monthly Returns 2015 (High-Weighted Companies)</i>	<i>Monthly Returns 2015 (Low-Weighted Companies)</i>
Mean	0.131726658	-0.085827723
Variance	0.404482912	0.008465931
Observations	120	120
df	119	119
F	47.77772481	
P(F<=f) one-tail	2.61948E-67	
F Critical one-tail	1.353610209	

t-Test: Two-Sample Assuming Unequal Variances

	<i>Monthly Returns 2015 (High-Weighted Companies)</i>	<i>Monthly Returns 2015 (Low-Weighted Companies)</i>
Mean	0.133108406	-0.085827723
Variance	0.407679684	0.008465931
Observations	119	120
Hypothesized Mean Difference	0	
df	123	
t Stat	3.702585286	
P(T<=t) one-tail	0.000160363	
t Critical one-tail	1.657336397	
P(T<=t) two-tail	0.000320726	
t Critical two-tail	1.979438685	