

# Competition in Complementary Transport Services

Derek John Clark<sup>a</sup>, Finn Jørgensen<sup>b</sup> and Terje Andreas Mathisen<sup>b\*</sup>

<sup>a</sup> Tromsø University Business School, University of Tromsø, NO-9037 Tromsø, Norway

<sup>b</sup> Bodø Graduate School of Business, University of Nordland, NO-8049 Bodø, Norway

\* Corresponding author: Email: [tam@uin.no](mailto:tam@uin.no), Tel.: +47 75 51 76 37, Fax.: +47 75 51 72 35

## Abstract

For passengers to reach the final destination of the trip it is often necessary to make use of the transport services provided by several firms. When these transport services follow in a natural transport chain they are characterized as complementarities and the firms providing the services can, as for substitutes, to some extent influence the demand facing the other firms by their own behaviour. A model is presented in this paper where two firms compete in complementary transport services differentiated by travel distance. Equilibria are derived for collusion and competition in price and quantity, and these are analyzed with respect to the degree of complementarity and distance. The analysis shows that the influence of type of competition on equilibrium price and quantity increases with the complementarity of the products. Moreover, it is discussed how marginal operating costs for the firms, marginal time cost for the passenger and the type of competition influences whether fares will increase with distance and which of the two firms will set the higher price. The commonly accepted ranking for complements that the collusive price is lower than the Bertrand price is not necessarily true. It is demonstrated that the collusive price of the shorter (longer) distance could be set above (lower) that of price competition. It is also addressed how mark-up of price over marginal cost is influenced by changes in own and competitors distance for the different types of competition.

## Keywords

Collusion, complementary competition, market equilibrium, passenger transport, equilibrium price

## 1. Introduction

Even though the liberalization of transport markets in industrialized countries has increased competition, many passenger transport routes are still served by one or two suppliers (e.g. Blauwens et al., 2008). Hence, the classic duopoly models discussed in the literature dating back to Cournot (1927) are still relevant to explain equilibria in passenger transport markets. The role of substitutability and complementarity with respect to equilibria has been addressed in economic literature (e.g. Economides and Salop, 1992; Häckner, 2000; Singh and Vives, 1984), but has rarely been related to the transport industry. A central characteristic of this industry is the role of transport distance which has been included in the literature on optimal design of scheduled passenger transport (e.g. Kraus, 1991; Mohring, 1972). Transport distance has also been addressed by Li et al. (2012) when discussing the optimal fares for a rail line. Founded on the model by Singh and Vives (1984), Clark et al. (2011) considered simultaneous and sequential duopoly competition and addressed specifically how transport distance influences the equilibrium prices when all firms maximize profit and compete on services between the same locations. However, these latter works focus purely on competition between alternative transport solutions, which thereby is regarded as substitutable services, over the same distance. Complementary services<sup>1</sup> are, however, often found in the passenger transport industry, and there is, therefore, a need to address them in the theoretical models.

The fact that cooperation and competition can be parts of one and the same was addressed by Nalebuff and Brandenburger (1996) using the concept co-opetition to describe such a relationship. Some studies on complementary services have used the computer industry as case. Packalen (2010) modelled a monopolist (Microsoft) acting in complementary markets and compared the equilibria to that of quantity competition (Cournot). Also, Casadesus-Masanell et al. (2008) studied competing complements using the relationship between the software provider Microsoft and CPU producers AMD and Intel as a case. As one of several industries, Economides and Salop (1992) addressed the transport industry when studying equilibrium prices under different forms of competition among complementary products. In fact, the situation where two firms both compete and cooperate is frequently found in the transport industry. An example is bus and train companies that compete on legs where both supply transport services, and complement each other by feeding passengers for further transport to the other provider.

---

<sup>1</sup> Complements are goods and services that are used together and defined by negative cross-price elasticity, where more negative value indicates a higher degree of complementarity (e.g. Hubbard and O'Brien, 2013).

In a congested transport corridor De Borger et al. (2007) studied pricing decisions when two links are controlled by different governments. Also using freight transport as a case, Rodrigue et al. (2009) argue that complementarity between transport modes can take place in different geographical markets, different transport markets and different levels of service. These criteria can clearly be transferred to passenger transport. The complementarity between air transport and high speed rail, and the possible advantages of integrating these two transport modes, has been addressed by Socorro and Viicens (2013). In fact, air transport is an obvious example of an industry where complementarity takes place both between different transport modes and within the same type of transport. First, passengers need to be transported between the city centre and the airport and vice versa. In this case, changes in price on the commuting services by train or bus can influence the demand for air transport. Second, due to the well established hub-and-spoke networks air transport does, however, also include complementarity between services with the same transport mode. When the route includes more than one flight, the trip from the airport of departure to the hub and further from the hub to the airport at the final destination can be provided by the same firm as well as by competing firms. Since there are relatively few suppliers of aircrafts, it can well be that legs served by different carriers are made by the same type of aircraft and thereby with somewhat similar costs. This is further actualised by the forming of alliances where a through ticket can include legs provided by more than one firm.

The most important aim of this paper is to analyse how fares relate to distance when two profit maximizing companies produce complementary transport services that are differentiated by distance. Taking the degree of competition addressed by Clark et al. (2009) and the role of trip length by Clark et al. (2011) the focus of this paper will be directed towards firms providing complementary services of different trip lengths. Hence, models applied in the earlier studies are changed and extended by assuming complementary services and introducing asymmetric distance for the services provided by the transport firms. Singh and Vives (1984) modelled duopoly under complements but did not solve for collusion or address quality differences which are relevant for the transport industry. Earlier studies have focused on differences in quality by factors such as frequency, capacity (e.g. De Borger and Van Dender, 2006) or congestion (e.g. Wan and Zhang, 2013; Wu et al., 2011). Moreover, it must be considered that, in the case of transport, competition often depends on strategic behaviour of governments rather than firms (see e.g. review by De Borger and Proost, 2012).

Equilibria are calculated under collusion and simultaneous and sequential competition on price or quantity. The model results demonstrate how fares for complementary transport services depend on distance under different regulatory policies and degrees of complementarity between the services

provided by the two firms. We present conditions under which fares are increasing and decreasing in distance, and for when the company with the largest travelled distance sets the highest price. Moreover, the influence of mark-up of price over marginal cost for changes in own and competitors distance is discussed for the different types of competition.

Section 2 presents the model and accounts for central assumptions. The equilibria under different forms of competition are derived in Section 3. Section 4 provides the analysis with focus on ranking of equilibrium fares and how they develop with respect to the degree of complementarity under different types of competition. Finally, conclusions and implications are presented in Section 5.

## 2. The model

Let us assume a transport route where passengers are required to make a mode change at place B in order to travel between A and C. The distances for the two legs are denoted  $D_1$  and  $D_2$  as illustrated in Figure 1. Two operators, denoted firm 1 and firm 2, provide transport services on the separate legs  $D_1$  and  $D_2$ , respectively. The two services are complements and total demand depends on their prices. The complementarity between the two legs of the service is, thereby, not perfect. The fact that demand could be transferred to other modes is not modelled here.

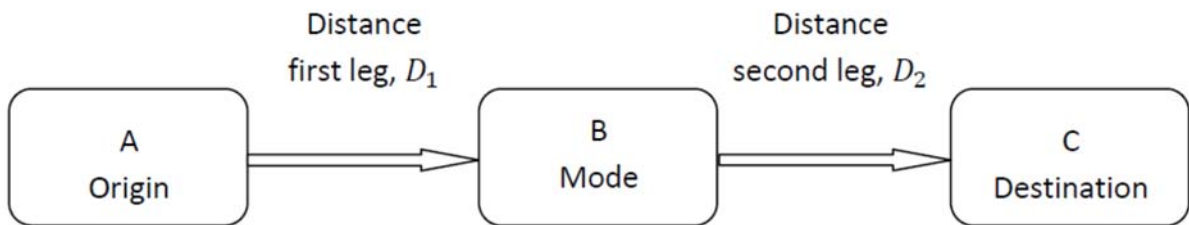


Figure 1: Complementary services on two legs.

The inverse demand functions in (1) follow the specifications by Clark et al. (2009) and Clark et al. (2011) based on how Singh and Vives (1984) incorporated in their model how passengers maximize their utility according to the quantities for the services provided by the two firms. In line with Singh and Vives (1984) we assume that a representative consumer maximizes consumer surplus based on the use of services  $X_i$  and  $X_j$  by  $CS = U(X_i, X_j) - G_i X_i - G_j X_j$  defined by the utility function

$U(X_i, X_j) = X_i + X_j - \frac{(X_i^2 + 2gX_iX_j + X_j^2)}{2}$ , with generalized costs  $G_i = P_i + k + bD_i$  (with the generalized cost of using service  $j$  defined similarly).

$$(1) \quad P_i = (1 - k - bD_i) - X_i - gX_j \text{ where } i, j = \{1,2\} \text{ and } i \neq j$$

In (1) the parameter  $D_i$  denotes distance in km for the services provided by firm  $i$ ,  $k$  is distance-independent time costs and  $(bD_i)$  is time costs when travelling by the mode. The  $k$  parameter depends on walking time, waiting time and time spent on boarding and alighting the mode (buses and trains) and/or transport time to airports (air transport), whilst the  $b$  parameter denotes each passenger's time costs of travelling an extra km by the mode and decreases when the mode's quality (for example speed) increases. The values of both  $k$  and  $b$  depend on the travellers' income and the trip purpose (e.g. Button, 2010) which are exogenously given in this model.

The parameter  $g \in (0, -1)$  in (1) measures the degree of complementarity between the services offered by the two firms. The services are perfect complements if  $g = -1$  and independent if  $g = 0$ . Hence, the higher proportion of the travellers starting and ending their travel at place B and the better services from other transport modes between A and B and B and C, the lower the value of  $g$ , in absolute terms. In Clark et al. (2011) this parameter was defined within the range of 0 and 1 when studying competition in substitutes. It has been demonstrated by Singh and Vives (1984) that the equilibrium for Cournot (Bertrand) when competing with substitutes is the dual of Bertrand (Cournot) when they compete in complements.

A rephrasing of (1) gives the direct demand for the service of firm  $i$  in (2).

$$(2) \quad X_i = \frac{1}{1-g^2} ((1-g)(1-k) - P_i + gP_j - bD_i + gbD_j) \text{ where } i, j = \{1,2\} \text{ and } i \neq j$$

In accordance with the model presented by Clark et al. (2011) the costs,  $(C_i)$ , in (3) are assumed to have a symmetric structure between the firms and linearly increase with the number of passengers  $(X_i)$  and the number of passenger km  $(X_iD_i)$  and with a fixed cost term  $f$ .

$$(3) \quad C_i(X_i) = f + hX_i + aX_iD_i, \text{ where } i = \{1,2\}$$

Such simple cost functions in transport are often good proxies of more advanced specifications (e.g. Pels and Rietveld, 2008), and the above functions in particular are supported from several empirical cost studies carried out for bus (Jørgensen and Preston, 2003) and ferry transport (Jørgensen et al., 2004) in Norway. For further discussions on properties of common functional forms for costs we refer to Baumol et al. (1988) and Coelli et al. (2005). The applicability of a symmetric cost structure can be argued by firms using the same type of transport mode and that there is no variation in efficiency between them.

From the cost function in (3) it follows that marginal costs,  $\partial C_i / \partial X_i$ , are

$$(4) \quad \frac{\partial C_i}{\partial X_i} = h + aD_i, \text{ where } i = \{1,2\}$$

In the linearly increasing relationship for marginal costs in (4) the parameter  $h$  represents distance-independent marginal costs, while  $a$  is costs inflicted on the transport firm when carrying a passenger an extra km.

The profit function of firm  $i$  is thus given by  $\pi_i = P_i X_i - C_i(X_i)$ .<sup>2</sup>

The reaction functions when choosing price (Bertrand) and quantity (Cournot) strategically can be commented on.<sup>3</sup> Under Bertrand the services are strategic substitutes ( $\partial P_i^B / \partial P_j^B < 0$ ) and increasing own distance gives a positive shift if  $a > b$ . Increasing opponent distance causes the reaction function to shift inwards. Under Cournot, services are strategic complements ( $\partial X_i^C / \partial X_j^C > 0$ ). An increase in own distance makes the reaction function shift inwards, while increasing opponent distance has no effect.

For the market to exist, it is required that the profit must be positive for carrying the passenger with the largest willingness to pay, meaning that  $1 - k - h - D_i(a + b) > 0$ . This expression is derived by taking the maximum willingness-to-pay minus the sum of fixed cost to the passenger of making the trip, the cost of production for the first passenger and the distance-related costs.

### 3. Market Equilibria

The equilibrium fare expressions will be derived to study how optimal fares are linked to operators' marginal costs ( $h$  and  $a$ ), the quality of supplied transport services ( $k$  and  $b$ ), the degree of complementarity between the services ( $g$ ) and, finally, the transport distance of services provided by the mode ( $D_i$ ). Focus will be directed towards the collusion (shared monopoly) case and traditional forms of simultaneous market competition where the firms compete in either quantity (Cournot) or price (Bertrand). Equilibrium prices, quantities and profits for collusion, Cournot and Bertrand are

---

<sup>2</sup> The use of pure profit maximization as goal is a question for debate. Transport firms are in many studies argued to put some weight on other goal such as consumer surplus, mainly due to the large proportion of shares owned by public bodies (see e.g. Clark et al., 2009; Jørgensen and Preston, 2007).

<sup>3</sup> The reaction functions for firm  $i$  are: Bertrand  $P_i^B = \frac{1}{2}((1 - g)(1 - k) + h + gP_j + D_i(a - b) + gD_j b)$  and Cournot  $X_i^C = \frac{1}{2}(1 - h - k - gX_j - D_i(a + b))$  where  $i, j = \{1,2\}$  and  $i \neq j$ .

denoted by superscripts *COLL*, *C* and *B*, respectively.<sup>4</sup> Sequential equilibria are examined in the subsequent analysis.

### 3.1 Simultaneous competition on price (Bertrand)

When the two transport firms maximize their profits in price competition, the prices, quantities and profits in equilibrium are

$$(5) \quad P_i^B = \frac{1}{4-g^2} ((2+g)(h+(1-g)(1-k)) + D_i(2a-b(2-g^2)) + gD_j(a+b))$$

$$X_i^B = \frac{((g+2)(1-g))(1-h-k) + (gD_j - (2-g^2)D_i)(a+b)}{(1-g)(g+1)(2-g)(g+2)}$$

$$X_1^B + X_2^B = \frac{2(1-h-k) - (D_1 + D_2)(a+b)}{(g+1)(2-g)}$$

$$\pi_i^B = (1+g)(1-g)(X_i^B)^2 - f$$

### 3.2 Simultaneous competition on quantity (Cournot)

When the two transport firms maximize profits in quantity competition, the prices, quantities and profits in equilibrium are

$$(6) \quad P_i^C = \frac{1}{(2+g)(2-g)} ((2-g)(h(1+g) + 1 - k) - D_i(2b - a(2-g^2)) + gD_j(a+b))$$

$$X_i^C = \frac{1}{(2+g)(2-g)} ((2-g)(1-h-k) + (a+b)(gD_j - 2D_i))$$

$$X_1^C + X_2^C = \frac{2(1-h-k) - (D_1 + D_2)(a+b)}{(2+g)}$$

$$\pi_i^C = (X_i^C)^2 - f$$

---

<sup>4</sup> For these equilibria to be valid, all price, quantity and profit expressions must be positive. This is assumed in the further analysis.

### 3.3. Collusion (shared monopoly)

When the two transport firms maximize total profit the equilibrium the prices, quantities and profits are

$$(7) \quad P_i^{COLL} = \frac{1}{2}(1 + h - k + D_i(a - b))$$

$$X_i^{COLL} = \frac{1}{2(1+g)(1-g)}((1 - g)(1 - h - k) - (D_i - gD_j)(a + b))$$

$$X_1^{COLL} + X_2^{COLL} = \frac{1}{2} \frac{2(1-h-k) - (D_1+D_2)(a+b)}{(1+g)}$$

$$\pi_1^{COLL} + \pi_2^{COLL} = (X_1^{COLL})^2 + (X_2^{COLL})^2 + 4gX_1^{COLL}X_2^{COLL} - 2f$$

### 3.4 Some brief comments on the equilibria

It can be seen from the quantities presented in equations (5), (6) and (7) that the firm with longest distance will have the lowest number of passengers and that the difference between the two firms is highest when they compete in quantities (Cournot) and least when they collude (see appendix A). Moreover, when looking at total quantity it is evident that the expressions have the same numerator and can easily be ranked to  $(X_i^{COLL} + X_j^{COLL}) > (X_i^B + X_j^B) > (X_i^C + X_j^C)$  by looking at the denominators. Hence, total quantity provided from the two firms will be highest when they collude and lowest when they compete in quantities which is in line with the results from earlier studies (e.g. Economides and Salop, 1992; Singh and Vives, 1984).

It can be determined that all equilibrium fares are increasing in the distance independent marginal costs ( $h$ ) and decreasing when the passengers time costs ( $k$  and  $b$ ) increase (see appendix B). The latter suggests that quality improvements of transport supply leading to reduced values of  $k$  and  $b$  will increase prices. Higher costs ( $h$ ) due to increased quality provision will strengthen this effect. It is also worth noting that exogenous decrease in passengers' time costs due to income reductions and/or higher proportion of leisure travels also will increase prices.

The distance dependent marginal costs ( $a$ ) influence fares in collusion positively, but gives ambiguous results for Cournot and Bertrand depending on the distances for the services provided by the two firms,  $D_1$  and  $D_2$ . It is, however, clear from the partial differentiations that it is more likely that optimal prices increase with  $a$  under Bertrand than under Cournot competition, and that price is positively related to own distance and negatively related to the rival's distance. Under Bertrand a



sufficient but not necessary condition that  $\partial P_i^B / \partial a > 0$  is  $D_i > 0.5D_j$ ; that is when firm  $i$  offers a transport service with distance more than half the length of firm  $j$ . The similar condition under Cournot for  $\partial P_i^C / \partial a > 0$  is  $D_i > D_j$  meaning that firm  $i$  provides a longer journey than firm  $j$ . The equilibrium prices are unaffected by the level of complementarity, ( $g$ ), between the services when the firms collude. The influence of  $g$  on equilibrium prices is ambiguous under Cournot and Bertrand competition. Hence, it is uncertain whether prices go up or down when the firms' services become closer complements.<sup>5</sup>

#### 4. Further analyses of equilibrium prices

##### 4.1 Fare and distance

The equilibria in Section 3 will be analyzed further with special attention given to the influence of trip length on the two sequential legs on optimal fares for the two firms. These relationships are studied by differentiation of equilibrium prices with respect to distance for the services provided by the firm itself and the competing firm.

It is evident from (8) that price in collusion increases only with the own distance of the services provided by a firm. Moreover, prices in collusion increase in distance if the marginal cost of transporting a passenger one extra kilometre for the firm, ( $a$ ), is larger than time costs for the passenger of travelling an extra kilometre, ( $b$ ), by the mode,  $a/b > 1$ . Furthermore, (8) provides the derivatives of equilibrium prices in the two competitive situations with respect to the distance provided by the company itself (indicated by  $i$ ).

$$(8) \quad \frac{\partial P_i^{COLL}}{\partial D_i} = a - b > 0 \text{ if } \frac{a}{b} > 1, \frac{\partial P_i^B}{\partial D_i} = \frac{2a-b(2-g^2)}{(2-g)(g+2)} > 0 \text{ if } \frac{a}{b} > \frac{(2-g^2)}{2} \text{ and}$$

$$\frac{\partial P_i^C}{\partial D_i} = \frac{(2-g^2)a-2b}{(2-g)(g+2)} > 0 \text{ if } \frac{a}{b} > \frac{2}{2-g^2} \text{ where } i = \{1,2\}$$

The conditions under which prices increase with own distance are illustrated in Figure 2 and show only partial overlap between the competitive regimes. It is evident that in Bertrand competition the relationship is always positive if the marginal cost for transporting the passenger is higher than the marginal time cost for the passenger,  $a/b > 1$ . For Cournot it is demonstrated that fares will always

---

<sup>5</sup> It can be verified (see appendix B) that a sufficient but not necessary condition for  $\partial P_i^B / \partial g < 0$  is  $D_j / D_i < -4g / (4 + g^2) \Rightarrow D_j < D_i$  where  $i, j = \{1,2\}$  and  $i \neq j$ .

decrease with distance if  $a < b$ . The fraction  $a/b$  is then less than 1 and marginal time cost for the passenger exceeds the marginal cost for transporting the passenger. In the area between the two curves prices will increase and decrease in own distance under Bertrand and Cournot competition, respectively.

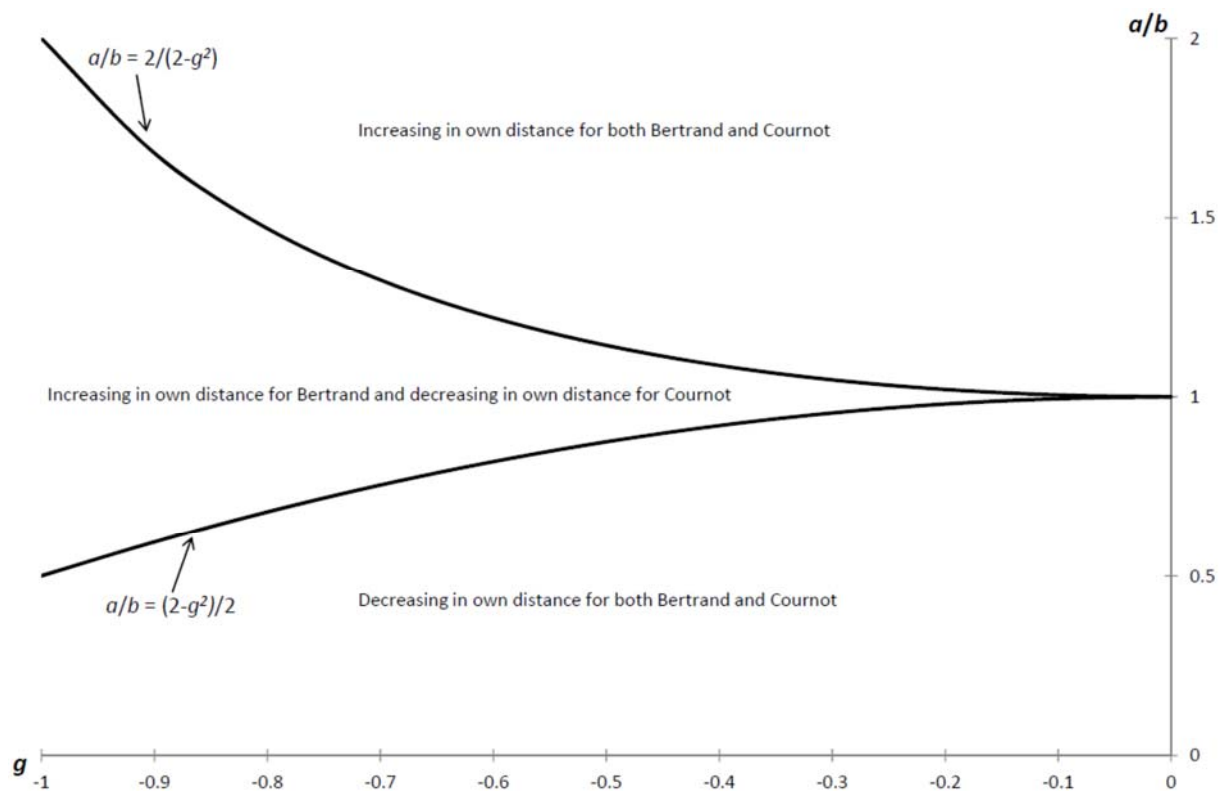


Figure 2: Conditions for increasing prices with own distance depending on type of competition.

Figure 2 shows that the type of competition can be vital for understanding the exact relationship between fares and distance. If the ratio  $a/b$  is very high or very low, then the type of competition does not affect this result. For values of  $a/b$  that are in between these two extremes then increasing distance will cause a fare increase if firms set fares strategically (Bertrand), and a decrease if capacity is chosen strategically (Cournot). As is demonstrated in the next section, Bertrand competition is more intense so that fares will be lower in this type of competition than Cournot. However, the  $a/b$  ratio at which the firms increase their fare in response to distance increases is quite low; a distance increase means that the firms incur larger costs per passenger and the already low price needs to be increased. For Cournot competition, the price includes a larger margin above costs, so that the fare does not increase in distance until the relative cost for the firm and passenger is larger.

The fraction  $a/b$  can be related to empirical evidence. In their study of the competition between air transport and high-speed rail in China, Yang and Zhang (2012) presented parameter values that enable us to calculate marginal costs for both operators and passengers. If we assume that variable unit costs per kilometre is a proxy for marginal costs and use time value for business passengers, the fraction  $a/b$  derives the values 2.75 and 0.58 for air transport and high-speed rail, respectively. Based on Norwegian cost data Jørgensen and Preston (2007) estimated  $a/b$  to 0.75 and 1.7 for ferry transport and bus transport, respectively. From these estimates and (8) we can conclude that operators of air transport (high-speed rail) in China and bus (ferry) operators in Norway under collusion would design fare schemes that increase (decrease) with distance. Seeing the estimates in the light of the results in Figure 2, suggest that air (high-speed rail) and bus (ferry) operators will design a fare scheme that increase (decrease) in own distance under Bertrand (Cournot) competition.<sup>6</sup> In intermediate cases, that is when air and bus firms compete in quantities and high-speed rail and ferry firms compete in prices, it is ambiguous how own distance influences fares; it depends on the magnitudes of the  $a/b$  and  $g$ .

Prices will, according to the derivative in (9), always decrease when the distance for complementary service increases. The condition is identical for the two forms of competition. This comparison is not relevant for the collusion case since optimal price in (7) is independent of the distance of the other firm. As the distance of one leg increases, the time cost associated with travel increases and, since the services exhibit a degree of complementarity, this leads to a reduction in the fare of the other leg of the journey.

$$(9) \quad \frac{\partial P_i^B}{\partial D_j} = \frac{\partial P_i^C}{\partial D_j} = g \frac{a+b}{(2-g)(g+2)} < 0 \text{ and } \frac{\partial^2 P_i^B}{\partial D_j \partial g} = \frac{\partial^2 P_i^C}{\partial D_j \partial g} = \frac{(g^2+4)(a+b)}{(g^2-4)^2} > 0 \text{ where } i, j = \{1,2\} \text{ and } i \neq j.$$

Moreover, it is shown in (9) that the change in price with respect to the distance for the complementary service is directly related to the degree of complementarity. When the level of complementarity is reduced ( $g$  increases and becomes less negative), then the reduction is less in optimal price for firm  $i$  caused by increased distance of the complimentary service provided by firm

---

<sup>6</sup> In Norway, the ferry fares increase linearly with travel distance. This may come from the fact that fares are not decided on in any form of complementary competition and that the Norwegian Public Roads Administration, who regulates the fares, pursues other goals than pure profit maximization, see Jørgensen and Preston (2007) for further discussion. Mixed goal functions in bus transport are discussed e.g. by Nash (1978) and the inclusion of users surplus in addition to profit is also addressed by Meunier and Quinet (2012).

*j*. Or put more simply, a higher value of  $g$  reduces the influence of the distance of the competing service on price.

#### 4.2 Mark-up of price over marginal cost

The mark-up of price over marginal cost ( $MC_i = h + aD_i, MC_j = h + aD_j$ ) can be studied for the types of competition. In the case of Bertrand competition, it is shown in (10) that the mark-up will decrease both in own and the competitor's distance. Even if the price should rise, the corresponding increase in MC outweighs this.

$$(10) \quad \frac{\partial(P_i^B - MC_i)}{\partial D_i} = (2 - g^2) \frac{a+b}{(g-2)(g+2)} < 0 \text{ and } \frac{\partial(P_i^B - MC_i)}{\partial D_j} = -g \frac{a+b}{(g-2)(g+2)} < 0$$

In the case of Cournot presented in (11), the mark-up always decreases in rivals distance since the price falls (see (9)). The effect on mark-up is uncertain with respect to the firm's own distance and it is clear from the condition in (11) that the required value of  $a/b$  increases with the complementarity of the services.

$$(11) \quad \frac{\partial(P_i^C - MC_i)}{\partial D_i} = a(1 - g^2) - 2b > 0 \text{ if } \frac{a}{b} < \frac{2}{1-g^2} \text{ and } \frac{\partial(P_i^C - MC_i)}{\partial D_j} = g(a + b) < 0$$

The relationship from (11) is illustrated in Figure 3 where mark-up increases in own distance for parameter combinations above the line  $a/b = 2/(1 - g^2)$ . When the cost to the firm of covering an extra km is relatively large compared to the cost of the consumer, the fare will increase in distance as will the marginal cost; however, the mark-up will still increase in own distance, increasing the profit of the service provider. Moreover, Figure 3 includes the condition from (8) ensuring that price increases in own distance. In the area between the two lines, an extra kilometre in own distance causes prices to rise, but mark-up to fall.

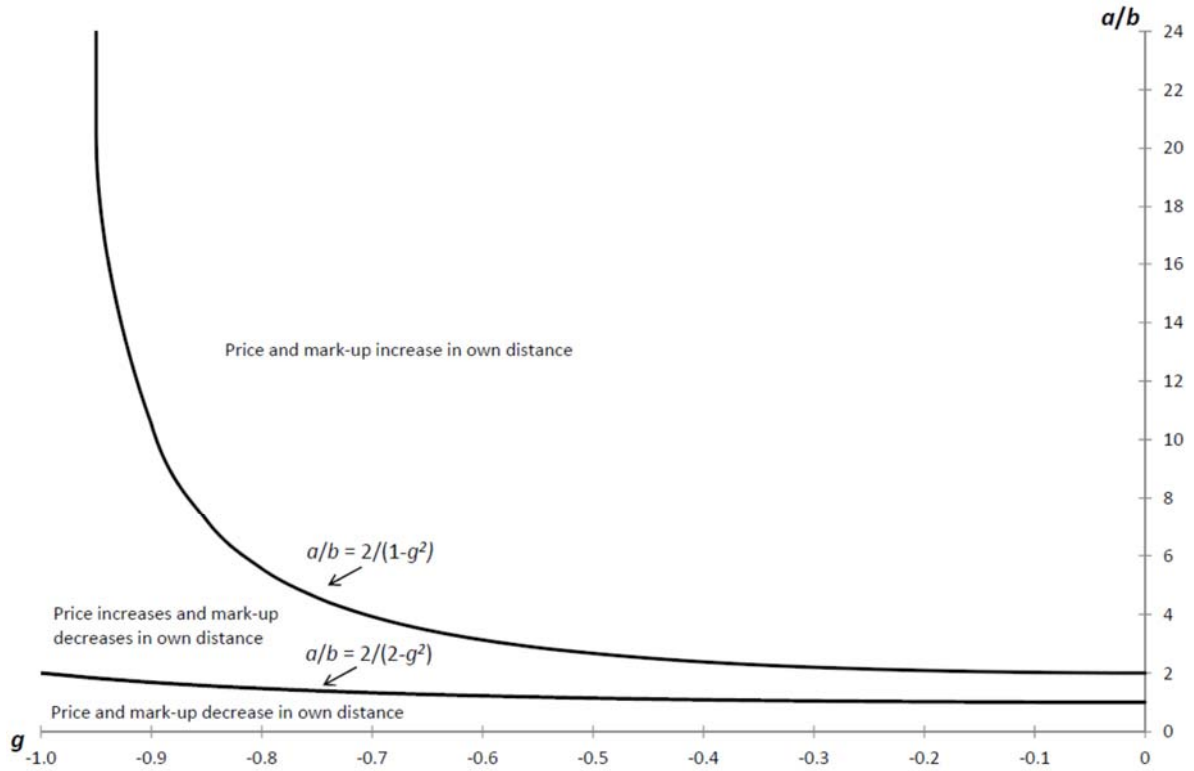


Figure 3: Mark-up over marginal costs with own distance for Cournot.

Finally, for collusion it is evident from (12) that own price increases in own distance but is always outweighed by the increase in marginal cost.

$$(12) \quad \frac{\partial(P_i^{COLL} - MC_i)}{\partial D_i} = -\frac{1}{2}(a + b) < 0$$

#### 4.3 Comparing fares for firms 1 and 2

The difference between the fares set by firms 1 and 2 in collusion is given in (13) and states that the longest distance has the higher fare as long as  $a/b > 1$ .

$$(13) \quad P_i^{COLL} - P_j^{COLL} = \frac{1}{2}(a - b)(D_i - D_j) \text{ where } i, j = \{1, 2\} \text{ and } i \neq j.$$

A comparison of optimal fares set by firm 1 and 2 in Bertrand competition is given in (14) where the condition for  $P_i^B$  being larger than  $P_j^B$ ,  $P_i^B - P_j^B > 0$ , depends on distances for the two complementary services. Consequently, for  $P_i^B$  to be larger than  $P_j^B$  then  $1 + g > (<) a/b$  if  $D_j > (<) D_i$ .

$$(14) \quad P_i^B - P_j^B = (D_j - D_i) \frac{(1+g)b-a}{2+g} \text{ where } i, j = \{1,2\} \text{ and } i \neq j.$$

Similarly, the comparison of optimal fares for Cournot renders the difference in (15). The condition for  $P_i^C$  to be larger than  $P_j^C$  is then  $a/b > (<) 1/(1-g)$  if  $D_i > (<) D_j$ .

$$(15) \quad P_i^C - P_j^C = (D_i - D_j) \frac{(1-g)a-b}{2-g} \text{ where } i, j = \{1,2\} \text{ and } i \neq j.$$

The conditions derived from (14) and (15) are illustrated in Figure 4. In area A (C), the operator with the longest distance sets the higher (lower) price, independent of whether they compete in quantities or prices. In area B, the type of competition is decisive. When competing in price (quantity) then the firm with the longer distance sets the highest (lowest) price. Again, the fares under price competition are pressed low due to more intense competition. Thereby, if the cost of transporting an extra passenger an extra kilometre rises, then the price has to be raised in order to cover costs. Hence distance has more weight in the pricing decision of firms that compete in price, making the firm with the longest distance set the higher fare for lower values of the relative marginal cost to the firm and the passenger,  $a/b$ . In a similar situation it was demonstrated by De Borger and Van Dender (2006) that increases in marginal cost of capacity at toll road may render increased profits in a congested Bertrand duopoly.

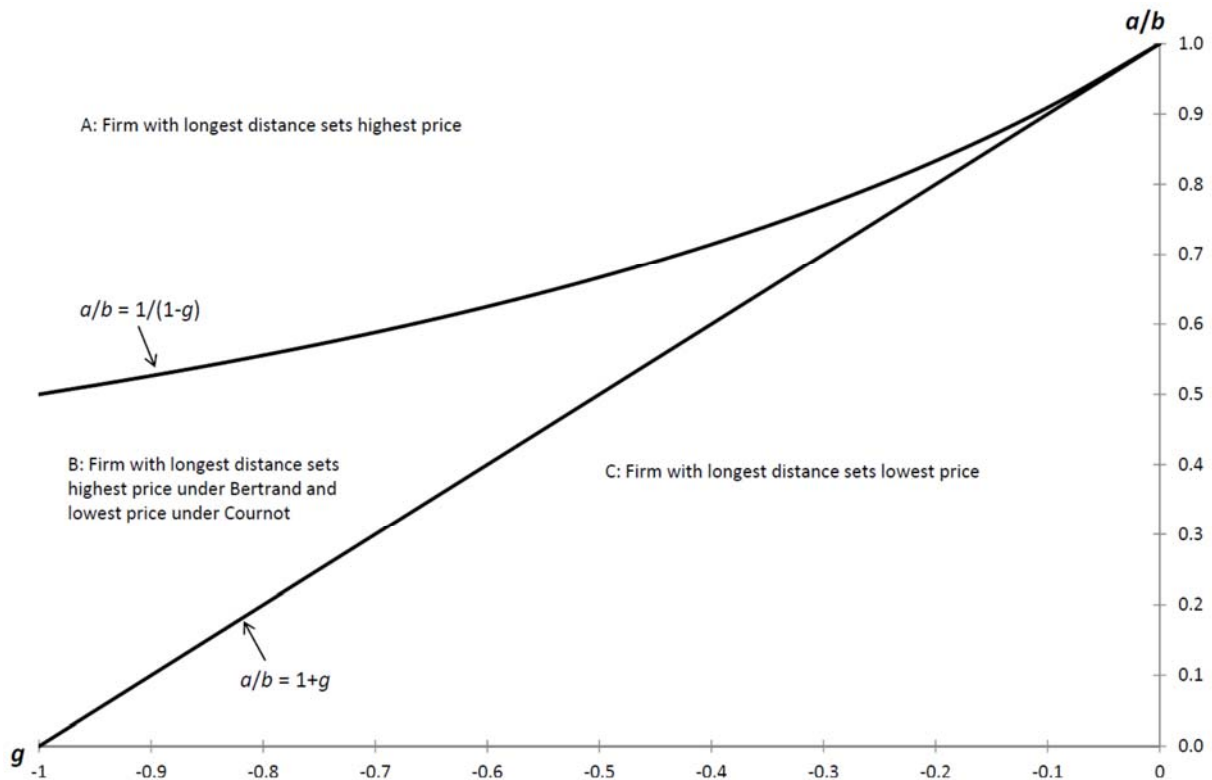


Figure 4: Relationship between ranking of prices and distance.

Hence, in collusion, the firm providing the service with the longest distance will set highest fare if the marginal cost for transporting the passenger relative to the marginal time cost for the passenger is at least 1. The required value of the fraction  $(a/b)$  to ensure that the longest distance has highest fare is reduced as the degree of complementarity, and thereby the degree of competition, increases. In the case of perfect complements, the fraction equals 0.5 and 0 for Cournot and Bertrand, respectively. The conditions presented in Figure 4 can be related to the empirical evidence given in 4.1 and demonstrates that air and bus operators with the longest distance always set highest fares whilst type of competition between the firms is decisive as far as high-speed rail and ferry transport is concerned.

#### 4.4 Comparing equilibrium prices between types of competition

The differences in equilibrium prices under Bertrand and Cournot competition are given in (16).

$$(16) \quad P_i^B - P_i^C = \frac{g^2}{(2-g)(2+g)} (D_i(a+b) - (1-h-k)) \text{ where } i = \{1,2\}$$

It can be derived from (16) that  $P_i^C > P_i^B$  if  $(1-h-k - D_i(a+b)) > 0$  which is true according to the positive profit restriction defined in Section 2. Consequently,  $P_i^C > P_i^B$ .

The difference between equilibrium prices in collusion and the competitive regimes can also be studied. A comparison relative to Cournot is given in (17).

$$(17) \quad P_i^{COLL} - P_i^C = \frac{g}{2} \frac{1}{(2+g)(2-g)} \left( (2-g)(1-h-k) + (a+b)(gD_i - 2D_j) \right) = \frac{1}{2} g X_j^C < 0$$

where  $i, j = \{1,2\}$  and  $i \neq j$ .

It thereby follows that  $P_i^{COLL} - P_i^C < 0$ . This is in line with well known result that mergers among complements (moving from Cournot to collusion or Monopoly) will reduce prices (e.g. Economides and Salop, 1992). It is also evident from (17) that the difference between the collusive price and Cournot prices will be reduced when the degree of complementarity between the services diminishes.

A comparison of collusion relative to Bertrand is given in (18).

$$(18) \quad P_i^{COLL} - P_i^B = \frac{g}{2} \frac{1}{(2+g)(2-g)} \left( (2+g)(1-h-k) - (2D_j + gD_i)(a+b) \right) \text{ where } i, j = \{1,2\}$$

and  $i \neq j$ .

When  $D_i \neq D_j$  it can be deduced from (18) that the condition for  $P_i^{COLL} < P_i^B$  is  $-g(1 - h - k - D_i(a + b)) < 2(1 - h - k - D_j(a + b))$  of which both sides of the equality can be seen to be positive due to the positive profit restriction in Section 2. In the special case when the distance for the two firms are equal,  $D_i = D_j = D$ , then the last element in (18) can be rephrased to  $(2 + g)(1 - h - k - D(a + b)) > 0$  by our previous assumption. Hence, at equal distance for the two services the difference in (18) is negative implying that  $P_i^{COLL} < P_i^B$ . In appendix C it is shown that the following possibilities exist for the relative comparison of the collusive and Bertrand prices when the providers traffic different distances: i) either both firms set their collusive prices under that of Bertrand, or ii) the firm with the largest distance sets its collusive price under the one it would set under price competition, and the firm with the shorter distance sets its collusive price above its Bertrand one.

The first case is the one that is usually recognized in standard models of industrial organization, where firms that supply complementary products use a collusive situation to reduce price, generating more demand for the products of both. However, it should be noted that this effect depends on the goals of the firms. It is demonstrated by Clark et al. (2009) that, when assuming services over equal distances, more weight placed on consumer surplus implies less reduction in prices when firms collude rather than compete. An additional effect that is captured by our model of a transport market is that distance plays a crucial role. The larger the distance, the less attractive a service is for travellers; a monopolist then reduces the price of this service (compared to price competition) in order to attract more passengers to this service, and via complementarity also to the other, initially more attractive service. The price of this service is marked up above the Bertrand price to take advantage of the extra demand.

#### 4.5 Some comments on sequential competition

The existence of dominant firms that largely define the price level in some markets makes it relevant to comment on sequential competition. The equilibria arising in this model in cases of sequential competition are characterised by complex expressions with few unambiguous relationships. This section elaborates on equilibrium prices in sequential competition on quantity and price, denoted by superscript  $SQ$  and  $SP$  respectively, and compare them to the results from the corresponding simultaneous model.



#### 4.5.1 Sequential quantity competition (Stackelberg)

The prices under sequential quantity competition (Stackelberg) are presented in (22) where  $i$  is leader and  $j$  is follower.

$$(19) \quad P_i^{SQ} = \frac{1}{4}(2(1 + h - k + D_i(a - b)) + g(h + k + D_j(a + b) - 1))$$

$$P_j^{SQ} = \frac{2g(1-h-k-D_i(a+b))+g^2(1+3h-k+D_j(3a-b))-4(1+h-k+D_j(a-b))}{4(g^2-2)}$$

In a standard sequential model with quantity competition and complementary products, the leader will set a lower price (fare) than in the case of simultaneous competition, whilst the follower will set a higher price. The same is true in this model with distance, since it is easily verified that  $P_i^{SQ} - P_i^C < 0$  and  $P_j^{SQ} - P_j^C > 0$ .

It is interesting to consider the effect of distance on the equilibrium fares. As commented on above, at equal distance  $0 > P_i^{SQ} - P_j^{SQ}$ . The magnitude of this difference in leader and follower fare depends on the distance; if  $\frac{a}{b} > \frac{g^2-g-2}{g^2+g-2}$ , then an increase in the distance covered by the leader will cause this difference to fall. This parameter area is above the line in Figure 5. As leader distance increases, the leader's fare is reduced more in relation to the follower (below the line in Figure 5). For sequential quantity competition an increase in rival distance reduces own fare for both the leader and follower firms. For the leader, an increase in own distance increases fares if  $a > b$ . This is also true for the follower, but in addition it is possible for own distance to increase own fare even when  $a < b$ . If  $3a > b > a$  then services must be sufficiently complementary (large absolute value of  $g$ ) and if  $b > 3a$  then services must be sufficiently independent (low  $g$ ) to ensure that the follower will increase fares in own distance.

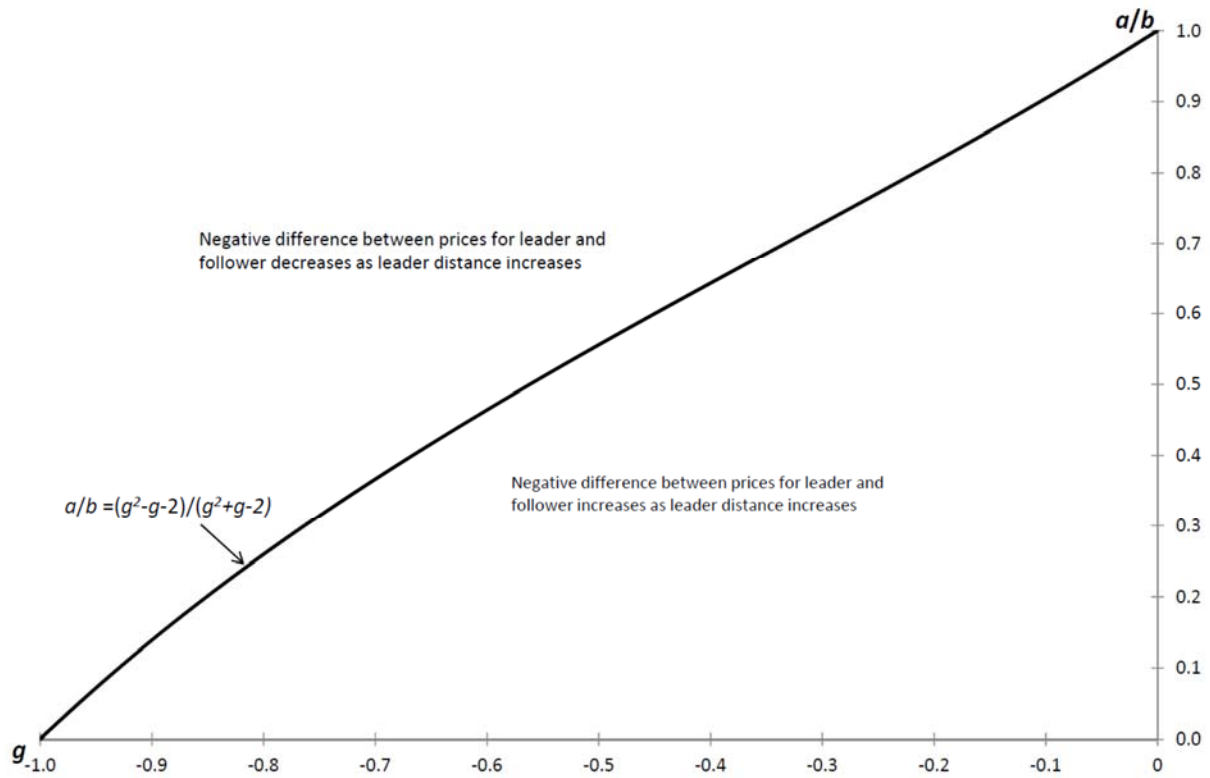


Figure 5: Condition for price difference between leader and follower in sequential quantity competition.

Considering the effect of how own distance affects the firms' mark-up over marginal cost reveals a difference compared to the case of simultaneous quantity competition. Whilst the effect of increasing own distance is parameter dependent for simultaneous quantity competition (see equation (11)), an increase in own distance causes both firms' mark-up to decrease with sequential competition:

$$(20) \quad \frac{\partial(P_i^{SQ} - MC_i)}{\partial D_i} = -\frac{1}{2}(a+b) < 0, \quad \frac{\partial(P_i^{SQ} - MC_i)}{\partial D_j} = \frac{1}{4}g(a+b) < 0$$

$$(21) \quad \frac{\partial(P_j^{SQ} - MC_j)}{\partial D_i} = \frac{1}{2} \frac{g(a+b)}{2-g^2} < 0, \quad \frac{\partial(P_j^{SQ} - MC_j)}{\partial D_j} = -\frac{1}{4} \frac{(2+g)(2-g)(a+b)}{2-g^2} < 0.$$

The effect of increasing rival distance on mark-up is negative whether competition is simultaneous or sequential.

#### 4.5.2 Sequential price competition

The prices under sequential price competition are presented in (22) where  $i$  is leader and  $j$  is follower.

$$(22) \quad P_i^{SP} = \frac{1}{2} \frac{(g^2-2)(1+h-k+D_i(a-b))+g(1-h-k-D_j(a+b))}{g^2-2}$$

$$P_j^{SP} = \frac{1}{4} \frac{2g(1-h-k-D_i(a+b))+g^3(h+k+D_i(a+b)-1)+g^2(3+h-3k+D_j(a-3b))-4(1+h-k+D_j(a-b))}{g^2-2}$$

In standard sequential price setting with complementary products, the leader will raise its price above the one set simultaneously, and the follower will lower its price (second-mover advantage). The same is true here since it can easily be verified that  $P_i^{SP} - P_i^B > 0$  and  $P_j^{SP} - P_j^B < 0$ . The gap between the leader and follower fare increases in the distance of the leader if  $a > b$ .

For mark-up, the results of increasing own or rival distance qualitatively mirror those of simultaneous price competition, since any increase in distance leads to a reduction in mark-up for both firms (see equation (10)).

### 5. Conclusions and Implications

By the time a passenger reaches the final destination of a trip it is often necessary to make use of the transport services provided by more than one firm. Transport services following in a natural order are complementary to some degree and can be provided by firms competing with each other. This paper studies prices and quantities in equilibrium for complementary services in the market for passenger transport, both when firms compete and collude. The model extends on previous research using oligopoly models to analyse transport markets by introducing complementary services differentiated by distance.

We may summarize the results derived from the model as follows:

1) It is demonstrated that the firm with longest distance will have the lowest number of passengers regardless of the type of competition as long as the services are not perfect complements. It is clear that distance is most important when the firms compete simultaneously in quantity and least when they collude. All equilibrium prices are increasing when the distance independent marginal cost ( $h$ ) increase and when passengers' time cost ( $k$  and  $b$ ) decrease. We can, however, not conclude unambiguously whether higher degree of complementarity between the services (higher  $g$  in

absolute terms) will lead to lower or higher prices under Cournot and Bertrand competition. For equal distance the leader will have lowest (highest) fares in sequential competition on quantity (price). The difference between fares for leader and follower when distances increase depends on the combination of parameter values.

2) The conditions for increasing prices with distance and the ranking of prices depend on 1) the cost of carrying one passenger an extra kilometre ( $a$ ) relative to the time cost for the passenger of travelling an extra kilometre ( $b$ ), 2) the type of competition and 3) the degree of complementarity ( $g$ ) between the transport services. The ratio of the costs of carrying the passenger an extra kilometre over the extra time cost for the passenger ( $a/b$ ) must be at least 1 in the case of collusion and when firms' compete, and the degree of complementarity between their services increases, then the required value of ( $a/b$ ) increases (decreases) from 1 for Cournot (Bertrand).

3) It can be derived from the model equilibrium that optimal Cournot price is always higher than optimal Bertrand price for complementary services. It is also proved that optimal price in Cournot is higher than in collusion and the difference increases when the services become closer complements. The ranking of prices in collusion and Bertrand cannot be unambiguously defined since distance can play a crucial role. Collusive prices are either both under the Bertrand ones, or the monopolist sets the collusive price of the longer distance lower than under price competition, and the price of the shorter journey above that arising from price competition. The leader (follower) of sequential quantity competition sets a lower (higher) price compared to simultaneous Cournot. The results are opposite for sequential price competition.

4) The mark-up over marginal costs will decrease both in own and a rival's distance in the case of Bertrand competition. In collusion, own price increases in own distance, but mark-up is reduced since the corresponding increase in marginal cost outweighs this. For Cournot, the mark-up always decreases in rival distance since the price falls and decreases in own distance if the ratio of the cost of carrying the passenger an extra kilometre to the extra time cost for the passenger ( $a/b$ ) is sufficiently high. With sequential competition, there is a subtle difference in the effect that distance has on mark-up, depending upon whether price or quantity is the choice variable. For price competition, the simultaneous and sequential models have similar effects. For quantity competition, the mark-up can increase in own distance if firms choose simultaneously, but there is an unequivocal fall in mark-up in the case of sequential competition.

The above results can aid regulators when aiming to meet politically decided objectives through regulation. If regulators are concerned about the welfare of the passengers they should endorse collusive behaviour rather than oppose it if the operators produce complementary services since it

ensures lower price and, correspondingly, higher quantity than when they compete. The presence of economies of scale would further enhance this conclusion since marginal costs then are reduced which paves the way for even lower prices. Regulators should be aware that type of competition has a stronger influence on market equilibrium when the degree of complementarity between the services increases. Hence, when the level of complementarity is high colluding service providers manage to set a low price, although it is still a monopoly price that exceeds marginal cost. It may then be tempting for the regulator to attempt to reduce this price further towards marginal cost. However, the regulator would need quite precise information on costs and demand, and how this may be affected by factors that we have not considered here such as other transport options for covering the same journey.<sup>7</sup>

Admittedly, there are several ways to expand the model to further take into consideration the characteristics of the transport market. First, goal functions extending beyond profit maximization could be included. Additional goals could be maximization of welfare or sales and they could be asymmetric between the firms. Second, a topic for further research would be to test the model results in practice by gathering actual data on fares and distance and relating them to the characteristics of competition and marginal cost for operators and passengers.

### **Acknowledgments**

We would like to thank the anonymous referees of this journal for valuable comments and suggestions.

### **References**

- Baumol, W.J., Panzar, J.C., Willig, R.D., 1988. *Contestable markets and the theory of industry structure*. Harcourt Brace Jovanovich, San Diego.
- Blauwens, G., Baere, P.D., Voorde, E.V., 2008. *Transport Economics*, 3rd ed. De Boeck Hoger.
- Button, K., 2010. *Transport Economics*, 3rd ed. Edward Elgar, Cheltenham, UK.
- Casadesus-Masanell, R., Nalebuff, B., Yoffie, D., 2008. Competing complements. Harvard Business School.

---

<sup>7</sup> This was pointed out to us by one of the referees.

- Clark, D.J., Jørgensen, F., Mathisen, T.A., 2011. Relationships between fares, trip length and market competition. *Transportation Research Part A* 45(7), 611-624.
- Clark, D.J., Jørgensen, F., Pedersen, P.A., 2009. Strategic interactions between transport operators with several goals. *Journal of Transport Economics and Policy* 43(3), 1-15.
- Coelli, T., Prasada Rao, D.S., O'Donnell, C.J., Battese, G.E., 2005. *An introduction to efficiency and productivity analysis*, 2nd ed. Springer, NY.
- Cournot, A., 1927. *Researches into the Mathematical Principles of the Theory of Wealth*. Originally published in French (1838), translated by Nathaniel Bacon, Macmillan, New York.
- De Borger, B., Dunkerley, F., Proost, S., 2007. Strategic investment and pricing decisions in a congested transport corridor. *Journal of Urban Economics* 62(2), 294-316.
- De Borger, B., Proost, S., 2012. Transport policy competition between governments: A selective survey of the literature. *Economics of Transportation* 1(1-2), 35-48.
- De Borger, B., Van Dender, K., 2006. Prices, capacities and service levels in a congestible Bertrand duopoly. *Journal of Urban Economics* 60(2), 264-283.
- Economides, N., Salop, S.C., 1992. Competition and integration among complements and network market structure. *The Journal of Industrial Economics* 40(1), 105-123.
- Hubbard, R.G., O'Brien, A.P., 2013. *Economics*, 4th ed. Pearson Education Ltd, Harlow, England.
- Häckner, J., 2000. A Note on Price and Quantity Competition in Differentiated Oligopolies. *Journal of Economic Theory* 93(2), 233-239.
- Jørgensen, F., Pedersen, H., Solvoll, G., 2004. Ramsey pricing in practice: the case of the Norwegian ferries. *Transport Policy* 11(3), 205-214.
- Jørgensen, F., Preston, J., 2003. Estimating bus operators' short-run, medium-term and long-run marginal costs. *International Journal of Transport Economics* 30(1), 3-24.
- Jørgensen, F., Preston, J., 2007. The relationship between fare and travel distance. *Journal of Transport Economics and Policy* 41(3), 451-468.
- Kraus, M., 1991. Discomfort externalities and marginal cost transit fares. *Journal of Urban Economics* 29(2), 249-259.
- Li, Z.-C., Lam, W.H.K., Wong, S.C., Sumalee, A., 2012. Design of a rail transit line for profit maximization in a linear transportation corridor. *Transportation Research Part E* 48(1), 50-70.
- Meunier, D., Quinet, E., 2012. Applications of transport economics and imperfect competition. *Research in Transportation Economics* 36(1), 19-29.
- Mohring, H., 1972. Optimization and scale economies in urban bus transportation. *The American Economic Review* 62(4), 591-604.
- Nalebuff, B.J., Brandenburger, A., 1996. *Co-opetition*. HarperCollins, London.
- Nash, C.A., 1978. Management objectives, fares and service levels in bus transport. *Journal of Transport Economics and Policy* 12(1), 70-85.

- Packalen, M., 2010. Complements and potential competition. *International Journal of Industrial Organization* 28(3), 244-253.
- Pels, E., Rietveld, P., 2008. Cost functions in transport, in: Button, K., Hensher, D.A. (Eds.), *Handbook of transport modelling*, 2nd ed. Pergamon, Amsterdam, pp. 381–394.
- Rodrigue, J.-P., Comtois, C., Slack, B., 2009. *The geography of transport systems*. Routledge, London.
- Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics* 15(4), 546-554.
- Socorro, M.P., Vicens, M.F., 2013. The effects of airline and high speed train integration. *Transportation Research Part A* 49, 160-177.
- Wan, Y., Zhang, A., 2013. Urban road congestion and seaport competition. *Journal of Transport Economics and Policy* 47(1), 55-70.
- Wu, D., Yin, Y., Yang, H., 2011. The independence of volume-capacity ratio of private toll roads in general networks. *Transportation Research Part B* 45(1), 96-101.
- Yang, H., Zhang, A., 2012. Effects of high-speed rail and air transport competition on prices, profits and welfare. *Transportation Research Part B* 46(10), 1322-1333.

## Appendices

### A. Firm with longest distance gets the lowest quantity

This can be studied by looking at the difference between the equilibrium quantities for the two firms under simultaneous competition and collusion. All expressions use the notation  $i, j = \{1, 2\}$  and  $i \neq j$ .

$$\text{Bertrand: } X_i^B - X_j^B = (a + b) \frac{D_j - D_i}{2 - g - g^2}$$

$$\text{Cournot: } X_i^C - X_j^C = (a + b) \frac{D_j - D_i}{2 - g}$$

$$\text{Collusion: } X_i^{COLL} - X_j^{COLL} = \frac{1}{2}(a + b) \frac{D_j - D_i}{1 - g}$$

For all expressions the difference is positive, meaning that  $X_i^m > X_j^m$  where  $m = \{COLL, B, C\}$ , if  $D_j > D_i$ . Moreover, it can be seen that

$$(X_i^{COLL} - X_j^{COLL}) - (X_i^C - X_j^C) = -\frac{1}{2}g(a + b) \frac{D_i - D_j}{g^2 - 3g + 2} < 0 \text{ if } D_i < D_j$$

$$(X_i^C - X_j^C) > (X_i^B - X_j^B) = g^2(a + b) \frac{D_i - D_j}{-g^3 + g^2 + 4g - 4} > 0 \text{ if } D_i < D_j$$

$$(X_i^{COLL} - X_j^{COLL}) - (X_i^B - X_j^B) = \frac{1}{2}g(a + b) \frac{D_i - D_j}{g^2 + g - 2} < 0 \text{ if } D_i < D_j$$

Hence, the influence on quantity by having the longest distance is highest for Cournot and least for Collusion.



## B. Partial differentiations of equilibrium prices in simultaneous competition and collusion

This section presents the differentiations with respect to  $h$ ,  $a$ ,  $k$ ,  $b$  and  $g$ . Differentiations wrt.  $D_1$  and  $D_2$  are presented in section 4. All expressions use the notation  $i, j = \{1, 2\}$  and  $i \neq j$ .

Bertrand:

$$\frac{\partial P_i^B}{\partial h} = \frac{1}{2-g} > 0$$

$$\frac{\partial P_i^B}{\partial a} = \frac{2D_i + gD_j}{(2+g)(2-g)} > 0 \text{ if } \frac{D_i}{D_j} > \frac{-g}{2}$$

$$\frac{\partial P_i^B}{\partial k} = -\frac{1-g}{2-g} < 0$$

$$\frac{\partial P_i^B}{\partial b} = \frac{1}{(2+g)(2-g)} (D_i(g^2 - 2) + gD_j) < 0$$

$$\frac{\partial P_i^B}{\partial g} = \frac{1}{(2+g)^2(2-g)^2} ((g+2)^2(h+k-1) + (D_j(4+g^2) + 4gD_i)(a+b)) \underset{<}{\geq} 0, \text{ where } \frac{\partial P_i^B}{\partial g} < 0 \text{ if } \frac{D_i}{D_j} >$$

$$\frac{4+g^2}{-4g} \Rightarrow D_j < D_i \text{ is sufficient}$$

Cournot:

$$\frac{\partial P_i^C}{\partial h} = \frac{g+1}{g+2} > 0$$

$$\frac{\partial P_i^C}{\partial a} = \frac{1}{(2+g)(2-g)} (D_i(2-g^2) + gD_j) > 0 \text{ if } \frac{D_i}{D_j} > \frac{-g}{(2-g^2)}$$

$$\frac{\partial P_i^C}{\partial k} = -\frac{1}{g+2} < 0$$

$$\frac{\partial P_i^C}{\partial b} = \frac{gD_j - 2D_i}{(2+g)(2-g)} < 0$$

$$\frac{\partial P_i^C}{\partial g} = \frac{1}{(2+g)^2(2-g)^2} ((g-2)^2(h+k-1) + (D_j(4+g^2) - 4gD_i)(a+b)) \underset{<}{\geq} 0$$

Collusion:

$$\frac{\partial P_i^{COLL}}{\partial h} = \frac{1}{2} > 0$$

$$\frac{\partial P_i^{COLL}}{\partial a} = \frac{1}{2} D_i > 0$$

$$\frac{\partial P_i^{COLL}}{\partial k} = -\frac{1}{2} < 0$$

$$\frac{\partial P_i^{COLL}}{\partial b} = -\frac{1}{2} D_i < 0$$

$$\frac{\partial P_i^{COLL}}{\partial g} = 0$$

C. Sign of  $P_i^{COLL} - P_i^B$

Denote  $P_i^{COLL} - P_i^B = \Delta_i$ ,  $P_j^{COLL} - P_j^B = \Delta_j$  and let  $t_i = 1 - h - k - D_i(a + b)$ ,  $t_j = 1 - h - k - D_j(a + b)$ . Note that  $t_i, t_j > 0$  by assumption. From (17), it is the case that

$$(C.1) \quad \Delta_i < 0 \text{ for } -gt_i < 2t_j$$

$$(C.2) \quad \Delta_j < 0 \text{ for } -gt_j < 2t_i$$

The following cases need to be considered:

Case 1:  $\Delta_i > 0$  and  $\Delta_j > 0$

Then by (C.1) and (C.2) it must be the case that

$$(C.3) \quad 0 > 2t_j + gt_i$$

$$(C.4) \quad 0 > 2t_i + gt_j$$

Summing these two inequalities gives

$$(C.5) \quad 0 > (2 + g)(t_i + t_j)$$

which is a contradiction. Hence  $\Delta_i > 0$  and  $\Delta_j > 0$  cannot be true.

Case 2:  $\Delta_i > 0$  and  $\Delta_j < 0$  for  $D_i > D_j$

The two  $\Delta$  inequalities imply that

$$(C.6) \quad 0 > 2t_j + gt_i$$

$$(C.7) \quad 0 > -2t_i - gt_j$$

Summing these two inequalities gives

$$(C.8) \quad 0 > (2 - g)(t_j - t_i)$$

which cannot hold when  $D_i > D_j$  since this implies  $t_j > t_i$ . Combined with  $2 - g > 0$ , this gives a contradiction. Similar reasoning means that one can also rule out the case in which  $\Delta_i < 0$  and  $\Delta_j > 0$  for  $D_j > D_i$ .

Case 3:  $\Delta_i > 0$  and  $\Delta_j < 0$  for  $D_j > D_i$

Case 4:  $\Delta_i < 0$  and  $\Delta_j > 0$  for  $D_i > D_j$

Case 5:  $\Delta_i < 0$  and  $\Delta_j < 0$

It is not possible to derive contradictions for the combinations in cases 3, 4 and 5 so these can characterize equilibrium behaviour, as discussed in the text.