UNDERLAY DEVICE-TO-DEVICE COMMUNICATIONS ON MULTIPLE CHANNELS

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ABSTRACT

Since the spectral efficiency of wireless communications is already close to its fundamental bounds, a significant increase in spatial efficiency is required to meet future traffic demands. Device-to-device (D2D) communications provide such an increase by allowing nearby users to communicate directly without passing their packages through the base station. To fully exploit the benefits of this paradigm, proper channel assignment and power allocation algorithms are required. The main limitation of existing schemes, which restrict D2D transmitters to operate on a single channel at a time, is circumvented by the joint channel assignment and power allocation algorithm proposed in this paper. This algorithm relies on convex relaxation to efficiently obtain nearly-optimal solutions to the mixed-integer program arising in this context. Numerical experiments corroborate the merits of the proposed scheme relative to state-of-the art alternatives.

Index Terms— Device-to-device communications, power allocation, channel assignment, convex relaxation.

1. INTRODUCTION

The exponentially increasing throughput demands of cellular communications [1, 2] can no longer be met by increasing the *spectral* efficiency of point-to-point links, e.g. through improvements in modulation and coding, since existing systems already approach the channel capacity [3, 4]. Hence, many contemporary research efforts aim at increasing *spatial* efficiency. Device-to-device (D2D) communications constitute a prominent example, where mobile users are allowed to communicate directly with each other without passing their messages through the base station (BS) [5–7]. Thus, users operating in D2D mode need half the time slots of those operating in the traditional cellular mode. Moreover, time slots used by D2D users can be simultaneously used by a traditional cellular user if both links do not interfere much, a technique termed *underlay*. To fully unlock the potential of underlay D2D communications, algorithms providing a judicious assignment of cellular sub-channels (e.g. resource blocks or time slots) to

D2D users and a prudent power control mechanism that limits interference to cellular users need to be devised.

Early works on D2D communications rely on simplistic channel assignment schemes, where each pair of D2D devices communicate through a cellular sub-channel (hereafter referred to as *channel*) selected uniformly at random. The impact of selecting a channel with poor quality has been counteracted by choosing among different modes of operation [8] or by sensing the selected channel [9]. Unfortunately, these approaches do not provide optimal throughput due to this random channel assignment and because no power control is effected to limit interference. To sidestep these limitations, [10] proposes a scheme where each D2D pair simultaneously transmits in all cellular channels and adjusts the transmit power at each of them. However, since every D2D pair adjusts power separately, important performance losses are expected when multiple D2D pairs operate in the same cell due to interference. Such a limitation is bypassed in [11, 12], where channels are jointly assigned by the BS to all D2D pairs. However, these works do not implement power control, which renders their channel assignments sub-optimal. This observation motivates joint channel assignment and power allocation as in [13–16]. Unfortunately, these schemes restrict D2D users to access at most one cellular channel. To sum up, no existing approach provides joint channel assignment and power allocation for the scenario where D2D users can operate on more than one cellular channel simultaneously, which is of high interest especially in crowded areas.

The present paper fills this gap by developing a joint channel assignment and power allocation scheme that allows each D2D pair to use more than one cellular channel. The adopted objective function involves throughput and promotes fair channel allocations through a regularizer, which is necessary to prevent most channels from being assigned to a small subset of D2D users. An efficient algorithm for approximately solving the resulting mixed-integer optimization problem is developed based on convex relaxation. A simulation study demonstrates the superior performance of the proposed method relative state-of-the-art alternatives.

The rest of this paper is structured as follows. Sec. 2 describes the system model. Sec. 3 introduces a novel channel assignment and resource allocation criterion and proposes an efficient solver. Finally, Sec. 4 provides the simulations and Sec. 5 summarizes conclusions.
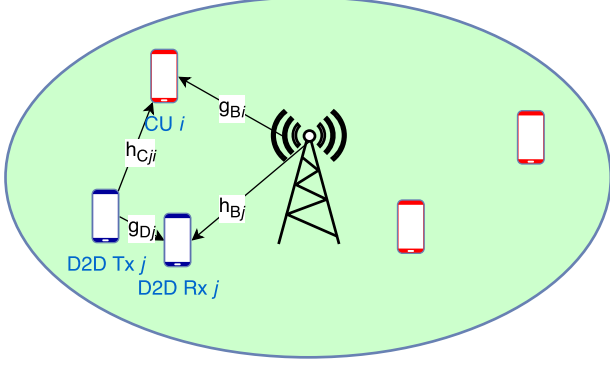
Fig. 1: Illustration of the system model.

2. SYSTEM MODEL

Consider a cell (or sector) where a BS communicates with N_C cellular users (CUs) through N_C downlink channels.[1] For convenience, the set of CUs (or, equivalently, channels) will be indexed by $\mathcal{C} = \{1, ..., N_C\}$. In this cell, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned downlink channels at the same time as the BS (underlay). The assignment of channels to D2D pairs will be represented by the indicators $\{\beta_{i,j}\}_{i \in \mathcal{C}, j \in \mathcal{D}}$, where $\beta_{i,j} = 1$ when D2D pair j uses channel i and $\beta_{i,j} = 0$ otherwise. It will be assumed that each D2D pair can access multiple channels at the same time, but no channel can be used by multiple D2D pairs, which implies that $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1, \forall i$. The transmission power used by the base station to communicate with the i-th CU is represented by P_{B_i} and is constrained to lie in the interval $0 \leq P_{B_i} \leq P_{B_{\max}}$. Similarly, $P_{D_{ji}}$ is the transmission power used by the j-th D2D pair when utilizing the i-th channel and is constrained as $0 \leq P_{D_{ji}} \leq P_{D_{\max}}$. Successful communications require that the signal-to-interference-plus-noise ratio (SINR) be greater than $\eta_{\min}^C$ for CUs and $\eta_{\min}^D$ for D2D receivers.

Fig. 1 illustrates the notation conventions for channel gains. Specifically, g_{B_i} denotes the gain between the BS and the i-th CU; g_{D_j} the gain of the j-th D2D link; $h_{C_{j,i}}$ the gain of the interference link between the transmitter of the j-th D2D pair and the i-th CU; h_{B_j} the gain of the interference link between the BS and the receiver of the j-th D2D pair; and N_0 the noise power.[2]

Given g_{B_i}, g_{D_j}, $h_{C_{ji}}$, h_{B_j} $\forall i, j$, as well as N_0, $\eta_{\min}^C$, $\eta_{\min}^D$, $P_{C_{\max}}$, and $P_{D_{\max}}$, the goal is to choose $\beta_{i,j}$, P_{B_i}, $P_{D_{ji}}$ $\forall i, j$ to maximize the aggregate throughput of the D2D pairs and CUs while ensuring fairness among multiple D2D pairs and preventing detrimental interference to CUs.

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[1]Recall that channel in this context may stand for resource blocks, time slots, and so on.

[2]Note that g_{D_j} and h_{B_j} should in principle depend also on i since the associated gains generally depend on the channel selected by the j-th pair; however, this subscript is dropped for simplicity since the proposed scheme carries over immediately to accommodate such dependence.

3. JOINT CHANNEL ASSIGNMENT AND POWER ALLOCATION

This section proposes a novel algorithm for channel assignment and power allocation that allows multiple D2D users in each cellular channel. Sec. 3.1 formulates the optimization problem and Sec. 3.2 proposes a solver. To simplify notation, collect the requested variables in vector-matrix form as $\boldsymbol{B} = [\beta_{i,j}]_{i,j} \in \mathbb{R}^{N_C \times N_D}$, $\boldsymbol{P_D} = [P_{D_{j,i}}]_{j,i} \in \mathbb{R}^{N_D \times N_C}$, and $\boldsymbol{p_B} = [P_{B_i}]_i \in \mathbb{R}^{N_C}$.

3.1. Channel Assignment and Power Allocation Criterion

This section formulates the problem of joint channel assignment and power allocation as an optimization problem. The first step is therefore to select a criterion that quantifies how desirable a given channel assignment and power allocation $(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D})$ is. As described next, the criterion adopted here equals the overall network binary rate plus a term that penalizes unfair channel assignments.

To obtain the overall network rate, let $\Gamma(z) := \log_2(1+z)$ and note that the total rate at channel i is given by $R_i := \sum_{j \in \mathcal{D}} \beta_{i,j}[R_{C_{i,j}} + R_{D_{j,i}}] + (1 - \sum_{j \in \mathcal{D}} \beta_{i,j})R_{C_{i,0}}$, where $R_{C_{i,j}} = \Gamma(P_{B_i}g_{B_i}/(N_0 + P_{D_{ji}}h_{C_{j,i}}))$ denotes the rate of the i-th CU when sharing the channel with the j-th D2D pair $(\beta_{ij} = 1)$; $R_{D_{j,i}} = \Gamma(P_{D_{ji}}g_{D_j}/(N_0 + P_{B_i}h_{B_j}))$ the rate of the j-th D2D pair when sharing the channel with the i-th CU $(\beta_{ij} = 1)$; and $R_{C_{i,0}} = \Gamma(P_{B_{max}}g_{B_i}/N_0)$ the rate of the i-th CU when it shares its channel with no D2D pair $(\beta_{ij} = 0 \ \forall j)$. The overall network rate is therefore $R := \sum_{i \in \mathcal{C}} R_i$.

The second term of the objective penalizes channel assignments where a small fraction of D2D pairs use a large part of the channels. To this end, the *unfairness* measure $\delta(\boldsymbol{B})$ from [11] will be used. It is given by $\delta^2(\boldsymbol{B}) = 1/(N_D x_0^2) \sum_{j=1}^{N_D} (x_j(\boldsymbol{B}) - x_0)^2$, where $x_j := \sum_{i=1}^{N_C} \beta_{i,j}$ is the number of channels assigned to the j-th D2D pair and $x_0 := N_C/N_D$. If N_C is an integer multiple of N_D, then $x_j = x_0 \ \forall j$ would be fairest channel assignment possible. $\delta(\boldsymbol{B})$ can be interpreted as the root mean deviation of $\{x_j\}_{j=1}^{N_D}$ from their fairest value x_0 and therefore is larger the more unevenly channels are assigned among D2D pairs.

The overall problem can then be formulated as:

$$\underset{\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}}{\text{maximize}} \quad R(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}) - \gamma \delta^2(\boldsymbol{B}) \tag{1a}$$

$$\text{subject to} \quad \beta_{i,j} \in \{0, 1\}, \ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1 \ \forall i \tag{1b}$$

$$0 \leq P_{B_i} \leq P_{B_{max}} \ \forall i \tag{1c}$$

$$0 \leq P_{D_{ji}} \leq P_{D_{max}} \ \forall j, i \tag{1d}$$

$$\forall i, j, \ \frac{P_{B_i} g_{Bi}}{N_0 + P_{D_{ji}} h_{C_{j,i}}} \geq \eta_{min}^C \ \text{if} \ \beta_{ij} = 1 \tag{1e}$$

$$\forall i, j, \ \frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{B_i} h_{B_j}} \geq \eta_{min}^D \ \text{if} \ \beta_{ij} = 1. \tag{1f}$$

Problem (1) is a mixed-integer program. Therefore it is non-convex and difficult to solve since it involves combinatorial complexity. The next section provides an efficient method to find an approximately optimal solution to (1).

3.2. Optimization via Convex Relaxation

This section presents an efficient method to approximate the solution to (1). Several tricks are applied to decompose (1) into multiple sub-problems of much lower complexity without any loss of optimality. One of these problems is an integer program, whereas the rest are problems that admit a closed-form solution. The proposed algorithm relies on convex relaxation to approximate the solution to the integer program.

The first step is to rewrite R in a simpler form. From the definitions of R and R_i in Sec. 3.1, it follows after rearranging terms that

$$R(\boldsymbol{B}, \boldsymbol{p_B}, \boldsymbol{P_D}) = \sum_{i \in \mathcal{C}} \left[\sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}}) + R_{C_{i,0}} \right],$$
(2)

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}} + R_{D_{j,i}} - R_{C_{i,0}}$ denotes the *rate increment* due to assigning the channel i to D2D pair j relative to the case where the channel i is only used by the CU.

It is next shown that (1) can be solved in two steps without loss of optimality: first, power allocation and, second, channel assignment. The trick is to replicate $\{P_{B_i}\}_i$ as described next. From (2), it follows that the objective of (1) can be written as $\sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}})$ plus some terms that do not depend on $\{P_{B_i}\}_i$. Clearly, an equivalent problem is obtained if P_{B_i} in each term $\beta_{i,j} v_{i,j}(P_{B_i}, P_{D_{ji}})$ is replaced with $P_{B_{i,j}}$ so long as the constraint $P_{B_{i,1}} = P_{B_{i,2}} = \ldots = P_{B_{i,N_D}}$ is enforced for all i. The resulting objective becomes $\sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}(P_{B_{i,j}}, P_{D_{ji}})$ plus terms that do not depend on $\{P_{B_{i,j}}\}_{i,j}$. One can similarly replace P_{B_i} with $P_{B_{i,j}}$ in (1e)-(1f) and also replace (1c) with $0 \leq P_{B_{i,j}} \leq P_{B_{max}} \, \forall i, j$ and the resulting problem will be equivalent to (1). Except for the recently introduced equality constraints, the objective and active constraints will only depend on at most one of the $\{P_{B_{i,j}}\}_j$ for each i. Thus, the equality constraint $P_{B_{i,1}} = \ldots = P_{B_{i,N_D}}$ can be dropped without loss of optimality. Similarly, one can also remove the condition "if $\beta_{i,j} = 1$" from (1e)-(1f). The resulting problem reads as

$$\underset{\boldsymbol{B}, \boldsymbol{P_B}, \boldsymbol{P_D}}{\text{maximize}} \quad \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \left[\beta_{i,j} v_{i,j}(P_{B_{ij}}, P_{D_{ji}}) \right] - \gamma \delta^2(\boldsymbol{B})$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\}, \ \sum_{j=1}^{N_D} \beta_{i,j} \leq 1 \, \forall i \quad (3)$$

$$\forall j, i \ \ 0 \leq P_{B_{ij}} \leq P_{B_{max}}, \ \ 0 \leq P_{D_{ji}} \leq P_{D_{max}}$$

$$\forall i, j \ \ \frac{P_{B_{ij}} g_{Bi}}{N_0 + P_{D_{ji}} h_{C_{j,i}}} \geq \eta_{min}^C, \ \frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{B_{ij}} h_{B_j}} \geq \eta_{min}^D$$

where $\boldsymbol{P_B} := [P_{B_{ij}}]_{i,j}$ and $\gamma > 0$ is a user-selected regularization parameter that balances the fairness-rate trade-off. To recover the optimal $\{P_{B_i}\}_i$ of (1) from the optimal $\{P_{B_{i,j}}\}_{i,j}$ of (3), one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $P_{B_i} = P_{B_{i,j}}$. If no such a j exists, i.e. $\beta_{i,j} = 0 \, \forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power $P_{B_i} = P_{B_{\max}}$.

Optimizing (3) with respect to $\boldsymbol{P_B}$ and $\boldsymbol{P_D}$ decouples across i and j into the $N_C N_D$ subproblems

$$\underset{P_{B_{ij}}, P_{D_{ji}}}{\text{maximize}} \quad v_{i,j}(P_{B_{ij}}, P_{D_{ji}}) \quad (4)$$

$$\text{subject to} \quad 0 \leq P_{B_{ij}} \leq P_{B_{max}}, \ 0 \leq P_{D_{ji}} \leq P_{D_{max}} \forall i, j$$

$$\frac{P_{B_{ij}} g_{Bi}}{N_0 + P_{D_{ji}} h_{C_{j,i}}} \geq \eta_{min}^C, \ \frac{P_{D_{ji}} g_{Dj}}{N_0 + P_{B_{ij}} h_{B_j}} \geq \eta_{min}^D, \ \forall i, j,$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. This power allocation subproblem coincides with the one arising in [13], which can be solved in closed-form as described therein.

Once (4) has been solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, it remains to substitute the optimal values of $v_{i,j}$ into (3) and minimize with respect to $\boldsymbol{B}$. If (4) is infeasible for a given (i, j), then set its optimal value to $v_{i,j} = -\infty$. The resulting channel assignment subproblem becomes:

$$\underset{\boldsymbol{B}}{\text{maximize}} \quad \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j} - \gamma \delta^2(\boldsymbol{B}), \quad (5)$$

$$\text{subject to} \quad \beta_{i,j} \in \{0,1\} \, \forall i, j, \quad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \, \forall i.$$

Problem (5) is an integer program of combinatorial complexity. Finding an exact solution is too computationally expensive and time consuming for sufficiently large $N_C N_D$, and therefore not suitable for real-time implementation as required by the application at hand. For this reason it is preferable to sacrifice some optimality if an approximately optimal solution can be found with a low computational complexity and therefore short processing time. To this end, one can leverage the notion of *convex relaxation* as described next.

The idea is that the source of non-convexity of (5) is the integer constraint $\beta_{i,j} \in \{0,1\}$. Replacing such a constraint with $\beta_{i,j} \in [0,1]$ will render (5) convex.[3] The resulting convexified problem can be efficiently solved e.g. through projected gradient descent [17]. Discretizing the solution $\{\tilde{\beta}_{i,j}\}_{i,j}$ to such a problem is expected to yield an approximately optimal optimum of (5). To this end, this paper considers two approaches: (A1) For every i, set $\beta_{i,j} = 1$ if $j = \arg\max_j \tilde{\beta}_{i,j}$. (A2) For each i, consider a random variable J_i taking values $1, \ldots, N_D$ with probabilities $P(J_i = j) = \tilde{\beta}_{i,j}$ (normalize $\{\tilde{\beta}_{i,j}\}_j$ to sum 1 if necessary). Then generate multiple realizations of $\{J_i\}_i$ and form the matrix $\boldsymbol{B}$, whose (i,j)-th entry is 1 if $J_i = j$ and 0 otherwise. Now evaluate the objective of (5) for all these realizations and select the realization with the highest objective value.

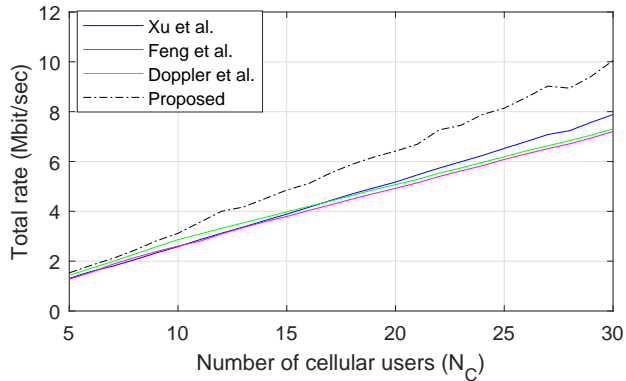[3]Strictly speaking, minimizing the negative of the objective of (5).

Fig. 2: Total rate R vs. N_C ($\gamma = 20$, $N_D = 10$, $N_0 = -70$ dBW, discretization via A2).
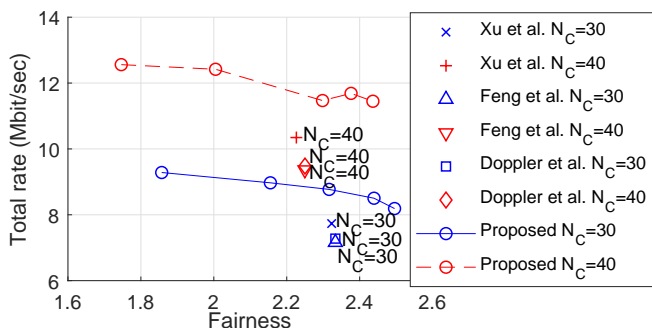

Fig. 3: Total rate R vs. fairness $\bar{\delta}$ for different values of N_C ($N_D = 10$, $N_0 = -70$ dBW, discretization via A2).

4. SIMULATIONS

This section compares the algorithm developed in Sec. 3 with state-of-the-art alternatives. The simulation setup comprises a circular cell with 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. Figures display averages over 100 independent realizations of the user locations with channels of 15 kHz. The proposed algorithm is compared with (i) the method by Xu et al. [11], which uses a price auction game for channel assignment without any power control, yet it allows D2D users to use multiple channels at the same time; (ii) the method by Doppler et al. [8], which randomly assigns a single channel to each D2D pair and selects among three modes of operation; and (iii), the method by Feng et al. [13], which jointly assigns a channel to each D2D pair and allocates power to maximize the total rate.

Fig. 2 depicts the total rate R of all four compared methods as a function of the number of cellular channels N_C. It is observed that the proposed method uniformly achieves the
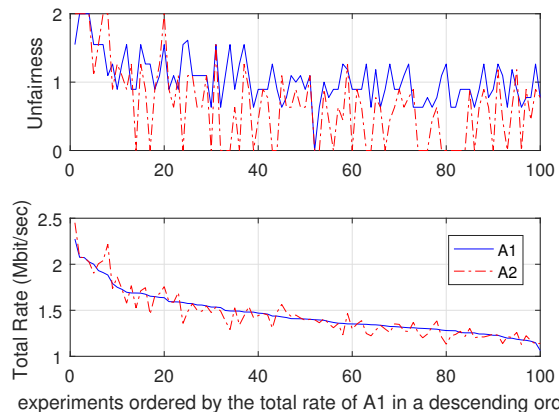

Fig. 4: Unfairness and rate for the discussed discretization approaches ($N_C = 5$, $N_D = 5$, $N_0 = -70$ dBW, $\gamma = 20$).

highest rate among all compared schemes; in particular, for $N_C = 30$, the rate of the proposed algorithm is approximately 25% more than the nearest competing alternative. In contrast to the methods by Feng et al. and Doppler et al., whose rates increments saturate for sufficiently large N_C since each D2D pair is only allowed to use at most one cellular channel, the rate of the proposed method steadily increases with N_C.

Fig. 3 represents the total rate vs. *fairness*, which is defined as $\bar{\delta}(\boldsymbol{B}) := \sqrt{N_D - 1} - \delta(\boldsymbol{B}) \in [0, \sqrt{N_D - 1}]$. Multiple points are obtained for the proposed method by varying γ between 40 and 200. In Fig. 3, the flexibility of the proposed method to adjust the desired point of the rate-fairness trade-off is manifest. Competing methods lack such flexibility. Moreover, over 10% increment in the total rate with respect to the nearest competing method is achieved with roughly the same fairness. This relative advantage increases further with N_C.

Finally, Fig. 4 compares the two discretization approaches provided at the end of Sec. 3 to recover the solution of (5) from the solution to its relaxed counterpart. Approach A2 is seen to yield nearly the same rate as A1 and an improved fairness. However, the computational complexity of A2 is significantly higher than that of A1.

5. CONCLUSIONS

This paper presented an algorithm for joint channel assignment and power allocation in underlay D2D cellular networks. The major novelty is to allow D2D pairs to operate on multiple cellular channels at the same time, which greatly increases throughput. After adopting a criterion that promotes high throughput and fairness, the resulting mixed-integer program is decomposed into multiple subproblems that are efficiently solved. Future research will develop distributed implementations, accommodate uncertainty in the channel gains, and incorporate user behavior models.

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