TOPOLOGY DESIGN TO REDUCE ENERGY CONSUMPTION OF DISTRIBUTED GRAPH FILTERING IN WSN

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ABSTRACT

The large number of nodes forming current sensor networks has made essential the use of distributed mechanisms in many traditional applications. In the emerging field of graph signal processing, the distributed mechanism of information potentials constitutes a distributed graph filtering process that can be used to solve many different problems. An important limitation of this algorithm is that it is inherently iterative, which implies that the nodes incur in a repeated communication cost along the exchange periods of the filtering process. Since the sensor nodes are battery powered and radio communications are energy demanding operations, in this work, we propose to redesign the network topology in order to reduce the total energy consumption of the filtering process. An accurate energy model is proposed and extensive numerical results are presented to show the efficiency of our methodology according to this energy model.

Index Terms— Topology design, graph filter, distributed algorithm, wireless sensor network

1. INTRODUCTION

Over the past years, distributed algorithms have been revealed as crucial mechanisms to efficiently solve many different problems appeared in increasingly complex wireless sensor networks (WSNs). Among many other advantages over centralized solutions, distributed algorithms reduce congestion around central entities and add robustness to the network in case of failures. As relevant examples, consensus algorithms [1–3] and information potentials [4] are two distributed algorithms that map the signal sensed in close proximity to each sensor device to a value more meaningful within the global

network context. This mapping presents an enormous potential as seen in [3] for consensus algorithms and recently in [4] for information potentials. In the emerging field of graph signal processing [5, 6], the information potentials algorithm is a distributed graph filter (i.e., potential kernel graph filter) that can be used to solve many problems, such as smoothing, greedy search and information discovery [4]. Recently, in order to detect the boundary and the region of an event of interest (e.g., oil or chemical spill), Loukas et al have proposed in [7] to filter the signal event in the spectral domain by subtracting two potential kernels or alternatively to apply the Laplacian after the signal has been smoothed by the potential kernel.

An important limitation of the proposed distributed filtering approach is that it is inherently iterative, which means that the nodes incur in a high energy consumption due to the need of performing repeated communications over time. Since radio communication is a very energy demanding operation in WSNs and nodes are usually battery powered, energy becomes a scarce resource that needs to be preserved.

In the past few years, there have been some works dedicated to design the network topology in order to reduce the energy consumption in WSNs when performing consensus algorithms [1–3], but none has dealt with the equivalent problem for the potential kernel graph filter. In this paper, we tackle this new problem, which significantly differs from the consensus case. In particular, the convergence time of the distributed potentials graph filter includes extra information of the network topology, which makes the optimal topology in terms of energy consumption to be, in general, very sparse. Since the problem is a complex combinatorial problem and can not be solved efficiently in polynomial time, we propose an efficient distributed heuristic algorithm to redesign the topology. We propose an accurate energy consumption model and show by extensive simulations that our methodology provides good performance in terms of the total energy consumption per filtering process. To the best of our knowledge, our work is the first one that focuses on the topological design problem in WSNs to reduce the energy consumption when applying a graph filter.

The remainder of this paper is organized as follows, Section 2 presents the main background on graph theory and

graph filters. In Section 3, the energy optimization problem is formulated. Section 4 and 5 present respectively the proposed heuristic algorithm and its performance evaluation. Finally, Section 6 states the conclusions of the work.

2. BACKGROUND

In this section, we review the main background related to the concepts of graph theory and graph filters.

Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ denote an undirected graph where $\mathcal{V}$ is a set of N vertices or nodes and $\mathcal{E}$ is a set of M links or edges such that if node i is connected to j, then $(i,j) \in \mathcal{E}$. For any given graph $\mathcal{G}$, we define the $N \times N$ adjacency matrix $\mathbf{A}$, where $[\mathbf{A}]_{ij} = 1$ if an only if $(i,j) \in \mathcal{E}$. Note that self-loops (i,i) are allowed. The set of neighbors of node i is defined by $\Omega_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. The degree of node i is $d_i = \sum_{j \in \Omega_i} [\mathbf{A}]_{ij}$ and $\mathbf{D}$ is the diagonal degree matrix including these degrees. Finally, we denote by $d_{\max}, d_{\text{avg}}, d_{\min}$ the maximum, the average and minimum network degree.

A graph signal, defined on the set of nodes of the graph, is a mapping $\mathbf{x} : \mathcal{V} \rightarrow \mathbb{R}$, and represented as a vector $\mathbf{x} = [x_1, ..., x_N]^T \in \mathbb{R}^N$. The i-th component x_i represents the signal value at the i-th vertex in $\mathcal{V}$.

A graph filter (GF) is a system $\mathbf{H}$ that takes a graph signal $\mathbf{x}$ as an input, processes it, and produces another graph signal $\mathbf{y}$ as an output. A graph filter $\mathbf{H} : \mathbb{R}^N \rightarrow \mathbb{R}^N$ is a map between graph signals, which is represented by an $N \times N$ matrix. GFs can be classified into two types [6, 8]: *Infinite Impulse Reponse (IIR)* GFs and *Finite Impulse Response (FIR)* GFs. Contrarily to IIR GFs, FIR GFs are designed such that their impulse responses are finite in the vertex domain. On the other hand, IIR GFs have the advantage of being less sensitive to graph and signal variation at the expense of higher communication cost.

The potential kernel is an IIR GF computed in [4] as:

$$y_i^{(t+1)} = (1 - \varphi) \sum_{j \in \Omega_i} \frac{y_j^{(t)}}{d_i} + \varphi \, x_i \qquad (1)$$

where φ is an inhibiting factor such that $0 < \varphi \leq 1$.

By introducing the matrix $\mathbf{P} = \mathbf{D}^{-1}\mathbf{A}$, the output of the potential kernel at iteration t can be rewritten as [4]:

$$\mathbf{y}^{(t+1)} = (1 - \varphi) \, \mathbf{P} \, \mathbf{y}^{(t)} + \varphi \, \mathbf{x} \qquad (2)$$

In the steady state (*i.e.* convergence), the output of the potential kernel $\mathbf{H}_\varphi$ is given by [7]:

$$\mathbf{y} = \mathbf{H}_\varphi \, \mathbf{x} = \mathbf{D}^{-\frac{1}{2}} \sum_{k=1}^{n} \left(\frac{1-\varphi}{\varphi} \lambda_k + 1 \right)^{-1} \langle \mathbf{D}^{\frac{1}{2}} \mathbf{x}, \boldsymbol{\psi}_k \rangle \, \boldsymbol{\psi}_k \qquad (3)$$

where $(\lambda_k, \boldsymbol{\psi}_k)$ are the eigenpairs of the normalized Laplacian $\mathcal{L}$ of the graph.

It has been shown in [4] that in time-invariant graphs and signals, the potential kernel converges ε-close to the steady state after n_{ex} exchange periods:

$$n_{ex} = \varphi^{-1} \log \left(\frac{c}{\varepsilon} \right) \qquad (4)$$

where $c = (q^2 + q)\|\mathbf{x}\|$ and $q = \frac{N}{2M} d_{\max} + (1 - \lambda_2)\sqrt{\frac{d_{\max}}{d_{\min}}}$.

The potential kernel disseminates the information contained in the signal over multiple hops, which is crucial for many information processing tasks in WSNs. However, the communication cost of this distributed mechanism is generally high for very energy constrained sensor devices since it needs to be computed iteratively. This rises the need to investigate efficient mechanisms to reduce the energy consumption, which should be distributed as the potential kernel itself to be useful in a WSN framework.

3. PROBLEM FORMULATION

Each of the N vertices included in the graph $\mathcal{G}$ models a sensor node with an omni-directional antenna, which has been deployed uniformly at random over certain area of interest. Each sensor i broadcasts its data, which means that a single transmission is sufficient to communicate with all the neighbors within its transmission range R_i. We assume that there is an underlying MAC protocol that resolves collisions and maintains the topology. Our model considers the fact that the graph filtering process is divided in n_{ex} exchange periods of communication. At each of these exchange periods, every node receives $(d_i - 1)$ packets of a certain size z from its neighbors, makes some local computations and broadcasts the new state to its neighbors.

The energy consumed by a node i in each exchange period is determined as follows:

$$E_i = (d_i - 1)(\alpha_R + \gamma) + \alpha_T \qquad (5)$$

where α_R and α_T are the energy spent by a sensor node to receive and transmit one message respectively and γ is the energy spent by a sensor node to make the needed computations for this packet in a single exchange period.

The energy spent by a sensor node to receive a message of size z in bits is given by [9]:

$$\alpha_R = E_{R_X elec} \, z \qquad (6)$$

where $E_{R_X elec}$ is the energy spent by the electronic circuitry at the receiver in J/bit.

The energy in joules required by a node i to transmit a packet of z bits to a node j located at a distance r_{ij}, is given by [9]:

$$\alpha_T = (E_{T_X elec} + E_{amp} r_{ij}^{\beta}) \, z \qquad (7)$$

where $E_{T_X elec}$ is the energy spent by the electronic circuitry at the transceiver in J/bit, E_{amp} is the energy dissipated at the

amplifier in J/bit/m^2 and β is the path loss exponent which is about 2 for free-space and higher in-door.

Finally, the energy spent by a sensor node on the computation at a single exchange period, when processing a single packet, is calculated as follows:

$$\gamma = E_{cp}\, z \tag{8}$$

where E_{cp} is the energy dissipated when processing a single packet and represented in J/bit/packet.

Therefore, the total energy E_{total} consumed in the network when applying the graph filter is given by:

$$E_{\text{total}} = n_{ex} \sum_{i=1}^{N} E_i \tag{9}$$

Since the sensor devices are often powered by batteries, implying a limited energy capacity, it is very important to reduce the energy consumption when applying the potential kernel in WSNs by re-designing the network topology.

Our problem is to determine the optimal topology (*i.e.* Adjacency matrix) that minimizes the energy consumption when applying the potential kernel. The corresponding optimization problem can be formulated as follows:

$$\text{minimize}_{\{\mathbf{A}\}} \quad n_{ex}(\mathbf{A}) \sum_{i=1}^{N} E_i(\mathbf{A})$$
$$\text{s. t.} \quad \xi \le \lambda_2(\mathcal{L}(\mathbf{A}))$$
$$[\mathbf{A}]_{ij} = [\mathbf{A}]_{ji} \ \forall i, j \in \mathcal{V}$$
$$[\mathbf{A}]_{ij} \in \{0,1\} \ \text{if} \ r_{ij} \le R_i \ \forall i, j \in \mathcal{V}$$
$$[\mathbf{A}]_{ij} = 0 \ \text{if} \ r_{ij} > R_i \ \forall i, j \in \mathcal{V}$$

where ξ is an arbitrary small positive constant that ensures that $\lambda_2(\mathcal{L}(\mathbf{A}))$ is larger than zero and the graph is connected. The second constraint guarantees that the resulting adjacency matrix is symmetric. The third and fourth constraints imply that the entries of the adjacency matrix are zeros or ones, depending on the distance between the two nodes involved and the transmission range at each node.

Our problem is a *combinatorial non convex* problem due to the objective function and binary variables constraints. This means that the problem cannot be solved efficiently in polynomial time. Therefore in this paper, instead of attempting to solve this problem in an optimal way, our goal is to propose a distributed solution that is efficient in reducing the total energy consumption and has a polynomial time complexity.

4. TOPOLOGY DESIGN FOR ENERGY EFFICIENT FILTERING

Our initial topology is formed from a randomly and densely deployed network of N nodes inside a $2D$ (sufficiently) large square area of a side L, where a link between two nodes is

established if their internode distance is shorter than their maximum transmission range R_{max}, i.e., the maximally connected topology.

Definition1: A strongly connected network is a network where exists a multi-hop path between every pair of nodes. A randomly deployed network is strongly connected with high probability (i.e., $1 - \frac{1}{N^2}$) if it is accomplished [10] that $R_{\text{max}} \ge L\sqrt{2\log N/N}$.

Definition2: A k-regular network is a network where all the nodes present the same degree k.

Lemma 1: The network topology with minimum filtering time (4) is a k-regular network with the maximum connectivity allowed by the maximum transmission range of the nodes, given by R_{max}.

Proof. The proof is based on the fact that for a k-regular network, it is accomplished that $k = d_{\text{avg}} = \frac{M}{N} = d_{\text{max}} = d_{\text{min}}$. This implies that the parameter q can be simplified to $q = \frac{1}{2} + (1 - \lambda_2(\mathcal{L}))$, which means that the convergence time only depends $\lambda_2(\mathcal{L})$. Finally, $\lambda_2(\mathcal{L})$ is a non-decreasing function of $\mathbf{A}$ [11], which implies that as larger the value of k is, the smaller the value of n_{ex}. $\qquad\square$

A direct conclusion from **Lemma 1** is that the fully connected topology is the one that minimizes the filtering time in (4). However, the maximum transmission range of the nodes R_{max} prevents in most situations to create such topology. Moreover, an important observation is that the fully connected topology is not generally optimal in terms of energy consumption, since it implies long links that may require a high energy consumption to be maintained. Finally, creating a k-regular network is not always easy in broadcast networks due to the different nodes densities in the deployment area, leading to a Poisson degree distribution around the parameter d_{avg}, as we explain next.

Suppose that the size of the square area is L^2. Since the deployment is generated uniformly random, the probability that there is a link between any two nodes is simply given by $p = \frac{\pi R^2}{L^2}$ and the average average degree d_{avg} that each node has is (approximately) $d_{\text{avg}} = Np = N\frac{\pi R^2}{L^2}$. According to [12], the degree distribution for a random network can be approximated by a binomial distribution, or in the limit for large scale networks, by a Poisson distribution. Thus, assuming a large-scale network, the probability of any node having degree k is:

$$p(k) = \frac{e^{-d_{\text{avg}}} d_{\text{avg}}^k}{k!} \tag{10}$$

and the degree distribution is simply $Np(k)$.

If a k-regular network can not be created due to one or several nodes having a degree d_i different to k, the links established by the nodes with $d_i > k$ must imply an increase
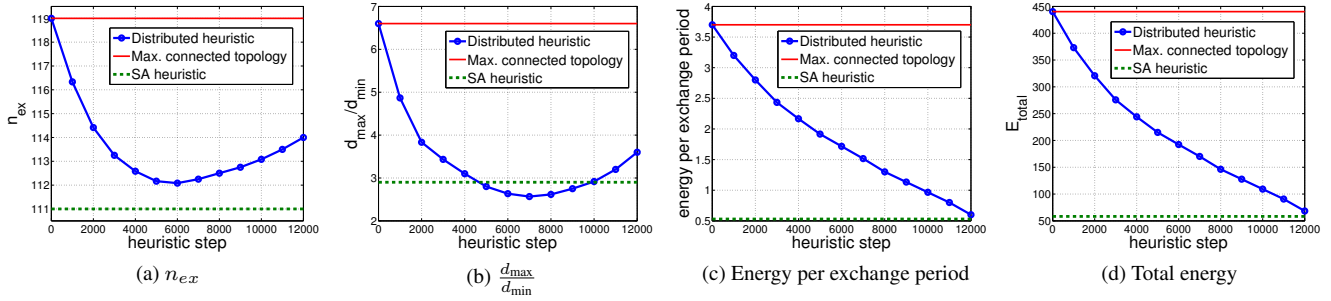
Fig. 1: Our distributed heuristic compared with the maximally connected topology and the meta-heuristic Simulated annealing (SA) in terms of the four parameters presented in this work.

on $\lambda_2(\mathcal{L})$) such that it compensates the irregularity[1]. In the opposite case, it is better to remove them such that $d_{\max} = k$.

In the most favourable irregular scenario and assuming an odd number of nodes, $N - 1$ nodes will have a degree equal to $k + 1$ and only one node will have a degree equal to k. In that case, the effect of the $2(N - 1)$ links on $\lambda_2(\mathcal{L})$ has to be at least[2] $\sqrt{\frac{1}{k}}\lambda_2(\mathcal{L})$. This can be easily obtained from the expression of q, where we have noted that for large scale networks the first term can be still approximated by $\frac{1}{2}$.

Given the degree distribution in (10), if we redesign the topology such that all the high degree nodes ($d_i > d_{\min}$) reduce their transmission range, we can make the network topology more regular, reducing both the value of $\frac{d_{\max}}{d_{\min}}$ and the energy consumption per round (5) of the high degree nodes.

A distributed heuristic with this philosophy is the following, starting from the maximally connected topology, nodes successively reduce an arbitrarily small amount their transmission range. Without altering the topology, during the first steps, all transmission ranges of the nodes go from $R_i = R_{\max} \ \forall i \in \mathcal{V}$ to $R_i = \max_{(i,j)\in\mathcal{E}} \{r_{ij}\}$.

After this first phase, the high degree nodes $d_i > d_{\min}$ start reducing their transmission range and removing the longest link each step. During the process, some low degree nodes may be forced to reduce their connectivity due to the high degree nodes not being able to reach them. Note that symmetry is forced in order to ensure convergence. Finally, each range reduction implies a decrease on the energy consumption per exchange period.

As an additional design procedure, note that links could be disabled between nodes able to communicate with each other, as proposed in [13]. This may reduce the quotient $\frac{d_{\max}}{d_{\min}}$ even more, reducing the convergence time in the majority of the cases. However, for simplicity, we evaluate only the effect of reducing the transmission range of the nodes.

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[1]The irregularrity can be quantified by the quotient $\frac{d_{\max}}{d_{\min}}$.

[2]For one level of irregularity (i.e., $d_{\max} - d_{\min} = 1$), the larger the number of links added, the easiest to compensate the irregularity.

5. NUMERICAL RESULTS

Extensive simulations were conducted in MATLAB to evaluate the performance of the proposed heuristic for optimizing the network topology, where the potential kernel is applied. A setup of $N = 1000$ sensor nodes randomly and uniformly distributed over a square area of a side $L = 200$ is considered in our simulations. The sensor nodes are assumed to inject in the network an input signal such that $\|\mathbf{x}\| = 1$. We also assume the following typical parameter values: z=200 Bytes, $\varepsilon = 0.0001$, $\varphi = 0.1$, $\beta = 2$, E_{cp}=5 $nJ/bit/packet$, E_{Rxelec}=E_{Txelec}=50 nJ/bit, E_{amp}=100 $pJ/bit/m^2$ [9].

The performance of the proposed heuristic algorithm is analyzed in term of the number of exchanges n_{ex}, the energy consumption per exchange period and the total energy consumption. We compare the different results to those obtained in the initial topology and Simulated Annealing (SA). The latter is a meta-heuristic algorithm that relies on randomness to generate good approximate solutions to combinatorial or NP-hard problems.

Fig. 1 (a) shows that when applying the proposed distributed heuristic, the number of exchanges can be significantly reduced as compared to the maximally connected network by reducing the network connectivity. This is something counter-intuitive and is explained by the behavior of the term $\frac{d_{\max}}{d_{\min}}$ (see Fig1 (b)), which is a measure of the extent of regularity in the network. In general, for a similar link density, as more regular the network is, the smaller number of exchange periods is needed to converge. Finally, since reducing connectivity always improves the energy consumption per exchange period (see Fig. 1 (c)), the total energy consumption is dramatically reduced by applying our distributed heuristic as shown in Fig. 1 (d). It can also be noticed from Fig. 1 that our distributed and low-complex heuristic provides almost the same results as the SA heuristic, which utilizes much more computational power to search for the optimal topology.

6. CONCLUSION

In this paper, we have formulated the problem of optimizing the network topology in order to minimize the total energy consumption of the potential kernel graph filter in WSNs. Since this problem is computationally too complex, we propose a distributed and low complex heuristic algorithm. The numerical results show that our algorithm provides good performance in terms of the total energy consumption required to execute the filtering process when compared to the maximally connected topology and the Simulated Annealing results.

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