

Tactical Asset Allocation in a Real-Life Setting

Vegard Hernæs

Supervisor

Valeriy Zakamulin

This master's thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

University of Agder, 2014

School of Business and Law

Abstract

This thesis will test one of the most popular market-timing strategies, using the longest available data set ranging from 1857 to 2012. The market-timing strategy has already proven to deliver superior results in the period 1926-2012 in a back-test. Which is why, the performance of the period pre-1926 will be compared to the post-1926 performance in a back-test. The performance of the two periods is similar, but period 1857-1925 is found to have the greatest improvements when the strategy is tested, which I find to be due to the lack of long consecutive bull markets. In both time periods the market-timing strategy is providing a small increase in returns while decreasing the volatility significantly when compared to a passive buy-and-hold strategy. In order to minimize the potential data-mining bias that all in-sample technical analysis struggle with, an out-of-sample simulation method is tested on the entire data-set and it is found that the performance is poorer than what was found in a back-test. The reason for the out-of-sample deterioration is mainly found to be due to changes of the optimal moving-average length.

Contents

Abstract	1
1. Introduction.....	3
2. Relevant literature and theory	5
2.1 Modern Portfolio Theory.....	5
2.2 Sharpe Ratio as a Performance Measure	8
2.3 The Efficient Market Hypothesis	9
2.4 Literature Review	10
3. Data Sources and Data Construction.....	13
4. In-Sample Tests.....	15
4.1 In-sample: Methodology	15
4.2 In-sample: Results	18
5. Out-of-Sample Tests	30
5.1 Out-of-Sample: Methodology	30
5.2 Out-of-sample: Results	35
6. Discussion	52
7. Conclusion	56
Bibliography.....	58
Appendix.....	61
R-kode:.....	61

1. Introduction

Investing in stocks has a reputation of being risky business, and as the latest decade has shown, it might be an entitled one. For stockholders the events of the dot.com crash and the financial crisis in 2007-08 were devastating, with losses cutting their wealth dramatically. Even the most well-diversified portfolios experienced rough times the last decade, as previously uncorrelated assets all started to plummet in value simultaneously. When investors needed the diversification advantages the most, it failed and it is not the first time it has happened. Throughout modern economic history the same pattern can be found. In times of financial crisis, a well-diversified portfolio is not enough to avoid large losses (Preis, Stanley, Helbing, & Ben-Jakob, 2012). Yet the potential profits outweigh the risk of losing money for the many investors. This thesis will explore the possibilities of implementing a market timing strategy based on technical analysis, in order to reduce market risk without having an adverse impact on the returns. The market timing systems seek to exploit potentially existing trend patterns in price series in order to forecast future realizations of the stock prices. Trend following strategies has gotten a lot of attention in the recent years, much because following a trend strategy would have resulted in astonishing results for the last decade, as it managed to time the exceptionally volatile market with great success.

In order to predict the future on the basis of trends, the trends need to be identified. This will be done by using a simple-moving-average (SMA) where the current price will be evaluated against the mean of the last “k” observation, if the current price is higher than the mean, the price is upward trending meaning the asset should be bought and held until the trend has turned negative, i.e. the current price is lower than the mean. There exist no clear guidelines on how the length of the moving-average should be chosen, the typical lengths used are 10-month mean on monthly data and 200-day mean on daily data. I will test several different lengths of the simple-moving-average in order to find what length results in the best performance in each of the periods tested.

There are numerous papers that have documented the superior performance of different trend-following strategies from the interwar period to present date, which is why I will test the strategy on a longer time period than any published article has to this point. The goal of this thesis is to find out if the SMA strategy works on a longer data-set than tested before. The data-sets used provide monthly data from 1857 to 2012. In order for the results to be comparable to what others have found the data will be divided into a “classical” period from 1926-2012 and another period

from 1857-1925, the performance of the two periods will be looked at separately and also compared to each other.

Even though the trend-following strategies has provided some very promising results, the market-timing strategy has been the subject of critique, where it is claimed that the performance found on historical data is better than one can expect if the strategy is implemented in real life. The reason is that this strategy is very exposed to data-mining issues, where only the best rules are reported in back-tests. Creating a potential of a substantial discrepancy between the performances reported on historical data and the performance one would get in a real life application. Some of the reason for this is when using a moving-average there is no way of knowing what length will give the best results, there is close to any documentation of the out-of-sample performance of this strategy. To address the data-mining critique and the lack of existing out-of-sample literature this thesis will have an out-of-sample simulation, where the choice of the optimal moving-average length will be based on the length that gives highest Sharpe ratio in a back-test. This will hopefully provide an unbiased picture of the real-life performance a user of the strategy can expect. The out-of-sample simulation will be tested on several different periods in the data set ranging from 1857 to 2012.

The goal of this thesis is to contribute to the ongoing research in-sample and also provide an out-of-sample simulation. The in-sample contribution will be to test the simple-moving-average strategy in a back-test on a longer data set than ever previously tested. The out-of-sample simulation will hopefully provide unbiased performance measures like what one can expect to achieve if implemented in real-life.

This paper will be built up in the following way; it will start with a concise part on portfolio theory and have an overview of relevant literature, before talking shortly about the data used in this paper, then first the methodology and the results of the in-sample study will be shown before the out-of-sample methodology and results will be shown. This will be followed by a discussion about the results, before the last section will contain a short summary and conclude the thesis.

2. Relevant literature and theory

2.1 Modern Portfolio Theory

Harry Markowitz introduced modern portfolio theory (MPT) in (Markowitz, 1952) and (Markowitz, 1959) where he proclaimed that individual stocks should not be selected based on its own merits, but rather on the basis of how each asset's individual price changes relative to the other asset prices in the portfolio. Through diversification and risk control MPT explains how an investor's portfolio should be composed in order to maximize the return given a level of risk, or minimize the volatility (risk) given a level of return. MPT uses the investor's set of beliefs for the expected return from each asset and the covariance between each pair of assets. Based on these probability beliefs all investors should create their portfolio to find what combination of assets provide the return and volatility that maximizes the perceived utility (often assumed to be a function of risk aversion). As a result MPT suggests that all portfolios should be efficient, (McInish, 2000) defines efficient portfolios as;

“A portfolio is efficient if it has the highest return for a given level of risk and the lowest risk for a given level of return. One portfolio dominates another if it has more return for a given level of risk or less risk for a given level of return”.

This means that for any given portfolio return, μ_p , the minimum portfolio variance, σ_p , can be found. If one assumes that there exists N different assets, and if the proportion of investment into each asset is w_i , where $i \in [1, \dots, N]$, so that,

$$\sum_{i=1}^N w_i = 1.$$

This makes the portfolio return, R_p , the weighted sum of the individual asset returns, R_i ,

$$R_p = \sum_{i=1}^N w_i R_i.$$

Where the expected portfolio return is given by,

$$\mu_p = E[R_p] = \sum_{i=1}^N w_i \mu_i, \text{ where } \mu_i = E[R_i].$$

The portfolio variance is equal to,

$$\sigma_p^2 = \text{var}(R_p) = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \text{cov}(R_i, R_j).$$

Then for some targeted portfolio return, μ^* , one can find the minimum variance by solving the following quadratic problem;

$$\min \frac{1}{2} \sigma_p^2, \text{ subject to } \mu_p = \mu^* \text{ and } \sum_{i=1}^N w_i = 1.$$

The solution to this problem gives the answer to what combination of assets that will provide the lowest risk for a given level of return. It is this combination of assets which is called efficient and it is lying on the efficient frontier. For every other feasible target return, there exist another set of asset weights that provide that return while having the minimal risk. By varying the target portfolio return the entire efficient frontier can be found. In (Tobin, 1958) he introduced a risk free asset in addition to the N risky assets. By introducing this, the efficient frontier will no longer consists of several alternative compositions, it will only be one single portfolio of risky assets and that portfolio is often called the tangency-portfolio. That tangency-portfolio (also called the optimal risky portfolio) is the portfolio that has the highest reward-to-risk ratio, and it will be held in combination with the risk free asset to best suit each individual investors risk tolerance. If the return of the optimal risky portfolio is denoted as R_j^* , the risk free return is R_f and w is the portfolio weight, the portfolio return will be,

$$R_p = wR_f + (1 - w)R_j^*.$$

This is often called the best capital-allocation line (CAL). Since the covariance between the risk free and risky asset is zero (in order to be risk free, R_f has to have a volatility equal to zero by definition), this new portfolio has a standard deviation of,

$$\sigma_p = (1 - w)\sigma_j.$$

Combining the expression for the portfolio return and standard deviation enables one to calculate the CAL as,

$$R_p = R_f + \left(\frac{R_j - R_f}{\sigma_j} \right) \sigma_p$$

An example of an efficient frontier with a capital-allocation line superimposed is shown in figure 1.

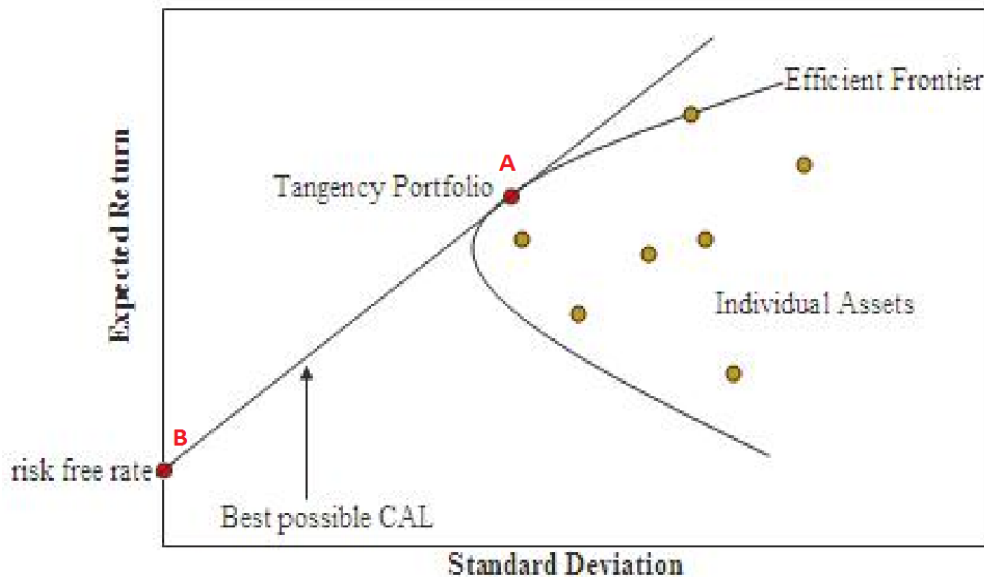


Figure 1: Efficient frontier with the capital-allocation line superimposed.

Modern portfolio theory dictates that all risk averse investors will adapt their portfolio somewhere along the CAL-line between point A and B¹ in Figure 1 to best suite their level of risk aversion.

The Capital-Asset-Pricing-Model (CAPM) was created by Jack Treynor, William Sharpe, John Lintner and Jan Mossin in the early 1960s. The model is an extension of the previous work of Markowitz, and it comes up with some interesting results about the properties of the tangency-portfolio. From the assumption that all investors are rational, have the same available information and are maximizing economic utility, it follows that they will all hold the exactly same optimal risky portfolio. When all market participants hold the same portfolio in combination with a risk

¹ Given that gearing is not allowed. If one allows for it, the feasible area of the CAL continues along the line past point A.

free asset, that portfolio has to be the market portfolio. Another interesting thing that follows from CAPM and the market tangency portfolio is that the market portfolio has to have the highest possible Sharpe ratio (defined in the section 2.2).

2.2 Sharpe Ratio as a Performance Measure

The slope of the capital-allocation line is a measure of reward-to-risk and is popularly called the Sharpe-ratio (SR) (Sharpe, 1966). It is one of the most commonly used measures of reward-to-risk, much because it is so easy to compute and interpret. The SR can be computed as the expected mean return in excess of the risk free rate of return, divided by the standard deviation of that excess return,

$$SR = \frac{E[R_j - R_f]}{Std[R_j - R_f]}.$$

Even though this measure is widely used and accepted, the measure has also been the subject of critique. The most relevant critique in data sets of the type I will use in this thesis is about the usage of the standard deviation of the excess return as a measure of risk (Zakamulin, 2013). The standard deviation takes the distance of each return from the mean, positive or negative. By doing this, large positive returns are also penalized in addition to the negative. So that in extreme cases, very large positive returns may actually lower the Sharpe ratio (Harding, 2002). Of course this problem has been encountered by others in the past and solutions have been proposed, for instance other measures have been developed, like the Sortino rate. That only uses the negative realizations to compute the standard deviation. Nevertheless, I will use the Sharpe ratio extensively and use it as an optimization criterion throughout this thesis. In order to test the statistical significance of changes found in the Sharpe ratios I will use the SR-test developed by (Jobson & Korkie, 1981) and later the test statistics was simplified by (Mommel, 2003). The null hypothesis is, $H_0 : SR_1 - SR_2 = 0$, giving the test statistic,

$$z = \frac{SR_1 - SR_2}{\sqrt{\frac{1}{T} \left[2(1 - \rho^2) + \frac{1}{2}(SR_1^2 + SR_2^2 - 2SR_1SR_2\rho^2) \right]}}$$

Where SR_1 and SR_2 are the Sharpe ratios of two different portfolios, in this application it will be for the active and passive strategy, the correlation coefficient, ρ , measures the correlation between the two Sharpe ratios. The test statistic, z , is thought to be asymptotically distributed as a standard normal distribution.

2.3 The Efficient Market Hypothesis

The efficient market hypothesis (EMH) is widely accepted amongst economists, is a concept developed by Eugene Fama amongst others in the 1960s. In (Fama, 1965a) he defined efficient markets as “a market where prices at every point in time represent best estimates of intrinsic values”, the efficient markets was only a few years later divided into three forms of market efficiencies that ultimately differs in what they consider to be “all available information”, (Bodie, Kane, & Marcus, 2011) defines the three forms as;

- Weak-form, asserts that all stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume, or short interest. This version of the hypothesis implies that trend analysis is fruitless.
- Semi-strong-form, states that all publically available information regarding the prospects of a firm must be reflected already in the stock price. Such information includes, in addition to past prices, fundamental data on the firm’s product line, quality of management, balance sheet composition, patents held, earning forecasts, and accounting practice.
- Strong-form, says that stock prices reflect all information relevant to the firm, even including information available only to company insiders.

In any of the three forms the EMH predicts that all technical analysis strategies should fail to provide any increase in performance, even in the weak-form there are no past price patterns that can be exploited. This means that all studies that have found strategies that can exploit past prices with success is a direct contradiction of the EMH.

At the same time it was (and still is) a large debate on whether or not stock prices follow a random walk or not. The random walk hypothesis says that stock price changes are identically distributed and are independent of each other, meaning that past movements or trends of a stock/market price cannot be used to predict its own future movements (Bodie, Kane, & Marcus,

2011). (Cootner, 1962), (Fama, 1965a) and (Granger & Morgenstern, 1963) all found evidence that stock market prices follows a random walk, while papers like (Alexander, 1961) and (Steiger, 1964) found that they do not follow a random walk. The link between the random walk debate and the efficient market concept is that in an efficient market, “the action of the many market participants should cause the actual price of a security to wander randomly about its intrinsic value” (Fama, 1965b). This is an important debate especially for the users of technical trading rules, as the whole technical trading business is built around the belief that the markets are in fact not efficient and that the prices do not follow a random walk, enabling the potential of systematic returns in excess of the market.

2.4 Literature Review

Technical analysis is far from a new concept and has been around for a long period. The earliest known use dates to the early 18th century in the Japanese rice market. Techniques based on psychology of the market made the developer Homma Munehisa (1724-1803), a successful financial trader at the time (Chen, 2010). The development of the Dow Theory in the first years of the 1900s is by many thought to be start of technical analysis in the western part of the world. This theory was published in a total of 255 “Wall Street Journal” editorials from 1900 to 1902 (Wikipedia, Dow Theory), all written by the Charles H. Dow. His theory is based on the use of different trends, where he distinguishes between three types of trends in order to predict future stock price movement.

During the next decades there were proposed a lot of different technical strategies, most of them used stock-charts and similar methods that do not rely on a vast number of computations as modern technical analysis often do. One of the first applications of a moving-average techniques can be found in the book “Profits in the Stock Market” written by (Gartley, 1935), where he uses a 200-day SMA (close to equivalent of a 10-month SMA on daily data). Modern technical analysis has developed a lot since the introduction of the Dow Theory for more than 110 years ago, today almost all technical analysis rely heavily on computer power. When the computers started making their entrance in the 1970s, it marked an important turning point in technical analysis (Burch, 2011). It enabled a whole set of new techniques that could be tested out, and many tried to find new strategies that could have prevented the losses experienced in connection with the financial crisis in the early 1970s, as a result the literature on technical analysis

exploded. Still, there were a lot of skeptics claiming that the performance found using different technical analysis techniques was due to randomness, data-snooping or even if the reason for the performance in fact had existed, it would surely disappear in an out-of-sample application. The paper by (Brock, Lakonishok, & LeBaron, 1992) is by many considered one of the most important papers in the field of technical analysis. The paper changed the academic view on technical analysis from something that does not work, to something that might actually have some relevance.

(Lo, Mamaysky, & Wang, 2000) try to provide some underlying statistical foundation to why technical analysis rules might work. An article by (Levy, 1966) summarize technical analysts' theory beliefs into four points;

1. Market value is determined solely by the interaction of supply and demand.
2. Supply and demand are governed by numerous factors, both rational and irrational. Included in these factors are those that are relied upon by the fundamentalists, as well as opinions, moods, guesses and blind necessities. The market weighs all of these factors continually and automatically.
3. Disregarding minor fluctuations in the market, stock prices tend to move in trends which persist for an appreciable length of time.
4. Changes in trend are caused by the shifts in supply and demand relationships. These shifts, no matter why they occur, can be detected sooner or later in the action of the market itself.

The trend-following part of the technical analysis literature has gotten very popular in the last few years, for one simple reason, the performance has been incredibly good since year 2000. Papers like (Faber, 2007) and (Park & Irwin, 2007) reported that following different trend techniques could provide superior performance both in terms of volatility and returns when compared to a passive buy-and-hold strategy. The articles by (Faber, 2007) and (Faber, 2009) tests a 10-month simple-moving-average (SMA(10)) technique on five different asset classes and finds that it provides “equity-like returns with bond-like volatility and drawdown”. (Gwilym, Clare, Seaton, & Thomas, 2010) investigate the performance of timing strategies for investing in 32 international equity markets and find that trend following strategies deliver superior risk-adjusted returns when compared to a buy-and-hold strategy. (Clare, Seaton, & Thomas, 2013) show how

transaction costs have a very limited impact on the performance of the moving-average strategies (often not even considered in other market-timing papers). All the different moving-average articles mentioned has as good as no out-of-sample parts, which they also acknowledge as being a problem as out-of-sample deterioration is a known problem in technical analysis, as argued by (Aronson, 2006). Aronson proposes several plausible explanations for the deterioration, where he partly claims it is due to what he calls a random component in any historic series which one can by chance find patterns that can exploit. But this random component will change for the next sample of history, making it impossible to exploit out-of-sample. He also points to data-mining biases, where randomness and “the logic of data-mining”, where the best performing rules are always selected. One of the very few papers trying to deal with the potential of out-of-sample deterioration is (Zakamulin, 2013), he implements an out-of-sample simulation strategy that has the objective of providing performance results free of potential data-mining biases. In addition to real-life simulation he also tests alternative realizations of the stock price movements in order to remove Aronson’s random component, where the resulting performance is substantially worse than others have reported in-sample.

There are a lot of alternative trend following strategies, the most common use different versions of moving-averages to determine when what assets should be held. The general moving-average (MA) rule creates buy and sell signals on the basis of;

$$P_t > MA_t(k) \rightarrow \text{Buy signal}$$

$$P_t < MA_t(k) \rightarrow \text{Sell signal.}$$

Where P_t is the observed price in period t and k is the number of months taken into consideration, the $MA_t(k)$ is generally computed as;

$$MA_t(k) = \frac{\sum_{i=0}^{k-1} w_i P_{t-i}}{\sum_{i=0}^{k-1} w_i}.$$

There exist several different weighting schemes for the way the past prices are used in the MA calculation. The most common is the simple-moving-average (SMA), which simply is the average price over a specified period,

$$SMA_t(k) = \frac{1}{k} \sum_{i=0}^{k-1} P_{t-i}.$$

Other popular versions can be the linearly weighted-moving-average (WMA) where the latest observations are given more weight. Computed as

$$WMA_t(k) = \frac{\sum_{i=0}^{k-1} (k-i) P_{t-i}}{\sum_{i=0}^{k-1} (k-i)}.$$

Another version who also reduce the weight for the prices furthest away from period t is the exponential-moving-average (EMA),

$$EMA_t(k) = \frac{\sum_{i=0}^{k-1} \lambda^i P_{t-i}}{\sum_{i=0}^{k-1} \lambda^i},$$

which often use all realizations up to time t in computation of the EMA (meaning that k equals t), where each price observation is weighed down exponentially by a factor of λ for each observation apart from t. It is the SMA weighting scheme that is the most commonly used, and in order for this study to more easily be comparable with findings of other papers it is the SMA who will be implemented. Moving averages are not the only trend following strategies out there, some of the more popular alternatives can be the momentum rules (MOM). The MOM rule prescribes to hold an asset if the difference between the prices at P_t and P_{t-k} is positive, and sell if it is negative, so that

$$MOM_t(k) = P_t - P_{t-k},$$

$$MOM_t(k) < 0 \rightarrow \text{Sell},$$

$$MOM_t(k) > 0 \rightarrow \text{Buy}.$$

3. Data Sources and Data Construction

In this thesis I will use two data sets, which will be compared against each other and combined. The first data set is from the work of (Schwert, 1990) who created a monthly return data set for the period 1802-1925, where the data is available online from Schwert's homepage

(schwet.ssb.rochester.edu). He created the data set from combining and splicing together several indices, as no complete records exist of a sufficient quality that far back, I will use the period 1857-1925 (828 months). This data set provides both the dividend return (calculated as the monthly 12-month moving sums of the dividends paid) and the capital return, which together makes the total return for the period. So that when later talking about the return, it is the total return that is meant (capital return adjusted for dividend return). The starting point of the period is determined on the basis of risk free rate, because that is how far we can get a reasonable estimate for it and I will stop in 1925 in order for the two compared data sets to approximately be equal length and not to overlap each other. For the second data set I use the updated data from a paper by (Goyal & Welch, 2008), which gives data on S&P500 for the period 1926-2012 (1044 months). The data contains index values and dividend returns, so the monthly total return can easily be calculated from it. The monthly risk free rate of return is also included in the data set.

There are no available data on risk free rates for the period prior to 1920, in the previously mentioned Goyal & Welch paper they use a regression based on the commercial paper rate in New York to get risk-free short-term debt back to 1872. The regression has a very high explanatory power in-sample (where it is possible to double check the estimated risk free rate and the actual risk free rate), so the estimation is most likely valid out-of-sample as well. For the period 1857-1925 I use the same regression in order to make a risk free rate prior to Goyal & Welch's work. The resulting linear regression is used;

$$\text{Risk free rate of return} = -0.004 + 0.886 * \text{Commercial Paper Rate}.$$

The commercial paper rate is obtained from "National Bureau of Economic Research" (nber.org).

In 1914 there were no trade for four months, due to the First World War, so a linear approximation has been used to estimate the changes, this is done by splitting the movement between closing and opening index for the closed period equally between each of the four closed months.

The period 1857-1925 will be called the first period and 1926-2012 will be called the second period.

Some key features of the period first and second period are summarized in Table 1 (all data are monthly). They look quite equal to each other, but there are some differences. The average total return is higher for period two as a result of a near doubling of the capital return. Period one has a smaller standard deviation, which also comes to show with smaller extreme values providing a smaller range of returns.

Period	1857-1925	1926-2012
Observations	828	1044
Average capital return	0.32%	0.61%
Average dividend yield	0.43%	0.33%
Average total return	0.75%	0.93%
Standard Deviation (total return)	4.48%	5.49%
Min Return (total return)	-24.37%	-29.20%
Max Return (total return)	17.59%	42.70%
Range	41.96%	71.91%

Table 1: Descriptive statistics for period one (1857-1925) and two (1926-2012), all data are monthly.

4. In-Sample Tests

4.1 In-sample: Methodology

I implement a widely used technical trading rule to decide when an investor's wealth should be invested either in a risky asset or in a risk free asset. As (Faber, 2007) argues the rules that determine the trade should be easy for investors to follow while being mechanical enough to remove emotion and subjective decision-making. This resulted in three criteria on which all trend-following rules should be based on,

- 1) Simple, purely mechanical logic.
- 2) The same model and parameters for every asset class.
- 3) Price-based only.

The rule I will use certainly meets the criteria proposed above. The short version of the rule I employ in this thesis says that if the risky asset at current time is above the k-month simple-moving-average the cash should be invested into the market, if not, the investor should exit the market and invest the wealth into a risk-free investment instead. Something means that one need

to compute the different SMA(k) values. I will limit the SMA(k) lengths to $k \in [8, 15]$, in order for the presentation of the results not to get too complex and chaotic.

The simple-moving-averages (SMA) are computed as,

$$SMA_t(k) = \frac{1}{k} \sum_{i=0}^{k-1} P_{t-i},$$

where (P_1, P_2, \dots, P_T) is the monthly observed closing index prices. When this is done for all observations between $t \in [\max[k], T - 1]$, (starting point determined by k in order to make all SMA(k) vectors equal length), the SMA(k) matrix and the price vectors can be presented as,

$$\begin{pmatrix} SMA_{\max[k]}(\min[k]) & \cdots & SMA_{\max[k]}(\max[k]) \\ \vdots & \ddots & \vdots \\ SMA_{T-1}(\min[k]) & \cdots & SMA_{T-1}(\max[k]) \end{pmatrix} \text{ and } \begin{pmatrix} P_{\max[k]} \\ \vdots \\ P_{T-1} \end{pmatrix}$$

It is this SMA matrix that will be evaluated against the monthly closing index price vector in order to determine what asset to hold for the coming month. Such that for each month one can determine if the index or the risk free asset should be held for the next month, which has to be done for all the different lengths of the SMA:

$$\text{if } SMA_t(k) > P_t \rightarrow \text{invest in index for } (t + 1)$$

$$\text{if } SMA_t(k) < P_t \rightarrow \text{Invest risk free for } (t + 1).$$

Once it has been established what months you should be invested in what, for each of the different SMA(k) vectors, you can use that information to assign the realized returns of that asset. This will create a new vector for each k for all periods of time between $t \in [\max[k], T]$ consisting of the returns of alternately the index and the risk free asset. It is these new active return vectors that will be evaluated against a benchmark to see how that SMA(k) length performs. The benchmark used will be a passive buy-and-hold strategy.

An in-sample analysis of this type can be done in a lot of different ways, and has been done in a lot of different ways already. This in-sample part will consist of two analyses where the rule described above will be put to work on. One part will be a comparison of the two periods 1857-1925 and 1926-2012, and the second part will analyze the entire period of 1857-2012. The in

depth in-sample research around, is one of the main reasons why I am going to do a comparison of the period 1857-1925 to 1926-2012 instead of a more standard performance evaluation (in addition to making the results comparable to previous research). The point being that the data from 1926 to present date has been very thoroughly examined by several previous papers, so my contribution to the ongoing in-sample research will focus on a performance evaluation of a longer data set than previously tested, as well as a time period comparison.

To illustrate how a moving-average looks like, a plot of the 10-month moving-average and the index are shown in Figure 2.

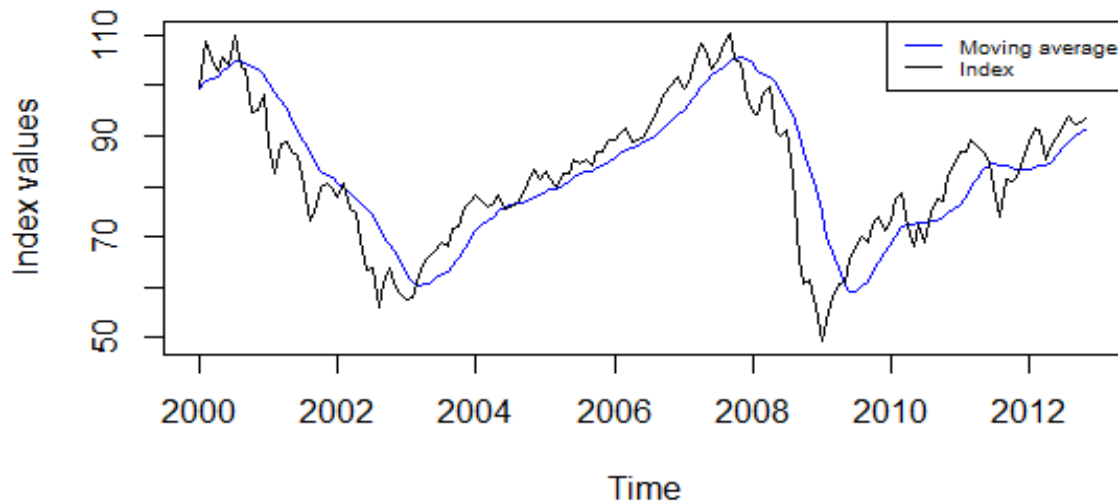


Figure 2: Illustrative example of how the SMA(10) vs. index looks like in the period 2000-2012.

There are many ways to do a performance evaluation, as already stated I will use the Sharpe ratio to rank the different versions. As a complement to the best performing SMA(k) lengths according to the Sharpe ratio, I will also report the geometric return as a measure of total end wealth for the investor. For after all, the goal of the investor is to maximize his or her wealth at the end of the investment period. The reason I will report both the arithmetic returns and the geometric returns are quite simple, the arithmetic return do not necessarily provide an unbiased picture of the observed wealth development. For instance if an extreme event happens and make a stock previously worth 100 \$, drop by 90 % in one day and the following day it increases by 90 %, the arithmetic return for the two days would be 0. But the geometric daily return would be - 55 %, as

the value would drop to 19 \$. In addition the drawdowns of the optimal rolling mean length will be inspected.

The maximum drawdown (MDD) in percent is calculated as the peak-to-trough decline in the asset value divided by the peak value (Burghardt, Duncan, & Liu, 2003) formally,

$$MDD_T = \max_{\tau \in (0, T)} \left\{ \frac{\max_{t \in (0, \tau)} X_t - X_\tau}{X_\tau} \right\}$$

For each decade, $\tau \in (0, 120)$, the maximum drawdown will be computed to show the strategy's ability to time markets and exit them in time. Below is an illustrative example (Figure 3) with the maximum drawdown superimposed for the index development from year 2000 to the end of 2012, with that period's maximum drawdown superimposed.

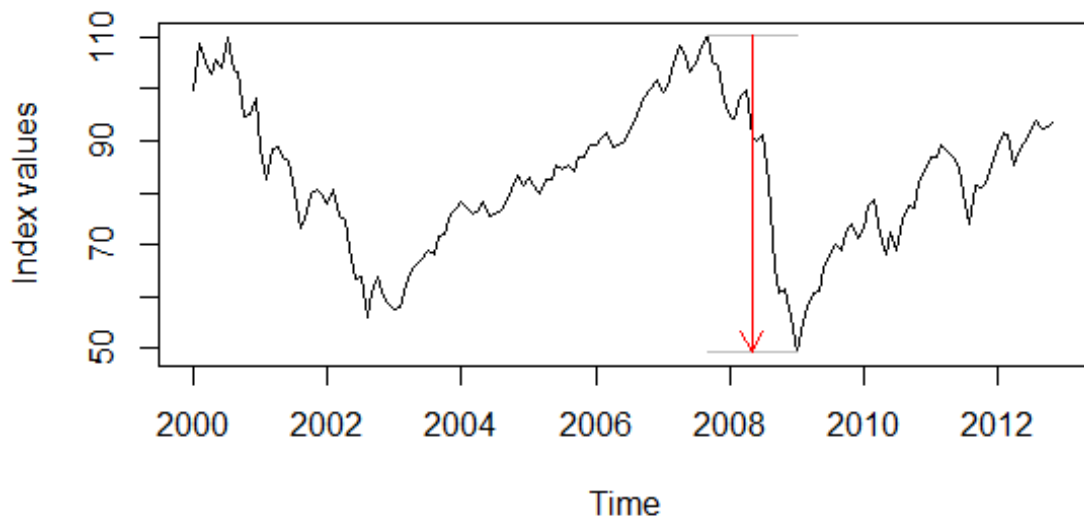


Figure 3: Illustrative example of the maximum drawdown for the period 2000-2012.

4.2 In-sample: Results

The results of the different SMA(k) lengths tested on period one and two, as well as the period as a whole are shown below in Table 2, 3 and 4.

Periode 1857-1925

K	8	9	10	11	12	13	14	15
Average monthly return	0.78%	0.76%	0.77%	0.79%	0.81%	0.78%	0.79%	0.78%
Geometric monthly return	0.75%	0.73%	0.74%	0.75%	0.78%	0.75%	0.75%	0.74%
Standard deviation	3.05%	3.04%	3.07%	3.02%	3.02%	3.09%	3.08%	3.12%
Min return	-13.1%	-13.1%	-13.1%	-13.1%	-13.1%	-20.3%	-20.3%	-20.3%
Max return	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%	15.7%
Range	28.8%	28.8%	28.8%	28.8%	28.8%	36.0%	36.0%	36.0%
Annualized Sharpe Ratio	0.442	0.416	0.424	0.448	0.482	0.437	0.440	0.423

Table 2: Performance of the different SMA(k) lengths for period one (1857-1925) in an in-sample test.

Periode 1926-2012

K	8	9	10	11	12	13	14	15
Average monthly return	0.82%	0.85%	0.87%	0.87%	0.88%	0.87%	0.86%	0.89%
Geometric monthly return	0.76%	0.79%	0.81%	0.81%	0.81%	0.80%	0.80%	0.82%
Standard deviation	3.45%	3.47%	3.43%	3.54%	3.55%	3.56%	3.56%	3.51%
Min return	-21.5%	-21.5%	-21.5%	-23.4%	-23.4%	-23.4%	-23.4%	-23.4%
Max return	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%
Range	37.8%	37.8%	37.8%	39.7%	39.7%	39.7%	39.7%	39.7%
Annualized Sharpe Ratio	0.523	0.550	0.579	0.561	0.568	0.554	0.550	0.582

Table 3: Performance of the different SMA(k) lengths for period two (1926-2012) in an in-sample test.

Periode 1857-2012

K	8	9	10	11	12	13	14	15
Average monthly return	0.80%	0.81%	0.83%	0.83%	0.85%	0.83%	0.83%	0.84%
Geometric monthly return	0.76%	0.76%	0.78%	0.78%	0.80%	0.78%	0.78%	0.79%
Standard deviation	3.28%	3.29%	3.27%	3.32%	3.33%	3.36%	3.36%	3.35%
Min return	-21.5%	-21.5%	-21.5%	-23.4%	-23.4%	-23.4%	-23.4%	-23.4%
Max return	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%	16.3%
Range	37.8%	37.8%	37.8%	39.7%	39.7%	39.7%	39.7%	39.7%
Annualized Sharpe Ratio	0.489	0.494	0.514	0.515	0.532	0.505	0.504	0.516

Table 4: Performance of the different SMA(k) lengths for the entire period (1857-2012) in an in-sample test.

As can be seen for period one (Table 2) the best performing simple-moving-average length is 12 months, when ranked according to the Sharpe ratios. The resulting SR ratio is 0.482. The eight different lengths tested all give results that are quite equal to each other, with average monthly returns ranging from 0.0076 to 0.0081². If we look at the geometric returns (computed from differences in end wealth), even small differences can have large impacts when the data series becomes this long. The difference in choosing the best performing length instead of the worst performing length, with respect to the end wealth, is more than 55%. Even though the performance is quite alike across the different lengths tested, there is one of them that stands out a bit, the SMA(12). When we compare the different Sharpe ratios the next best realized Sharpe ratio it is 0.032 lower, which is quite a lot when we take into consideration that the entire range of Sharpe ratios is only 0.066.

The results for period two are shown in Table 3, where the best performing SMA(k) length is 15 months. For this period we can't see the same evidence of one length being superior compared to the others, here we have several Sharpe ratios in the neighborhood of the maximum one. Both 10 and 12 month SMA seem to perform about equal as the 15 month version, respectively with SRs at 0.579 and 0.568 compared to 0.582. The second period's Sharpe ratios are a bit higher than what the strategy achieved in period one, which is simply due to the fact that there were a higher mean return for the latter period. The standard deviation is very similar to each other in all of the SMA(k)'s tested. In close to all of the cases, ranking according to either mean return, end wealth or Sharpe ratios give the same ranking, but not all. According to mean return and Sharpe ratio's the best strategies for period two is 15, 10, 12 starting with the best, but for end wealth its 15, 12, 10.

If the two periods are treated as one (Table 4), we find that the best length is SMA(12), with a Sharpe ratio of 0.532. Else, most of what is said about period one and two is also true for the entire period. In addition since the two periods are about the same length, the period as a whole

² When talking about the average mean return it is the arithmetic mean return that is meant and not the geometric.

$$\text{Arithmetic return} = \frac{1}{n} \sum_{j=1}^n r_j.$$

$$\text{Geometric return} = \left((1 + r_1)(1 + r_2) \dots (1 + r_n) \right)^{\frac{1}{n}} - 1.$$

will almost be an average of the two. To avoid repeating myself, there will only be limited comments on the results for the whole period.

Strategy type:	Passive	Timing	% change	Passive	Timing	% change	Passive	Timing	% change
Periode	1857-1926	1857-1926	% change	1926-2012	1926-2012	% change	1857-2012	1857-2012	% change
K		12			15			12	
Average monthly return	0.76%	0.81%	7%	0.93%	0.89%	-5%	0.86%	0.85%	-1%
Geometric monthly return	0.68%	0.78%	15%	0.78%	0.82%	6%	0.74%	0.80%	9%
Standard deviation	4.29%	3.02%	-30%	5.49%	3.51%	-36%	5.00%	3.33%	-33%
Min return	-20.29%	-13.08%	-36%	-29.20%	-23.38%	-20%	-29.20%	-23.38%	-20%
Max return	15.72%	15.72%	0%	42.70%	16.28%	-62%	42.70%	16.28%	-62%
Range	36.01%	28.80%	-20%	71.91%	39.65%	-45%	71.91%	39.65%	-45%
Annualized Sharpe Ratio	0.2952	0.4821	63%	0.3993	0.5816	46%	0.3573	0.5321	49%

Table 5: Performance of the best in-sample length for period one, two and the entire period compared to the performance of a passive strategy

If we compare the best SMA(k) length in a back test with its passive counterpart as done in the Table 5 above, there are several interesting things worth noting. The most obvious might be the drastic improvements in Sharpe ratios, for period one it has increased by a staggering 63% and for period two 46%. The large improvements are mainly due to the large drop in standard deviation for each of the two periods, which is about 1/3 less in both periods for the active strategy. The mean return have improved by about 7% for period one, while dropping by 5% for period two. As mentioned in the methodology part, looking at the average returns may give a biased picture of what is really going on. If we instead look at the geometric returns, the improvements are even larger than for the average monthly returns. This is due to how high volatility diminishes geometric returns (Chambers & Zdanowics, 2014), or said in a different way, increase the difference between average return and geometric return. Period one's average return increase was 7%, its geometric return was at 15%. While period two experienced a drop in the average return of 5%, the geometric return increased by 6%. The same story goes for the period as a whole, with a decrease in average return of 1% and an increase in geometric return of 9%. These results show why market timing rules, like SMA, have gotten such a resurgence of popularity in the recent years, simply because the results the techniques can provide is exceptional when used in a back-test. The range of the observed returns, have drastically dropped for the active strategy as well, when compared to the passive, with drops of 20% and 45% for period one and two. When we look at the range, it is not only the minimum return that has improved as a result of the active trading strategy, which is something good. But the maximum returns have also decreased, which of course is bad in the eyes of the investor. If we look at the Sharpe's test statistics for period one they are presented in Table 6:

	1857-1925	1926-2012	1857-2012
SMA(k)	12	15	12
z Value	-1.9971	-2.1496	-2.6379
Probability	4.58%	3.16%	0.83%

Table 6: Test for Sharpe ratio change significance for the best SMA(k) in a back-test for period one, two and the entire period.

As Table 6 shows, the best performing SMA(k) in all the periods lead to statistically significant higher Sharpe ratios at a 5% confidence level. This means that we can reject H_0 and say that the data support the alternative hypothesis that says that the two Sharpe ratios are not equal. The

active strategy does provide a statistically significant higher Sharpe ratio than the passive for all of the three periods tested.

To inspect the improvements of the drawdowns, the maximum drawdowns for each 10-year period in period one and two for the best active strategy and the passive strategy will be shown, see Table 7 and 8.

10-year period	Passive Strategy		Active Strategy, k=12	
	Months from period start	Drawdown	Months from period start	Drawdown
1859-1868	64-75	-22%	22-24	-22%
1869-1878	86-102	-35%	8-9	-8%
1879-1888	45-66	-23%	84-88	-7%
1889-1898	38-55	-36%	80-84	-16%
1899-1908	95-106	-35%	85-90	-12%
1909-1918	94-107	-25%	84-85	-7%
1919-1925	10-30	-22%	10-11	-10%
Average Length / Drawdown	15.57	-28%	2.57	-12%

Table 7: Maximum drawdowns for each 10-year period in period one for the active strategy and passive.

10-year period	Passive Strategy		Active Strategy, k=15	
	Months from period start	Drawdown	Months from period start	Drawdown
1926-1935	44-77	-83%	44-46	-23%
1936-1945	14-27	-50%	14-83	-51%
1946-1955	5-11	-22%	5-25	-17%
1956-1965	72-78	-22%	7-20	-13%
1966-1975	84-105	-43%	114-117	-11%
1976-1985	59-79	-17%	59-68	-9%
1986-1995	20-23	-30%	20-22	-23%
1996-2005	56-81	-45%	56-59	-13%
2006-2012	22-38	-51%	52-56	-17%
Average Length / Drawdown	15.89	-40%	13.88	-20%

Table 8: Maximum drawdowns for each 10-year period in period two for the active strategy and passive.

The drawdown tables above are to be read as following, the first number in the “Months from period start” is the months from the period start to the month of the peak, and the second number is the month of the trough away from the periods start. So for instance, the first line in the table for period one, month 64 after 1859, means that the peak was in April 1864 and the trough was hit 11 months later (in the 75th month after 1859) meaning March 1865 and the asset decline resulted in a 22% value drop. There are specially two things that stand out when looking at the two tables above, the average drawdown of the active strategy is a lot lower than if an investor followed a passive buy and hold strategy. In addition the average length of the drawdown period is lower for the active than the passive strategy.

For period one (Table 7), the average 10-year drawdown has been reduced from 28% to 12% and the average length of the maximum drawdowns has been reduced from an average of 15.57 months to only 2.57 months. Period two also have some improvements, reducing the average maximum 10-year drawdown from 40% to 20%. The average length of the drawdown period has “only” been reduced from 15.89 to 13.88 months, some of the reason for this, is due to what happened in the period 1936-1945. Where the passive strategy uses 13 months to hit the trough and while the active strategy use 69 months, pulling the average significantly up. If that 10-year period is excluded, the average length would be almost half the length.

The drawdowns experienced in period two (Table 8) is on average a bit larger than the ones experienced in period one, especially the periods during the Great Depression are huge. The index experienced a drop of a staggering 83% from the autumn 1929 to the middle of 1932, in addition to 4 other sub-periods with more than 40% drops. Period one had no 10-year periods with drawdowns larger than 40%. But an important point is that this should not necessarily be put to much weight into, as there are a good possibility that choosing a different year to start each 10-year period might alter the results quite a bit. If a drawdown is “cut in half” by the sub periods start or end points it can fail to get reported as the maximum drawdown, even though it should have been in one of the periods. This is some of the disadvantage with a setup like this, but it still illustrates the point, the active strategy has on average far smaller maximum drawdowns when compared to a passive buy-and-hold strategy, something that is true for both periods.

Some key numbers of the trades following the active strategy for period one and two are added in Table 9 below.

Period	1857-1925	1926-2012	1857-2012
K	12	15	12
% time long index	56.58%	67.53%	62.84%
Number of round-trips in total	48	44	97
Average round-trips per year	0.696	0.506	0.622
Average length of index holding, in months	9.760	16.023	12.128
Average length of risk free holding, in months	7.490	7.705	7.171

Table 9: Key numbers following the trade of the optimal active strategy found in a back-test for period one, two and the entire data-set.

We can see some interesting differences between the trades in the two periods (Table 9), period one's trade led to being long the risky index for about 57% of the time, period two were long almost 68% of the time. Period one also has more round-trips than period two, even though period one is slightly shorter than period two. This resulted in considerably higher average round-trips per year number for period one. Since period one has more trades per year, one would assume that the average holding length of the index and the risk free would be shorter. The length of the index holding is considerably lower for period one, with 9.76 months and 16.02 months for period one and two, but the same cannot be said for the risk free holding. The average lengths of the risk free holding are almost identical, which means that the reason for the fewer trades in period two must be seen in correlation with the longer index holding periods.

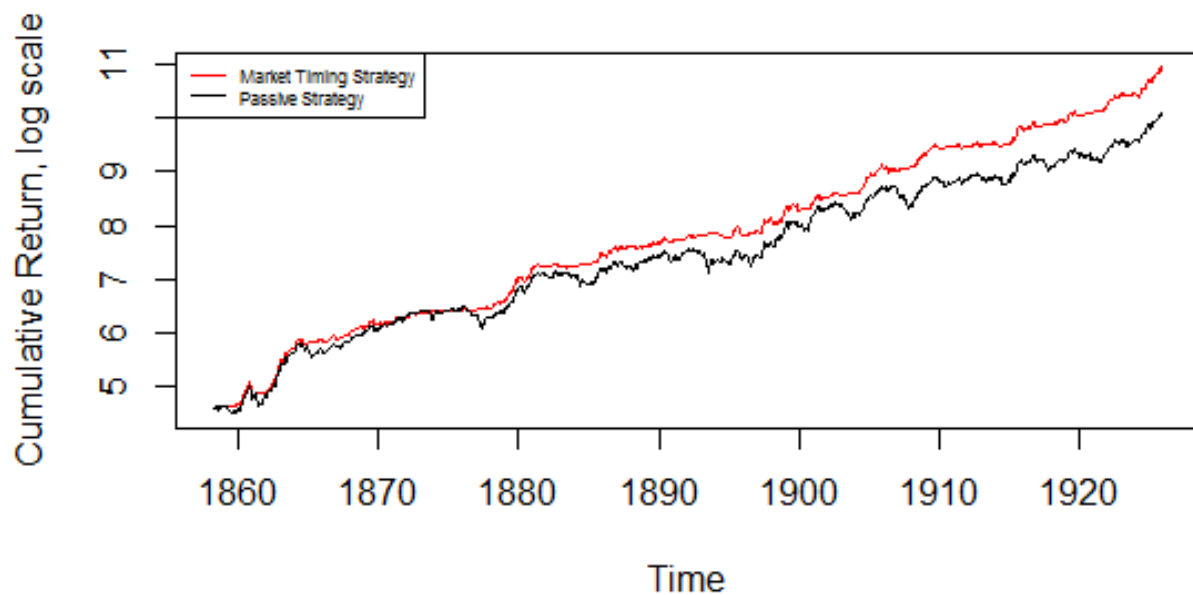


Figure 4: SMA(12) and passive index development for period one on a logarithmic scale.

As we can see in Figure 4 the market timing strategy is above the passive for most of the time and ends a fair amount above it. Even though it ends well above the passive index, we can see that for a long period of time, it did not perform better, it is actually about the same for the first 20-25 years. Especially the years between 1900 and 1925 the active strategy performs very well, we can see that this is a period filled with lots of relatively short periods where the market either goes up or down, and only very few periods with a flat development.

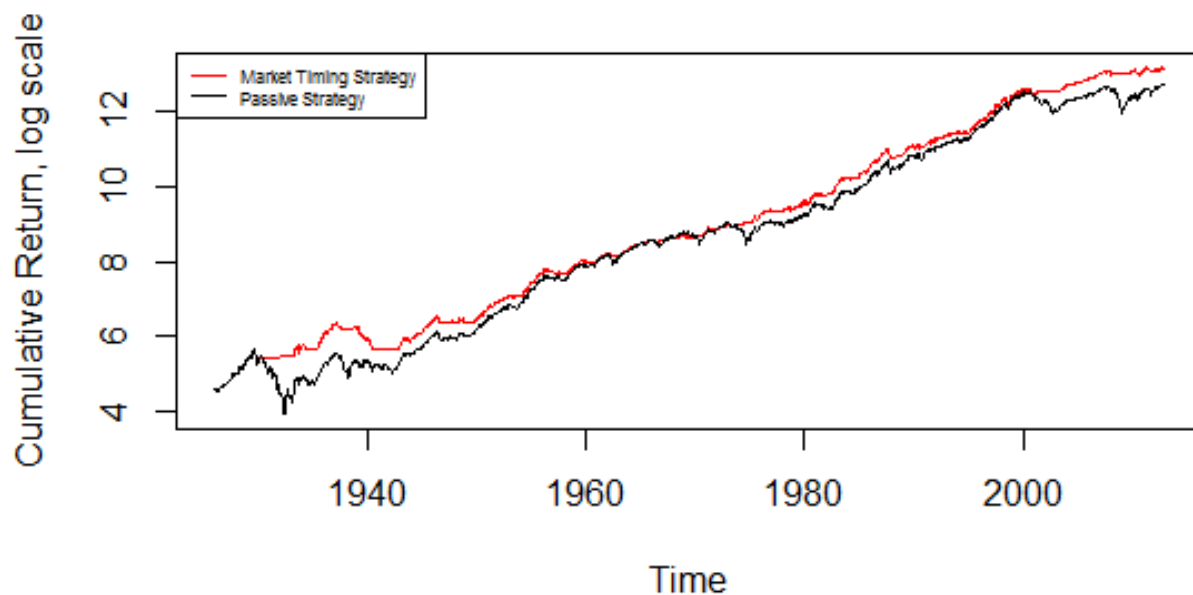


Figure 5: SMA(12) and passive index development for period two on a logarithmic scale.

The second period has quite a different development than the first period as we can see from Figure 5. This period is dominated by long bull and bear markets, especially the events around the Great Depression in the 1930s stand out. The active strategy performs exceptionally well during that period, bringing the market timing strategy value significantly higher than the passive index value. But the lead that was built up during the Great Depression is gradually eaten away in the 20-year period starting from 1940, when the accumulated wealth is as good as equal once again in 1960. The active and passive strategy follows each other quite close in the 40-year period between 1960 and 2000, the active is slightly ahead after the bad years in the early 1970s, but that lead gradually decreased as the almost 20-year long bull market from 1980 to 2000 started. By the time the downturn in the 2001 started, the active and passive strategy was once again almost equal, but the active quickly gained ground as yet another persistent bear market started. The happenings in 2007 further increased the discrepancy of the strategies, ensuring that the active strategy ends up above the passive strategy in terms of wealth, but still not as much above as it were under the Great Depression.

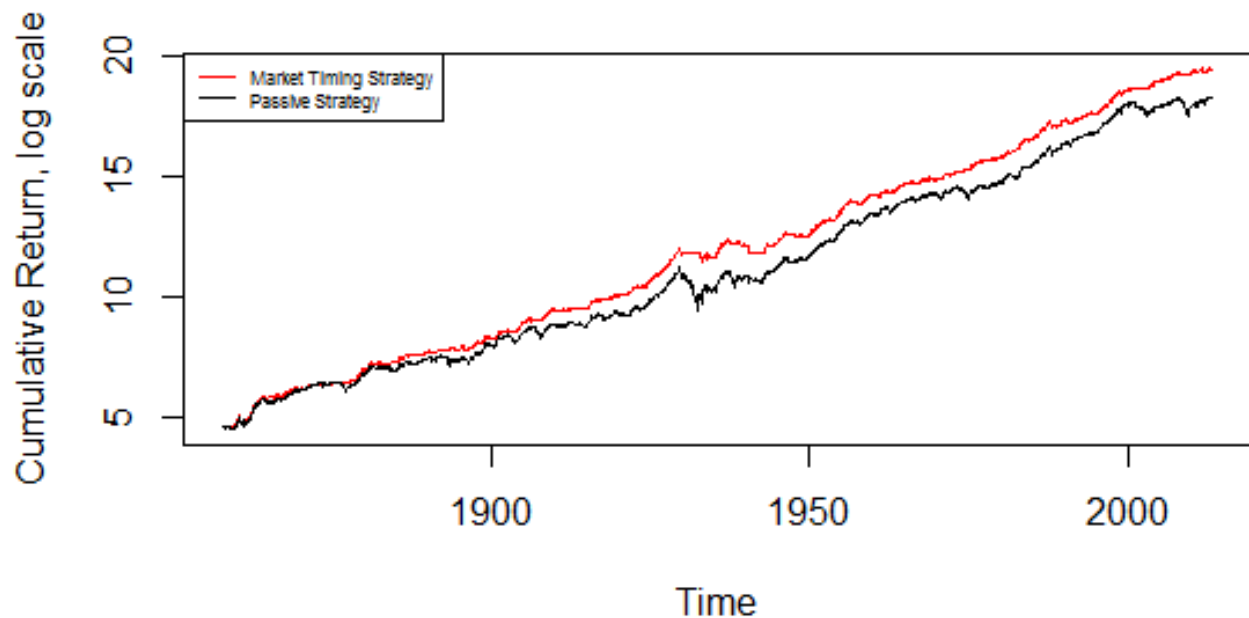


Figure 6: SMA(12) and passive index development for the spliced data-set, 1857-2012.

If the two periods are treated as one, the active strategy would look like shown in Figure 6. Even though this uses the SMA(12) for the second period as well (instead of SMA(15)), it definitely resembles the two previous graphs just spliced together, which it also of course what it essentially is. We can see that the values following the active strategy is above the passive index for almost all of the years, but with a varying magnitude. What has already been said separately for period one and two still applies when the periods are treated as one.

The results shown for the in-sample back test part clearly shows that the SMA(k) can perform exceptionally well under certain circumstances, as it do outperform our passive benchmark in a back test in both the periods tested. A further discussion and comparison of the results will take place in the section 6 of this thesis, the discussion part.

5. Out-of-Sample Tests

5.1 Out-of-Sample: Methodology

As mentioned earlier this type of back testing different technical trading strategies, like the SMA rule, can be exposed to serious data-mining issues, which in turn can lower the credibility of the results in some cases. Where the risk is in that when you are trying out several different strategies or rules, they will vary in performance and most likely only the best performing strategies looked at will be reported. This especially applies to examples where there are no academic grounds to suspect that one version of the strategy should outperform another, like what SMA(k) length to use. As (Zakamulin, 2013) also points out, a lot of literature on this subject has obviously tested several different moving average lengths and only reported the best performing length for the periods tested. Like the paper by (Faber, 2007), where he only reports the results of using a SMA(10). Even though the length he used was the best performing choice for the periods he looked at, that do not mean that that length will be the best for other periods. In fact, if we look at what length of the simple-moving average that would have resulted in the best SR for each 10-year period in the data set (first observation is for the period 1860,1-1870,1, then the second observation is for the period 1860,2-1870,2 and so on), we can see that what SMA(k) length gives the best trading signals is very volatile (Figure 7).

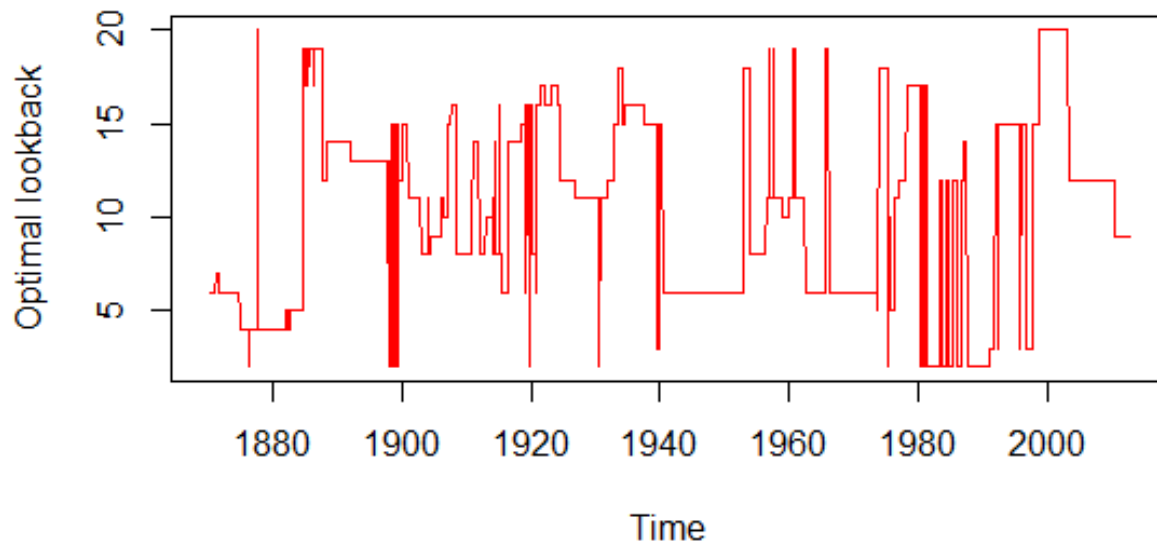


Figure 7: The optimal SMA(k) length changes for a 10-year rolling window in the period 1860-2012.

The optimal length k has a mean of 10.21, a median of 11 and a standard deviation of 4.97.

This is the reason why the results of several SMA(k) lengths was inspected and reported for the in-sample part of this thesis and is also the reasons to why an out-of-sample simulation with both a rolling and an expanding approach will be tested. As there are no restricting reporting concerns to take into consideration, the different SMA(k) length to be tested will be increased from $k \in [8,15]$ for the in-sample part to $k \in [2,20]$ for the out-of-sample part.

This out-of-sample simulation is a method that has the intention of simulating the choices an investor has to make in real-life. In this paper these choices would be to find the optimal SMA(k) length, and use that to decide whether or not he wants to be invested in the risky asset or the risk free asset for the next month. From the investor's point of view, he is trying to decide if he thinks that the market will go up or down in the next month and that is all he is interested in. Of course if he thinks it will go up, he will stay long and if not, he will want to put his money in the bank instead. As previously said he will base that decision on the rule that if the last observation of the closing price or the index is above the SMA(k), he will be invested in the index and if not, he will

choose a risk free investment instead. Since the different lengths of the SMA(k) may lead to different decisions, he needs to choose a k for each single month and use that length of SMA(k) to evaluate against the closing index prices. Which leads to the next big question, how to choose what SMA(k) length to base that decision on?

The way this will be done is to take an initial in-sample period and check the performance of each SMA(k) length and whatever length that performs the best in that period, according to some optimization criteria (SR still used), and use that in the rule to decide the capital allocation problem. This exercise will then be repeated for each single month that passes. If we let the total number of observations range from 1 to T, and τ is denoting the split point between what is considered in-sample and out-of-sample. So the initial in-sample period or rolling window length (dependent on whether the expanding or rolling strategy is used) is denoted as $[1, \tau]$ and then when deciding where to invest the first period, $[\tau + 1]$, first optimize according to the Sharpe ratio for the period initial in-sample/rolling window to find what SMA(k) length to use the following month,

$$\max_{k \in [2, 20]} SR(r_1, r_2, \dots, r_\tau).$$

Then use that optimal SMA length to evaluate against the current closing index price and base the decision on what asset you want to hold for the coming month. Then the next month the process can be repeated to make a decision for period $[\tau + 2]$. But first the investor needs to make a choice of what type of a look-back window he wants to use, either an expanding or a rolling window. Two strategies that are quite similar, but that can lead to different results. The reason both an expanding and a rolling strategy is tested is that they are both viable options, there is no “right or wrong” choice. For the expanding window type, for every iteration the sample that is used to decide what the optimal SMA(k) length is increased by one. So that if you initially started following the rule with $[1, \tau]$ as start and split points, after n iterations you would find the k that maximize according to the optimization criteria for the period with start and split as $[1, \tau + n - 1]$. Then the value of that optimal SMA(k) will be evaluated against the passive closing index price the $[\tau + n - 1]$ period to decide which of the two assets are to be held for period $[\tau + n]$.

The only thing that changes for the rolling window type is that you keep the period you optimize the SMA(k) length from, a constant. So that after n iterations the period would be ranging between $[n, \tau + n - 1]$.

As for what optimization criteria to use, once again the Sharpe ratio is the logical choice, although it has some drawbacks as previously discussed, it is all in all a very good measure of reward-to-risk. So that the general maximization criteria for the nth iteration of the expanding window would look like

$$\max_{K \in [2, 20]} SR(r_1, r_2, \dots, r_{\tau+n-1}).$$

And for the rolling window with a length of τ ,

$$\max_{K \in [2, 20]} SR(r_n, r_{n+1}, \dots, r_{\tau+n-1}).$$

There are still some considerations that need to be thought through, for instance how long should the initial in-sample part be and how large should the rolling window be? I will test several different lengths and compare the performance. The different lengths from start to split considered, will range from 5 to 50 years for both the expanding and the rolling strategy. For the comparison of the Sharpe ratios to be unbiased the periods compared to each other need to be exactly the same length and be for the same period. Something that isn't ideal when testing such a variety of different lengths of initial in-sample periods and rolling windows, meaning that even if we only test for a 5 year length, the first observation that can be used are still after the maximum length tested, in this case 50 years. So when holding the split date and end date fixed, the variable that needs to be changed is the starting point. In this thesis I have got index and risk free rate of return data ranging from 1857-2012, meaning that this performance comparison can only get out-of-sample simulation data for approximately 1910-2012 (you lose 20 observations due to rolling mean calculations in addition to the 600 from maximum initial in sample size tested).

The best length for both the expanding and the rolling, ranked by Sharpe ratio, will be further evaluated, with an examination of the drawdowns and the indices produced following the active and the passive strategies will be shown. In addition I will show plots of the performance of the active strategies versus the passive strategies, which will be used as a tool to show what periods the active strategies performs well and what periods it does not. This will be computed as the

logarithmic values of the active index values divided by a risk adjusted version of the passive index values or put formally, where σ_A, σ_P are the standard deviations of the active and passive strategy and the r_t^P, r_t^f are the passive and risk free returns for period t. This makes $r_t^{P'}$ the risk adjusted passive returns, which have the same standard deviation as the active strategy,

$$r_t^{P'} = \frac{\sigma_A}{\sigma_P} * r_t^P + \left(1 - \frac{\sigma_A}{\sigma_P}\right) * r_t^f,$$

$$\sigma_A = \sigma_{P'}.$$

If the active returns (the returns resulting from following either the expanding or the rolling strategy) and the risk adjusted passive returns are made to an index.

$$I_t^A = I_0 e^{r_1^A + r_2^A + \dots + r_t^A} = I_0 e^{\sum_{i=1}^t r_i^A}$$

$$I_t^{P'} = I_0 e^{r_1^{P'} + r_2^{P'} + \dots + r_t^{P'}} = I_0 e^{\sum_{i=1}^t r_i^{P'}}$$

It is these indices that are divided by and taken the natural logarithmic value of to create the relative strength, V_t , for each month.

$$V_t = \log\left(\frac{I_t^A}{I_t^{P'}}\right)$$

If the expression is swapped with the one found earlier we get

$$V_t = \log\left(\frac{e^{\sum_{i=1}^t r_i^A}}{e^{\sum_{i=1}^t r_i^{P'}}}\right)$$

$$V_t = \log\left(e^{\sum_{i=1}^t r_i^A}\right) - \log\left(e^{\sum_{i=1}^t r_i^{P'}}\right)$$

$$V_t = \left(\sum_{i=1}^t r_i^A - \sum_{i=1}^t r_i^{P'}\right)$$

This rewriting illustrates what this really is, the cumulative difference in performance for the active and the risk adjusted passive strategy. This means that if V_t increases, the active strategy is outperforming the passive strategy and vice versa if V_t decrease.

5.2 Out-of-sample: Results

Below are the results of the different lengths tested for both strategies, with different starting points, but with equal length of the period between split and end. I will begin showing the performance of all lengths tested, before showing the changes of the best performing length vs. the passive, the best performing length will also have the trades, drawdowns and cumulative wealth analyzed.

Rolling Strategy, lbs 2-20

Rolling window size, in months	60	120	180	240	300	360	420	480	540	600
Start	1905	1900	1895	1890	1885	1880	1875	1870	1865	1860
Split	1910	1910	1910	1910	1910	1910	1910	1910	1910	1910
End	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012
Average monthly return	0.84%	0.81%	0.82%	0.80%	0.83%	0.83%	0.85%	0.82%	0.82%	0.78%
Standard Deviation	3.44%	3.33%	3.40%	3.34%	3.37%	3.39%	3.37%	3.41%	3.38%	3.36%
Range	40.56%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%
Annualized Sharpe Ratio	0.53672	0.52039	0.52528	0.51444	0.5406	0.5328	0.55311	0.51586	0.52295	0.4812

Table 9: Performance of different rolling-window sizes using a rolling out-of-sample simulation strategy, equal length of the time between split and end (1910-2012).

Expanding Strategy, lbs 2-20

Initial in sample, in months	60	120	180	240	300	360	420	480	540	600
Start	1905	1900	1895	1890	1885	1880	1875	1870	1865	1860
Split	1910	1910	1910	1910	1910	1910	1910	1910	1910	1910
End	2012	2012	2012	2012	2012	2012	2012	2012	2012	2012
Average monthly return	0.84%	0.80%	0.82%	0.84%	0.86%	0.86%	0.85%	0.86%	0.85%	0.84%
Standard Deviation	3.40%	3.42%	3.42%	3.43%	3.43%	3.43%	3.45%	3.44%	3.45%	3.47%
Range	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%	39.65%
Annualized Sharpe Ratio	0.5403	0.49881	0.51413	0.53902	0.56009	0.56009	0.5459	0.55195	0.54601	0.53486

Table 10: Performance of different initial-in-sample sizes using an expanding out-of-sample simulation strategy, equal length of the time between split and end (1910-2012).

From the performance of the rolling strategy (Table 9) it is a rolling window size of 420 months, or 35 years, that performs the best in terms of Sharpe ratio. The range of the Sharpe ratios is relatively narrow, with observations between the best performing window size of 0.553 and the worst size achieves 0.481. Also here I find that the main reason for the differences in the Sharpe ratios has to do with the differences in the numerators and only in a limited degree with the denominator. There are several of the lengths tested that are in the neighborhood of the 420 month version, but the bottom observation, the 600 month, is the definitive worst performing length tested. The range of the maximum and minimum return observation is also close to equal for all of the window sizes.

The expanding strategy, Table 10, have much of the same features like the rolling window has, with low variation in the both the average return and the volatility, but of course there are still some variability. For the expanding strategy the best performing length is a tie between having an initial-in-sample part of 300 and 360 months, the resulting Sharpe ratio is 0.560. I find that using a 120-month initial-in-sample result in the worst performance, with a SR of 0.499.

For the rolling strategy, there might be a vague pattern that the longest window sizes tested perform a bit worse than the ones tested in the other end of the scale and for the expanding strategy, there might be poorer performances for the smallest initial-in-sample sizes, but this analysis do not look deeply enough into the matter to really claim such a relationship in either of the cases. It is hard to get a feel that one of the lengths are more appropriate than the other based on this, the “relationship” between the different lengths and the performance can just as likely be coincidental or be caused by something else. Even though the differences in the Sharpe ratios are small, most of them are still statistically significant, the probability and the test statistics of the best vs. the worst performing length for both strategies are presented in Table 11.

Period	1857-2012	1857-2012
Strategy	Rolling	Expanding
Lengths tested against each other	420 and 600	120 and 300
z Value	-2.501	-2.261
Probability	1.24%	2.37%

Table 11: Statistical significance of the difference found in the SR of the best vs. the worst initial in-sample periods and rolling-windows tested.

In short, table 11 says there are clear evidence supporting that there are differences in the Sharpe ratios for the different lengths of both the initial in-sample and the rolling window.

If we further look at the best performing lengths for the expanding and rolling strategy in comparison with the passive index, we find similar results as we did for the in-sample-part.

Period	1910-2012	1910-2012	% Change	1910-2012	% Change
Strategy	Passive	Rolling	-	Expanding	-
Window/initial-in-sample	-	420	-	300	-
Average return	0.90%	0.85%	-6%	0.86%	-4%
Geometric return	0.76%	0.79%	4%	0.80%	6%
Standard deviation	5.27%	3.37%	-36%	3.43%	-35%
Range	71.91%	39.65%	-45%	39.65%	-45%
Sharpe ratio	0.3878	0.5531	43%	0.5601	44%

Table 12: Key figures of the best performing window/initial-in-sample period for the OOS-simulation compared with the passive strategy.

In Table 12 we can see that the Sharpe ratios have increased a fair amount for the two strategies in comparison with the passive performance, 43% and 44% increase for the rolling and expanding strategy. The passive index performed very well in the period as well, with a Sharpe ratio of 0.388. Also here we can see some good examples of how the geometric return and average return may give a different picture of the increase/decrease in end wealth. It is also worth noting that the only reason the Sharpe ratio is increasing, is due to the drop in standard deviation of 36% and 35% for the two strategies. The results of a SR test of the changes between the Sharpe ratios for the two active strategies and the passive are added below in Table 13.

Period	1910-2012	1910-2012
Strategy	Rolling	Expanding
Lengths tested against passive	420	300
z Value	-1.910	-2.024
Probability	5.61%	4.30%

Table 13: Statistical significance of SR changes for the two active strategies vs. passive for different starting points but with split 1910 and end 2012.

In both cases we are about 5% certain that the H_0 (that the SRs are of equal size) is correct, but the rolling strategy is strictly speaking not significant at a 5% level. We can with close to 95% certainty say that there is a difference between the Sharpe ratios for the two active strategies and the passive strategy.

The rolling and the expanding strategy seem very equal to each other based on Table 12 above, but if we inspect some key numbers of trade, Table 14, we can find some differences for the trades following the strategies.

Period	1910-2012	1910-2012
Strategy	Rolling	Expanding
Window/Initial in sample	420	300
% time long index	64%	65%
Number of round-trips in total	79	55
Average round-trips per year	0.767	0.534
Average length of index holding in months	9.945	14.539
Average length of risk free holding in months	5.701	7.934

Table 14: Key figures of the trade following the best performing OOS-simulation for the rolling and the expanding strategy for different starting points but with split 1910 and end 2012.

Even though they have a very similar time long the risky asset, they have some significant differences in the amount of trades. Something that is also shown in the average holding length of the two different assets. The rolling strategy had 79 round-trips in total, compared to only 55 for the expanding strategy, a difference of almost 44%. The average length of index holding is about 10 months for the rolling, compared to 14.5 months for the expanding. For the risk free average holding length the same pattern can be seen, with a shorter average for the rolling than the expanding. The relative decrease in the average holding length is about the same for both the risky and the risk free asset.

The maximum drawdowns of the rolling and the expanding strategy compared with the passive strategy are shown in Table 15.

10-year period	Passive Strategy		Expanding, initial in sample 300		Rolling, window size 420	
	Months from period start	Drawdown	Months from period start	Drawdown	Months from period start	Drawdown
1910-1919	83-96	-25%	118-119	-10%	118-119	-10%
1920-1929	117-120	-33%	116-118	-23%	116-118	-23%
1930-1939	4-30	-79%	86-119	-36%	86-119	-38%
1940-1949	78-84	-22%	77-97	-17%	3-35	-29%
1950-1959	92-97	-15%	79-92	-13%	79-92	-9%
1960-1969	108-116	-13%	107-114	-11%	107-119	-9%
1970-1979	37-58	-43%	104-106	-9%	36-47	-15%
1980-1989	93-96	-30%	92-94	-23%	92-104	-26%
1990-1999	103-105	-15%	102-104	-15%	102-104	-15%
2000-2009	95-111	-51%	94-97	-11%	8-11	-13%
2010-2012	17-22	-16%	4-8	-17%	4-8	-17%
Average Length / Drawdown in months	9.82	-31%	8.09	-17%	11.36	-19%

Table 15: Maximum drawdowns of every 10-year period for the best performing out-of-sample length for the expanding and rolling strategy for different starting points, but with split 1910 and end 2012.

Both strategies have smaller maximum drawdowns on average when compared to the passive. The passive had an average maximum drawdown for 10-year periods of 31%, while the rolling and expanding had 19% and 17% respectively. There is not very large changes in average length of the drawdown periods, the passive averaged about 10 months, while the rolling had close to 11.4 months on average and for the expanding they lasted about 8 months. We can see that the two active strategies has a lot of the exact same drawdown periods, actually in 6 of the 11 decades, which also backs up the fact that these two strategies only marginally differ from each other.

The graphical plot of the wealth following the expanding strategy with a 300 month initial-in-sample part and the rolling strategy with a 420 month rolling window is shown in Figure 8.

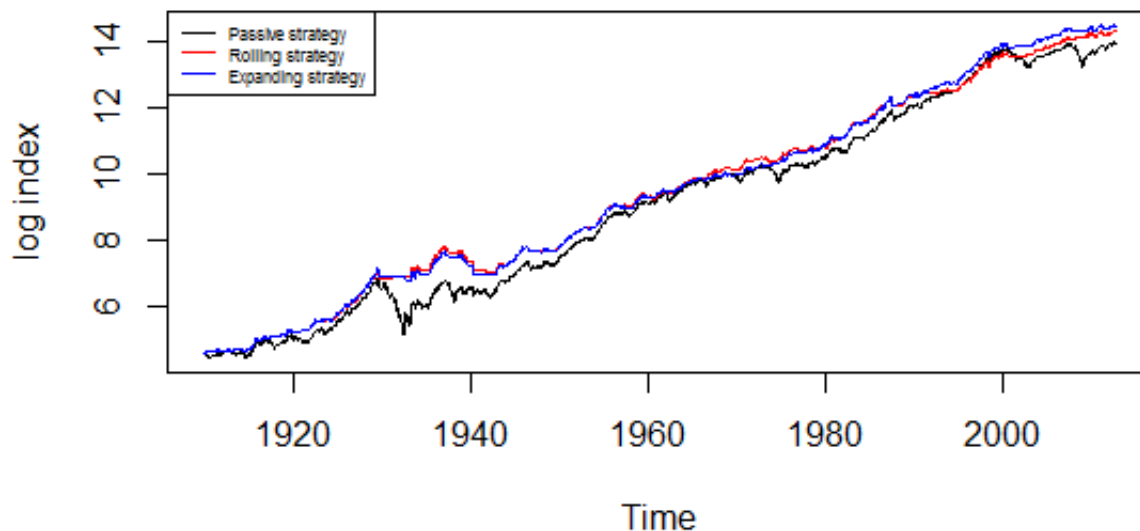


Figure 8: Value development of the rolling-, expanding- and passive-strategy for period 1910-2012

This graph in Figure 8 resembles the one presented for the second period of the in-sample-part (Figure 5) of this thesis quite a bit. We can see how the performance in the period around the Great Depression also here is extremely good, almost avoiding the drawdown completely. Also in this plot, once the passive index really begins to recover from the turbulent 1930s, the two active

strategies lose ground to the passive. In the late 1960s the three strategies are almost dead even once again, but they maintain the small lead that they have built up during the 1970s, until they once again outperform the passive significantly in the start of the 21st century. We can see that the two active strategies really follow each other closely for most of the time, where they are never really far apart from each other. The largest discrepancy is starting to occur in the beginning of 1990s, where the expanding strategy seems to exit the risky asset too early, missing out on a period with some good return.

It can be hard to really see how the performance of the two active strategies develops over time, which is why I'll show the relative strength of the indices next in Figure 9. As explained in the methodology part it is defined as the logarithmic values of the active index values divided by a risk adjusted version of the passive index values.

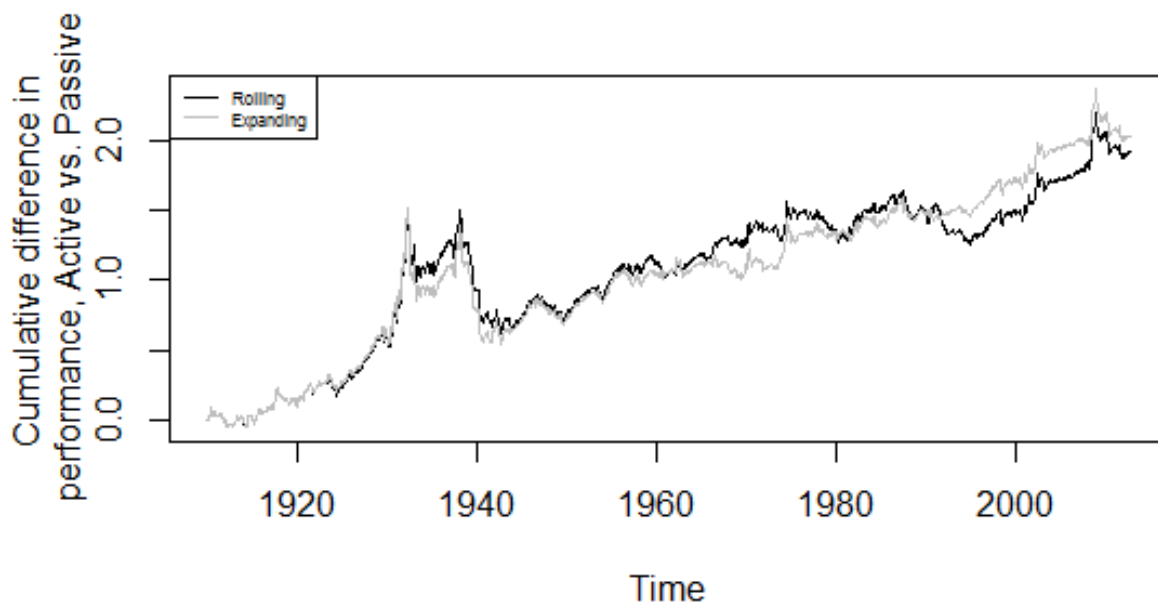


Figure 9: Cumulative difference in performance between the active and the passive strategies, period 1910-2012.

We can see from Figure 9, that the general trend is that the difference between of the cumulative wealth are increasing, which means that active strategies are outperforming the risk adjusted passive strategy with respect to cumulative wealth. Both the rolling and expanding have, except

for some short periods, almost a linear increase, ending close to the value 2. A value of 2 means that the difference in cumulative wealth is $e^2 = 7.39$, so both the active strategies are in the neighborhood of having 7 times the value as the risk adjusted passive strategy would have. Which sounds like a lot, but it is important to keep in mind that this is in fact the risk adjusted passive strategy, which in order to have the same standard deviation as the active strategy, consist of approximately 1/3 risk free asset and 2/3 index. Both active strategies have periods where the performance is deviating from the trending increase, the biggest is the increase around 1929. Although, it was later reversed back towards the trend level in the second half of the 1930s. The active strategies performed poorly in 1990s as well, especially the rolling strategy. We can also see periods of varying performance around 2007-08, where the strategies performance increased right before and decreased right after. It is no coincidence that all of the periods just mentioned are results of either periods with extremely high returns or low returns. The active strategies are performing poorly in very good times and relatively good in bad times.

Another important thing that needs to be taken into consideration is the choice of starting points. This is of course of great importance to almost any time series analysis, but because the strategy varies a whole lot in performance it is of even greater importance as just illustrated. The active strategies do not gain its strength by consistently outperforming the passive strategy, but rather in its ability to avoid major bear markets. Something which is a rare event, in the last 150 years there has been between 3-35 periods, depending on how they are measured and how big a drop need to be to be classified as a “bear market”. There have been 32 periods of bear markets with a 20% drop or more in the last 114 years (Anspach, 2013), but if the requirements are only increased by a little, the number drops drastically. The shorter the period inspected is, the greater the impact of choosing a starting point is. This can be another major source of data-mining issues, where only the periods where the strategy works best is reported. As mentioned in the literature review, this whole SMA(k) strategy has gained a real boost in popularity in the last five years or so. The decade following year 2000 has been one of the most turbulent decades in modern economic history, consisting of periods with extreme returns, as well as the two major crashes (bear markets) following the “IT-boom” and the “subprime-bubble”. During which the SMA(k) strategy performed extremely good, if an investor followed the SMA(k) instead of following a passive buy-and-hold strategy, he would have ended up with a staggering 2.33 times more

compounded wealth than its passive counterpart in the period between 2000 and 2010. This just stresses the importance of determining the right time periods to inspect, as an answer to this issue I will test the out-of-sample simulation techniques on several different periods in addition to the main analysis previously done, additional periods tested will be 1885-2012, 1935-2012 and 1975-2012. So, what rolling window and initial-in-sample size should be used for these periods? Even though the best answer was found for the period with split 1910 and end 2012, it is of course not automatically the same for the other three periods. I will use the length that gave the best combined Sharpe ratio for the two strategies, which was 300 months as the rolling window and initial-in-sample period.

The results for 1885-2012, using a 300 month initial-in-sample/rolling window size (Table 16)

Strategy type:	Expanding		Rolling		Passive
Periode	1885-2012	% change vs. Passive	1885-2012	% change vs. Passive	1885-2012
Initial in sample/Window	300		300		NA
Average monthly return	0.82%	-6%	0.79%	-9%	0.87%
Geometric monthly return	0.76%	3%	0.74%	0%	0.74%
Standard deviation	3.38%	-34%	3.30%	-36%	5.15%
Min return	-23.38%	-20%	-23.38%	-20%	-29.20%
Max return	16.28%	-62%	16.28%	-62%	42.70%
Range	39.65%	-45%	39.65%	-45%	71.91%
Annualized Sharpe Ratio	0.5152	38%	0.5037	35%	0.3742

Table 16: Key numbers of the trade, OOS-simulation using 300 rolling window/initial-in-sample for period. Start 1860, split 1885 and end 2012.

Period	1885-2012	1885-2012
Strategy	Rolling	Expanding
Lengths tested against passive	300	300
z Value	-1.692	-1.889
Probability	9.06%	5.89%

Table 17: Sharpe test for changes between the active and passive strategies for period 1885-2012 using 300 as both the initial-in-sample and the rolling-window.

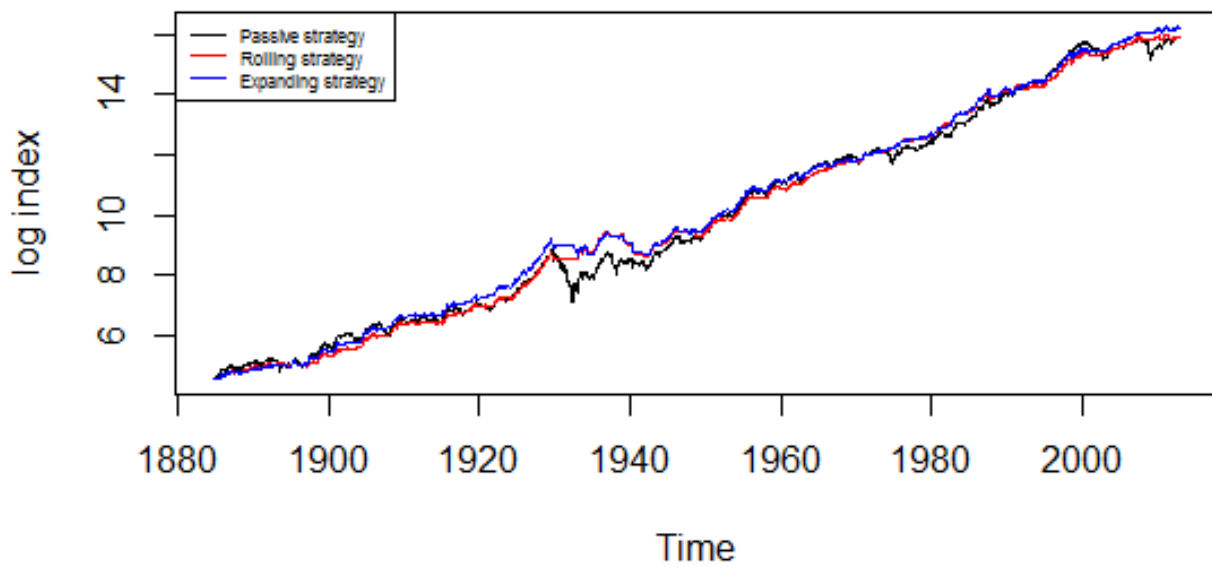


Figure 10: Values of the rolling-, expanding- and passive-strategy for period 1885-2012, using a 300 month initial in-sample/rolling window.

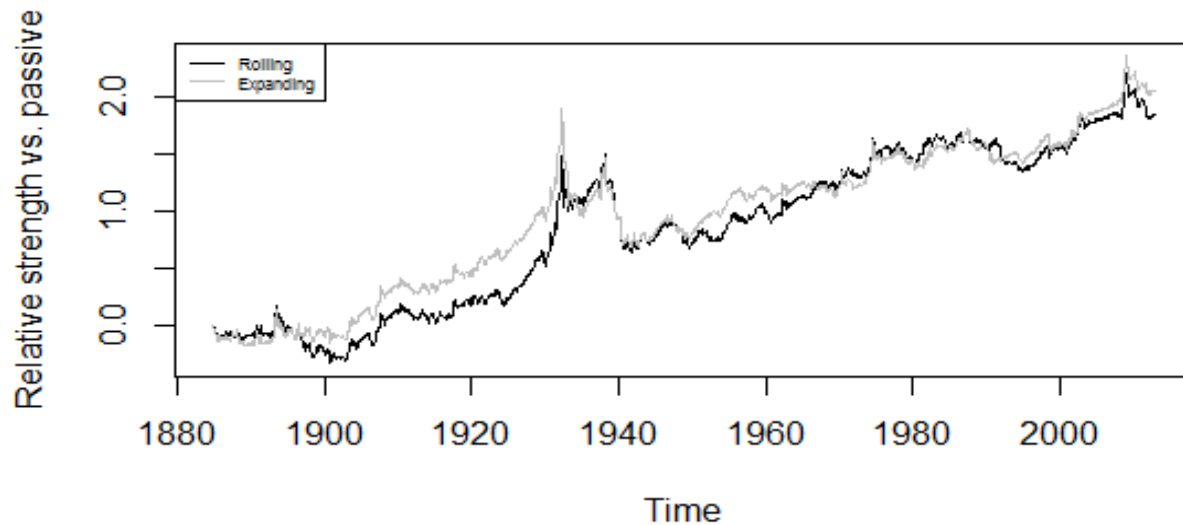


Figure 11: Cumulative difference in performance between the active and the passive strategies, period 1885-2012.

The two active strategies both end up with almost the same end value as the index following the passive strategy, but have an increase in the SR of around 35% (Table 16). When testing the statistical significance (Table 17), both strategies fail to provide enough certainty that there in fact exist a difference in the observed Sharpe ratio of the active vs. the passive strategies on a 5% level, the expanding strategy comes very close with a probability of 5.89% and the rolling strategy is at 9.06%. Besides that one cannot say with certainty that there is a difference in the SRs, not that much interesting can be found in this period. The expanding approach seems to be doing slightly better than the rolling approach for most of the time, but there is something interesting in the relative strength graph (Figure 11). In the period 1900-1930 the expanding strategy seems to be performing a considerable amount better than the rolling strategy, so that the difference between the two is rather big. This difference for that period did not really exist when the strategy was tested with different start to split lengths earlier in the paper. Although it has to be said that the difference only seem to be temporary, as the two strategies follow each other closely once again after 1930. It may seem odd that the results of the expanding are not the same as when tested earlier, but even though the length of the initial-in-sample period is the same, it is not optimizing k for the same period as before. Different k 's can create different trading signals.

Something that seem to be the most reasonable explanation to why the difference can be observed. The relative graph ends up with about the same values as it did earlier, close to 2.

Results of the period 1935-2012

Strategy type:	Expanding		Rolling		Passive
Periode	1935-2012	% change vs. Passive	1935-2012	% change vs. Passive	1935-2012
Initial in sample/Window	300		300		NA
Average monthly return	0.83%	-13%	0.82%	-14%	0.95%
Geometric monthly return	0.78%	-9%	0.77%	-10%	0.85%
Standard deviation	3.35%	-27%	3.27%	-28%	4.57%
Min return	-23.38%	-4%	-23.38%	-4%	-24.28%
Max return	13.43%	-47%	13.43%	-47%	25.24%
Range	36.80%	-26%	36.80%	-26%	49.52%
Annualized Sharpe Ratio	0.5373	11%	0.5381	11%	0.4853

Table 18: Key numbers of the trade, OOS-simulation using 300 rolling window/initial-in-sample for period. Start 1910, split 1935 and end 2012.

Period	1935-2012	1935-2012
Strategy	Rolling	Expanding
Lengths tested against passive	300	300
z Value	-0.564	-0.573
Probability	57.27%	56.64%

Table 19: Sharpe test for changes between the active and passive strategies for period 1935-2012 using 300 as both the initial-in-sample and the rolling-window.

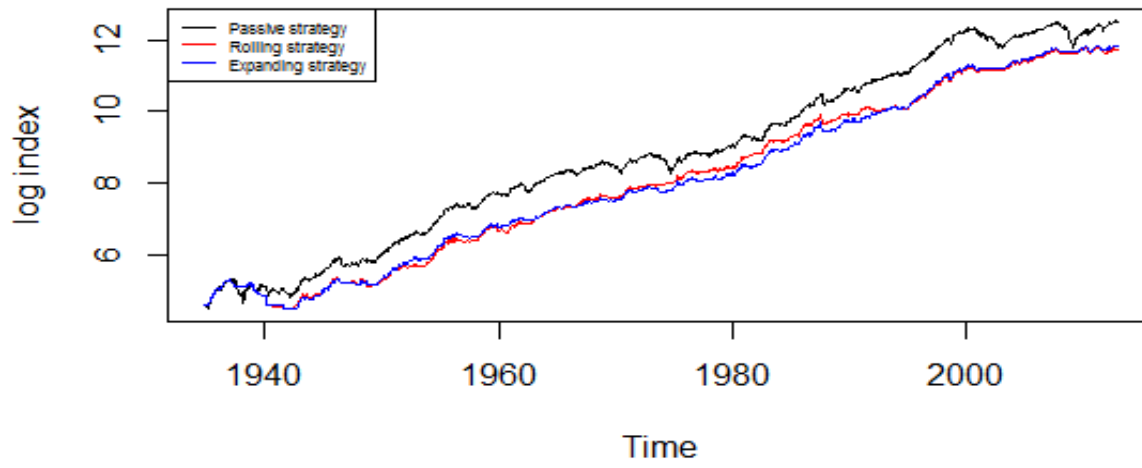


Figure 12: Values of the rolling-, expanding- and passive-strategy for period 1935-2012, using a 300 month initial in-sample/rolling window.

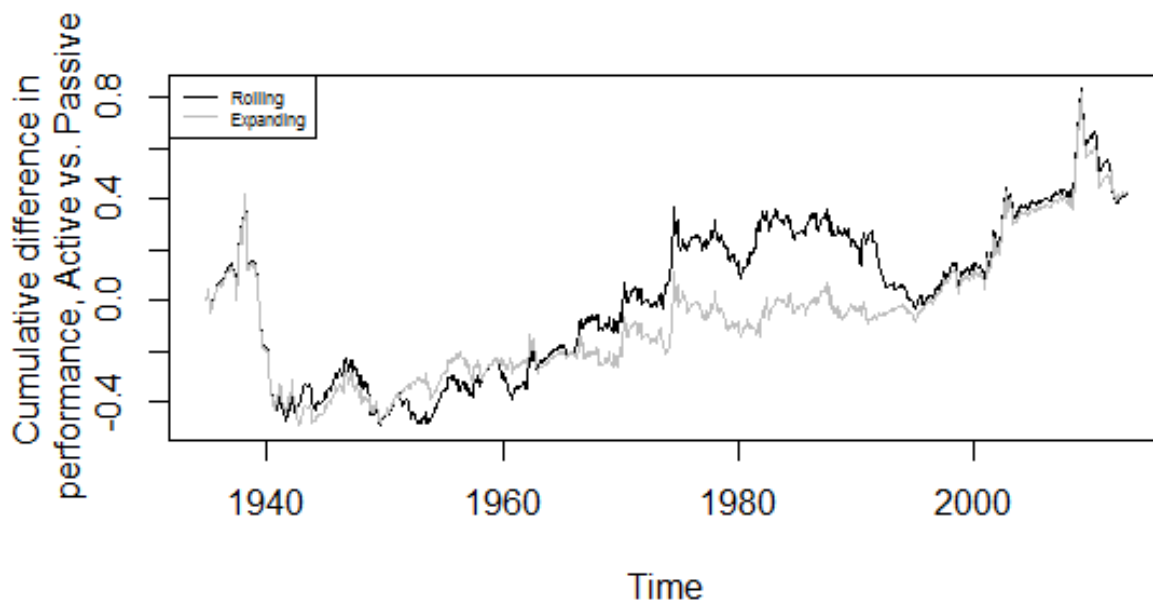


Figure 13: Cumulative difference in performance between the active and the passive strategies, period 1935-2012.

The performance of this period is not very impressive, the average return has decreased with 13% and 14% for the expanding and rolling strategy (Table 18). While the standard deviation has “only” decreased with about 27-28%, all of the previous periods tested had decreases of more than 30%. These two factors both contribute to the very low Sharpe ratio improvement of only 11% for both the strategies, neither of the strategies SR changes come close to being significant in the test performed in Table 19. This performance can also easily be seen on the index plot, where the passive index is well above both the rolling and expanding strategy for close to the entire period. There seems to be very little difference between the rolling and expanding strategy for this period, except for some time between 1970 and 1990, where the rolling is some above the expanding. If we look at the relative strength graph (Figure 13), it looks quite different than the ones previously presented. It has no clear upward trending element, it is below zero for much of time, it actually crosses zero for the final time just before year 2000. We can also see the difference between the values of the indices in the 20-year period from 1970, where the difference is close to 0.4 at its maximum and if we exploit this relationship,

$$0.4 = V_t^R - V_t^E$$

$$0.4 = \log\left(\frac{I_t^R}{I_t^{P'}}\right) - \log\left(\frac{I_t^E}{I_t^{P'}}\right)$$

$$e^{0.4} = \frac{I_t^R}{I_t^E} = 1.492$$

It means that if an investor followed the rolling strategy instead of the expanding strategy he would have had 49.2% more wealth at that point. The two strategies are at most at 0.8 in the relative graph, but the bull market to follow the crash in 2007-08 ensures the relative graph ends up around the value 0.4.

The last period tested was from 1975 to 2012, this is the results.

Strategy type:	Expanding		Rolling		Passive
Periode	1975-2012	% change vs. Passive	1975-2012	% change vs. Passive	1975-2012
Initial in sample/Window	300		300		NA
Average monthly return	0.91%	-10%	0.88%	-14%	1.01%
Geomtric monthly return	0.85%	-7%	0.82%	-10%	0.91%
Standard deviation	3.35%	-24%	3.36%	-24%	4.42%
Min return	-21.47%	0%	-21.47%	0%	-21.47%
Max return	13.43%	0%	13.43%	0%	13.43%
Range	34.90%	0%	34.90%	0%	34.90%
Annualized Sharpe Ratio	0.4913	8%	0.4597	1%	0.4555

Table 20: Key numbers of the trade, OOS-simulation using 300 rolling window/initial-in-sample for period. Start 1950, split 1975 and end 2012.

Period	1975-2012	1975-2012
Strategy	Rolling	Expanding
Lengths tested against passive	300	300
z Value	-0.188	-0.467
Probability	85.10%	64.02%

Table 21: Sharpe test for changes between the active and passive strategies for period 1975-2012 using 300 as both the initial-in-sample and the rolling-window.

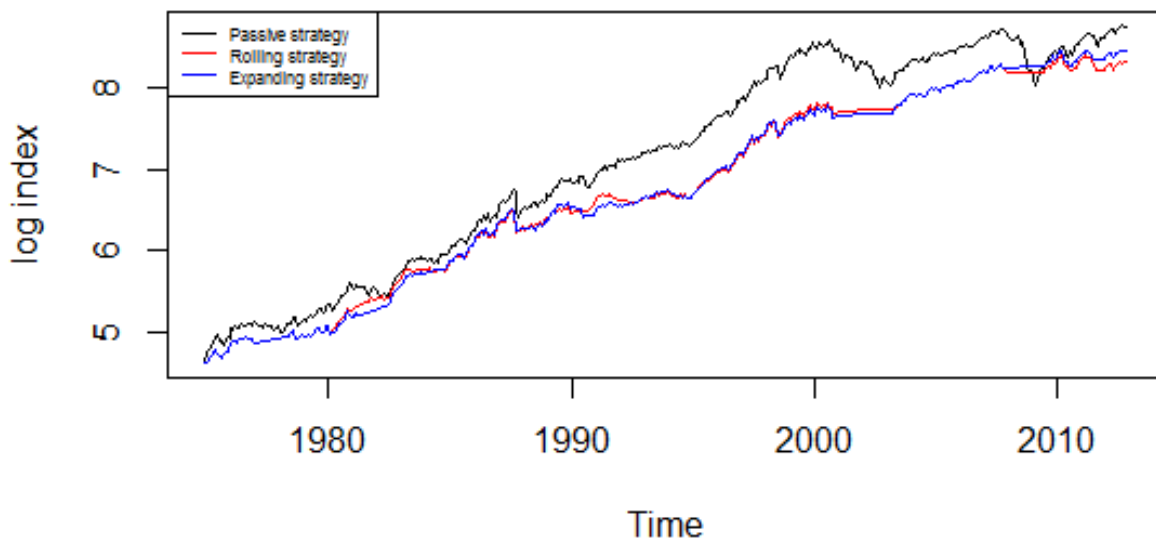


Figure 14: Values of the rolling-, expanding- and passive-strategy for period 1975-2012, using a 300 month initial in-sample/rolling window.

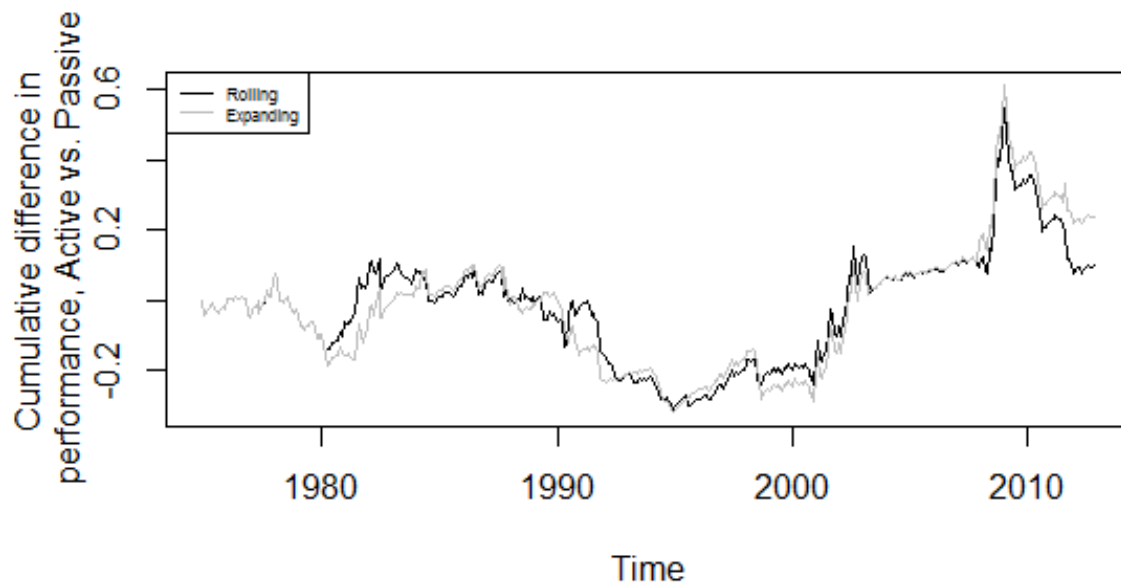


Figure 15: Cumulative difference in performance between the active and the passive strategies, period 1975-2012.

Also for this period, as we can see from Table 20 that both the average and the geometric returns are smaller for the active strategies than for the passive. With average return drops of 10% and 14% for the expanding and the rolling. The standard deviation dropped with 24% in both cases. Despite having a drop in the standard deviations, the Sharpe ratio increase is only 8% and 1% for the two strategies. If we look at the Sharpe test in Table 21, there is no evidence to support the claim that there is a difference in the Sharpe ratios for either strategy. For the first time in all of the periods tested, the range is as large for the active as for the passive strategy. If we look at the indices created following the active and passive trades, it really shows the reason to why the performance reported is so poor, the active indices just can't keep up with the passive in the bull market that almost lasted from the periods start to year 2000. Especially in the years between 1990 and 2000 the difference grew by a lot, where in hindsight the active strategies produced too many exit signals making it miss out on important return. Both the active strategies actually catch up with the passive during the financial crisis in 2007-08 for a short period of time, but they quickly fall behind once the market improves. The relative graph (Figure 15) has almost a flat development for the first 30 years, where it stays pretty close to zero until the turbulent times in

the beginning of the millennia starts, first increasing from -0.2 to 0.2 in the first round and then increasing another 0.4 in the second round, before decreasing some again.

6. Discussion

As the results showed the market timing strategy has large fluctuations in the performance, with both periods of extreme over- and under-performance. This discussion part will try to explain the reasons for the differences we can observe for the different strategies and for the different periods tested.

After inspecting the cumulative wealth and cumulative difference in performance figures it is pretty evident, the market timing strategy works well for bear periods and badly for bull periods. This is due to the timing strategy's ability to switch between the risky and the risk free asset. But it is also this ability that makes it underperform in bull periods when compared to a passive buy-and-hold benchmark. The motivation for exchanging one asset for the other is the belief that it will yield a higher return for the coming month, but that belief is not always realized. In fact most of the times the risky asset is swapped with the risk free asset it is the result of a "false signal", where the investor would have been better off sticking with the risky asset. In order for the active strategies to end up with a higher cumulative wealth, the average return of the periods where the risk free asset is held has to be higher for the risk free than that of the risky asset. A market timing strategy is definitely a two way street, where surely exiting the market can prevent large losses, but it can also prevent large gains by not being in the market at the right time. This is due to the way the simple-moving-average works, where the SMA becomes a lagged version of previous realizations giving it the ability to exit the market if there is a negative trend, but that also means that it takes time for the signal to switch back to buy. So when the trend has turned positive we can already have missed out on important return. This also means that if the events are sudden drops in a very limited number of observations the strategy will perform poorer than if a drop of equal size is spread out in longer period of time. When talking about trends, what is actually implied is that there exist a correlation between the months (autocorrelation), so on the basis of previous realizations one can make a qualified guess for the next unrealized month. There have been established that there exist significant autocorrelation for monthly return data previously, I find the first order auto-correlation to be 0.076 for the period of 1857-2012, it is not

a very high correlation, but it is found to be statistically significant at a $\alpha = 1\%$ level using Pearson's product moment correlation test (Pearson, 1895). This correlation shows that there are price patterns that can potentially be exploited with the right setup, as according to MPT and the random walk hypothesis there should not exist such correlation.

From both the in-sample and out-of-sample analysis we could see drops in the standard deviation for the active strategy of approximately 25-35%, this is the result of the time invested in the risk free asset. The reduction in volatility comes through when one looks at the reduced drawdowns, but it is important to remember that this reduction also means that one has an increased risk of missing out on return and ultimately end up with a lower wealth. There is a risk change from potentially large drawdowns to potentially lower end wealth, this change might be obvious, as economic theory dictate; reduced risk always comes at the price of reduced expected returns, but from the results shown we can see several examples of reduced volatility while not having an adverse impact on the returns. My point being that there is still a considerable risk that a market timing strategy user will end up with less cash than a passive strategy, not because of drawdowns or market drops, but as a result of not being in the market at the right time and that may not be reflected when using volatility as a measure of risk.

For the in-sample part, period one (1857-1925) had a substantially larger increase in Sharpe ratio than period two (1926-2012) had, a 63% increase compared to only 46%. As already stated, the market timing strategy gains its strength from its ability to avoid bear markets, and when we know that the average 10-year maximum drawdown for period one was 28% and 40% for period two, one would think it would be period two that should see the bigger increase in performance relative to the passive. I believe the fact that it do not is due to the lack of the extreme bull periods one could observe in the second period (1945-1955, 1980-2000 for example). Even though there are no periods of really exceptional performance, there are very few periods of underperformance as well, making the discrepancy between the market timing and the passive strategy's cumulative wealth a little bit bigger for every drawdown the market experienced. It is these long bull periods in the second data set which I find to be the main reason to why the strategy seems to work better in period one than two, but the timing strategy would certainly had provided better results than a buy and hold strategy could have in both periods, both in terms of cumulative wealth and reward-to-risk measurement.

The in-sample results found in period two are very similar to what others have found before, both in terms of average return and volatility. In the literature review I said that it was the SMA(10) that was the most popular length in the literature, and still I do not find it to be the optimal length in the in-sample analysis which may seem a bit odd. (Zakamulin, 2013) use data from 1926-2012 for the S&P500 and finds the SMA(10) to be the optimal length, which is identical to my period two where I find that it is the SMA(15). The reason for this is that one loses some observations at the start of the data-set due to simple-moving-average calculations (lose the number of the highest k tested), and since my complete data set it ranging from 1857 and not 1926, I only lose the k first observations in 1857 and not in 1926 as well. If I try the strategy only for the data set from 1926-2012 (meaning that the first SMA results are for 1926 + k months ahead) I also find that it is the SMA(10) that is the optimal choice.

As the out-of-sample simulation showed the active strategy performed very well in some of the periods tested and rather poorly in other. In the main out-of-sample analysis which was for the period 1910-2012, where the different initial in-sample periods tested were put accordingly in the time before the split point in 1910, the resulting performance for the rolling and the expanding had a 43% (0.553) and a 44% (0.560) increase in the Sharpe ratio when compared to what a passive strategy (0.388) would have achieve in the same period. If the same period was tested and treated as in-sample the best performing SMA(k) length would give a SR of 0.575, this difference between the out-of-sample and the in-sample SR ratio is the size of the out-of-sample deterioration of the Sharpe ratio. It can be calculated as,

$$Deterioration\ in\ \% = \left(\frac{SR_{BT} - \frac{SR_R + SR_E}{2}}{\frac{SR_R + SR_E}{2}} \right),$$

SR_{BT} is the Sharpe ratio of the optimal SMA(k) length in a back-test, SR_R , SR_E are the Sharpe ratios of the rolling and expanding strategy. The out-of-sample deterioration for the different periods tested are given in Table 22.

Period	1910-2012	1885-2012	1935-2012	1975-2012
Deterioration	3.32%	9.38%	19.25%	16.08%

Table 22: Out-of-sample deterioration for the different OOS-simulation time periods tested

The deterioration is quite different in the periods tested, this clearly show that there is a risk of getting substantially lower Sharpe ratios than what can be found using a standard in-sample back test. For the period 1910-2012 the optimal length of the initial in-sample and the rolling window was found, but it was not found for the other three periods. This can account for some of the deterioration, but it seems unlikely that that is the only reason, the magnitude is bigger than the range of Sharpe ratios found using the different lengths of the initial in-sample and rolling window in the period 1910-2012 (range of SR for the expanding strategy was 0.061 and for the rolling it was 0.071). And even if the differences is a result of not having the optimal initial in-sample size or rolling window length, one cannot on beforehand know what that length is, which is why I cannot really see any reason not to expect an out-of-sample deterioration of similar magnitude of what is shown in Table 22 if one were to implement this timing strategy in the real world. Although the in-sample results suggests that this trend following strategy can provide superior performance to what a buy-and-hold strategy can, it is not enough evidence to claim that the efficient market hypothesis is wrong (meaning that there are still past price patterns that can be exploited in the stock market prices). In my opinion there are two problems with making such a claim. One, the out-of-sample performance is a substantial amount poorer than what the in-sample performance is. Two, the strategy do not provide consistent performance, where the performance is reliant on some short and rare periods of time in order to work well. Since the strategy seems to be working for some of the periods inspected, and not at all for others, it might at best be evidence that there are periods where markets are not efficient. But further research is needed to be able to conclude with anything.

From the empirical results obtained in this thesis it is not obvious whether an expanding or a rolling window is the more appropriate to use. From a theoretical point of view it depends on the data, if the data are drawn from the same distribution in all 156 of the years used, an expanding window are the more appropriate choice, but if it changes throughout the years one should use a rolling window instead. Trying to decide if all data are “drawn” from the same distribution for the entire sample can be very hard to determine, one simplification of the problem can be to split the data in two and compare the 78 first years to the 78 last years and for instance use a two-sample Kolmogorov-Smirnov test³ to see if the two sets are coming from the same distribution or not, but

³ The point is not to go into the details of the test, but to have some statistical foundation to say that the data has changed/not changed in the period looked at. The test was developed by Andrey Kolmogorov and Nikolai

if implemented the results are not strong enough to say that they are from the same distribution or from different distributions. In Figure 7, the optimal lookback period for a 10-year rolling window was shown, and it changed a lot, having a standard deviation of almost 5 months. The world has definitely changed a lot the last 150 years, so it would not be very surprising if the supposed underlying distribution function of the returns have changed as well, but as just stated the first half is not statistically different from the second half. The expanding strategy seems to perform slightly better than the rolling strategy, having the highest cumulative wealth of the two strategies in all periods tested. Another thing that suggests the expanding strategy might be better is that it generates fewer trades, making the transaction costs lower. So from a conceptual point of view I would say the rolling window is more appropriate, and based on the results found here an expanding window would be better. None of the arguments are in my opinion strong enough to conclude with what strategy is best, further research would be necessary to determine which is the better to use or under what conditions one outperform the other.

7. Conclusion

This thesis has inspected the performance of a popular trend following strategy on a longer time series than previously tested. The complete data set was divided into two parts (1857-1925, 1926-2012), where the performance of the two periods were found and compared using a back-test. In addition to the standard back-test, a simulated out-of-sample approach was used to eliminate some of the potential data-mining bias that exists. The choice of optimal length of the SMA was simulated in four different time periods in order to see how the performance differs in the periods tested. I find this trend following strategy to perform very well in severe bear-markets and very poorly in major bull-markets.

The in-sample part revealed that the simple-moving-average trend following strategy worked well in both periods tested, with substantially decreased volatility without it compromising the returns and therefore giving rise to enhanced risk-to-reward returns. It seems like the strategy worked a bit better in period one (1857-1925) than two (1926-2012), despite period two having

Vasilyevich Smirnov, in the 1930- and 1940s through a series of published articles. Further explanation, test-statistics and critical value tables can be found at (Wikipedia, Kolmogorov–Smirnov test)

on average having larger 10-year maximum drawdowns. The reason for this seems to be the lack of extreme bull-markets that period two experienced more of than period one.

The out-of-sample analysis revealed that the risk of out-of-sample deterioration is definitely present, the amount of deterioration seems highly dependent of the period it is tested on. If this simple-moving-average technique is to be tested out in real-life, one can expect to have a reduction in standard deviation of about 1/3, it is not as obvious what one can expect for the returns, but one should not be surprised if the wealth development following the active strategy is less than what one would get following a passive buy-and-hold strategy. Yet, the drop in standard deviation should ensure that one ends up with a higher risk-to-reward return than what a passive strategy achieves.

Bibliography

- Alexander, S. (1961). Price movements in speculative markets: Trends or random walks. *Industrial Management Review*, Vol.2, No.2, 7-26.
- Anspach, D. (2013). Bear Markets - and Their Subsequent Recoveries. *Whitepaper*, Available at "<http://moneyover55.about.com/od/howtoinvest/a/bearmarkets.htm>".
- Aronson, D. (2006). *Evidence-Based Technical Analysis: Applying the Scientific Method and Statistical Inference to Trading Signals*. John Wiley & Sons, Ltd.
- Bodie, Z., Kane, A., & Marcus, A. (2011). *Investments and Portfolio Management: Global Edition*. New York: McGraw-Hill/Irwin.
- Brock, W., Lakonishok, J., & LeBaron, B. (1992). Simple Technical Trading Rules and the Stochastic Properties of Stock Returns. *The Journal of Finance*, Vol.47, No.5, 1731-1764.
- Burch, C. (2011). Stock Market Technical Analysis. *Whitepaper*, available at "<https://www.simple-talk.com/dotnet/.net-framework/stock-market-technical-analysis>".
- Burghardt, G., Duncan, R., & Liu, L. (2003). Understanding Drawdowns. *Whitepaper*, available at "<http://www.intelligentedgefundinvesting.com/pubs/rb-bdl.pdf>".
- Chambers, D., & Zdanowics, J. (2014). The Limitations of Diversification Return. *Whitepaper*, available at "<http://www.hec.ca/finance/Fichier/Chambers2014.pdf>".
- Chen, J. (2010). *Essentials of Technical Analysis for Financial Markets*. Hoboken, New Jersey: John Wiley & Sons, Inc.
- Clare, A., Seaton, J., & Thomas, S. (2013). BREAKING INTO THE BLACKBOX: Trend Following, Stop Losses, and the Frequency of trading: the case of the S&P500. *Journal of Asset Management*, Vol. 14, No. 3, 182-194.
- Cootner, P. (1962). Stock prices: Random vs. Systematic changes. *Industrial Management Review*, Vol. 3, No.2, 24-45.
- Faber, M. (2007). A Quantitative Approach to Tactical Asset Allocation. *The Journal of Wealth Management*, Vol. 9, No. 4, 69-79.
- Faber, M. (2009). A Quantitative Approach to Tactical Asset Allocation—An Update. *Cambria Investment Management*.
- Fama, E. (1965a). The Behavior of Stock-Market Prices. *Journal of Business*, Vol. 38, No.1, 34-105.
- Fama, E. (1965b). Random Walks in Stock Market Prices. *Financial Analysts Journal*, Vol.21, No.5 , 55-59.
- Gartley, H. (1935). *Profits in the Stock Market*. Lambert Gann Pub.

- Goyal, A., & Welch, I. (2008). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *Review of Financial Studies, Society of Financial Studies, Vol.21, No. 4*, 1455-1508.
- Granger, C., & Morgenstern, O. (1963). Spectral analysis of New York stock market prices. *Kyklos, Vol. 17*, 1-30.
- Gwilym, O. a., Clare, A., Seaton, J., & Thomas, S. (2010). Price and Momentum as Robust Tactical Approaches to Global Equity Investing. *The Journal of Investing, Vol.19, No.3*, 80-91.
- Harding, D. (2002). A Critique of the Sharpe Ratio. *Whitepaper, available at <http://performance-measurement.org/Harding2002.pdf>*.
- Investopedia, Efficient Frontier.* (n.d.). Retrieved from <http://www.investopedia.com/terms/e/efficientfrontier.asp>
- Jobson, J. D., & Korkie, B. M. (1981). Performance Hypothesis Testing with the Sharpe and Treynor Measures. *The Journal of Finance, Vol.36, No.4*, 889–908.
- Kirkpatrick, C., & Dahlquist, J. (2010). *Technical Analysis - The complete Resource for Financial Market Technicians*. New Jersey: Pearson Education, Inc.
- Levy, R. (1966). Conceptual Foundations of Technical Analysis. *Financial Analysts Journal, Vol. 22, No.4*, 83-89.
- Lo, A., Mamaysky, H., & Wang, J. (2000). Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation. *The Journal of Finance, Vol. 55, No.4*, 1705-1770.
- Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, Vol. 7, No. 1*, 77-91.
- Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investment*. New York: John Wiley & Sons.
- McInish, T. (2000). *Capital Markets - a gloabl perspective*. Oxford: Blackwell Publishers.
- Memmel, C. (2003). Performance Hypothesis Testing with the Sharpe Ratio. *Finance Letters, Vol.1, No.1*.
- nber.org.* (n.d.). Retrieved from <http://nber.org/databases/macroeconomy/contents/chapter13.html>
- Park, C.-H., & Irwin, S. (2007). What do we know about the profitability of technical analysis? *Journal of Economic Surveys, Vol.21, No. 4*, 786-826.
- Pearson, K. (1895). Notes on regression and inheritance in the case of two parents. *Royal Society Proceedings, Vol. 58*, 240-242.
- Preis, T., Stanley, H., Helbing, D., & Ben-Jakob, E. (2012). Quantifying the Behavior of Stock Correlation under Market Stress. *Scientific Reports, Vol.2, No. 752*.

Schwert, W. (1990). Indexes of U.S. Stock Prices from 1802 to 1987*. *Journal of Business*, Vol. 63. No. 3, 399-426.

schwert.ssb.rochester.edu. (n.d.). Retrieved from <http://schwert.ssb.rochester.edu/mstock.htm>

Sharpe, W. (1964). Capital Asset Prices: A Theory of Market Equilibrium under conditions of risk. *Journal of Finance*, Vol. 19, No.3, 425-442.

Sharpe, W. (1966). Mutual Fund Performance. *Journal of Business*, Vol. 39, No. 1, 119-138.

Steiger, W. L. (1964). A test of Non randomness in stock price changes. *The MIT Press*, 303-312.

Tobin, J. (1958). Liquidity Preference as Behavior Towards Risk. *Review of Economic Studies*, Vol. 25, No.2, 65-86.

Treynor, J. (1961). Market Value, Time, and Risk. *Unpublished working paper, available at "https://faculty.fuqua.duke.edu/~charvey/Teaching/BA453_2005/French_Treynor_CAPM.pdf"*.

Treynor, J. (1962). Toward a Theory of Market Value of Risky Assets. *Unpublished until 1999, available at "http://papers.ssrn.com/sol3/papers.cfm?abstract_id=628187"*.

Wikipedia, Dow Theory. (n.d.). Retrieved from http://en.wikipedia.org/wiki/Dow_theory

Wikipedia, Kolmogorov–Smirnov test. (n.d.). Retrieved from http://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test

Wikipedia, Modern Portfolio Theory. (n.d.). Retrieved from http://en.wikipedia.org/wiki/Modern_portfolio_theory

Zakamulin, V. (2013). The Real-Life Performance of Market Timing with Moving Average and Time-Series Momentum Rules. *Working paper, Available at "http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2242795"*.

Appendix

R-kode:

```
## Packages used
library ("zoo")
library ("tseries")

### Functions
SharpeTest <- function(ex1, ex2) {
  # test for equality of two Sharpe ratios
  # ex1 and ex2 are excess returns to two portfolios/strategies
  # returns the p-value of the test
  if (length(ex1) != length(ex2))
    stop("Different lengths of two returns!")

  SR1 <- mean(ex1)/sd(ex1)
  SR2 <- mean(ex2)/sd(ex2)
  ro <- cor(ex1,ex2)
  n <- length(ex1)
  z <- (SR2-SR1)/sqrt( (2*(1-ro)+0.5*(SR1^2+SR2^2-2*SR1*SR2*ro^2))/n )
  pval <- 2*pnorm(-abs(z))
  return(pval)
}

#Rollmean, change lbs freely before program line
rollk <- function(x, lbs=2:20){
  roll <- list()
  ft <- lbs
  for (i in seq_along(ft)){
    roll[[i]]<- rollmean(x, ft[i])}
  return(roll)
}

#Assign appropriate return based on rollm vs. index (us after rollk)
aret <- function(index, rollm, ret, rfree){
  n <- length(index)-1
  aret <- rep(0,n)
  for (i in 1:n){
    if (rollm[i] < index[i])(aret[i] <- ret[i+1])
    else (aret[i] <- rfree [i+1])
  }
  return(aret)
}

#Annual Sharpe-Ratio created from monthly data
SRannual <- function(er){
  Sharpe <- SR(er)*sqrt(12)
  Sharpe
}

#SR creator
SR <- function(er) {
```

```

# computes the Sharpe ratio
return(mean(er, na.rm=TRUE)/sd(er, na.rm=TRUE))
}

#Expanding window, OOS-simulation
smak.e <- function (roll,rfree, split=120){
  smak <- vector()
  ft <- (length(roll[,1]))
  for (i in 1:(ft-split-1)){
    part.r<- (roll[1:(i+split),])
    part.rf <- (rfree[1:(i+split)])
    er.part <- part.r -part.rf
    SRperiod <- apply(er.part, 2, SR)
    endperiod <- as.numeric(SRperiod)
    k <- which.max(endperiod)
    smak[i+split]<- roll[i+split+1,k]
  }
  return(smak)
}

#Expanding window, OOS-simulation
k.e <- function (roll,rfree, split=120){
  smak <- vector()
  ft <- (length(roll[,1]))
  for (i in 1:(ft-split-1)){
    part.r<- (roll[1:(i+split),])
    part.rf <- (rfree[1:(i+split)])
    er.part <- part.r -part.rf
    SRperiod <- apply(er.part, 2, SR)
    endperiod <- as.numeric(SRperiod)
    smak[i+split] <- which.max(endperiod)+min(lbs)-1
  }
  return(smak)
}

#Rolling window, shows how k change
smak.r <- function (roll, rfree, split=120){
  smak <- vector()
  ft <- (length(roll[,1]))
  for (i in 1:(ft-split-1)){
    part.r<- (roll[i:(i+split),])
    part.rf <- (rfree[i:(i+split)])
    er.part <- part.r -part.rf
    SRperiod <- apply(er.part, 2, SR)
    endperiod <- as.numeric(SRperiod)
    k <- which.max(endperiod)
    smak[i+split]<- roll[i+split+1,k]
  }
  return(smak)
}

#Rolling window, shows how k change
k.r <- function (roll, rfree, split=120){
  smak <- vector()

```

```

ft <- (length(roll[,1]))
for (i in 1:(ft-split-1)){
  part.r<- (roll[i:(i+split),])
  part.rf <- (rfree[i:(i+split)])
  er.part <- part.r -part.rf
  SRperiod <- apply(er.part, 2, SR)
  endperiod <- as.numeric(SRperiod)
  smak[i+split] <- which.max(endperiod)+min(lbs)-1
}
return(smak)
}

#Assigns dates to a data set
part <- function(data, hele.data=c(2012,12), start=c(2000,12),
end=c(2012,12), freq=12){
  ts.data <- ts(data, end=hele.data, freq=freq)
  ts.data2 <- window(ts.data, start=start, end=end, freq=freq)
  return(ts.data2)
}

#Shows value development of a return vector
cumproduct <- function(tsaktiv, start=100){
  S <- vector("numeric", length(tsaktiv))
  S[1] <- start
  for (i in 1:length(tsaktiv)){
    S[i+1] <- S[i]*(1+tsaktiv[i+1])}
  return(S)}

#Summary function, use on returns, if er not available use r x2
my.summary <- function(r,er) {
  cat("\n***** \n")
  cat(" SUMMARY STATISTICS \n")
  cat("***** \n")
  cat("Number of observations = ", length(r), "\n")
  cat("Mean = ", mean(r, na.rm=TRUE), "\n")
  cat("Median = ", median(r, na.rm=TRUE), "\n")
  cat("Variance = ", var(r, na.rm=TRUE), "\n")
  cat("Standard deviation = ", sd(r, na.rm=TRUE), "\n")
  cat("Min = ", min(r, na.rm=TRUE), "\n")
  cat("Max = ", max(r, na.rm=TRUE), "\n")
  cat("Range = ", max(r, na.rm=TRUE)-min(r, na.rm=TRUE), "\n")
  cat("SR= ", SRannual(er), "\n")
}

#####

# Create the data file
# Period 1, 1857-1925
STKDATM <- read.table("~/STKDATM.dat",header=TRUE)

#drop data for 1802-1856,12 = 660 months
STKDATM1 <- STKDATM[-(1:660),]

cprates <- read.table("~/paperrates.dat", header=FALSE)

```



```

kuncprates <- cprates$V3

#Estimated tbill rate from commercial paper rates 1857-1972
tbillhat <- tbillhat (kuncprates)

#Data as whole percent to numbers 5% = 0,05 tall
rfree.1<- annualtomonthly (tbillhat)
rfree.1 <- window(rfree.1[1:828])
rfree.1 <- as.numeric(rfree.1)

#Estimated tbill rate from commercial paper rates 1857-1926
#tsrfree <- ts(monthlycprates, start=c(1857,1), end=c(1925,12), freq=12)

#Index og dividender
rret <- STKDATM1$bla
rdiv <- STKDATM1$bla1
a.ret <- rret+rdiv

#####
predict <- read.csv("~/PredictorData.csv",header=TRUE, sep=",")
predict <- predict[-(1:660),]
ret2 <- vector("numeric")
for (i in 1:(length(predict$Index))){
  ret2[i+1]<- (predict$Index[i+1]-predict$Index[i])/predict$Index[i]
}
ret2 <- head(ret2, -1)
ret2[1]<- 0.02247
div2 <- (predict$D12/predict$Index)/12
df2 <- data.frame(ret2, div2)
a.ret2 <- rowSums(df2, na.rm=T)
rfree2 <- predict$Rfree

capital.return <- c(rret, ret2)
dividend.return <- c(rdiv, div2)
rfree <- c(rfree.1, rfree2)
total.return <- c(a.ret, a.ret2)

index <- cumproduct(total.return, start=100)
index<- head(index, -1)
date <- seq(as.Date("1857-1-1"), by="month", length=1872)

new.dates <- data.frame(dates = as.Date(date, by="month"))

df <- cbind(new.dates, capital.return, dividend.return, total.return,
rfree, index)

write.table(df, file="data.dat")

# Calculation

data <- read.table("~/data.dat",header=TRUE)
lbs <- 8:15 #Set wanted SMAK to test for

```

```

tds <- max(lbs)-1
est <- length(lbs)-1

index <- cumproduct(data$capital.return, start=100) # Choice what returns
the rollk should be based on, either capital.return or total.return
index <- head(index, -1)
rolling <- rollk(index, lbs)
for (i in 1:est){
  rolling[[i]]<- (rolling[[i]][-(1:(length(lbs))-i)])
}
rolling <- matrix(unlist(rolling), ncol=length(lbs))
data <- data[-(1:tds),]
index <- index[-(1:tds)]

aktiv.ret <- matrix(ncol=length(lbs) , nrow=(nrow(data)-1))
for (i in 1:length(lbs)){
  aktiv.ret[,i] <- aret(index, rolling[,i], data$total.return,
data$rfree)}
data <- data[-1,]

er.aktiv.ret <- aktiv.ret-data$rfree
er.aktiv.ret <- ts(er.aktiv.ret, end=c(2012,12), freq=12)

# In sample, set time for start and end period, ONLY for in sample
start=c(1926,1)
end=c(2012,12)
del.periode <- window(er.aktiv.ret, start=start, end=end, freq=12)
SRperiode <- apply(del.periode, 2, SR)
endperiod <- as.numeric(SRperiode)
best.v <- which.max(endperiod)
best.k <- best.v+min(lbs)-1
best.k

SRannual(del.periode[,best.v])
ts.aktiv <- ts(aktiv.ret, end=c(2012,12), freq=12)
del.aktiv <- window(ts.aktiv, start=start, end=end, freq=12)
te <- best.v
my.summary(del.aktiv[,te],del.periode[,te])
cum.w <- cumproduct(del.aktiv[,te], start=100)
cum.w[length(cum.w)-1]
del.periode.pret <- part(data$total.return, start=start, end=end)
ts.rfree <- ts(data$rfree, start=start, end=end, freq=12)
er.del.periode.pret <- del.periode.pret - ts.rfree
cum.w.p <- cumproduct(del.periode.pret, start=100)
cum.w.p[length(cum.w.p)-1]

SharpeTest(del.periode[,te], er.del.periode.pret)

active.ind <- ts(cum.w, start=start, end=end, freq=12)
passive.ind <- ts(cum.w.p, start=start, end=end, freq=12)
range <- range(c(log(active.ind), log(passive.ind)), na.rm=TRUE)
plot(log(active.ind), ylab="Cumulative Return, log scale", col="red",
ylim=range)
lines(log(passive.ind), col="black")

```

```

legend("topleft", c("Market Timing Strategy", "Passive Strategy"),
      cex=0.5, col=c("red","black"), lty =1)

# OOS simulating, set time for start and end period, split = rolling
start=c(1950,1)
end=c(2012,12)
split=300 #initial in sample period from start to n periods after start
and size of rolling window
del.periode.r <- part(aktiv.ret, start=start, end=end)
del.periode.rf <- part(data$rfree, start=start, end=end)
del.periode.pret <- part(data$total.return, start=start, end=end)

# Expanding
expanding <- smak.e(del.periode.r, del.periode.rf, split=split)
er.expanding <- expanding- del.periode.rf[-1]
er.expanding <-ts(er.expanding, end=end, freq=12)
SRannual(er.expanding[split:(length(er.expanding))])
k.expanding <- k.e(del.periode.r, del.periode.rf, split=split)
k.expanding <- k.expanding[split:length(k.expanding)]
k.expanding <- ts(k.expanding, end=end, freq=12)
roundtrips.expanding <-
sum(rle(er.expanding[(split+1):length(er.expanding)])$values==0)
time.long.expanding <-
(sum(er.expanding[(split+1):(length(er.expanding))]!=0))/length(er.expanding[(split+1):(length(er.expanding))])
my.summary(expanding, er.expanding)
#Relative graph
del.pret <- (del.periode.pret[(split+1):(length(del.periode.pret))])
del.rf <- (del.periode.rf[(split+1):(length(del.periode.rf))])
del.expanding <- expanding[split:(length(expanding))]
relativ.sd <- sd(expanding, na.rm=TRUE)/sd(del.pret)
mkt <- del.pret*relativ.sd+(1-relativ.sd)*del.rf
ind.mkt <- cumproduct(mkt, start=100)
ind.exp <- cumproduct(del.expanding, start=100)
relativ.ind.exp <- log(ind.exp/ind.mkt)
relativ.ind.exp <- ts(relativ.ind.exp, end=c(2012,12), freq=12)
plot(relativ.ind.exp)

# Rolling
rolling <- smak.r(del.periode.r, del.periode.rf, split=split)
er.rolling <- rolling- del.periode.rf[-1]
er.rolling <-ts(er.rolling, end=end, freq=12)
SRannual(er.rolling[split:(length(er.rolling))])
k.rolling <- k.r(del.periode.r, del.periode.rf, split=split)
k.rolling <- k.rolling[split:length(k.rolling)]
k.rolling <- ts(k.rolling, end=end, freq=12)
roundtrips.rolling <-
sum(rle(er.rolling[(split+1):length(er.rolling)])$values==0)
time.long.rolling <-
(sum(er.rolling[(split+1):(length(er.rolling))]!=0))/length(er.rolling[(split+1):(length(er.rolling))])
my.summary(rolling, er.rolling)
#Relative graph
del.pret <- (del.periode.pret[(split+1):(length(del.periode.pret))])

```

```

del.rf <- (del.periode.rf[(split+1):(length(del.periode.rf))])
del.roll <- rolling[split:(length(rolling))]
relativ.sd <- sd(rolling, na.rm=TRUE)/sd(del.pret)
mkt <- del.pret*relativ.sd+(1-relativ.sd)*del.rf
ind.mkt <- cumproduct(mkt, start=100)
ind.roll <- cumproduct(del.roll, start=100)
relativ.ind.roll <- log(ind.roll/ind.mkt)
relativ.ind.roll <- ts(relativ.ind.roll, end=c(2012,12), freq=12)
plot(relativ.ind.roll)

#Max drawdown calculations
max <- 120
index <- index.active #Put name of whatever index is to be inspected
ac.in <- index[-length(index)]
x <- seq_along(ac.in)
ac.in <- as.numeric(ac.in)
test <- split(ac.in, ceiling(x/max))
res <- list()
lengde <- length(test)
for (i in seq(lengde))(
  res[[i]] <- MDD((test[[i]])))

#Correlation inspection
data <- read.table("~/data.dat",header=TRUE)
capital.return <- ts(data$capital.return, end=c(2012,12), freq=12)
dividend.return <- ts(data$dividend.return, end=c(2012,12), freq=12)
total.return <- ts(data$total.return, end=c(2012,12), freq=12)

start=1860
end=2010
cor3 <- vector()
for (i in start:end){
  start[i]= (i)
  end[i]=(i+1)

  del.total.return <- window(total.return, start=c(start[i],1),
end=c(end[i],12), freq=12)

  cor1 <- del.total.return[2:(length(del.total.return))]
  cor2 <- del.total.return[1:(length(del.total.return)-1)]
  cor3[i] <- cor(cor1, cor2)
  cor3
}
cor3<- ts(cor3[1860:2011], end=2010, freq=1)
plot(cor3)

cor11 <- capital.return[2:(length(capital.return))]
cor22 <- capital.return[1:(length(capital.return)-1)]
cor(cor11, cor22)
cor.test(cor11, cor22)

```