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Research Article

Delay-Probability-Distribution-Dependent $\mathscr{H}_\infty$ FIR Filtering Design with Envelope Constraints

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This paper studies the problem of $\mathscr{H}_\infty$ finite-impulse response (FIR) filtering design of time-delay system. The time-delay considered here is time-varying meanwhile with a certain stochastic characteristic, and the probability of delay distribution is assumed to be known. Furthermore, the requirement of pulse-shape is also considered in filter design. Employing the information about the size and probability distribution of delay, a delay-probability-distribution-dependent criterion is proposed for the filtering error system. Based on a Lyapunov-Krasovskii functional, a set of linear matrix inequalities (LMIs) are formulated to solve the problem. At last, a numerical example is used to demonstrate the effectiveness of the filter design approach proposed in the paper.

1. Introduction

In the studies about filtering problem, one most significant approach frequently applied in the past decades is Kalman filtering, the main idea of which is to minimize the variance of the estimation error assuming considered system dynamics to be exactly known and the external disturbances to be stationary Gaussian noises with known statistical properties [1, 2]. However, in many practical engineering applications, the statistical details about external noise are not available [3–8]. In these cases, many approaches are introduced to improve systems' robustness, such as $\mathscr{H}_\infty$, $\mathscr{H}_2$, and mixed $\mathscr{H}_\infty/\mathscr{H}_2$ filtering [2, 9–13]. In this paper, the $\mathscr{H}_\infty$ filtering approach is utilized.

On the other hand, time-delays are frequently encountered in practical engineering systems, such as manufacturing systems, power systems, and networked control systems [14–17]. Existence of delay makes the analysis and synthesis of systems a much more difficult task; meanwhile it is also the source of instability and poor performance in many cases [13, 18–20]. The main approaches to solve delay problems can be classified into delay-dependent approach and delay-independent approach. It has been shown in [21, 22] that the results obtained using delay-dependent approaches are generally less conservative than the delay-independent approaches ones [23]. Acknowledging this fact, the delay-dependent approach is applied in this paper.

In fact, the variation of delay may often stick to some probability distribution in spite of its varying and underivable property [24, 25]. Furthermore, in many real systems such as networked control systems, the time-varying delay may have some abrupt burst, leading to very large delay with a very small probability [26]. In this sense, the discussion about time-delay should not only depend on its size but also on its probability distribution. In this paper, a new filter design approach and new stability criteria for the filtering error system taking the stochastic characteristic of time-varying delay into account is proposed.

While an $\mathscr{H}_\infty$ optimal filter can catch the frequency-domain property, the time-domain constraints such as envelope constraints or bounds on signals cannot be handled by this frequency-domain approach [27]. Among various time-domain specifications, envelope constraints, which make requirement on the pulse-shape, have significant applications in many practical engineering systems, such as communication systems, radar, sonar systems, and signal processing systems [28–31]. For instance, in deconvolution filtering and data channel equalization problems, it is extremely important to achieve a desired pulse-shape through designing an appropriate filter [27].

Therefore, aiming at incorporating both frequency-domain and time-domain constraints into the problem, we intend to design a filter satisfying the $\mathscr{H}_\infty$ performance and subject to envelope constraints in outputs. Meanwhile, time-varying delays with certain stochastic characteristics in the transmission channel are also taken into account. With the proposed filter design approach, a more general condition of time-varying delay problem can be solved. As in most situations, although detailed and exact information about delay cannot be achieved, the delay's probability distribution characteristics can be predicted or observed relatively easily. Once the probability information is gotten, the filter design approach can be developed.

In this paper, based on a Lyapunov-Krasovskii functional, we first present an $\mathscr{H}_\infty$ optimal solution to the design of a finite-impulse response (FIR) filter using information about the range of time-varying delay and its probability distribution. Then, the envelope constraints are taken into consideration. The resultant filter is called an $\mathscr{H}_\infty$ optimal Envelope-Constrained FIR (ECFIR) filter. We obtain the solution via solving an LMI optimization problem. At last, a numerical example is presented to illustrate the effectiveness of the proposed filtering design approach.

2. Problem Formulation and Preliminaries

Consider a filtering system shown in Figure 1, where Σ_l represents a linear dynamic system with state-space realization given by

$$\Sigma_l : \begin{cases} x_l(k+1) = A_l x_l(k) + B_l w(k) \\ s(k) = C_l x_l(k), \end{cases} \tag{1}$$

where $x_l(k) \in \mathfrak{R}^{n_l}$ is the model state vector, $w(k) \in \mathfrak{R}^{n_w}$ is the input signal, $s(k) \in \mathfrak{R}^{n_s}$ is the source signal generated by the model, and A_l, B_l, C_l are known constant matrices with appropriate dimensions. Then the output $s(k)$ is transmitted through a channel with time-varying delay modeled by

$$\Sigma_c : \begin{cases} x_c(k+1) = A_c x_c(k) + A_d s(k-d(k)) + B_c v(k) \\ y(k) = C_c x_c(k) + C_d s(k-d(k)) + D_c v(k), \end{cases} \tag{2}$$

where $x_c(k) \in \mathfrak{R}^{n_c}$ is the channel state vector, $d(k) \in [0, d_2]$ is the time-varying delay with an upper bound of d_2, $y(k)$ is the output of the channel, and $v(k)$ is the disturbance input; $A_c, A_d, B_c, C_c, C_d, D_c$ are all known constant system matrices with appropriate dimensions. As is shown in (2), the source signal $s(k)$ suffers from influence of time-varying delay $d(k)$ and disturbance from the environment represented by $v(k)$. The output of transmission channel is $y(k)$, which is also the input signal of the filter. We are going to use the corrupted signal $y(k)$ to reconstruct original source signal.

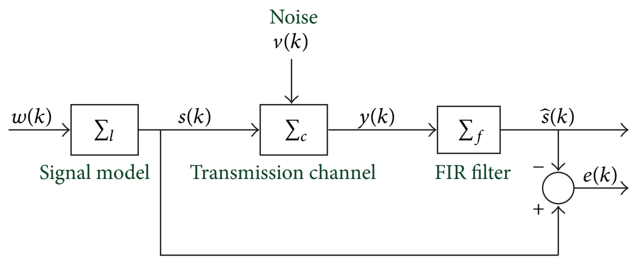

Figure 1: Filtering system.

Assumption 1. $d(k)$ changes randomly and for a constant $d_1 \in [0, d_2]$, and the probability of $d(k) \in [0, d_1)$ and $d(k) \in [d_1, d_2]$ can be known. The following sets and functions are defined:

$$\begin{aligned} \Omega_1 &= \{k : d(k) \in [0, d_1)\}, \\ \Omega_2 &= \{k : d(k) \in [d_1, d_2]\}, \\ d_1(k) &= \begin{cases} d(k), & \text{for } k \in \Omega_1 \\ 0 & \text{for } k \notin \Omega_1, \end{cases} \\ d_2(k) &= \begin{cases} d(k), & \text{for } k \in \Omega_2 \\ d_1, & \text{for } k \notin \Omega_2. \end{cases} \end{aligned} \tag{3}$$

Obviously, it can be seen from the definition that $k \in \Omega_1$ is equal to the occurrence of event $d(k) \in [0, d_1)$ and $k \in \Omega_2$ means that the event $d(k) \in [d_1, d_2]$ occurs. Therefore, a stochastic variable $\beta(k)$ can be defined as

$$\beta(k) = \begin{cases} 1, & k \in \Omega_1 \\ 0, & k \in \Omega_2. \end{cases} \tag{4}$$

Assumption 2. $\beta(k)$ is a Bernoulli distributed sequence with

$$\begin{aligned} \text{Prob}\{\beta(k) = 1\} &= \mathbb{E}\{\beta(k)\} = \beta_0, \\ \text{Prob}\{\beta(k) = 0\} &= 1 - \mathbb{E}\{\beta(k)\} = 1 - \beta_0, \end{aligned} \tag{5}$$

where $0 \le \beta_0 \le 1$ is a constant.

Remark 3. From Assumption 2, it is easy to see that $\mathbb{E}\{\beta(k) - \beta_0\} = 0$ and $\mathbb{E}\{(\beta(k) - \beta_0)^2\} = \beta_0(1 - \beta_0)$. As $\text{Prob}\{d(k) \in [0, d_1)\} = \text{Prob}\{\beta(k) = 1\} = \beta_0$ and $\text{Prob}\{d(k) \in [d_1, d_2]\} = \text{Prob}\{\beta(k) = 0\} = 1 - \beta_0$, β_0 and $1 - \beta_0$ also denote the probability of $d(k)$ taking values in $[0, d_1)$ and $[d_1, d_2]$, respectively.

According to Assumptions 1 and 2, the system model described by (2) can be rewritten as

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + \beta(k) A_d s(k - d_1(k)) \\ &\quad + (1 - \beta(k)) A_d s(k - d_2(k)) + B_c v(k), \\ y(k) &= C_c x_c(k) + \beta(k) C_d s(k - d_1(k)) \\ &\quad + (1 - \beta(k)) C_d s(k - d_2(k)) + D_c v(k). \end{aligned} \tag{6}$$

At the receiving end, we are interested in designing a linear filter with state-realization as follows:

$$\Sigma_f : \begin{cases} x_f(k+1) = A_f x_f(k) + B_f y(k) \\ \hat{s}(k) = C_f x_f(k) + D_f y(k), \end{cases} \quad (7)$$

where $x_f(k) \in \Re^{n_f}$ is the filter state vector, $\hat{s}(k)$, is the estimated signal of source signal $s(k)$ and A_f, B_f, C_f, D_f have the following form:

$$A_f = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n_f \times n_f}, \qquad B_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n_f \times 1}, \quad (8)$$

$$C_f = \left[f(n_f) \ \ f(n_f - 1) \ \ \ldots \ \ f(1) \right] D_f = f(0).$$

The transfer function of the filter is given by

$$\begin{aligned} \Phi_f(z) &= C_f \left(zI - A_f\right)^{-1} B_f + D_f \\ &= f(0) + f(1) z^{-1} + f(2) z^{-2} + \cdots + f(n_f) z^{-n_f}, \end{aligned} \quad (9)$$

where $f(0)$, $f(1), \ldots$, and $f(n_f)$ are parameters to be determined. Define the filtering error as $e(k) = s(k) - \hat{s}(k)$. Then, via augmenting the models Σ_l and Σ_c, the filtering error system is given as follows:

$$\Sigma_e : \begin{cases} x_e(k+1) = A_e x_e(k) \\ \quad + \beta(k) A_{ed} x_e(k - d_1(k)) \\ \quad + (1 - \beta(k)) A_{ed} x_e(k - d_2(k)) + B_e w_e(k) \\ e(k) \\ = C_e x_e + \beta(k) C_{ed} x_e(k - d_1(k)) \\ \quad + (1 - \beta(k)) C_{ed} x_e(k - d_2(k)) + D_e w_e(k), \end{cases} \quad (10)$$

where

$$\begin{aligned} x_e^T(k) &= \left[x_l^T(k) \ \ x_c^T(k) \ \ x_f^T(k) \right]^T, \\ w_e(k) &= \left[w^T(k) \ \ v^T(k) \right]^T, \\ A_e &= \begin{bmatrix} A_l & 0 & 0 \\ 0 & A_c & 0 \\ 0 & B_f C_c & A_f \end{bmatrix}, \qquad B_e = \begin{bmatrix} B_l & 0 \\ 0 & B_c \\ 0 & B_f D_c \end{bmatrix}, \\ C_e &= \left[C_l \ \ -D_f C_c \ \ -C_f \right], \\ D_e &= \left[0 \ \ -D_f D_c \right], \\ A_{ed} &= \begin{bmatrix} 0 & 0 & 0 \\ A_d C_l & 0 & 0 \\ B_f C_d C_l & 0 & 0 \end{bmatrix}, \qquad C_{ed} = \left[-D_f C_d C_l \ \ 0 \ \ 0 \right]. \end{aligned} \quad (11)$$

Before giving the main results, we need following definitions at first.

Definition 4. For a given function $V(x(k))$, its stochastic difference operator is defined as

$$\Delta V(x(k)) = \mathbb{E}\{V(x(k+1)) \mid x(k)\} - V(x(k)). \quad (12)$$

Definition 5 (see [32]). The filtering error system in (10) is said to be stochastically stable if for any initial condition $x_e(0)$ and zero exogenous noise $w_e(k)$, there exists a positive definite W independent of $x_e(0)$, such that the following condition is satisfied:

$$\mathbb{E}\left\{ \sum_{k=0}^{\infty} \left| x_e(k) \right|^2 \mid x_e(0) \right\} < x_e^T(0) W x_e(0). \quad (13)$$

Definition 6. System (10) is said to be stochastically stable with an $\mathscr{H}_\infty$ norm bound γ, if the following conditions hold.

(1) The filtering error system with $w_e(k) = 0$ is stochastically stable.

(2) For all nonzero $w_e(k) \in l_2[0, \infty)$ and under zero initial conditions, the following inequality holds:

$$\|e(k)\|_2 \le \gamma \|w_e(k)\|_2. \quad (14)$$

Now, with the definitions above, we present the objective of this paper.

Given the filtering system shown in Figure 1, we are interested in designing a filter in the form of (7)-(8) such that

(a) the filtering error system (10) is asymptotically stable in the stochastic sense;

(b) the filtering error system (10) possesses a minimized $\mathscr{H}_\infty$ performance level γ;

(c) a time-domain envelope constraint is imposed on the output signal $\hat{s}(k)$ as follows:

$$l(k) \le \hat{s}(k) \le u(k), \quad (15)$$

where $l(k)$ and $u(k)$ are the lower and upper bounds of the time-domain mask, respectively.

3. Main Results

In this section, based on the Lyapunov-Krasovskii stability theorem, a delay-probability-distribution-dependent approach is proposed to solve the $\mathscr{H}_\infty$ FIR filter design problem subject to envelope constraints described in (15). First, a stability criterion for the filtering error system described in (10) is proposed. Then the envelope constraints are taken into consideration. An $\mathscr{H}_\infty$ optimal ECFIR filter design approach is given at last.

Theorem 7. *Given the system in Figure 1, for some given constants $0 \le d_1 \le d_2$, β_0, and γ, the filtering error system* (10) *is stochastically stable with $\mathscr{H}_\infty$ performance γ if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ of appropriate dimensions such that the following optimization problem has solutions,*

$$\min_{P>0,\,Q_1>0,\,Q_2>0,\,R_1>0,\,R_2>0,\,f} \gamma, \tag{16}$$

subject to the following LMI constraint:

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{gathered}
\Xi_{11} = \begin{bmatrix} -P & PA_e & \beta_0 PA_{ed} \\ * & Q - P - \dfrac{1}{d_1}R_1 & \dfrac{1}{d_1}R_1 \\ * & * & -Q_1 - \dfrac{1}{d_1}R_1 - \dfrac{1}{d_2 - d_1}R_2 \end{bmatrix}, \\
\Xi_{12} = \begin{bmatrix} (1-\beta_0) PB_e & PB_e & 0 \\ 0 & 0 & \sqrt{d_1}\left(A_e^T - I\right) R_1 \\ \dfrac{1}{d_2 - d_1}R_2 & 0 & \sqrt{d_1}\beta_0 A_{ed}^T R_1 \end{bmatrix}, \\
\Xi_{13} = \begin{bmatrix} 0 & 0 \\ \sqrt{d_2 - d_1}\left(A_e^T - I\right) R_2 & C_e^T \\ \beta_0 \sqrt{d_2 - d_1} A_{ed}^T R_2 & \beta_0 C_{ed}^T \end{bmatrix}, \\
\Xi_{22} = \begin{vmatrix} -Q_2 - \dfrac{1}{d_2 - d_1}R_2 & 0 & \sqrt{d_1}(1-\beta_0) A_{ed}^T R_1 \\ * & -\gamma^2 I & \sqrt{d_1} B_e^T R_1 \\ * & * & -R_1 \end{vmatrix}, \\
\Xi_{23} = \begin{bmatrix} \sqrt{d_2 - d_1}(1-\beta_0) A_{ed}^T R_2 & (1-\beta_0) C_{ed}^T \\ \sqrt{d_2 - d_1} B_e^T R_2 & D_e^T \\ 0 & 0 \end{bmatrix}, \\
\Xi_{33} = \begin{bmatrix} -R_2 & 0 \\ * & -I \end{bmatrix}, \\
Q = \left(1 + d_1\right) Q_1 + \left(d_2 - d_1 + 1\right) Q_2,
\end{gathered} \tag{18}$$

and A_e, A_{ed}, B_e, C_e, C_{ed}, and D_e are defined in (11).

Proof. First, define a Lyapunov-Krasovskii functional as follows:

$$V(k) \triangleq V_1(k) + V_2(k) + V_3(k) + V_4(k), \tag{19}$$

where

$$\begin{aligned}
V_1(k) &\triangleq x_e^T(k) P x_e(k), \\
V_2(k) &\triangleq \sum_{i=k-d_1(k)}^{k-1} x_e^T(i) Q_1 x_e(i) + \sum_{i=k-d_2(k)}^{k-1} x_e^T(i) Q_2 x_e(i), \\
V_3(k) &\triangleq \sum_{i=-d_1+2}^{-1} \sum_{j=k+i-1}^{k-1} x_e^T(j) Q_1 x_e(j) + \sum_{i=-d_2+2}^{-d_1+1} \sum_{j=k+i-1}^{k-1} x_e^T(j) Q_2 x_e(j), \\
V_4(k) &\triangleq \sum_{i=k-d_1}^{k-1} \sum_{j=i}^{k-1} \delta^T(j) R_1 \delta(j) + \sum_{i=k-d_2}^{k-d_1-1} \sum_{j=i}^{k-1} \delta^T(j) R_2 \delta(j), \\
\delta(j) &\triangleq x_e(j+1) - x_e(j),
\end{aligned} \tag{20}$$

and $P = P^T > 0$, $Q_1 = Q_1^T > 0$, $Q_2 = Q_2^T > 0$, $R_1 = R_1^T > 0$, and $R_2 = R_2^T > 0$ are Lyapunov matrices to be determined.

Then using the stochastic difference operator defined in (12), we obtain

$$\begin{aligned}
\Delta V_1(k) = {} & \left[x_e^T(k) A_e^T + \beta_0 x_e^T(k - d_1(k)) A_{ed}^T \right. \\
& \left. + (1-\beta_0) x_e^T(k - d_2(k)) A_{ed}^T + w_e^T(k) B_e^T\right] \\
& \times P \left[A_e x_e(k) + \beta_0 A_{ed} x_e(k - d_1(k)) \right. \\
& \left. + (1-\beta_0) A_{ed} x_e(k - d_2(k)) + B_e w_e(k)\right] \\
& - x_e^T(k) P x_e(k), \\
\Delta V_2(k) = {} & x_e^T(k) \left(Q_1 + Q_2\right) x_e(k) - x_e^T(k - d_1(k)) \\
& \times Q_1 x_e(k - d_1(k)) \\
& - x_e^T(k - d_2(k)) Q_2 x_e(k - d_2(k)) \\
& + \sum_{i=k+1-d_1(k+1)}^{k-1} x_e^T(i) Q_1 x_e(i) \\
& - \sum_{i=k-d_1(k)+1}^{k-1} x_e^T(i) Q_1 x_e(i) \\
& + \sum_{i=k+1-d_2(k+1)}^{k-1} x_e^T(i) Q_2 x_e(i) \\
& - \sum_{i=k-d_2(k)+1}^{k-1} x_e^T(i) Q_2 x_e(i)
\end{aligned}$$

$$
\begin{aligned}
&\leq x_e^T(k)(Q_1+Q_2)x_e(k) - x_e^T(k-d_1(k)) \\
&\quad\times Q_1 x_e(k-d_1(k)) \\
&\quad - x_e^T(k-d_2(k))Q_2 x_e(k-d_2(k)) \\
&\quad + \sum_{i=k-d_1+1}^{k} x_e^T(i)Q_1 x_e(i) + \sum_{k-d_2+1}^{k-d_1} x_e^T(i)Q_2 x_e(i), \\
\Delta V_3(k) &= d_1 x_e^T(k)Q_1 x_e(k) + (d_2-d_1)x_e^T(k)Q_2 x_e(k) \\
&\quad - \sum_{i=k-d_1+1}^{k} x_e^T(i)Q_1 x_e(i) - \sum_{i=k-d_2+1}^{k-d_1} x_e^T(i)Q_2 x_e(i), \\
\Delta V_4(k) &= d_1\delta^T(k)R_1\delta(k) + (d_2-d_1)\delta^T(k)R_2\delta(k) \\
&\quad - \sum_{i=k-d_1}^{k-1}\delta^T(i)R_1\delta(i) - \sum_{i=k-d_2}^{k-d_1-1}\delta^T(i)R_2\delta(i) \\
&\leq d_1\delta^T(k)R_1\delta(k) + (d_2-d_1)\delta^T(k)R_2\delta(k) \\
&\quad - \sum_{i=k-d_1(k)}^{k-1}\delta^T(i)R_1\delta(i) - \sum_{i=k-d_2(k)}^{k-d_1-1}\delta^T(i)R_2\delta(i).
\end{aligned} \tag{21}
$$

Using the Jensen inequality [33], the following expressions are obtained:

$$
\begin{aligned}
&-\sum_{i=k-d_1(k)}^{k-1}\delta^T(i)R_1\delta(i) \\
&\quad\leq -\frac{1}{d_1(k)}\left(\sum_{i=k-d_1(k)}^{k-1}\delta^T(i)\right)R_1\left(\sum_{i=k-d_1(k)}^{k-1}\delta(i)\right) \\
&\quad\leq -\frac{1}{d_1}\left(\sum_{i=k-d_1(k)}^{k-1}\delta^T(i)\right)R_1\left(\sum_{i=k-d_1(k)}^{k-1}\delta(i)\right), \\
&-\sum_{i=k-d_2(k)}^{k-d_1-1}\delta^T(i)R_2\delta(i) \\
&\quad\leq -\frac{1}{d_2(k)-d_1}\left(\sum_{i=k-d_2(k)}^{k-d_1-1}\delta^T(i)\right)R_2\left(\sum_{i=k-d_2(k)}^{k-d_1-1}\delta(i)\right) \\
&\quad\leq -\frac{1}{d_2-d_1}\left(\sum_{i=k-d_2(k)}^{k-d_1-1}\delta^T(i)\right)R_2\left(\sum_{i=k-d_2(k)}^{k-d_1-1}\delta(i)\right).
\end{aligned} \tag{22}
$$

Thus, we have

$$
\begin{aligned}
\Delta V_4(k) &\leq \delta^T(k)\left[d_1R_1 + (d_2-d_1)R_2\right]\delta(k) \\
&\quad - \frac{1}{d_1}\left[x_e^T(k) - x_e^T(k-d_1(k))\right] \\
&\quad \times R_1\left[x_e(k) - x_e(k-d_1(k))\right] \\
&\quad - \frac{1}{d_2-d_1}\left[x_e^T(k-d_1(k)) - x_e^T(k-d_2(k))\right] \\
&\quad \times R_1\left[x_e(k-d_1(k)) - x_e(k-d_2(k))\right].
\end{aligned} \tag{23}
$$

Thus, we obtain

$$
\begin{aligned}
\Delta V(k) &= \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \\
&\quad + \Delta V_4(k) \leq \eta^T(k)\Upsilon\eta(k),
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
\eta^T(k) &= \left[\, x_e^T(k) \;\; x_e^T(k-d_1(k)) \;\; x_e^T(k-d_2(k)) \;\; w_e^T(k) \,\right], \\
\Upsilon &= \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} \\ * & * & \Upsilon_{33} & \Upsilon_{34} \\ * & * & * & \Upsilon_{44} \end{bmatrix}, \\
\Upsilon_{11} &= A_e^TPA_e + (A_e^T - I)R(A_e - I) + Q - P - \frac{1}{d_1}R_1, \\
\Upsilon_{12} &= \beta_0 A_e^TPA_{ed} + \beta_0(A_e^T - I)RA_{ed} + \frac{1}{d_1}R_1, \\
\Upsilon_{13} &= (1-\beta_0)A_e^TPA_{ed} + (1-\beta_0)(A_e^T - I)RA_{ed}, \\
\Upsilon_{14} &= A_e^TPB_e + (A_e^T - I)RB_e^T, \\
\Upsilon_{22} &= \beta_0^2 A_{ed}^T(P+R)A_{ed} - Q_1 - \frac{1}{d_1}R_1 - \frac{1}{d_2-d_1}R_2, \\
\Upsilon_{23} &= \beta_0(1-\beta_0)A_{ed}^T(P+R)A_{ed} + \frac{1}{d_2-d_1}R_2, \\
\Upsilon_{24} &= \beta_0 A_{ed}^T(P+R)B_e, \\
\Upsilon_{33} &= (1-\beta_0)^2 A_{ed}^T(P+R)A_{ed} - Q_2 - \frac{1}{d_2-d_1}R_2, \\
\Upsilon_{34} &= (1-\beta_0)A_{ed}^T(P+R)B_e, \\
\Upsilon_{44} &= B_e^T(P+R)B_e, \\
R &= d_1R_1 + (d_2-d_1)R_2.
\end{aligned} \tag{25}
$$

By Schur complement, it can be concluded from (17) that $\Upsilon < 0$. By similar lines as in [32], the stochastic stability can be guaranteed if condition (17) holds.

Then, define the performance index as follows:

$$
J = \sum_{k=0}^{\infty}\left[e^T(k)e(k) - \gamma^2 w_e^T(k)w_e(k)\right]. \tag{26}
$$

Considering the fact that $V(k) \geq 0$, under the zero initial condition, we have

$$\begin{aligned} J &\leq \sum_{k=0}^{\infty}\left[e^{T}(k)e(k)-\gamma^{2}w_{e}^{T}(k)w_{e}(k)\right]+V(\infty)-V(0) \\ &= \sum_{k=0}^{\infty}\left[e^{T}(k)e(k)-\gamma^{2}w_{e}^{T}(k)w_{e}(k)+\Delta V(k)\right]. \end{aligned} \tag{27}$$

Thus, $J < 0$ is equal to

$$\eta^{T}(k)(\Theta+\Upsilon)\eta(k) < 0, \tag{28}$$

where

$$\Theta = \begin{bmatrix} C_e^T C_e & \beta_0 C_e^T C_{ed} & (1-\beta_0) C_e^T C_{ed} & C_e^T D_e \\ * & \beta_0^2 C_{ed}^T C_{ed} & \beta_0(1-\beta_0) C_{ed}^T C_{ed} & \beta_0 C_{ed}^T D_e \\ * & * & (1-\beta_0)^2 C_{ed}^T C_{ed} & (1-\beta_0) C_{ed}^T D_e \\ * & * & * & D_e^T D_e - \gamma^2 I \end{bmatrix}. \tag{29}$$

Through applying Schur complement, it is shown that $(\Theta + \Upsilon) < 0$ can be guaranteed by condition (17). That is to say, once (17) is satisfied, the $\mathscr{H}_\infty$ performance can be guaranteed to be less than γ. Thus, the proof is completed.

At this point, the second desired property of the system will be considered, which is the envelope constraints demand. First, some notations are introduced [34]:

$$y = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(m) \end{bmatrix}, \qquad l = \begin{bmatrix} l(0) \\ l(1) \\ \vdots \\ l(n) \end{bmatrix},$$
$$u = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(n) \end{bmatrix}, \qquad f = \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(n_f) \end{bmatrix}, \tag{30}$$
$$Y = \begin{bmatrix} y(0) & 0 & \cdots & 0 \\ y(1) & y(0) & \cdots & 0 \\ \vdots & y(1) & \cdots & \vdots \\ y(m) & \vdots & & y(0) \\ 0 & y(m) & \vdots & y(1) \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & y(m) \end{bmatrix},$$

where Y is an $n \times (n_f + 1)$ matrix, $n = m + n_f + 1$,

$$\{y(0)\ \ y(1)\ \ \cdots\ \ y(m)\ \ 0\ \ 0\ \ \cdots\} \tag{31}$$

is a given signal, and

$$\begin{gathered} \{l(0)\ \ l(1)\ \ \cdots\ \ l(m)\}, \\ \{u(0)\ \ u(1)\ \ \cdots\ \ u(m)\} \end{gathered} \tag{32}$$

Are, respectively, the upper and lower bounds. Therefore, the constraint of (15) is equal to

$$\operatorname{diag}(l) \leq \operatorname{diag}(Yf) \leq \operatorname{diag}(u), \tag{33}$$

where diag(•) denotes a conversion from a vertical vector to a diagonal matrix.

Based on Theorem 7 and (33), we can establish another theorem to determine the filter that satisfies the envelope constraint meanwhile possessing optimal $\mathscr{H}_\infty$ performance. □

Theorem 8. *An $\mathscr{H}_\infty$ optimal filter of the form* (7)-(8) *satisfying envelope constraint in* (15) *can be obtained by solving the following LMI optimization problem:*

$$\min_{P>0,\, Q_1>0,\, Q_2>0,\, R_1>0,\, R_2>0,\, f} \gamma, \tag{34}$$

subject to

$$\begin{gathered} \Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix} < 0, \\ \operatorname{diag}(l) \leq \operatorname{diag}(Yf), \\ \operatorname{diag}(Yf) \leq \operatorname{diag}(u), \end{gathered} \tag{35}$$

where Ξ is defined in (17).

4. An Illustrative Example

In this section, an example is given to support the filter design method proposed in the paper. Consider a filtering system as shown in Figure 1. The parameters for Σ_l are given by

$$\begin{gathered} A_l = \begin{bmatrix} -2.3060 & -2.9625 & -2.2590 & -1.0922 & -0.3009 & -0.0325 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\ B_l = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ C_l = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.0062 & 0.2170 \end{bmatrix}. \end{gathered} \tag{36}$$

The parameters for the delay channel Σ_c are given by

$$\begin{gathered} A_c = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad A_d = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \\ C_c = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad C_{cd} = 0.2, \qquad D_c = 1, \\ d_1 = 2, \qquad d_2 = 3, \qquad \beta_0 = 0.7. \end{gathered} \tag{37}$$

Using Theorem 8, the $\mathscr{H}_\infty$ optimal filter is obtained via using the LMI toolbox of MATLAB with n_f chosen to be 5. The

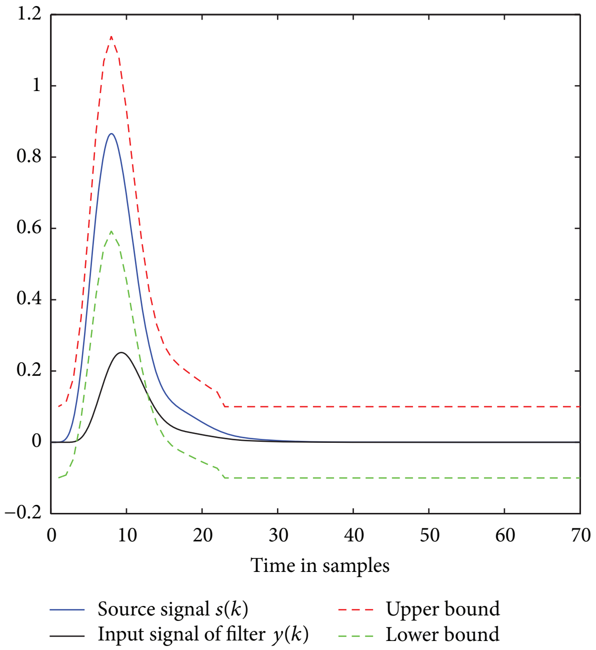

FIGURE 2: Source signal $s(k)$, filter input signal $y(k)$, and envelope bounds.

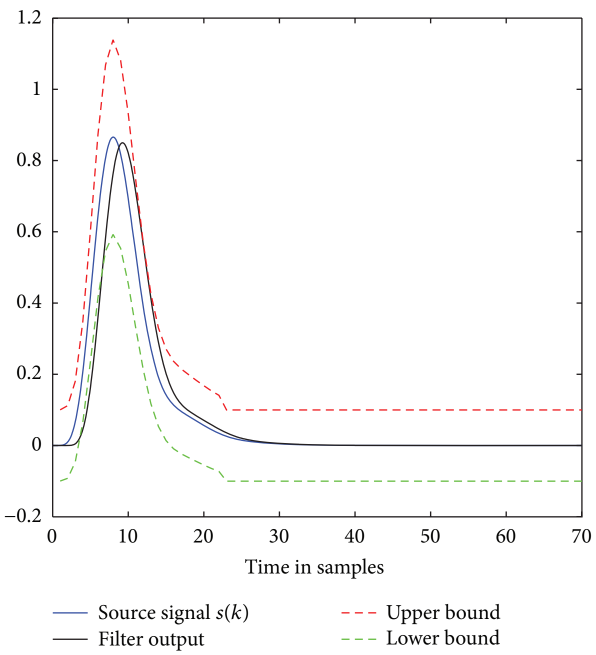

FIGURE 3: Output of the filter without disturbance.

resultant optimal γ is 8.5057 and filter gains are given as follows:

$$C_f = \begin{bmatrix} 0.0437 & -0.3344 & -0.4023 & 0.2045 & -0.5089 \end{bmatrix}, \qquad D_f = 4.3664. \tag{38}$$

The expected envelope constraints and $s(k)$ (the output of Σ_l) corresponding to a particular case where input signal

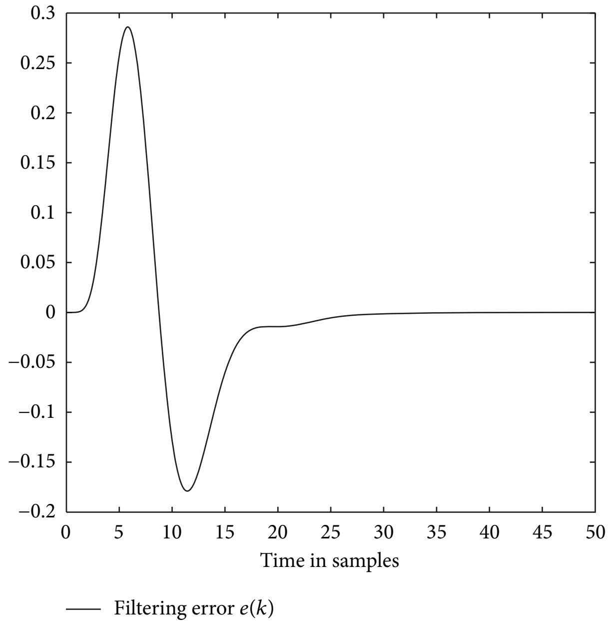

FIGURE 4: Filtering error $e(k)$ without disturbance.

$w(k)$ is chosen to be unit impulse signal are shown in Figure 2. The transmitted signal $y(k)$ through Σ_c which is generated with no noise added is also given in the figure. The filter output $\hat{s}(k)$ and filtering error $e(k)$ are given in Figures 3 and 4, respectively.

Furthermore, to illustrate the performance of the designed filter, we add the disturbance signal $v(k)$ chosen as white noise with mean of zero and variance of 1×10^{-3} into the system. The resultant filter output and filtering error are shown in Figures 5 and 6, respectively. It is shown that the designed filter is effective.

5. Conclusions

In this paper, we have solved the filtering design problem of time-delay system. The time-delay considered here is time-varying meanwhile with a certain stochastic characteristic, and the probability of delay distribution is assumed to be known. Furthermore, the envelope constraints are also considered in the process of filtering design. The delay-distribution-dependent criterion is formed for the filtering error system, employing the information about not only the size of delay but also its probability distribution. A set of linear matrix inequalities (LMIs) are formulated to solve the problem. Through solving the LMI optimization problem, the $\mathscr{H}_\infty$ performance is minimized and pulse-shape demand imposed by envelope constraints is satisfied. Finally, an illustrative example is presented to demonstrate the effectiveness of the filtering design approach. For future research directions, extending the filter design approach proposed in this paper to networked control systems and distributed systems is an interesting issue. Besides, more general filter

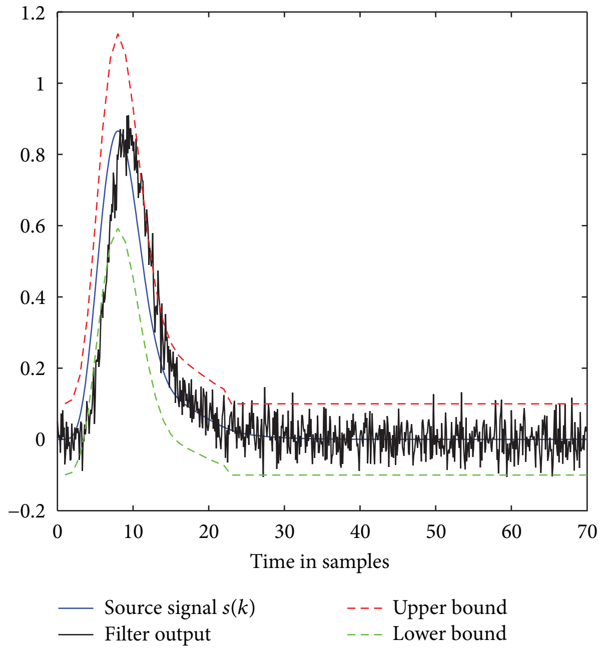

FIGURE 5: Output of the filter with disturbance.

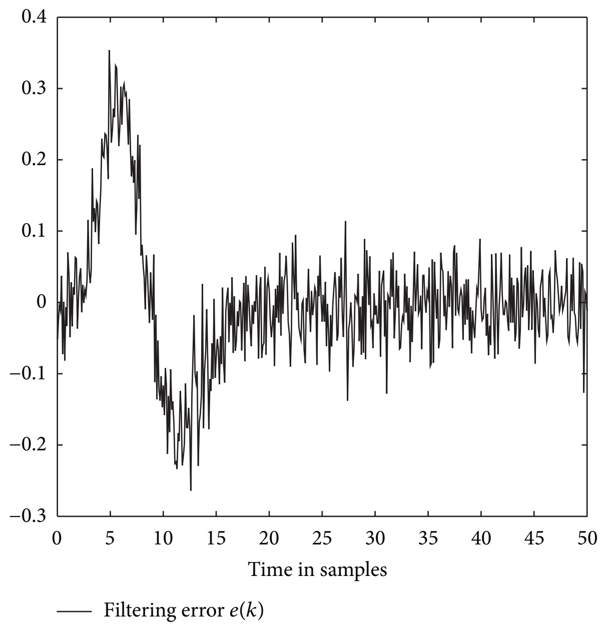

FIGURE 6: Filtering error $e(k)$ with disturbance.

design approaches considering delays in different forms with different characteristics also deserve further investigation.

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