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ORIGINAL ARTICLE

Kindergarten teachers' accounts of their developing mathematical practice

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Abstract This study explores kindergarten teachers' accounts of their developing mathematical practice in the context of their participation in a developmental research project. Observations and interviews were analysed to elaborate the accounts as regards orchestrating mathematical activities in the kindergarten. A co-learning agreement was established as collaboration between the kindergarten teachers and researchers. The study reveals that the kindergarten teachers argue that they have been empowered in developing an inquiry stance towards mathematics and mathematical activities. Taking an inquiry stance, they claim, has increased their awareness of the mathematics involved in activities, and enabled them to be more explicit when communicating mathematical ideas to children. An adjusted didactic triangle within the kindergarten setting is proposed based on these results.

Keywords Didactic triangle · Empowering · Inquiry · Kindergarten teachers · Mathematical practice

1 Introduction

This paper reports from a study where we investigated kindergarten teachers' accounts of their developing mathematical practice. By the term *account* we mean the kindergarten teachers' expressed personal views regarding their developing mathematical practice as these are made explicit in interviews. Our study is part of a larger developmental research project, subsequently referred to as "the project", which included teachers at all educational levels from kindergarten to upper secondary school. In our case, the aim was to work alongside the kindergarten teachers supporting their endeavour to improve their practice and offering opportunities for them to assume their agency in developing their mathematical and didactical competencies. Furthermore, our aim was to explore the extent to which the kindergarten teachers consider inquiry to be important in empowering their *mathematical epistemology* (van Oers 2002). The notion of empowerment is used in accordance with Villarreal et al. (2010), who described how the use of technology empowered students' engagement with modelling. We explore how appropriation of inquiry empowered the kindergarten teachers in mediating mathematics through pedagogical activities in the kindergarten. Specifically, we use the notion of empowerment to describe the kindergarten teachers' reported process regarding improvement of their mathematical competencies and their increased mathematical agency.

The kindergarten in Norway is part of the national educational system, and has increasingly been regarded as an important institution in which children (ages 1–6) are educated. In the current framework, mathematics has been included as a learning domain (Ministry of Education and Research 2006). Norwegian kindergartens are situated within a social pedagogy tradition (OECD 2006), where care, play and learning comprise the core enterprise. In the contemporary kindergarten teacher education, a bachelor education, mathematics has gained increased emphasis (zero ECTS credits before 1994, six credits in the period 1994–2003, and currently ten credits. ECTS is the European Credit Transfer and accumulation System. Ten credits means a work load of 250–300 hours and 60 credits is a full year of study at university level, 1500–1800 hours of work). Since mathematics was introduced as a learning domain in Norwegian kindergartens, in 2006, there has

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been a situation among practising kindergarten teachers characterised by uncertainty and lack of confidence regarding mathematics at the kindergarten level.

With this background a 3-year project focusing on orchestrating mathematical activities was initiated by the University of Agder in which schools and kindergartens participated. The aim of the project was, among others, to develop communities of inquiry in order to improve the teaching and learning of mathematics. Didacticians (mathematics education researchers) and kindergarten teachers collaborated in workshops at the University and in kindergartens. Our collaboration focused on issues of engaging children in mathematical inquiry, designing of mathematical tasks and activities. The notion of *inquiry* was emphasised in order to treat mathematical ideas in a playful manner within the kindergarten. Together with the kindergarten teachers we discussed questions such as: What does mathematics in the kindergarten look like? What activities are appropriate in the kindergarten? How do we orchestrate such activities? The following research question has been formulated for our study:

> What justifications do kindergarten teachers give in their accounts of their implementation of mathematical activities?

In order to address this question, interview data from kindergarten teachers and video data from observations of mathematical activities in the kindergarten were collected and analysed. In the following section we describe our theoretical framework within which the research was conducted, followed by a section describing developmental research as our adopted methodology. In Sect. 4 analyses of interviews with kindergarten teachers, followed by a mathematical learning activity, are presented. The paper concludes with a discussion in which we suggest an adjusted didactic triangle for the kindergarten setting.

2 Theoretical framework

In this study we adopt a sociocultural perspective on learning to examine kindergarten teachers' accounts of their developing mathematical practice. This elaborated Vygotskian perspective on learning and development (Rogoff 1990; Vygotsky 1986; Wertsch 1998) is activity oriented and asserts activity as a medium through which people gain experience and appropriate tools and actions. From a sociocultural view, mathematics is seen as a cultural activity in which people engage and contribute with ideas and arguments. Participation in this cultural activity, through collaboration and interaction with others, results in processes of appropriating mathematical tools and actions.

2.1 Inquiry as an approach to mathematics and mathematical activities

We use the term inquiry to describe a general approach to mathematics and implementation of mathematical activities in the kindergarten. "Inquiry as a way of being" (Jaworski 2005, 2007) is characterised by wondering, investigating, and exploring in our case the mathematical issues involved in orchestrating mathematical activities in the kindergarten. According to Wells (1999), inquiry is about "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (p. 121). Inquiry is closely related to the long tradition of researching collaborative problem solving (Lester 1994; Mason et al. 1982; Polya 1957; Schoenfeld 1985). Lindfors (1999) describes "authentic inquiry acts" to involve reasoning regarding seeking and connecting information, investigations, and clarification of opinions. In the kindergarten context inquiry thus involves a disposition to investigate mathematical topics such as number, geometry and measuring, and in orchestrating children's engagement with these topics in practical activities. As an example, elaborated in Sect. 4, a measuring activity related to weight was orchestrated by inquiring into issues of comparing the weights of toys.

Following Cochran-Smith and Lytle (1999), inquiry is approached through the notion of *an inquiry stance*. The consequences of adopting this stance are that didacticians as well as kindergarten teachers critically, but positively, investigate the practice in which we take part—the practice of orchestration and appropriation of mathematical tools. This stance also requires asking critical questions in order to get to the core of this practice. By taking an inquiry stance one opens oneself to various kinds of suggestions, ideas and thoughts that contribute to developing shared meanings of mathematical activities in the kindergarten.

In their study of professional development of mathematics teachers, Farmer et al. (2003) describe key issues involved if teachers are to change their practice as a result of participating in professional development projects. For our purpose here, we want to draw attention to the issue of *ownership*. Our assumption is that if any change in practice is likely to occur the teachers will need to take ownership of the changes by actively creating, designing and carrying them out. In our case, this means that the kindergarten teachers need to acknowledge and take ownership for what it means to take an inquiry approach towards mathematical activities in the kindergarten. Farmer et al. (2003) discuss the possibilities for teachers to appropriate mathematical tools and actions as well as issues of mathematics didactics in professional development projects, i.e. the teachers' "(a) mathematical knowledge, (b) view of mathematics

learning and teaching, (c) attitudes toward mathematics and mathematics learning, and (d) beliefs about the nature of mathematics, mathematics learning and mathematics teaching" (p. 334). The developmental research in which we engage is consistent with the study conducted by Farmer et al. (2003), that is to say, participation in developmental research offers opportunities for the kindergarten teachers to appropriate mathematical and didactical tools.

2.2 The didactic triangle in kindergarten versus school

Kindergarten teachers are concerned with the interrelationships and interactions among themselves, the children and the mathematics. These relations may be theorised by the notion of the didactic triangle. However, it is our opinion that the interrelationships within the didactic triangle in the kindergarten context are different from their parallels in the school setting. In Fig. 1 we illustrate what the didactic triangle looks like in a school context. The three most important elements as regards the teaching and learning of mathematics are: mathematics as an anchoring point, for instance the Pythagorean theorem; the teacher whose role it is to mediate mathematics through orchestrating mathematical activities in class, for instance drawing a right-angled triangle at the board, squares on the sides of the triangle, measuring lengths and areas, comparing sums, and asking questions while doing this; and the student who is exposed to goal-directed actions in order to experience and appropriate the mathematical tools and actions, for instance making their own right-angled triangles in their notebook, copying the teacher, doing exercises, and solving practical problems where application of the Pythagorean theorem is focused. The school context with its actors may very well be described through the notion of a *content oriented mathematical epistemology* (van Oers 2002).

This didactic triangle, we argue, is transformed when applied in the kindergarten context (see Fig. 2). The anchor point 'mathematics' is transformed into 'pedagogical activities', which might include some mathematics issues like counting and number. This is the major difference between the kindergarten setting and the school setting. In Norwegian kindergartens children take part in several activities every day, both adult-initiated and self-initiated. These activities are typically carried out in play.

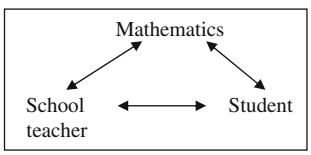

Fig. 1 The didactic triangle in a school context

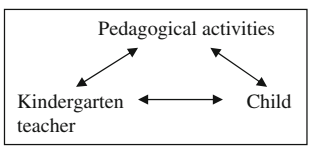

Fig. 2 The didactic triangle in a kindergarten context

Another contrast between the school context and the kindergarten context is in how children engage with the mathematics. In the kindergarten, children very often play with each other and experience issues involved in mathematical activities together with others. It is not usual that children engage with mathematical issues on an individual basis. An individual approach to learn mathematics is more apparent in the school context, where students often work individually with their textbook and tasks. Yet another difference between the two contexts is that the school context is very much an institution where writing and reading mathematics are emphasised actions. In the kindergarten, interaction is marked by verbal instructions and questions. Furthermore, the children engage *in* the activities, making the appropriation of mathematics closely interconnected with the activity.

From the outset, there is an asymmetrical relationship between adult(s) and child(ren) in educational settings. The teacher has responsibility to lead interactional and learning processes. This asymmetry, we argue, is more apparent in the kindergarten than in school, since according to Kansanen (1999) the asymmetrical nature of interaction is stronger the younger the children are. This is due to the more informal nature of interactions in kindergarten compared with school. Finally, there is a political and cultural contrast between schools and kindergartens in Norway as regards the emphasis on learning mathematics (cf. OECD 2006). Up to some years ago, children's subject matter learning was not focused as part of the kindergarten's enterprise. Politically it was emphasised that the school was the institution in which organised subject matter learning should take place. This has changed in the last few years, with a new framework for Norwegian kindergartens (Ministry of Education and Research 2006). Within this framework it is explicitly mentioned that the enterprise of the kindergarten must emphasise 'process goals'. That means that the children should experience mathematical challenges, engage with mathematical issues and ideas, and meet mathematical concepts and words, without demanding that the children have to appropriate those ideas and concepts as conventionally defined scientific concepts (Vygotsky 1986). Due to these differences with the school context, it is our opinion that the centre of gravity in the kindergarten didactic triangle is in the vertex labelled *pedagogical activities* rather than in *mathematics* as in the school context. Pedagogical activities are the focus of attention when kindergarten

teachers approach mathematics in their practice. Instead of reasoning about what mathematical subject to focus on, kindergarten teachers reason about what activity to offer, and then secondly what mathematical issues may be "touched" when participating in the activity.

2.3 Empowering through appropriation

From a sociocultural perspective, learning is seen as a process of appropriation, in other words as a process where individuals are "taking something that belongs to others and making it one's own" (Wertsch, 1998, p. 53). In accordance with Rogoff (1990) and Moschkovich (2004) the process of appropriating cultural tools such as mathematical concepts, ideas and strategies encompasses five elements. In order for a person to become a cultural knower of a tool, he or she needs to: (1) collaborate with others and put effort into participating in a joint activity; (2) establish a joint focus of attention with his/her collaborators; (3) develop and establish shared meanings with his/her collaborators; (4) transform others' ideas and contributions and be able to use them in his/her ongoing activities; and (5) attend to established mathematical and didactical knowledge and make connections between these and his/her own experience. In our case the process of appropriation thus may be described as a collaborative endeavour where kindergarten teachers collaborate with each other and didacticians. These parties negotiate and agree about what mathematical tools to focus on, the didactical and mathematical issues involved and how to orchestrate mathematical activities in the kindergarten involving these issues. Following Farmer et al. (2003), the kindergarten teachers' appropriation of mathematical and didactical tools may nurture the process of empowerment. Through their own developing competence within mathematics and developing expertise in orchestrating mathematical activities in the kindergarten, they may be empowered in their practice.

We assert that inquiry plays an essential role in this process of appropriation in that by adopting inquiry as an approach towards the mathematical tools, the process of making these tools one's own is nurtured and energised. As explained above, the process of appropriation is constituted by five elements, in which inquiry has a significant role in every one of them. Taking an inquiry stance towards the emerging mathematical and the didactical issues significantly contributes to the involvement and development of shared foci and meanings.

2.4 The role of inquiry in empowerment

We believe that inquiry empowers the kindergarten teachers in their implementation and orchestration of mathematical activities in the kindergarten, in several ways:

- inquiry is useful for the kindergarten teachers in appropriating the mathematics themselves;
- inquiry is useful for the kindergarten teachers in realising the mathematical potential of diverse activities;
- inquiry is useful for the kindergarten teachers in orchestrating mathematical activities;
- inquiry is useful for the children in their processes of appropriating mathematical tools and actions.

In our collaboration with the kindergarten teachers, an important aim was to develop a sustained impact among them with respect to appropriating inquiry as a tool and as a way of being (Jaworski 2005, 2007). By focusing the workshops around inquiry into mathematical areas such as geometry, algebra, functions, probability, and number, opportunities were offered to support the kindergarten teachers' awareness and appropriation of (a) mathematical tools and actions, (b) ways of engaging children in mathematical activities and (c) an approach towards mathematics and mathematics learning.

2.5 Mathematical epistemology and appropriation

The study of teachers' mathematical development and their mathematical learning processes is fundamentally about mathematical epistemology. The learning process results from educational experience and teaching practice. Thus, we interpret mathematical epistemology to be both personally and contextually dependent. That is, the mathematical epistemology has to be explored with respect to each of the kindergarten teachers' view of mathematics, what mathematics looks like in the kindergarten context, and their implementation of mathematical activities in the kindergarten. In his study, van Oers (2002) demonstrated that the teachers' practice was characterised by a content oriented epistemology and an activity oriented epistemology. The latter of these epistemologies, we argue, is most prevalent when describing Norwegian kindergarten teachers' practice. As in the Dutch situation described by van Oers, the Norwegian kindergartens' enterprise is governed by a play-based framework. This framework has an important role when the kindergarten teachers are to implement and orchestrate mathematical activities. A play-based framework and activity oriented epistemology are, in our view, compatible and thus provide consistent and coherent markers that direct the kindergarten teachers' practice. Thus, the framework and epistemological orientation are useful characterisations of the kindergarten teachers regarding their mathematical engagement and practice. An activity oriented mathematical epistemology, we argue, is characterised by the view that mathematics is a cultural activity where problem solving, interaction, and mathematising are important elements.

The above characterisation of a mathematical epistemology is consistent with inquiry as a way of being towards mathematics—a stance which emphasises the playful engagement and interaction with mathematical ideas and tools, by children as well as kindergarten teachers. Inquiry thus plays an essential role in developing each of the kindergarten teachers' mathematical epistemology. Additionally, the kindergarten teachers themselves, by signing up for participation in the project, signalled that they wanted to develop their mathematical epistemology. These participants thus are involved in a process of developing their mathematical epistemology, characterised as "a gradual process going from mixed epistemologies in the beginning towards a more coherent mathematical epistemology that is more in accordance with the activity oriented epistemology favoured by the play-based curriculum approach" (van Oers 2002, p. 24).

Furthermore, as argued by Farmer et al. (2003), professional development of kindergarten teachers aims at appropriation at several levels. According to these authors, there are three levels of appropriation, cumulative in nature, in which teachers participating in professional development projects engage. At *level one*, the focus is on appropriating specific mathematical and didactical tools, as well as particular mathematical problems or tasks that can be used in the kindergarten. At *level two*, the kindergarten teachers are appropriating new attitudes and beliefs concerning mathematics and mathematical orchestrations in the kindergarten. These appropriations may be described as professional principles and ideas that allow the kindergarten teachers to establish a more coherent picture of mathematics and what it means to orchestrate mathematical activities in the kindergarten. At this level the kindergarten teachers are both learners and improving professionals. Concrete problems and activities met at workshops are used as exemplifying general principles and categories. At *level three*, kindergarten teachers participating in professional development consider themselves as mathematical and didactical learners alongside the children. They are struggling to appropriate "how their students are thinking and why, and how to pose interesting worthwhile tasks" (Farmer et al. 2003, p. 342). At this level the kindergarten teachers are appropriating inquiry as a way of being, as an attitude when approaching new situations and challenges, both mathematically and didactically. The appropriated tools and principles are used as tools of inquiry at this level.

3 Methodology and research methods

The project in which this study is based adopted a developmental research methodology (Freudenthal 1991; Goodchild 2008; Gravemeijer 1994) aiming at making an impact on practitioners and the arena (kindergarten) in which they create settings to mediate mathematics. Additionally, the project was intended to make an impact on didacticians' arena of mathematics education research. In the project, there was a cyclical relationship between the work and activities arranged for the mathematics teachers' professional development and the research conducted by didacticians and teachers. Freudenthal (1991) argues that developmental research means "experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience" (p. 161). The methodology of developmental research thus simultaneously studies the research process and the process of development. By engaging in developmental research, we are able to study the complexity of orchestrating mathematical activities in kindergarten settings as well as possibly making contributions to taking an inquiry approach towards the mathematics. Developmental research can thus be viewed as both charting and promoting development (Jaworski 2010).

3.1 Co-learning between participants

In the project the notions of *co-learning* and *co-learning agreement* were important when portraying the collaboration between kindergarten teachers and didacticians. We take these notions from Wagner (1997):

> In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling. (Wagner 1997, p. 16)

We acknowledge that the two groups of participants bring different expertise when engaging in collaboration. The kindergarten teachers, the practitioners, are experienced professionals in orchestrating activities in the kindergarten; while didacticians, the researchers, are experienced professionals in conducting mathematics education research. This is, however, not to say that the kindergarten teachers are not researchers. The kindergarten teachers are seen as co-researchers, collaborators and participants both in the developmental process and the research process. Our point is to explicate the main responsibility of the two groups, respectively. This notion of co-learning agreement is of significance when it comes to the research methodology chosen in the project, developmental research, where both kindergarten teachers and didacticians are aware of their different roles. We are

aware that there might be an issue of power involved when three male didacticians are to collaborate with mostly female kindergarten teachers in co-researching mathematics education in the kindergarten. However, we have collaborated over 3 years in the project and come to know each other. Our meetings were thus characterised by trust and support, and both groups participated on equal ground in the process.

3.2 Data material and participants

The empirical basis we are drawing upon in this study is the interviews of eleven kindergarten teachers at three kindergartens (10 women and 1 man). The interviews have been transcribed in detail. The majority of these kindergarten teachers were educated before 1994, i.e. they have no formal education in mathematics. The kindergartens are approximately equal as regards number of staff altogether (25) as well as number of children (70–80). In Norwegian kindergartens three main groups of adults are working, assistants (adults with no pedagogical education), child and youth workers (adults with pedagogical education at the level of upper secondary school), and kindergarten teachers (bachelor education at University level) (Table 1).

Additionally, we report an activity from Duckling Pre-school Centre related to the mathematical theme of measuring. In this case the focus is comparing weights of various toys. The activity illustrates how one kindergarten teacher, Unni, orchestrated a measuring activity related to the weight of toys of different size and weight by the use of a pair of scales. The activity included a mixed-aged (3–4 years old) and mixed-gender group of six children.

3.3 Methods of data analysis

In this study we draw on data resources collected through field notes and reflection notes from observations and video recordings of kindergarten visits as well as audio recordings of three semi-structured focus group interviews. With respect to the analysis of the transcribed interviews, we took a systematic and iterative approach. Firstly, we made an analysis of the interviews based on the questions asked and answers given (see Appendix). Through several iterations we analysed the interviews and discussed their contents. This analytical approach resulted in codes such as *views of mathematics in general*, *views of mathematics in the kindergarten*, *experience from participating in the project*, *the role of framework* and *kindergarten teacher's role in orchestration of mathematical activities*. Secondly, the kindergarten teachers' accounts were compared based on emerging issues, commonalities and relevance for our research question. The unit of analysis in our study is thus kindergarten teachers' justifications for their developing practice.

We use the term *account* when referring to the kindergarten teachers' responses to questions in the interviews. According to Cohen et al. (2007), "accounts serve to explain our past, present, and future oriented actions" (p. 385). These accounts are thus seen as the kindergarten teachers' articulations of their experience from participating in the project and, as Cohen et al., we argue that these accounts offer a rationalisation of the kindergarten teachers' practice. The accounts the kindergarten teachers articulate in the focus group interviews carry similarities with storytelling as described by Cohen et al. (2007). We agree with these authors in that "stories…offer an opportunity for the researchers to gather authentic, rich and 'respectable' data" (p. 395). We wanted to know how the kindergarten teachers made sense of their own developing mathematical practice. To be more specific, we deal with the kindergarten teachers' accounts by tracing the themes elaborated as the two results communicated below (in Sects. 4.1 and 4.2, respectively). Our methodological approach is thus about making the kindergarten teachers talk about their practice, i.e. to expose their rationalisations about their own practice. Our approach bears characteristics similar to the study by Cooper and McIntyre (1996), in that the focus group interviews were set to enable the kindergarten teachers to articulate their personal views of orchestrating mathematical activities in the kindergarten context.

Due to the identification of the finding reported in Sect. 4.2, we made analyses of mathematical activities from the kindergartens. One of these activities we analysed in detail to make explicit the point made by kindergarten teachers in interviews concerning their argued improvement in making mathematical ideas more explicit. This particular activity was chosen to elaborate on the kindergarten teachers' utterances regarding making mathematical ideas explicit in interaction with the children. In a workshop, kindergarten teachers and didacticians discussed measuring as a mathematical topic and how to implement measuring activities in the kindergarten context. The observed activity was planned by the particular kindergarten teacher and

Table 1 Overview of participating kindergarten teachers in the interviews

Name of kindergarten	Naturbarnehagen Pre-school Centre	Pinocchio Pre-school Centre	Duckling Pre-school Centre
Name of the of kindergarten teachers in the interview	Else, Sam, Marit, Ronny (male)	Julie, Maiken, Lotte	Vilde, Line, Unni, Venke

didacticians were invited to observe the activity. Similar cycles of collaboration on mathematical topics in workshops and implementations of those topics in mathematical activities in kindergartens were typical throughout the project. This activity also illustrates how the didactic triangle is applicable in the kindergarten context, with an emphasis on experiencing mathematical concepts and ideas through activity. Furthermore, the activity exemplifies how one kindergarten teacher uses inquiry as a tool in order to make mathematical ideas more explicit. Regarding our methods of analysing one mathematical activity as naturally occurring talk-in-interaction, the video recordings were transcribed in detail to serve an in-depth analysis of the mathematical focus of the orchestrated activity.

4 Analysis and results

From our analysis of audiotaped interviews and videotaped observations, we will emphasise two findings, reported in Sects. 4.1 and 4.2, respectively.

4.1 The kindergarten teachers argue that they have been empowered through increased awareness regarding mathematics in the kindergarten setting

When the kindergarten teachers were asked to make explicit their experience from participating in the project, we found evidence in their accounts regarding the nature of mathematics and how mathematics unfolds in the kindergarten practice, including their personal view of mathematics.

Lotte at Pinocchio argued that through her participation in the project she has changed her way of thinking about mathematics in the kindergarten: "It is much easier for me to open up for more mathematical aspects to include in the activities and to think about how I can include these aspects. I have several mathematical ideas now that I will develop further." Several of the kindergarten teachers argued that they have been empowered mathematically as a consequence of their participation in the project. This argument is exemplified by Unni at Duckling: "We have all developed experience making us more aware and ready to exploit the mathematical potentials when they appear in activities. Perhaps we observed similar things before too, but then we were unconscious of the mathematics." Ronny at Naturbarnehagen said: "It has been fun to see that colleagues, who have had problems to see the meaning of including mathematics as a learning domain in the kindergarten, now have started to change their view of mathematics." These utterances exemplify the kindergarten teachers' accounts regarding their increased awareness of mathematics and in unfolding mathematical activities.

Furthermore, Lotte added to this picture by claiming that: "I believe it is important to move beyond the stage 'mathematics is everywhere' and 'we are counting'. I believe we have to move further than that. Of course, we are counting, but we have to be more thorough, to be more systematic, and justify what we are doing as regards mathematics." We interpret Lotte's utterance to communicate her accounts of an increased awareness regarding how to work mathematically in the kindergarten. In order to cope with mathematics in the kindergarten, Lotte finds it necessary for kindergarten teachers to be more systematic and thorough in their approach. Mathematical ideas implicitly present in pedagogical activities should be made explicit and emphasised in interaction with the children. We interpret Lotte's comment that she argues for a need to orchestrate mathematical activities focused on particular mathematical ideas.

The kindergarten teachers generally expressed an earlier lack of enthusiasm as regards mathematics, but due to their participation in the project they claim this to have changed. Marit at Naturbarnehagen said: "We see that the project has contributed in making mathematics less fearful to us." Such changed personal relationship with mathematics is also expressed by Venke at Duckling: "I have been one of those who have disliked mathematics, but now I very much appreciate it." We interpret these utterances as exemplifying the kindergarten teachers' accounts of their changed personal view of and relationship to mathematics. Their increased enthusiasm, we believe, carries opportunities for orchestrating meaningful mathematical activities in the kindergarten. The kindergarten teachers claim they have changed their view of the nature of mathematics in the kindergarten as well as their individual view of mathematics. We interpret their utterances as exemplifying their accounts of their processes of changing attitudes towards mathematics and orchestration of mathematical activities. These changes, they argue, are due to their participation in the professional development project. These claimed changes are in accordance with the findings reported by Farmer et al. (2003). The kindergarten teachers, we interpret, have taken ownership with respect to their personal approach to mathematics in the kindergarten setting.

4.2 The kindergarten teachers made mathematical ideas explicit when communicating with the children

The kindergarten teachers argue that their increased awareness and changed individual relationship with mathematics have led them to change their interaction with children. In the interviews the kindergarten teachers emphasised that they have progressed in their orchestration of mathematical activities. They valued the workshops in

the project as important settings to discuss and share mathematical and didactical ideas. Line at Duckling reflected: "During the workshops, my brain starts to work and I am thinking on how I can use this, whether it is too difficult to implement or if I can use parts of it. In the workshop on symmetry [in a plenary session, pictures of symmetric leaves and flowers were presented], I thought that this is useful and possible because we are, in our kindergarten, always focusing on outdoor activities." This utterance exemplifies how the kindergarten teacher seeks to establish connections between mathematical ideas introduced at a workshop and how to possibly implement and make explicit those mathematical ideas in the kindergarten context. As Lotte's utterance above shows, she argues that the kindergarten teachers currently "are more thorough and more systematic" in their implementation of mathematical activities in the kindergarten. This utterance, we argue, also indicates that Lotte is concerned to make the mathematical issues involved in the activities explicit in her interaction with the children.

In order to document explicitness in communicating mathematical ideas in interaction with children, we will analyse an episode designed by one of the kindergarten teachers in the project. The following episode illustrates how Unni attends to particular mathematical ideas in a learning activity using a pair of scales, a toy crocodile, and a number of small plastic bears of different sizes and weights. Unni and six children were sitting around a table, talking about the function of the pair of scales, the weights of toys, and comparing weights. During a 30 min activity, the group of children together with Unni investigated and discussed weight issues. Our focus in these episodes is on the kindergarten teacher and teaching and not on the children's appropriation of the concept of weight. This activity we divide into four phases.

4.2.1 *Phase 1: introduction to how a pair of scales works—comparisons of weights*

The mathematical scope of the activity was about inquiring into the weights of the different plastic bears, comparing their weights, and realising which bear is heavier. The children compared the weights of the plastic bears by holding them in their hands to experience their different weights. The children also compared the weights of the small bears and the weight of a toy crocodile made of fabric by using the pair of scales. This crocodile thus had a much bigger volume compared with the plastic bears, but it weighed less than the bears. Unni asked the question: "What is heavier, the crocodile or the box [the box contained a number of plastic bears]?" She let the children hold the two objects in their hands. Some of the children argued that the crocodile was heavier (which was incorrect) because it was bigger (Fig. 3).

Fig. 3 Illustration of the situation in phase 1—a toy crocodile versus a number of bears

When placing the crocodile in one of the scale pans and the box of plastic bears in the other, the pan with the crocodile moved upwards and the pan with bears moved downwards. In this situation, by seeking to establish equilibrium between the scales, Unni made explicit the difference between the comparison words "big" and "heavy" through the use of questions. At this point we are not convinced that the children have realised that equilibrium means equal weights. In particular, one of the boys wanted to put another plastic bear into the heaviest scale in order to establish equilibrium of the scales. However, the fact that the children experienced the comparing of weights of the plastic bears in their own hands gave them opportunities to become aware of the functionality of the pair of scales.

Unni's actions in this phase indicated that she had identified two main goals for the activity: children's exploration of the functionality of the pair of scales and of the difference between volume and weight. She wanted the children to experience that two objects may weigh the same, independent of volume, and that two sets with different number of objects may weigh the same. Unni confirmed these interpretations of her actions when we asked her about this in our interview with her immediately after the session.

4.2.2 *Phase 2: systematic inquiry into weight relationships*

During this phase Unni introduced small plastic bears with different size and weight, and the children started to investigate the internal relationships of the bears' weight. The plastic bears came in three sizes: big, middle sized and small. One big bear has the same weight as three small bears, and one middle sized bear has the same weight as two small bears. Unni set up an equilibrium situation with four small bears in one of the scale pans and one big bear together with a small bear in the other pan (Fig. 4).

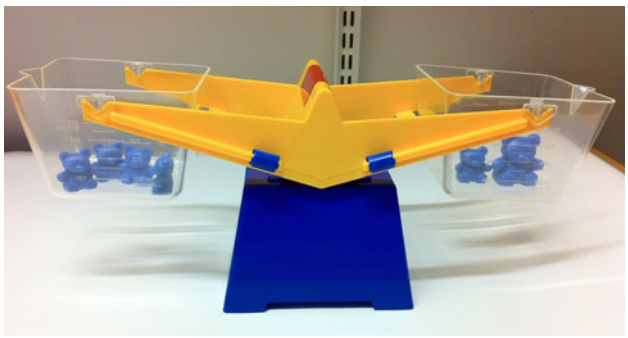

Fig. 4 Illustration of the situation in phase 2: four small bears versus one big + one small bear

Unni asked questions such as: "Is it possible to find out how many small bears are needed in order to make them as heavy as one big bear?" and "If we want to have the big bear alone in this scale, and they are to weigh the same, what do we then have to do?" The children seemed unsure what to do in order to cope with these questions. Unni then suggested to Geir: "If the big bear is going to be alone, we have to take away the small bear, don't we?" When he removed the small bear, this pan went up. Tore observed this and immediately took away one small bear from the other pan. Equilibrium was then re-established. The following dialogue then took place:

Unni: That was a smart thing to do, Tore. Now the scales became level. What did you do now?
Tore: We removed these two.
Unni: You removed these two. But did they come from the same pan?
Geir: No.
Tore: No.
Unni: Are they of different size?
Geir: Yes.
Tore: Yes.
Unni: Are they? Look at them (Tore and Geir compare the sizes of the bears they have removed from the scales.)
Unni: They do have the same size. So, if we remove two bears of the same size, one from each of the scales, we get back to the pans being level.

In this dialogue Unni gave the children opportunities to re-establish equilibrium of the scales through removing the same from both of them. Unni concluded that this happened because the two bears that were removed were of equal size. From a mathematical point of view this phase concerns the concept of equation, where the goal is to find—the unknown x—the weight of one big bear: $x + 1 = 4$. When the boys removed one small bear from each of the scales, the new situation revealed that one big bear has the same weight as three small bears, $x = 3$. When participating in such an activity, opportunities were given for the children to experience basic principles of equations in a kindergarten setting. Unni's actions during this phase exemplify her activity oriented mathematical epistemology (van Oers 2002).

4.2.3 Phase 3: continuing inquiry into weight relationships

This phase included situations established by Unni in order to emphasise the weight relationships between the different sized bears. Two new challenges were given. In the first one, Unni put two big bears into one of the scale pans and asked: "How can we make them (the pans) level?" Julie put two small bears into the other pan, but equilibrium was not obtained. Tore then helped Julie and instead put two big bears into the pan. By doing that, equilibrium was established.

Then Unni introduced the second challenge, where she told Tore to put three small bears into the right pan (currently there are two big bears in the left pan and three small bears in the right pan). Unni asked: "What do you have to do now to make them level?" We interpret Unni's intention by this question to make Tore put three additional small bears into the right pan. However, Tore put one middle sized and one small bear into the right pan. Despite this, equilibrium was established. Unni then took the opportunity to start a third challenge, and she asked the children: "How can we find the weight of one middle sized bear?" After a while Tore put one middle sized bear in one pan and two small bears in the other pan. Unni then concluded, together with Tore, that one middle sized bear has the same weight as two small ones. The interaction between Unni and the children in this phase shows that even though the children took unintentional directions in the activity Unni took the opportunity to guide the inquiry process in this case and make the mathematical ideas explicit.

4.2.4 Phase 4: summing up and concluding

Unni summarised the activity by being explicit concerning the mathematical knowing emerging in the activity. She initiated the phase by saying: "Now I am going to show you something." She placed one big bear in front of three small bears and one middle sized bear in front of two small bears. By pointing with her finger and emphasising words she said: "*That* one (pointing at the big bear) weighs the same as *those* three (pointing at the small ones), and *this* one (pointing at the middle sized bear) weighs the same as *those* two (pointing at the small ones)." The children responded to this by saying that "they are equal", meaning that the big bear has the same weight as three small ones and the middle sized bear has the same weight as two small bears.

4.2.5 Summary of the activity

Unni is explicit as regards the mathematics involved, by emphasising the emerging mathematical knowing in a structured way. During the first three phases she asked questions and suggested actions in order for the children to experience mathematical principles and ideas related to algebra. These actions led to the concluding phase where the children together with Unni established shared meanings as regards the internal weight relationships between the bears.

5 Discussion

In this study, we set out to answer the question: "What justifications do kindergarten teachers give in their accounts of their implementation of mathematical activities?" Through our analysis we have reported two main results that provide answers to this question. The kindergarten teachers report that they have increased their awareness with respect to identifying mathematical ideas implicitly included in pedagogical activities in the kindergarten context. Furthermore, the kindergarten teachers express that they make mathematical ideas explicit in interaction with the children.

On the basis of these results we argue that participation in the project has offered opportunities for the kindergarten teachers to appropriate mathematical knowing (cf. Rogoff 1990). Moreover, their participation in the project, taking an inquiry stance (Cochran-Smith and Lytle 1999), has led them to develop their competence in engaging with mathematics in the kindergarten both for children and themselves. The kindergarten teachers seem to have developed their views with respect to mathematical content in activities, children's possibilities to appropriate mathematical tools and actions, and how to mediate mathematics in the kindergarten context. They have in this way taken ownership in developing their own practice. Our findings thus correspond with the results of Farmer et al. (2003), however at a kindergarten level.

Both by their statements in focus group interviews as well as in practical activities, we have seen how the kindergarten teachers express a raised awareness of mathematical ideas. They have themselves been involved in processes of appropriating mathematical tools and actions (Rogoff 1990; Wertsch 1998). The kindergarten teachers claim they have developed their competence in orchestrating meaningful mathematical activities by making the mathematical ideas involved more explicit. The mathematics involved in phase 2 above illustrates the fact that mathematics through an inquiry approach, met by the kindergarten teachers in the project, gave them possibilities to implement mathematical ideas in kindergarten through practical activities.

The children's appropriation processes originate in the activities (Moschkovich 2004; Rogoff 1990), and the mathematical learning goals are contextually formulated in close relationship with the activities. The activities are hence crucial elements with respect to creating opportunities for the children to make mathematical experience. Thus, the kindergarten teachers may be said to adopt an activity oriented mathematical epistemology (cf. van Oers 2002). By mathematical activities we mean activities in which the children participate and engage, and when participating in such activities the children come into contact with mathematical tools and words, mathematical relationships and ideas.

The kindergarten teachers' activity oriented mathematical epistemologies have been empowered by their appropriation of inquiry as a stance towards mathematics in the kindergarten setting. They make mathematical ideas and actions more explicit in interaction with children due to increased awareness as regards mathematics. With the notion of "levels of appropriation" (Farmer et al. 2003), the kindergarten teachers have appropriated mathematical skills, concepts and pedagogical techniques (level 1). As an example, we see that Unni has appropriated knowing within measuring, i.e. what measuring is all about, what mathematical words to emphasise, the possible miscommunications involved as well as how to engage the children in the measuring, what and how to ask questions and how to involve the children in an inquiry process. The kindergarten teachers have also developed their views of mathematics and the orchestration of mathematical activities in the kindergarten (level 2). In the focus group interviews we saw examples of utterances where this point is emphasised. For instance, Ronny's utterance communicates this: "It has been fun to see that colleagues, who have had problems to see the meaning of including mathematics as a learning domain in the kindergarten, now have started to change their view of mathematics." The kindergarten teachers have become interested in mathematical and pedagogical principles and ideas that may guide their orchestration of mathematical activities.

In approaching level 3, the kindergarten teachers are about to view themselves as mathematical learners alongside their children while engaging in mathematical activities. Moreover, they have been empowered in their processes of taking inquiry as stance towards mathematics and mathematical activities. The very essence of taking inquiry as a way of being (cf. Jaworski 2005, 2007) is an encouragement to get to know more of the mathematics, to know more with respect to engaging their children in mathematical activities, and to know more of how to emphasise the mathematical content within these activities.

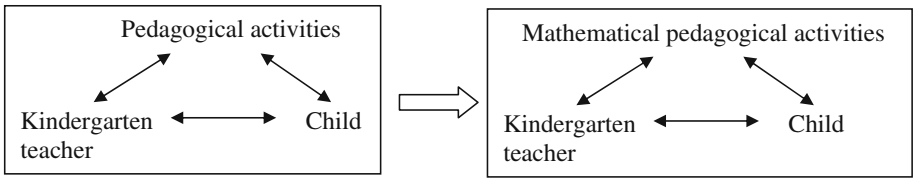

Fig. 5 Development of a modified didactic triangle in a kindergarten context

Drawing on our results, we suggest that the didactical relationships illustrated in Fig. 2 may be described differently. The upper vertex of the triangle labelled 'Pedagogical activities', in which mathematical ideas were included or emphasised in an ad hoc way, may be transformed to encompass a more focused view regarding mathematical content within the activities. The kindergarten teachers express an increased awareness with respect to implement mathematical ideas and make them explicit within their orchestrated activities. We therefore argue to label this vertex 'Mathematical pedagogical activities' (Fig. 5).

In this modified didactic triangle, mathematical ideas and goals are at the centre and they are the motivation for the orchestration of the activity. The current situation in the kindergartens thus can be described by both these didactic triangles. These triangles seem to co-exist, where mathematical ideas are possibly included in pedagogical activities while the mathematical ideas are set as the focal point in mathematical pedagogical activities.

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Appendix: Questions for focus group interview

1.
   a. If you were asked to tell a group of teachers or kindergarten teachers at another school or kindergarten about the project, what would you say to them?
   b. What does it mean to you to be part of the project?
2.
   a. What do you think we have succeeded with?
   b. Do you have any suggestions as to what to possibly improve?
3.
   a. What do you consider to be affordances and constraints regarding your participation in the project?
   b. What do you have to do at your kindergarten in order for you to be where you want to be in 1 year?
4. Do you as a kindergarten team have any questions for the didacticians?

Information: the questions were meant as a guide for the conversation. Duration was about 1 h.

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