Statistical Properties of the Capacity of Double Nakagami-m Channels

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***Abstract*—In this article, we have presented an extensive statistical analysis of the capacity of double[1] Nakagami-m channels. The double Nakagami-m channel model has applications in keyhole channels and amplify-and-forward relay based dualhop communication systems in cooperative networks. We have derived exact analytical expressions for the probability density function (PDF), the cumulative distribution function (CDF), the level-crossing rate (LCR), and the average duration of fades (ADF) of the capacity of double Nakagami-m channels. Moreover, the influence of severity of fading on the statistical properties of the channel capacity has been studied. It is observed that an increase in the severity of fading in one or both links in dualhop communication systems decreases the mean channel capacity, while it results in an increase in the ADF of the channel capacity. Moreover, this effect decreases the LCR of the channel capacity at lower signal levels. The results presented in this paper also reveal that an increase in the maximum Doppler frequencies of the wireless nodes in a dualhop communication system increases the LCR of the channel capacity, while it has an opposite influence on the ADF of the channel capacity. The results presented in this article are useful for mobile communication system engineers for the design and optimization of dualhop communication systems.**

I. Introduction

The design and analysis of cascaded fading models has been an active area of research in recent years due to its applications in numerous real world scenarios such as keyhole channels [1], [2], and multihop communication systems [3]–[6]. It is shown in [7], [8] that in the presence of a keyhole, the fading between each transmit and receive antenna pair in a multi-input multi-output (MIMO) system can be characterized using a double Rayleigh process. Afterwards, this model has been extended to the double Nakagami-m fading model in [9]. In [1], authors have listed a few real world scenarios which give rise to the keyhole effect. Two such scenarios include diffraction through the street edges in urban microcellular environments [7] and traversal of the propagation paths through a narrow space for the case when the distance between the rings of scatterers around the transmitter and receiver is large [10].

Multihop communication systems on the other hand fall under the category of cooperative diversity techniques [11], [12]. In such systems, the wireless nodes (in a cooperative network) assist each other by relaying the information from the source mobile station (SMS) to the destination mobile station (DMS), hence improving the network coverage quite significantly. The statistical analysis of the received signal envelope under non-line-of-sight (NLOS) propagation conditions in an amplify-and-forward based dualhop communication system can be found in [13], where the overall channel between the transmitter and the receiver is modeled using a double Rayleigh process. This model is then extended to the double Rice channel model in [6], by taking the line-of-sight propagation conditions into account. The statistical properties of the capacity of double Rice channels have been analyzed in [14]. However, the Nakagami-m process is considered to be a more general channel model as compared to the Rice and Rayleigh channel models. Hence, to generalize all the aforementioned works in the regime of multihop communication, the authors of [4] have presented the statistical analysis of the N*Nakagami-m model (i.e., a product of N Nakagami-m processes). Moreover, second order statistics for the double Nakagami-m process can be found in [2]. Though a lot of papers have been published in the literature employing the cascaded fading channel model, the statistical properties of the capacity of double Nakagami-m channels have not been investigated so far, which finds applications both in keyhole channels and dualhop communication systems [2].

In this article, we present the statistical properties of the capacity of double Nakagami-m channels. Specifically, the influence of the severity of fading on the statistical properties of the capacity of double Nakagami-m channels is analyzed. We have derived exact analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity. Here, the LCR and ADF of the channel capacity are important characteristic quantities which provide insight into the temporal behavior of the channel capacity [15], [16]. Our analysis has revealed that if the fading severity in one or both links of double Nakagami-m channels decreases (i.e., increasing the value of the severity parameter m in one or both of the cascaded

[1]Throughout this paper, we will refer to a double process as the product of two independent but may not necessarily identical processes.

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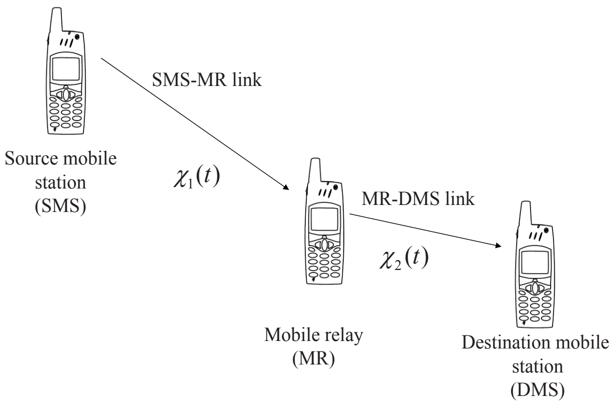

Fig. 1. The propagation scenario describing double Nakagami-m fading channels.

Nakagami-m processes), the mean channel capacity increases, while the ADF of the channel capacity decreases. Moreover, this effect results in an increase in the LCR of the channel capacity at lower signal levels.

The rest of the paper is organized as follows. Section II briefly describes the double Nakagami-m channel model and some of its statistical properties. In Section III, the statistical properties of the capacity of double Nakagami-m channels are presented. Numerical results are analyzed in Section IV. Finally, the concluding remarks are given in Section V.

II. The Double Nakagami-m Channel Model

A typical example of the transmission link from the SMS to the DMS via a mobile relay (MR) in an amplify-and-forward relay-based dualhop communication system is depicted in Fig. 1. In such a scenario, the channel between the SMS and the DMS via an MR can be represented as a concatenation of the SMS-MR and MR-DMS channels [6], [13]. In this article, we have assumed that the fading in the SMS-MR link and the MR-DMS link is characterized by Nakagami-m processes denoted by $\chi_1(t)$ and $\chi_2(t)$, respectively. Hence, the overall fading channel describing the SMS-DMS link can be modeled as a double Nakagami-m process given by [2], [17]

$$\Xi(t) = A_{\text{MR}}\,\chi_1(t)\chi_2(t) \qquad (1)$$

where A_{MR} is a real positive constant representing the relay gain. For the case of keyhole channels, A_{MR} equals unity. The PDF $p_{\chi_i}(z)$ of the Nakagami-m process $\chi_i(t)$ $(i = 1, 2)$ is given by [18]

$$p_{\chi_i}(z) = \frac{2m_i^{m_i} z^{2m_i-1}}{\Gamma(m_i)\Omega_i^{m_i}} e^{-\frac{m_i z^2}{\Omega_i}}, \quad z \geq 0 \qquad (2)$$

where $\Omega_i = E\left\{\chi_i^2(t)\right\}$, $m_i = \Omega_i^2 \big/ \text{Var}\left\{\chi_i^2(t)\right\}$, and $\Gamma(\cdot)$ represents the gamma function [19]. The parameter m_i controls the severity of the fading. Increasing the value of m_i, decreases the severity of fading and vice versa.

The PDF of double Nakagami-m processes $\Xi(t)$ is given by [4]

$$p_\Xi(z) = \frac{4z^{m_1+m_2-1}}{\prod_{i=1}^{2}\Gamma(m_i)\left(\acute{\Omega}_i \big/ m_i\right)^{(m_1+m_2)/2}} \times K_{m_1-m_2}\left(2z\prod_{i=1}^{2}\sqrt{\frac{m_i}{\acute{\Omega}_i}}\right), \quad z \geq 0 \qquad (3)$$

where $\acute{\Omega}_1 = A_{\text{MR}}^2\Omega_1$, $\acute{\Omega}_2 = \Omega_2$, and $K_n(\cdot)$ denotes the modified Bessel function of the second kind of order n [19, Eq. (8.432/1)]. In order to derive the expressions for the PDF, CDF, LCR, and ADF of the capacity of double Nakagami-m channels, we need the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ of the squared process $\Xi^2(t)$ and its time derivative $\dot{\Xi}^2(t)$, as well as the PDF $p_{\Xi^2}(z)$ of $\Xi^2(t)$. The joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ can be found by following the procedure presented in [2] for the joint PDF $p_{\Xi\dot{\Xi}}(z,\dot{z})$ and then by using the concept of transformation of random variables [20, Eq. (7-8)], which results in

$$\begin{aligned} p_{\Xi^2\dot{\Xi}^2}(z,\dot{z}) &= \frac{1}{4z}\, p_{\Xi\dot{\Xi}}\left(\sqrt{z}, \frac{\dot{z}}{2\sqrt{z}}\right) \\ &= \frac{z^{m_2-3/2}}{\sqrt{2\pi}}\left[\prod_{i=1}^{2}\frac{m_i^{m_i}}{\acute{\Omega}_i^{m_i}\Gamma(m_i)}\right]\int_0^\infty \frac{x^{2m_1-2m_2-1}}{\sqrt{\frac{z\beta_1}{x^2}+x^2\beta_2}}\, e^{-\frac{zm_2}{x^2\Omega_2}} \\ &\quad \times e^{-\left(\frac{\dot{z}^2}{8z\left(\frac{z\beta_1}{x^2}+x^2\beta_2\right)}+\frac{x^2 m_1}{\acute{\Omega}_1}\right)} dx, \quad z \geq 0, |\dot{z}| < \infty \end{aligned} \qquad (4)$$

where

$$\beta_1 = \frac{\acute{\Omega}_1\pi^2}{m_1}\left(f_{\max_1}^2 + f_{\max_2}^2\right) \qquad (5a)$$

and

$$\beta_2 = \frac{\acute{\Omega}_2\pi^2}{m_2}\left(f_{\max_2}^2 + f_{\max_3}^2\right). \qquad (5b)$$

Here, $f_{\max_1}$, $f_{\max_2}$, and $f_{\max_3}$ represent the maximum Doppler frequencies of the SMS, MR, and DMS, respectively. The expression for the PDF $p_{\Xi^2}(z)$ can be obtained by integrating the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ over $\dot{z}$. Alternatively, the PDF $p_{\Xi^2}(z)$ can also be found from the PDF $p_\Xi(z)$ in (3) using the relationship $p_{\Xi^2}(z) = \left(1/\,2\sqrt{z}\right)\, p_\Xi(\sqrt{z})$.

III. Statistical Properties of the Capacity of Double Nakagami-m Channels

The instantaneous capacity $C(t)$ of double Nakagami-m channels is defined as [21]

$$\begin{aligned} C(t) &= \frac{1}{2}\log_2\left(1+\gamma_s\left|\Xi(t)\right|^2\right) \\ &= \frac{1}{2}\log_2\left(1+\gamma_s\,\Xi^2(t)\right) \;\; \text{(bits/sec/Hz)} \end{aligned} \qquad (6)$$

where γ_s denotes the average received signal-to-noise ratio (SNR) at the DMS. The factor $1/2$ in (6) is due to the fact that the MR in Fig. 1 operates in a half-duplex mode, and hence the signal transmitted from the SMS is received at the DMS in two time slots. Equation (6) can be considered as a mapping of a random process $\Xi(t)$ to another random process

$C(t)$. Hence, the expressions for the statistical properties of the channel capacity $C(t)$ can be found by using the results for the statistical properties of the process $\Xi(t)$ obtained in the previous section. The PDF $p_C(r)$ of the channel capacity $C(t)$ can be found in closed form with the help of the PDF $p_{\Xi^2}(z)$ and by applying the concept of transformation of random variables [20, Eq. (7-8)] as

$$\begin{aligned} p_C(r) &= \left(\frac{2^{2r+1}\ln(2)}{\gamma_s}\right) p_{\Xi^2}\left(\frac{2^{2r}-1}{\gamma_s}\right) \\ &= \frac{2^{2r+2}\ln(2)\left(\left(2^{2r}-1\right)/\gamma_s\right)^{(m_1+m_2)/2}}{(2^r-1)\prod\limits_{i=1}^{2}\Gamma(m_i)\left(\acute{\Omega}_i/m_i\right)^{(m_1+m_2)/2}} \\ &\quad \times K_{m_1-m_2}\left(2\sqrt{\frac{2^{2r}-1}{\gamma_s}}\prod_{i=1}^{2}\sqrt{\frac{m_i}{\acute{\Omega}_i}}\right), \quad r \geq 0. \end{aligned} \tag{7}$$

The CDF $F_C(r)$ of the channel capacity $C(t)$ can now be derived by integrating the PDF $p_C(r)$ and by making the use of relationships in [19, Eq. (9.34/3)] and [22, Eq. (26)] as

$$\begin{aligned} F_C(r) &= \int_0^r p_C(x)dx \\ &= \frac{1}{\prod\limits_{i=1}^{2}\Gamma(m_i)} G_{1,3}^{2,1}\left[\frac{2^{2r}-1}{\gamma_s}\prod_{i=1}^{2}\left(\frac{m_i}{\acute{\Omega}_i}\right)\middle|\begin{matrix}1\\ m_1, m_2, 0\end{matrix}\right], \\ &\qquad r \geq 0 \end{aligned} \tag{8}$$

where $G[\cdot]$ denotes the Meijer's G-function [19, Eq. (9.301)]. The LCR $N_C(r)$ of the channel capacity describes the average rate of up-crossings (or down-crossings) of the capacity through a certain threshold level r. In order to find the LCR $N_C(r)$, we first need to find the joint PDF $p_{C\dot{C}}(z,\dot{z})$ of $C(t)$ and its time derivative $\dot{C}(t)$. The joint PDF $p_{C\dot{C}}(z,\dot{z})$ can be obtained by using the joint PDF $p_{\Xi^2\dot{\Xi}^2}(z,\dot{z})$ given in (4) as

$$\begin{aligned} p_{C\dot{C}}(z,\dot{z}) &= \left(\frac{2^{2z+1}\ln(2)}{\gamma_s}\right)^2 p_{\Xi^2\dot{\Xi}^2}\left(\frac{2^{2z}-1}{\gamma_s}, \frac{2\dot{z}\ln(2)}{\gamma_s/2^{2z}}\right) \\ &= \frac{\left(2^{2z+1}\ln(2)\right)^2\left(2^{2z}-1\right)^{m_2-\frac{3}{2}}}{\sqrt{2\pi\gamma_s}\,\gamma_s^{m_2}\left[\prod\limits_{i=1}^{2}\left(\frac{\acute{\Omega}_i}{m_i}\right)^{m_i}\Gamma(m_i)\right]}\int_0^\infty \frac{x^{2m_1-2m_2-1}}{\sqrt{\frac{(2^{2z}-1)\beta_1}{\gamma_s x^2}+x^2\beta_2}} \\ &\quad \times e^{-\left(\frac{\left(2^{2z+1}\ln(2)\dot{z}\right)^2}{8\gamma_s\left(2^{2z}-1\right)\left(\frac{(2^{2z}-1)\beta_1}{\gamma_s x^2}+x^2\beta_2\right)}+\frac{x^2 m_1}{\acute{\Omega}_1}+\frac{\left(2^{2z}-1\right)m_2}{\gamma_s x^2\acute{\Omega}_2}\right)} dx \end{aligned} \tag{9}$$

for $z \geq 0$ and $|\dot{z}| < \infty$. Finally, the LCR $N_C(r)$ can be found as follows

$$\begin{aligned} N_C(r) &= \int_0^\infty \dot{z}\, p_{C\dot{C}}(r,\dot{z})d\dot{z} \\ &= \sqrt{\frac{8}{\pi}}\left(\frac{2^{2r}-1}{\gamma_s}\right)^{m_2-\frac{1}{2}}\left[\prod_{i=1}^{2}\frac{m_i^{m_i}}{\acute{\Omega}_i^{m_i}\Gamma(m_i)}\right]\int_0^\infty e^{-\frac{\left(2^{2r}-1\right)m_2}{\gamma_s x^2\acute{\Omega}_2}} \\ &\quad \times \frac{\sqrt{\frac{(2^{2r}-1)\beta_1}{\gamma_s x^2}+x^2\beta_2}}{x^{1+2m_2-2m_1}} e^{-\frac{x^2 m_1}{\acute{\Omega}_1}} dx, \quad r \geq 0. \end{aligned} \tag{10}$$

The ADF $T_C(r)$ of the channel capacity $C(t)$ denotes the average duration of time over which the capacity is below a given level r [16], [23]. The ADF $T_C(r)$ of the channel capacity can be expressed as [23]

$$T_C(r) = \frac{F_C(r)}{N_C(r)} \tag{11}$$

where $F_C(r)$ and $N_C(r)$ are given by (8) and (10), respectively.

IV. Numerical Results

In this section, we will discuss the analytical results obtained in the previous section. The validity of the theoretical results is confirmed with the help of simulations. For comparison purposes, we have also shown the results for double Rayleigh channels, which represent a special case of double Nakagami-m channels. In order to generate Nakagami-m processes $\chi_i(t)$, we have used the following relationship [24]

$$\chi_i(t) = \sqrt{\sum_{l=1}^{2\times m_i}\mu_{i,l}^2(t)} \tag{12}$$

where $\mu_{i,l}(t)$ $(l = 1, 2, ..., 2m_i;\ i = 1, 2)$ are the underlying independent and identically distributed (i.i.d.) Gaussian processes, and m_i is the parameter of the Nakagami-m distribution associated with the ith link of the dualhop communication systems. The Gaussian processes $\mu_{i,l}(t)$, each with zero mean and variances σ_0^2, were simulated using the sum-of-sinusoids model [25]. The model parameters were computed using the generalized method of exact Doppler spread (GMEDS$_1$) [26]. The number of sinusoids for the generation of Gaussian processes $\mu_{i,l}(t)$ was chosen to be $N = 29$. The parameter Ω_i was chosen to be equal to $2m_i\sigma_0^2$. Unless stated otherwise, the values of the maximum Doppler frequencies $f_{\max_1}$, $f_{\max_2}$, and $f_{\max_3}$ were taken to be 0, 91, and 125 Hz, respectively. The SNR γ_s was set to 15 dB. The parameters A_{MR} and σ_0 were chosen to be unity. Finally, using (12), (1), and (6), the simulation results for the statistical properties of the channel capacity were found.

The PDF and CDF of the channel capacity of double Nakagami-m channels are presented in Figs. 2 and 3, respectively. Both figures illustrate the fact that increasing the value of the severity parameter m_i (i.e., a decrease in the level of the severity of fading) in one or both links of the double Nakagami-m channels results in an increase in the mean channel capacity. This result is specifically presented in Fig. 4, where the mean channel capacity is studied for different values of the severity parameter m_i $(i = 1, 2)$. It can also be seen that double Rayleigh channels $(m_i = 1;\ i = 1, 2)$ have a lower mean channel capacity as compared to the mean channel capacity of double Nakagami-m channels $(m_i = 2;\ i = 1, 2)$. Moreover, it can also be observed from Figs. 2 and 3 that increasing the value of the severity parameter m_i decreases the variance of the channel capacity.

Figure 5 presents the LCR $N_C(r)$ of the capacity $C(t)$ of double Nakagami-m channels. It is observed that an increase

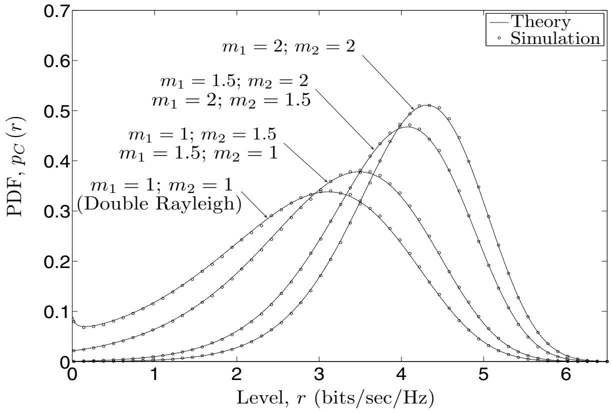

Fig. 2. The PDF $p_C(r)$ of the capacity of double Nakagami-m channels.

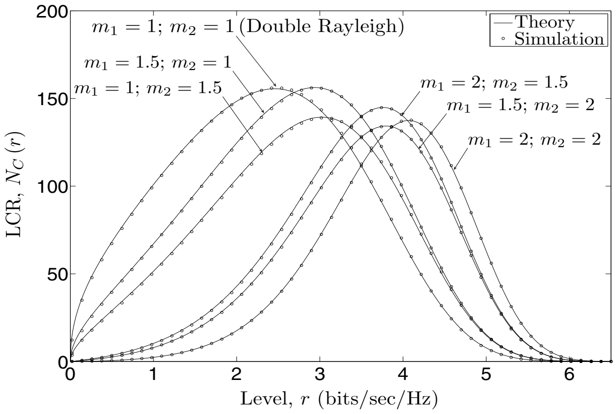

Fig. 5. The LCR $N_C(r)$ of the capacity of double Nakagami-m channels.

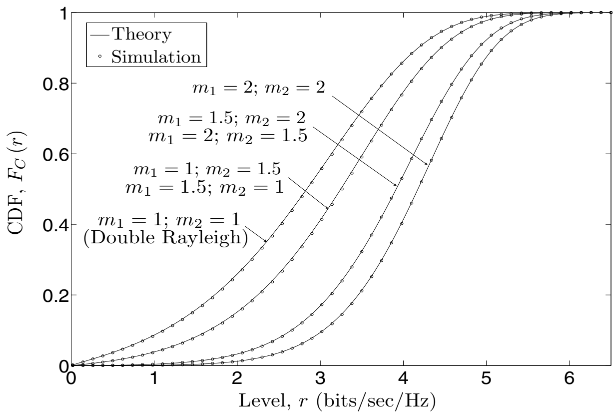

Fig. 3. The CDF $F_C(r)$ of the capacity of double Nakagami-m channels.

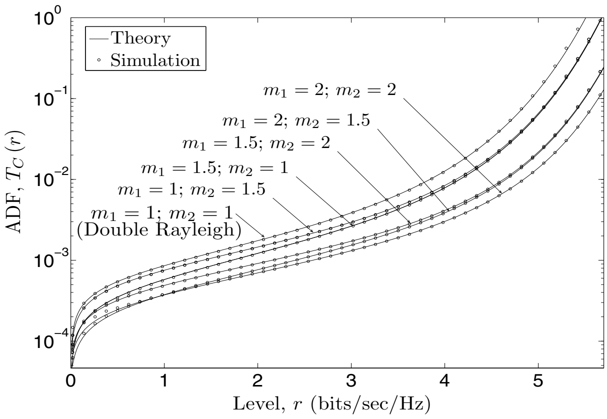

Fig. 6. The ADF $T_C(r)$ of the capacity of double Nakagami-m channels.

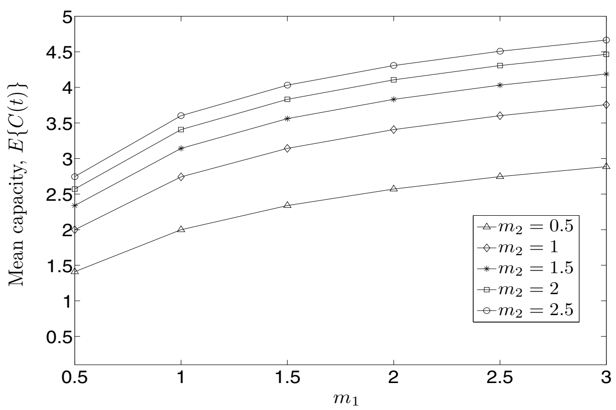

Fig. 4. The mean channel capacity of double Nakagami-m channels for different levels of fading severity.

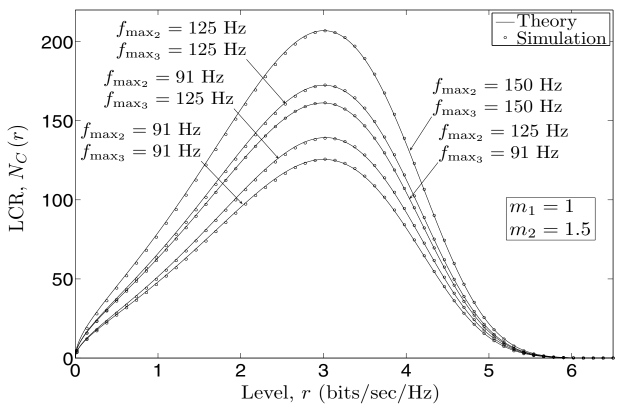

Fig. 7. The LCR $N_C(r)$ of the capacity of double Nakagami-m channels.

in the level of severity of fading in one or both links of double Nakagami-m channels increases the LCR $N_C(r)$ of the channel capacity at low levels r. Hence, at low levels r, the LCR $N_C(r)$ of the capacity of double Rayleigh channels ($m_i = 1;\ i = 1, 2$) is higher as compared to that of double Nakagami-m channels ($m_i = 2;\ i = 1, 2$). However, the converse statement is true for higher levels r. The ADF of the capacity of double Nakagami-m channels is shown in Fig. 6. It is evident from this figure that the ADF of the capacity decreases with an increase in the value of the severity parameter m_i ($i = 1, 2$).

Figures 7 and 8 study the influence of the maximum Doppler frequencies of the MR and the DMS on the LCR and ADF of the channel capacity. It can clearly be observed in Figs. 7 and 8 that the LCR and ADF are strongly dependent on the Doppler

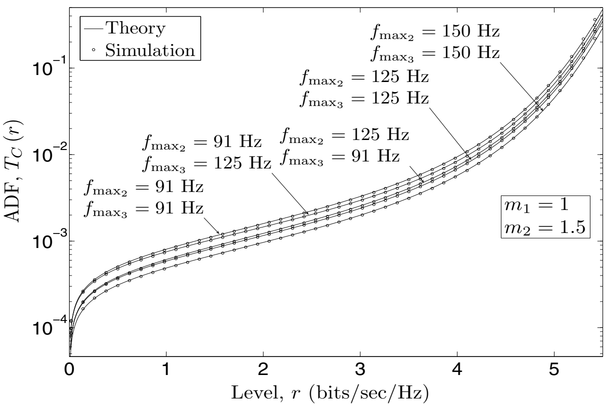

Fig. 8. The ADF $T_C(r)$ of the capacity of double Nakagami-m channels.

frequencies of the MR and the DMS. This means that the mobility of the MR and the DMS has a significant influence on the LCR and ADF of the channel capacity. It is observed that increasing the maximum Doppler frequencies $f_{\max_2}$ and $f_{\max_3}$ results in a significant increase in the LCR. However, the ADF decreases by increasing the maximum Doppler frequencies of the MR and the DMS.

V. Conclusion

This article presents the derivation of exact analytical expressions for the statistical properties of the capacity of double Nakagami-m channels, which finds applications in cooperative networks and keyhole channels. We have studied the influence of the severity of fading on the PDF, CDF, LCR, and ADF of the channel capacity. It is observed that an increase in the severity of fading in one or both links of double Nakagami-m channels decreases the mean channel capacity, while it results in an increase in the ADF of the channel capacity. Moreover, at lower signal levels, this effects increases the LCR of the channel capacity. Results also show that the mobility of the MR and DMS has a significant influence on the LCR and ADF of the channel capacity. Specifically, an increase in the maximum Doppler frequencies of the MR and DMS increases the LCR, while it has an opposite influence on the ADF of the channel capacity. The presented exact results are validated with the help of simulations, whereby a very good fitting is observed.

Acknowledgment

The contribution of G. Rafiq and Prof. M. Pätzold in this paper was partially supported by the Research Council of Norway (NFR) through the project 176773/S10 entitled "Optimized Heterogeneous Multiuser MIMO Networks – OptiMO".

The contribution of Dr. B. O. Hogstad was partially supported by the Spanish Ministry of Education and Science as well as by the European Regional Development Fund through the program CONSOLIDER-INGENIO 2010 CSD2008-00010 COMONSES.

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