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On the Statistical Properties of Equal Gain Combining over Mobile-to-Mobile Fading Channels in Cooperative Networks

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***Abstract*— This article deals with the statistical analysis of equal gain combining (EGC) over mobile-to-mobile (M2M) fading channels in a dual-hop amplify-and-forward relay network. Here, we analyze narrowband M2M fading channels under non-line-of-sight (NLOS) propagation conditions. It is assumed that there exist K diversity branches between the source mobile station and the destination mobile station via K mobile relays. The received signal envelope at the output of the equal gain (EG) combiner is thus modeled as a sum of K double Rayleigh processes. It has been shown that the evaluation of the probability density function (PDF) of this sum process using the characteristic function (CF) is rather intractable. However, the target PDF can efficiently be approximated by the gamma distribution. Exploiting the properties of the gamma distribution, the cumulative distribution function (CDF), the level-crossing rate (LCR), and the average duration of fades (ADF) of the sum process are also approximated. The approximation of the mentioned sum process by a gamma distributed process makes it possible to provide simple and closed-form analytical expressions for the aforementioned statistical quantities. The validity of the obtained analytical expressions is confirmed by simulations. The presented results can easily be utilized in the performance analysis of EGC over relay-based M2M fading channels.**

I. Introduction

Recent studies in the field of wireless communications have demonstrated the potential of cooperative relaying to meet the high data-rate and coverage requirements of future wireless communication systems [1], [2]. The advantage of such relaying is the attainment of spatial diversity gain by utilizing the existing resources of the network. From a variety of cooperative diversity schemes [3]–[5], the study presented in this article revolves around amplify-and-forward-type relaying. In this scenario, several mobile stations assist the source mobile station by amplifying and forwarding its information signal to the destination mobile station.

Here, a dual-hop amplify-and-forward configuration has been taken into account, where there exist K mobile relays between the source mobile station and the destination mobile station. Such a configuration in turn gives rise to K diversity branches. Thus, the previously mentioned spatial diversity gain is achieved by combining the signal received from the K diversity branches at the destination mobile station. Among the most important diversity combining techniques [6], maximal ratio combining (MRC) has been proved to be the optimum one [6]. It is widely acknowledged in the literature that a suboptimal and less complex combining technique, referred to as EGC, performs very close to MRC [6]. Studies regarding the statistical properties of EGC and MRC in non-cooperative networks over Rayleigh, Rice, and Nakagami fading channels are reported in [7]–[9]. Furthermore, performance analysis of the said schemes in terms of the bit error and outage probability over Rayleigh, Rice, and Nakagami fading channels can be found in [10]–[12]. In addition, the performance analysis of cooperative diversity using EGC and MRC over Rayleigh and Nakagami-m fading channels is presented in [13]–[15].

In recent years, M2M communications has also gained considerable attention for its potential utilization in cooperative networks, ad hoc networks, and vehicle-to-vehicle (V2V) communications. Statistical studies of M2M fading channels in [16] have revealed that such channels behave quite differently from the conventional cellular and land mobile terrestrial channels like, e.g., Rayleigh, Rice, and Suzuki channels. In relay-based cooperative networks, M2M fading channels under NLOS propagation conditions can be modeled as double Rayleigh stochastic processes [17]. Under line-of-sight (LOS) propagation conditions, double Rice processes effectively model M2M fading channels [18]. The authors of [19] have recently studied the performance of digital modulation over double Nakagami-m fading channels with MRC diversity. However, to the best of our knowledge, the statistical analysis of double Rayleigh and/or double Nakagami-m fading channels with EGC has not been carried out so far.

This article deals with the derivation and analysis of the statistical properties of EGC over M2M fading channels in cooperative networks. Although, on the one hand, the PDF can efficiently characterize a fading channel, the LCR and the ADF, on the other hand, provide vital information about how fast the fading channel is changing with time. Thus, studies pertaining to the PDF, the CDF, the LCR, and the ADF along with the CF of the received signal envelope at the output of the EG combiner are included in this article. The output signal of the EG combiner is modeled as a sum of K double Rayleigh processes. It has been illustrated that the computation of the PDF of the sum of K double Rayleigh processes is rather intractable when it comes to the evaluation of the inverse

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Fourier transform of the CF. An alternate approach is therefore presented in which the PDF is either approximated by another but a simpler expression or by a series.

Depending upon the purpose for which the approximated PDF has to be used, several methods of approximation have been proposed so far [20]–[23]. Here, we follow the approximation approach using orthogonal series expansion. Specifically, we introduce an approximation approach based on the Laguerre series expansion [21]. It turns out that the first term in the Laguerre series equals gamma distribution. This enables us to show that the PDF of the sum of K double Rayleigh processes can efficiently be approximated by the gamma distribution. Exploiting the properties of the gamma distribution, closed-form approximations are presented for the CDF, the LCR, and the ADF of the sum process. Furthermore, the close fitting of the approximated theoretical results with those of the exact simulation results shows that the approximation approach followed here is valid. We have demonstrated the influence of the number of diversity branches K on the statistical properties of the received signal envelope at the output of the EG combiner.

The organization of the remaining part of the paper is as follows. Section II describes the M2M fading channel model associated with amplify-and-forward relay networks. Section III deals with the derivation of the analytical expressions for the CF, the PDF, the CDF, the LCR, and the ADF of the received signal envelope at the output of the EG combiner. Section IV validates the correctness of the analytical expressions presented in Section III by simulations. Finally, some concluding remarks are presented in Section V.

II. EGC over M2M Fading Channels

In this section, we describe the system model of EGC over M2M fading channels in a dual-hop cooperative network, where there are K mobile relays connected in parallel between the source mobile station and the destination mobile station. We aim to investigate frequency non-selective M2M fading channels under NLOS propagation conditions in isotropic scattering conditions. All mobile stations in the network, i.e., the source mobile station, the destination mobile station, and the K mobile relays do not transmit and receive a signal at the same time in the same frequency band. The propagation scenario considered is illustrated in Fig. 1, where it can be seen that the mobile relays form K diversity branches.

We have taken into account the time-division multiple-access (TDMA) based amplify-and-forward relay protocols proposed in [24], [25]. Thus, the signals from the K diversity branches in different time slots can be combined at the destination mobile station using EGC. Assuming perfect channel state information (CSI) at the receiver, the received signal envelope at the output of an EG combiner can be written as [6]

$$\Xi(t) = \sum_{k=1}^{K} \left|\varsigma^{(k)}(t)\right| = \sum_{k=1}^{K} \chi^{(k)}(t) \quad (1)$$

where $\varsigma^{(k)}(t)$ $(k = 1, 2, \ldots, K)$ describes the fading process in the kth subchannel from the source mobile station to the destination mobile station via the kth mobile relay. Here, we model the fading process $\varsigma^{(k)}(t)$ as a weighted zero-mean complex double Gaussian process, i.e.,

$$\varsigma^{(k)}(t) = \varsigma_1^{(k)}(t) + j\varsigma_2^{(k)}(t) = A_{\mathrm{R}(k)}\, \mu^{(2k-1)}(t)\, \mu^{(2k)}(t) \quad (2)$$

for $k = 1, 2, \ldots, K$. In (2), $\mu^{(i)}(t)$ $(i = 1, 2, \ldots, 2K)$ represents a complex circular Gaussian process with zero mean and variance $2\sigma^2_{\mu^{(i)}}$. These Gaussian processes are mutually independent, where the spectral characteristics of each one are described by the classical Jakes Doppler power spectral density. The Gaussian process $\mu^{(i)}(t)$ defines for $i = 2k-1 = 1, 3, \ldots, (2K-1)$ the scattered component of the subchannel between the source mobile station and the kth mobile relay. Similarly, the Gaussian process $\mu^{(i)}(t)$ represents for $i = 2k = 2, 4, \ldots, 2K$ the scattered component of the subchannel between the kth mobile relay and the destination mobile station. In (2), $A_{\mathrm{R}(k)}$ is the relay gain of the kth relay. It is worth mentioning that the relay gain $A_{\mathrm{R}(k)}$ can be considered as a scaling factor for the variance of the complex Gaussian process $\mu^{(i)}(t)$, i.e., $\mathrm{Var}\left\{A_{\mathrm{R}(k)}\mu^{(i)}(t)\right\} = 2(A_{\mathrm{R}(k)}\sigma_{\mu^{(i)}})^2$, where $i = 2, 4, \ldots, 2K$. The absolute value of $\varsigma^{(k)}(t)$ is denoted by $\chi^{(k)}(t)$ in (1), where each $\chi^{(k)}(t)$ is a double Rayleigh process.

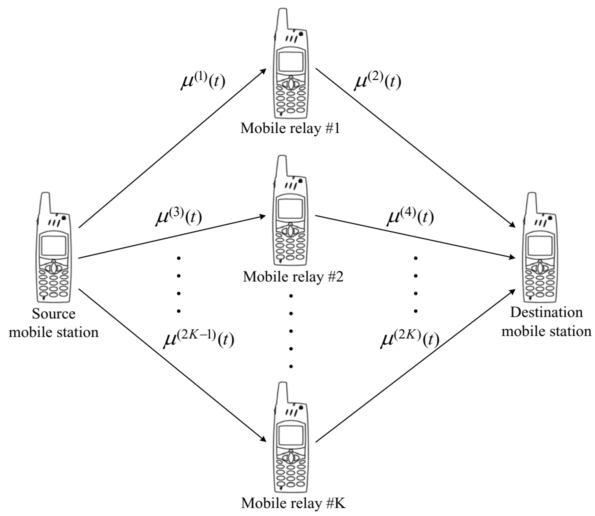

Fig. 1. The propagation scenario describing K-parallel dual-hop relay M2M fading channels.

III. Statistical Analysis of EGC over M2M Fading Channels

This section aims at presenting a thorough study of the statistical properties of EGC over M2M fading channels. The derivation and analysis of the closed-form analytical expressions for the CF, the PDF, the CDF, the LCR, and the ADF of $\Xi(t)$ is included in the discussion below.

A. PDF of a Sum of Double Rayleigh Processes

In order to express the PDF $p_\Xi(x)$ of $\Xi(t)$, we discuss two independent approaches, namely the CF-based exact solution and the approximate solution obtained using an orthogonal series.

1) CF-based Exact Solution: Mobile radio fading channels are usually characterized with the help of statistical quantities, such as the PDF of the envelope and the phase of the received signal. Another, important statistical quantity is the CF, which is related to the corresponding PDF through the Fourier transform.

Since the received signal envelope at the output of the EG combiner $\Xi(t)$ given in (1) is a sum of K independent but not necessarily identical double Rayleigh processes $\chi^{(k)}(t), (k = 1, 2, \ldots, K)$. Therefore, we can express the CF $\Phi_{\Xi}(\omega)$ of $\Xi(t)$ as a product of the CFs $\Phi_{\chi^{(k)}}(\omega)$ of the stochastic processes $\chi^{(k)}(t)$, i.e.,

$$\Phi_{\Xi}(\omega) = \prod_{k=1}^{K} \Phi_{\chi^{(k)}}(\omega) . \tag{3}$$

By definition, the PDF of a stochastic process and its CF are related through the Fourier transform [26] as

$$\Phi_{\chi^{(k)}}(\omega) = \int_{-\infty}^{\infty} p_{\chi^{(k)}}(x)\, e^{j x \omega}\, dx \tag{4}$$

where $p_{\chi^{(k)}}(x)$ represents the double Rayleigh distribution [17], i.e.,

$$p_{\chi^{(k)}}(x) = \frac{x}{\sigma^2_{\mu^{(2k-1)}}\sigma^2_{\mu^{(2k)}}} K_0\left(\frac{x}{\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}}}\right). \tag{5}$$

Substituting (5) in (4) and solving the integral over x using [27, Eq. (6.621-3)] allows us to write $\Phi_{\chi^{(k)}}(\omega)$ as [28]

$$\Phi_{\chi^{(k)}}(\omega) = \frac{4}{3}\frac{1}{(1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}})^2} \times {}_2F_1\left[2, \frac{1}{2}; \frac{5}{2}; \frac{-1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}}}{1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}}}\right] \tag{6}$$

$\forall\, k = 1, 2, \ldots, K$. In (6), ${}_2F_1[\cdot,\cdot;\cdot;\cdot]$ is the Gauss hypergeometric function [27]. Finally, substituting (6) in (3) provides us with the required CF $\Phi_{\Xi}(\omega)$ of $\Xi(t)$, i.e.,

$$\Phi_{\Xi}(\omega) = \prod_{k=1}^{K} \frac{4}{3}\frac{1}{(1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}})^2} \times {}_2F_1\left[2, \frac{1}{2}; \frac{5}{2}; \frac{-1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}}}{1 - j\omega A_{R^{(k)}}\sigma_{\mu^{(2k-1)}}\sigma_{\mu^{(2k)}}}\right] . \tag{7}$$

Given the CF $\Phi_{\Xi}(\omega)$ of $\Xi(t)$, the corresponding PDF $p_{\Xi}(x)$ can be obtained by computing the (complex conjugate of the) inverse Fourier transform of the CF $\Phi_{\Xi}(\omega)$. Unfortunately, the complicated form of the CF $\Phi_{\Xi}(\omega)$ makes it difficult to obtain a simple and closed-form expression for the PDF $p_{\Xi}(x)$.

Assuming $\chi^{(k)}(t)$ $(k = 1, 2, \ldots, K)$ are independent and identically distributed (i.i.d.) stochastic processes, implies that $\Phi_{\chi}(\omega) = \Phi_{\chi^{(k)}}(\omega)$, with $\sigma^2_{\mu^{(2k-1)}} = \sigma^2_{\mu^{(1)}}$, $\sigma^2_{\mu^{(2k)}} = \sigma^2_{\mu^{(2)}}$, and $A_R = A_{R^{(k)}}$ $\forall\, k = 1, 2, \ldots, K$. This in turn allows us to express the CF $\Phi_{\Xi}(\omega)$ in (3) as

$$\Phi_{\Xi}(\omega) = \{\phi_{\chi}(\omega)\}^K = \left\{\frac{4}{3}\frac{1}{(1 - j\omega A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}})^2} \times {}_2F_1\left[2, \frac{1}{2}; \frac{5}{2}; \frac{-1 - j\omega A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}}{1 - j\omega A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}}\right]\right\}^K . \tag{8}$$

By substituting (8) in [26, Eq. (5-94)], the PDF $p_{\Xi}(x)$ can be obtained. However, note that the CF $\Phi_{\Xi}(\omega)$ in (8) is still quite complex to be evaluated using [26, Eq. (5-94)].

2) Approximate Solution Using an Orthogonal Series Expansion: It has been shown in Section III-A.1 that it is rather complicated to compute the PDF $p_{\Xi}(x)$ of $\Xi(t)$ by taking the inverse Fourier transform of the CF $\Phi_{\Xi}(\omega)$. An alternate approach is to approximate the PDF $p_{\Xi}(x)$ of $\Xi(t)$ either by another but a simpler expression or by a series. Here, we follow the approximation approach using orthogonal series expansion. From various options of such series, like, e.g., the Edgeworth series and the Gram-Charlier series, we apply in our analysis the Laguerre series expansion [21]. The Laguerre series provides a good approximation for PDFs that are unimodal (i.e., having single maximum) with fast decaying tails and positive defined random variables. Furthermore, the Laguerre series is often used when the first term of the series provides a good enough statistical accuracy [21].

The PDF $p_{\Xi}(x)$ of $\Xi(t)$ can then be expressed using the Laguerre series expansion as [21]

$$p_{\Xi}(x) = \sum_{n=0}^{\infty} b_n\, e^{-x} x^{\alpha_L} L_n^{(\alpha_L)}(x) \tag{9}$$

where

$$L_n^{(\alpha_L)}(x) = e^x \frac{x^{(-\alpha_L)} d^n}{x!\, dx^n}\left[e^{(-x)} x^{n+\alpha_L}\right], \alpha_L > -1 \tag{10}$$

denote the Laguerre polynomials. The coefficients b_n can be given as

$$b_n = \frac{n!}{\Gamma(n + \alpha_L + 1)} \int_0^{\infty} L_n^{(\alpha_L)}(x)\; p_{\Xi}(x)\, dx \tag{11}$$

where $x = y/\beta_L$ and $\Gamma(\cdot)$ is the gamma function [27].

The parameters α_L and β_L can be computed by solving the system of equations in [21, p. 21] for $b_1 = 0$ and $b_2 = 0$. The solution of the mentioned system of equations yields

$$\alpha_L = \frac{(\kappa_1^{\Xi})^2}{\kappa_2^{\Xi}} - 1, \qquad \beta_L = \frac{\kappa_2^{\Xi}}{\kappa_1^{\Xi}} \tag{12a,b}$$

where κ_1^{Ξ} and κ_2^{Ξ} are the first and second cumulant, respectively, of the stochastic process $\Xi(t)$. Note that the first and second cumulant of $\Xi(t)$ are merely the mean value and the variance, respectively. Furthermore, assuming that the power of the sum process $\Xi(t)$ is normalized by the number of diversity branches K and the underlying double Rayleigh processes are i.i.d. stochastic processes, we can express the nth cumulant of $\Xi(t)$, κ_n^{Ξ}, in terms of the nth cumulant of $\chi(t)$, κ_n^{χ}, as

$$\kappa_n^{\Xi} = \frac{\kappa_n^{\chi}}{K^{(n-1)}} \tag{13}$$

where $\chi(t)$ denotes a double Rayleigh process. The first two cumulants of $\chi(t)$ can be expressed as [29]

$$\kappa_1^{\chi} = \frac{A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}\pi}{2}, \quad \kappa_2^{\chi} = \frac{1}{4}A_R^2\sigma^2_{\mu^{(1)}}\sigma^2_{\mu^{(2)}}\left(16 - \pi^2\right). \tag{14a,b}$$

Given κ_n^{χ}, the value of κ_n^{Ξ} $(n = 1, 2)$ can easily be computed using (13), which directly leads to the evaluation of α_L and β_L in (12a,b). Substituting the obtained quantities α_L and β_L in the Laguerre series expansion, the first term of the series can be identified as the gamma distribution

$$p_{\Gamma}(x) = \frac{x^{\alpha_L}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L + 1)} e^{-\frac{x}{\beta_L}} . \tag{15}$$

The PDF $p_{\Xi}(x)$ of $\Xi(t)$ can thus be approximated as

$$p_{\Xi}(x) \approx p_{\Gamma}(x) = \frac{x^{\alpha_L}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} e^{-\frac{x}{\beta_L}} . \quad (16)$$

The usefulness of the PDF $p_{\Xi}(x)$ of $\Xi(t)$ lies in the fact that it can be utilized in the performance analysis at the link level of EGC systems. Such performance studies require the knowledge of the PDF of the received signal-to-noise ratio (SNR). Making use of (16) and applying the concept of transformation of random variables [26, p. 244], the required PDF of the SNR can be obtained.

In the following, we derive the analytical expressions of the CDF, the LCR, and the ADF assuming that the double Rayleigh processes $\chi^{(k)}(t)$ are i.i.d. stochastic processes. In addition, we would exploit the properties of the gamma distribution.

B. CDF of a Sum of Double Rayleigh Processes

The CDF $F_{\Xi_-}(r)$ of $\Xi(t)$ is the probability that $\Xi(t)$ remains below the threshold level r [26]. Substituting (16) in $F_{\Xi_-}(r) = \int_0^r p_{\Xi}(x)\,dx$ and solving the integral over x using [27, Eq. (3.381-3)] allows us to express the CDF $F_{\Xi_-}(r)$ in closed form as

$$F_{\Xi_-}(r) \approx 1 - \frac{1}{\Gamma(\alpha_L+1)}\Gamma\left(\alpha_L, \frac{r}{\beta_L}\right) \quad (17)$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function [27].

C. LCR of a Sum of Double Rayleigh Processes

The LCR $N_{\Xi}(r)$ of $\Xi(t)$ represents the average rate of up-crossings (or down-crossings) of the stochastic process $\Xi(t)$ per second through a certain threshold level r. The LCR $N_{\Xi}(r)$ can be computed using the formula [30]

$$N_{\Xi}(r) = \int_0^{\infty} \dot{x}\, p_{\Xi\dot{\Xi}}(r,\dot{x})\,d\dot{x} \quad (18)$$

where $p_{\Xi\dot{\Xi}}(r,\dot{x})$ is the joint PDF of the stochastic process $\Xi(t)$ and its corresponding time derivative $\dot{\Xi}(t)$ at the same time t. Throughout this paper, the overdot represents the time derivative.

In Section III-A.2, we have shown that the PDF $p_{\Xi}(x)$ of $\Xi(t)$ can efficiently be approximated by the gamma distribution $p_{\Gamma}(x)$. Based on this fact, we assume that the joint PDF $p_{\Xi\dot{\Xi}}(r,\dot{x})$ is approximately equal to the joint PDF $p_{\Gamma\dot{\Gamma}}(r,\dot{x})$ of a gamma process and its corresponding time derivative at the same time t, i.e.,

$$p_{\Xi\dot{\Xi}}(r,\dot{x}) \approx p_{\Gamma\dot{\Gamma}}(r,\dot{x}) . \quad (19)$$

A gamma distributed process is equivalent to a squared Nakagami-m distributed process [31]. Thus, applying the concept of transformation of random variables [26, p. 244], we can express the joint PDF $p_{\Gamma\dot{\Gamma}}(x,\dot{x})$ in terms of the joint PDF $p_{N\dot{N}}(y,\dot{y})$ of a Nakagami-m distributed process and its corresponding time derivative at the same time t as

$$p_{\Gamma\dot{\Gamma}}(x,\dot{x}) = \frac{1}{4x} p_{N\dot{N}}\left(\sqrt{x}, \frac{\dot{x}}{2\sqrt{x}}\right) . \quad (20)$$

After substituting $p_{N\dot{N}}(y,\dot{y})$ as given in [32, Eq. (13)] in (20), the joint PDF $p_{\Gamma\dot{\Gamma}}(x,\dot{x})$ can be written as

$$p_{\Gamma\dot{\Gamma}}(x,\dot{x}) = \frac{1}{2\sqrt{2\pi x}\dot{\sigma}} \frac{x^{(m-1)}}{(\Omega/m)^m\,\Gamma(m)} e^{-\frac{x}{(\Omega/m)} - \frac{\dot{x}^2}{8\dot{\sigma}^2 x}} \quad (21)$$

where m, Ω, and $\dot{\sigma}$ are the parameters associated with the Nakagami-m distribution. The result in (21) can be expressed in terms of the parameters of the gamma distribution, i.e., α_L and β_L, as

$$p_{\Gamma\dot{\Gamma}}(x,\dot{x}) = \frac{1}{2\sqrt{2\pi\beta x}} \frac{x^{\alpha_L}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} e^{-\frac{x}{\beta_L} - \frac{\dot{x}^2}{8\beta x}} \quad (22)$$

where $\beta = 2\left(\pi A_{\rm R}\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}\right)^2\left(f_{\rm S_{max}}^2 + 2f_{\rm R_{max}}^2 + f_{\rm D_{max}}^2\right)/K$. Here, $f_{\rm S_{max}}$, $f_{\rm R_{max}}$, and $f_{\rm D_{max}}$ are the maximum Doppler frequencies caused by the motion of the source mobile station, mobile relays, and the destination mobile station, respectively.

Numerical investigations show that $p_{\Xi\dot{\Xi}}(r,\dot{x}) \approx \frac{1}{\sqrt{3A_{\rm R}}} p_{\Gamma\dot{\Gamma}}(r,\dot{x})$. Substituting $p_{\Xi\dot{\Xi}}(r,\dot{x})$ in (18) allows us to express the LCR $N_{\Xi}(r)$ in closed form as

$$N_{\Xi}(r) \approx \int_0^{\infty} \frac{\dot{x}}{\sqrt{3A_{\rm R}}} p_{\Gamma\dot{\Gamma}}(r,\dot{x})\,d\dot{x} = \sqrt{\frac{2r\beta}{3\pi A_{\rm R}}} \frac{r^{\alpha_L} e^{-\frac{r}{\beta_L}}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} = \sqrt{\frac{2r\beta}{3\pi A_{\rm R}}} p_{\Xi}(r) . \quad (23)$$

D. ADF of a Sum of Double Rayleigh Processes

The ADF $T_{\Xi_-}(r)$ of $\Xi(t)$ is the expected value of the time intervals over which the stochastic process $\Xi(t)$ remains below a certain threshold level r. Mathematically, the ADF $T_{\Xi_-}(r)$ is defined as the ratio of the CDF $F_{\Xi_-}(r)$ and the LCR $N_{\Xi}(r)$ of $\Xi(t)$ [6], i.e.,

$$T_{\Xi_-}(r) = \frac{F_{\Xi_-}(r)}{N_{\Xi}(r)} . \quad (24)$$

By substituting (17) and (23) in (24), we can easily obtain an approximate solution for the ADF $T_{\Xi_-}(r)$.

IV. Numerical Results

The purpose of this section is twofold. Firstly, to illustrate the important theoretical results by evaluating the expressions in (16), (17), (23), and (24). Secondly, to validate the correctness of the theoretical results with the help of simulations. The exact simulation results have been obtained by applying the sum-of-sinusoids (SOS) concept [33] on the uncorrelated Gaussian noise processes making up the received signal envelope at the output of the EG combiner. The model parameters of the channel simulator were computed by the generalized method of exact Doppler spread (GMEDS_1) [34]. Each Gaussian process $\mu^{(i)}(t)$ $(i = 1, 2, \ldots, 2K)$ was simulated using $\mathrm{N}_l^{(i)} = 14$ for $i = 1, 2, \ldots, 2K$ and $l = 1, 2$, where $\mathrm{N}_l^{(i)}$ is the number of sinusoids required to simulate the inphase $(l = 1)$ and quadrature components $(l = 2)$ of $\mu^{(i)}(t)$. The authors of [33] have shown that with $\mathrm{N}_l^{(i)} \geq 7$ $(l = 1, 2)$, the simulated distribution of $\left|\mu^{(i)}(t)\right|$ closely approximates the Rayleigh distribution for all $i = 1, 2, \ldots, 2K$. The maximum Doppler frequencies caused by the motion of the source mobile station, the mobile relays, and the destination mobile station, denoted by $f_{\mathrm{S}_{\max}}$, $f_{\mathrm{R}_{\max}}$, and $f_{\mathrm{D}_{\max}}$, respectively, were set to 91 Hz, 125 Hz, and 210 Hz. The variances $\sigma^2_{\mu^{(i)}}$ were chosen to be $\sigma^2_{\mu^{(i)}} = 1/K \; \forall \; i = 1, 2, \ldots, 2K$. The relay gains $A_{\mathrm{R}^{(k)}}$ were selected to be unity, i.e., $A_{\mathrm{R}^{(k)}} = A_{\mathrm{R}} = 1 \; \forall \; k = 1, 2, \ldots, K$ unless stated otherwise.

The results presented in Figs. 2–5 show a good fitting of the approximated analytical and the exact simulation results considered as the true results. Figure 2 illustrates the theoretical results of the PDF $p_\Xi(x)$ of $\Xi(t)$ described by the approximation in (16). For the purpose of validation of the theory, the simulation results obtained by evaluating the statistics of the waveforms generated by using the SOS-based channel simulator are included in Fig. 2. The PDF $p_\Xi(x)$ of $\Xi(t)$ for a different number of diversity branches K, keeping the relay gain A_{R} constant is shown in Fig. 2. For $K = 1$ and $A_{\mathrm{R}} = 1$, the PDF $p_\Xi(x)$ of $\Xi(t)$ maps to the double Rayleigh distribution, confirming that our approximation in (16) is valid. Furthermore, the PDF $p_\Xi(x)$ of $\Xi(t)$ tends to a Gaussian distribution if K increases. This observation is in accordance with the central limit theorem (CLT) [26].

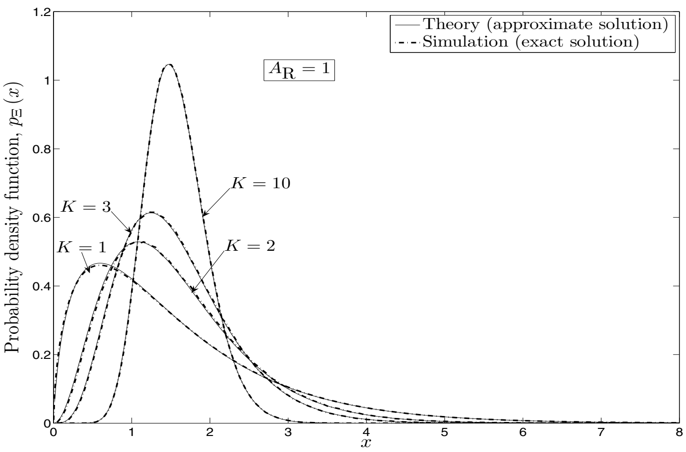

Fig. 2. The PDF $p_\Xi(x)$ of the received signal envelope at the output of the EG combiner $\Xi(t)$ for a different number of diversity branches K.

In Fig. 3, the theoretical results of the CDF $F_{\Xi_-}(r)$ of $\Xi(t)$ described by the approximation in (17) are illustrated. Here, the CDF $F_{\Xi_-}(r)$ of $\Xi(t)$ for a different number of diversity branches K keeping the relay gain A_{R} in each branch constant is shown. A close agreement can be observed between the presented approximate solution and the exact simulation results.

The LCR $N_\Xi(r)$ of $\Xi(t)$ described by (23) is evaluated along with the exact simulation results in Fig. 4. This figure illustrates the LCR $N_\Xi(r)$ of $\Xi(t)$ for a different number of diversity branches K while the relay gain A_{R} is kept constant. It is quite clear from the graphs that for $K = 1$ and $A_{\mathrm{R}} = 1$, (23) provides us with a very close approximation to the exact LCR of a double Rayleigh distributed process given in [17]. In addition, increasing K while keeping A_{R} constant, result in a decrease in the LCR $N_\Xi(r)$ for both lower and higher signal levels r.

Figure 5 reveals the theoretical results of the ADF $T_{\Xi_-}(r)$ of $\Xi(t)$ described by (24) along with the exact simulation results. It can be observed in Fig. 5 that keeping the relay gain

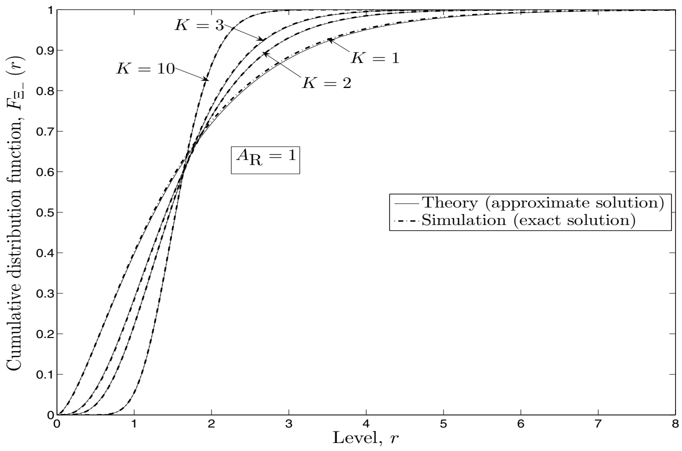

Fig. 3. The CDF $F_{\Xi_-}(r)$ of the received signal envelope at the output of the EG combiner $\Xi(t)$ for a different number of diversity branches K.

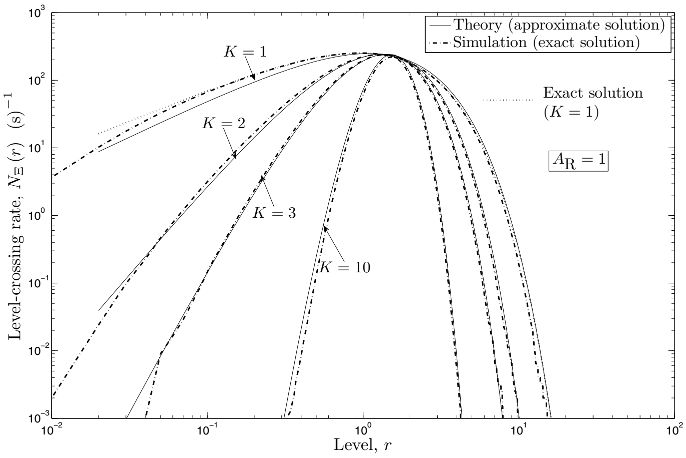

Fig. 4. The LCR $N_\Xi(r)$ of the received signal envelope at the output of the EG combiner $\Xi(t)$ for a different number of diversity branches K.

A_{R} constant and increasing the number of diversity branches K result in an increase of the ADF $T_{\Xi_-}(r)$ at higher values of r. However, at lower values of r, the ADF $T_{\Xi_-}(r)$ decreases with increasing K.

V. Conclusion

In this article, we have studied the statistical properties of EGC over frequency non-selective M2M fading channels under NLOS propagation conditions in a dual-hop amplify-and-forward relay network. It is assumed that K diversity branches are present between the source mobile station and the destination mobile station via K mobile relays. The received signal envelope at the output of the EG combiner has therefore been modeled as a sum of K double Rayleigh processes. Here, we have thoroughly analyzed the PDF, the CDF, the LCR, and the ADF along with the CF of this sum process. We have proposed an approximation approach using an orthogonal series expansion in the form of the Laguerre series to approximate the PDF of the sum of K double Rayleigh processes. The approximation of the target PDF using the Laguerre series makes it possible to approximate the PDF of the sum process by a gamma distribution. It has been shown that the approximation provided by the Laguerre series is remarkably good. Exploiting the properties of the gamma distribution, the

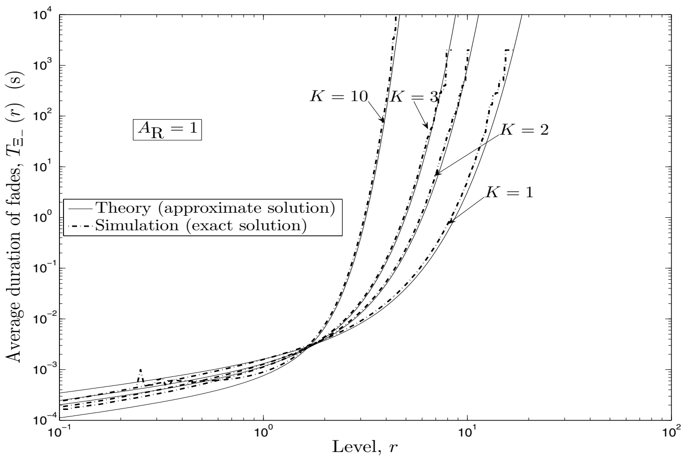

Fig. 5. The ADF $T_{\Xi_-}(r)$ of the received signal envelope at the output of the EG combiner $\Xi(t)$ for a different number of diversity branches K.

CDF, the LCR, and the ADF of the sum process are also approximated. With the help of this approximation, we are able to present simple and closed-form expressions for the aforementioned statistical quantities. Furthermore, the close agreement of the approximated theoretical results with those of the exact simulation results shows that the approximation approach followed here is valid. We have demonstrated the influence of the number of diversity branches K on the statistical properties of the received signal envelope at the output of the EG combiner. The results can easily be utilized in the performance analysis studies of EGC over relay-based M2M fading channels.

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