Research Article

Fault-Reconstruction-Based Cascaded Sliding Mode Observers for Descriptor Linear Systems

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This paper develops a cascaded sliding mode observer method to reconstruct actuator faults for a class of descriptor linear systems. Based on a new canonical form, a novel design method is presented to discuss the existence conditions of the sliding mode observer. Furthermore, the proposed method is extended to general descriptor linear systems with actuator faults. Finally, the effectiveness of the proposed technique is illustrated by a simulation example.

1. Introduction

With the development and applications of modern control techniques, the safety and reliability of control systems are becoming increasingly important. Therefore, the fault diagnosis has become one of the most important techniques to ensure the safety and reliability of control systems [1, 2]. During the last two decades, many significant results have been obtained for the analysis and observer design of fault diagnosis of the regular systems, such as unknown input observers [3, 4], eigenstructure assignment method [5], H_∞ filtering [6–9], parity space approach [10], and parameter identification approach [11].

Just like regular systems, the fault diagnosis for descriptor systems has recently attracted increasing attention due to their importance in real-world systems. In [12], a parametric approach is proposed to design unknown input observers to realize fault detection of descriptor linear multivariable systems with unknown disturbances. By directly identifying parity space, a model-free approach for fault detection is developed, which can be applied if the model of descriptor systems is unknown [13]. In [14], the factorization

approach for robust residual generation is extended to descriptor systems, and then a post-filter is added to ensure the robustness of fault diagnosis. In [15], H_∞ filter is utilized for providing disturbance rejection and robustness properties of the fault detection and isolation schemes of linear time-invariant descriptor systems. In [16], several sufficient conditions of existence of unknown input observers are obtained for Takagi-Sugeno descriptor systems, which are affected by unknown inputs. Unfortunately, although these methods can successfully generate residuals, they fail to reconstruct fault signals.

Recently, fault reconstruction is a promising alternative for fault detection. Instead of generating residuals, a number of methods, such as sliding mode observers (SMOs) [17–23], descriptor observer method [24–26], and PI observer [27–29], can be used to reconstruct fault signals. The sliding mode control is employed in the situations including state estimation and fault detection, since it is insensitive to matched uncertainties, nonlinearity, or disturbances [30]. Edwards et al. [17] firstly used the concept of the equivalent output error injection signals to reconstruct faults. Tan and Edwards [19] extended this work for robust reconstruction of sensor and actuator faults by minimizing the effect of uncertainty on the reconstruction in an L_2 sense. Some well-studied works, aiming at reducing the system constraints associated with the results in [17, 19], have recently appeared in the literature [18, 20–23]. In order to relax the matching conditions, the cascaded sliding mode observer method was proposed to deal with a class of systems with relative degree higher than one [20, 21]. In [22], the auxiliary outputs are defined such that the conventional sliding mode observer in [17] can be used for systems without the observer matching condition. In order to obtain those auxiliary outputs, high-order sliding-mode observers are constructed to act as exact differentiators using a super-twisting algorithm. Inspired by Floquet et al. [22], high-gain approximate differentiators and high-order sliding-mode robust differentiators were proposed to generate auxiliary outputs for the design of sliding mode observers [18, 23].

Although there are many achievements in regular systems, few results have been reported to the descriptor case despite its importance in real-world systems. In [31, 32], the sliding mode observer method was employed to detect and isolate faults and to reconstruct the faults for descriptor systems. However, the uncertainty was not considered in these results. In [33], the sliding mode observer was proposed to minimize the effect of uncertainly on the reconstruction of faults for descriptor systems. Unfortunately, the fault detection filter based sliding mode observer has to satisfy the strict condition in [31–33], which severely limits the applicability of these approaches for a wide range of practical systems.

Motivated by the above discussion, in this paper, we develop a cascaded sliding mode observer method to reconstruct actuator faults for a class of descriptor linear systems. The main contribution of this paper can be summarized as follows: (1) we present a novel cascaded sliding mode observer method to reconstruct actuator faults for a class of descriptor linear systems; (2) in the design process, we remove this restrictive assumption and extend the cascaded sliding mode observer approach of Tan et al. [20, 21] to descriptor systems; (3) a novel cascaded sliding mode observer is designed for reconstructing actuator faults for a class of descriptor linear systems.

The paper is organized as follows. In Section 2, the problem is formulated, and appropriate coordinate transformations are introduced to exploit the system structure. In Section 3 the design algorithm of cascaded sliding mode observer for linear descriptor systems is given. In Section 4, a design method of cascaded sliding mode observer and fault reconstruction for general descriptor systems are presented. In Section 5, an example is given to support the effectiveness of the proposed approach. Finally, the conclusions are drawn.

2. Problem Statement and System Analysis

Consider a descriptor linear system described by

$$E\dot{x} = Ax + Bu + Df$$
$$y = Cx,$$

$$(2.1)$$

where $x \in R^n$ is the state variable, $u \in R^k$ is the input vector, $y \in R^p$ is the output variable, and $f \in R^q$ is unknown but bounded so that

$$\|f\| \leq \beta,$$

$$(2.2)$$

where the positive scalar β is known. The signal f models the actuator fault within the system. $A \in R^{m \times n}$, $B \in R^{m \times k}$, $C \in R^{p \times n}$, and $D \in R^{m \times q}$ are known constant real matrices. Without loss of generality, it is assumed that $\text{rank}(D) = q$, $\text{rank}(C) = p$, and E is full row rank.

In [32], a sliding mode observer is given in the following form:

$$\dot{z} = Fz + T_1 Bu + K_1 y + K_2 y + G_n v$$
$$\hat{x} = z + T_2 y$$
$$\hat{y} = C\hat{x},$$

$$(2.3)$$

where $z \in R^{\tilde{n}}$ is the state vector of the SMO, $\hat{x}$ is the estimation of the state vector x, and v is the discontinuous output error injection vector defined by

$$v = \begin{cases} -\eta \dfrac{P_0 e_y}{\|P_0 e_y\|} & e_y \neq 0 \\ 0 & \text{other}, \end{cases}$$

$$(2.4)$$

where $e_y = \hat{y} - y$, $\eta > 0$, F, T_1, T_2, K_1, K_2, G_n, and P_0 are parameters to be designed.

For the descriptor system (2.1), the sufficient conditions for the existence of the sliding mode observer (2.3) are as follows:

$$\text{rank}\begin{bmatrix} E & D \\ C & 0 \end{bmatrix} = n + q$$

$$(2.5)$$

$$\text{rank}\begin{bmatrix} sE - A & D \\ C & 0 \end{bmatrix} = n + q, \quad \text{Re}(s) \geq 0.$$

$$(2.6)$$

It is well known that condition (2.5) is quite restrictive and may not apply to a wide range of systems. In the following, we give two more relaxed conditions:

$$\text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n, \tag{2.7}$$

$$\text{rank}\begin{bmatrix} E & D \\ C & 0 \end{bmatrix} = n + l, \tag{2.8}$$

where $l \leq q$.

Before presenting the main results, some lemmas are given as follows.

Lemma 2.1. *If the conditions* (2.7) *and* (2.8) *hold, there exists a nonsingular matrix U such that*

$$\text{rank}\begin{bmatrix} E & D_1 \\ C & 0 \end{bmatrix} = n + l, \tag{2.9}$$

$$\text{rank}\begin{bmatrix} E & D_2 \\ C & 0 \end{bmatrix} = n, \tag{2.10}$$

where $[D_1 \ D_2] = DU$, and $D_1 \in R^{m \times l}$, $D_2 \in R^{m \times (q-l)}$.

Proof. if l is equal to q, the conclusion is obviously true. So the following is to prove the case that l is less than q.

Obviously, there exists a nonsingular matrix U_1 so that $DU_1 = [D_1 \ \overline{D}_2]$ and (2.9) hold. Then,

$$\text{rank}\begin{bmatrix} E & D \\ C & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} E & D_1 & \overline{D}_2 \\ C & 0 & 0 \end{bmatrix} = n + l. \tag{2.11}$$

So there exists a matrix $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ so that

$$\begin{bmatrix} \overline{D}_2 \\ 0 \end{bmatrix} = \begin{bmatrix} E & D_1 \\ C & 0 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}. \tag{2.12}$$

Thus, we have $\overline{D}_2 = EY_1 + D_1Y_2$ and $CY_1 = 0$.

Setting

$$U_2 = \begin{bmatrix} I & -Y_2 \\ 0 & I \end{bmatrix} \tag{2.13}$$

and $U = U_1U_2$, we have

$$\text{rank}\begin{bmatrix} E & D_2 \\ C & 0 \end{bmatrix} = \text{rank}\begin{bmatrix} E & EY_1 \\ C & CY_1 \end{bmatrix} = \text{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n. \tag{2.14}$$

$\square$

Lemma 2.2. *If the following conditions*

$$\text{rank}\begin{bmatrix} E & D_1 \\ C & 0 \end{bmatrix} = n + l,$$

$$\text{rank}\begin{bmatrix} sE - A & D_1 \\ C & 0 \end{bmatrix} = n + l, \quad \text{Re}(s) \geq 0$$

$$(2.15)$$

hold, there exist two nonsingular matrices P and Q such that

$$PEQ = \begin{bmatrix} 0 & E_{12} \\ I_{\tilde{n}-\tilde{p}} & E_{22} \end{bmatrix}, \qquad PAQ = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$PD_1 = \begin{bmatrix} D_I \\ 0 \end{bmatrix}, \qquad CQ = [0 \ I_p],$$

$$(2.16)$$

where $E_{12} \in R^{(m-n+p)\times p}$, $E_{22} \in R^{(n-p)\times p}$, $A_{11} \in R^{(m-n+p)\times(n-p)}$, $A_{12} \in R^{(m-n+p)\times p}$, $A_{21} \in R^{(n-p)\times(n-p)}$, $A_{22} \in R^{(n-p)\times p}$, $D_I = [0 \ I_l]^T \in R^{(m-n+p)\times l}$, and the subblock A_{11} has the structure

$$A_{11} = \begin{bmatrix} A_{111} \\ A_{112} \end{bmatrix},$$

$$(2.17)$$

in which $A_{111} \in R^{(m-n+p-l)\times(n-p)}$, $A_{112} \in R^{l\times(n-p)}$, and the pair (A_{21}, A_{111}) is detectable.

    It can be established easily by Lemma 2 in [33], and hence the proof is omitted.

Lemma 2.3. *If the conditions (2.6), (2.7), and (2.8) hold, there exist nonsingular matrices P, Q, and U such that*

$$PEQ = \begin{bmatrix} 0 & E_{12} \\ I_{n-p} & E_{22} \end{bmatrix}, \qquad PAQ = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$(2.18)$$

$$PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad CQ = [0 \ I_p]$$

$$(2.19)$$

$$PDU = \begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix},$$

$$(2.20)$$

where $E_{12} \in R^{(m-n+p)\times p}$, $E_{22} \in R^{(n-p)\times p}$, $A_{11} \in R^{(m-n+p)\times(n-p)}$, $A_{12} \in R^{(m-n+p)\times p}$, $A_{21} \in R^{(n-p)\times(n-p)}$, $A_{22} \in R^{(n-p)\times p}$, $B_1 \in R^{(m-n+p)\times k}$, $B_2 \in R^{(n-p)\times k}$, $D_{11} = [0 \ I_l]^T \in R^{(m-n+p)\times l}$, $D_{22} \in R^{(n-p)\times(q-l)}$, and the subblock A_{11} has the structure

$$A_{11} = \begin{bmatrix} A_{111} \\ A_{112} \end{bmatrix},$$

$$(2.21)$$

where $A_{111} \in R^{(m-n+p-l)\times(n-p)}$, $A_{112} \in R^{l\times(n-p)}$, and (A_{21}, A_{111}) is detectable.

Proof . By Lemma 2.1, there exists a nonsingular matrix U such that (2.9) and (2.10) hold, where $DU = [D_1 \ D_2]$.

Obviously,

$$\text{rank} \begin{bmatrix} sE - A & D_1 \\ C & 0 \end{bmatrix} = n + l, \quad \text{Re}(s) \geq 0. \tag{2.22}$$

By Lemma 2.2, there exist two nonsingular matrices P and Q such that (2.18) and (2.19) hold and

$$PD_1 = \begin{bmatrix} D_{11} \\ 0 \end{bmatrix}. \tag{2.23}$$

Setting $PD_2 = \begin{bmatrix} D_{21} \\ D_{22} \end{bmatrix}$, we have

$$
\begin{aligned}
\text{rank} \begin{bmatrix} E & D_2 \\ C & 0 \end{bmatrix} &= \text{rank} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} E & D_2 \\ C & 0 \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & I \end{bmatrix} \\
&= \text{rank} \begin{bmatrix} 0 & E_{12} & D_{21} \\ I_{n-p} & E_{22} & D_{22} \\ 0 & I_p & 0 \end{bmatrix} \\
&= \text{rank}(D_{21}) + n.
\end{aligned}
\tag{2.24}
$$

Combining (2.10) and (2.24), we have $\text{rank}(D_{21}) = 0$. Obviously, $D_{21} = 0$. □

By Lemma 2.3, it can be assumed without loss of generality that system (2.1) has the following form:

$$
\begin{aligned}
\begin{bmatrix} 0 & E_{12} \\ I_{n-p} & E_{22} \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \\
&\quad + \begin{bmatrix} D_{11} \\ 0 \end{bmatrix} f_1 + \begin{bmatrix} 0 \\ D_{22} \end{bmatrix} f_2 \\
y &= x_2,
\end{aligned}
\tag{2.25}
$$

where $x = [x_1^T \ x_2^T]^T$, $x_1 \in R^{n-p}$, $x_2 \in R^p$ and

$$f \longrightarrow Uf = [f_1^T \ f_2^T]^T. \tag{2.26}$$

The descriptor system (2.25) may be considered as the system with the fault f_1 and the disturbance f_2. Using the fault reconstruction method in [33], the fault f_1 can be reconstructed and the L_2 gain from the f_2 to reconstruction error of fault f_1 can be minimized. But the fault f_2 and the state x_1 cannot be estimated. Inspired by Tan et al. [20, 21], the cascaded sliding mode observer is applied to estimate both the state x and fault f in the following.

3. Design of Cascaded Sliding Mode Observer

The primary sliding mode observer for system (2.25) is

$$\dot{z} = (T_1 A - K_1 C)z + T_1 Bu + K_1 y + FT_2 y + G_n \upsilon$$

$$\hat{x} = z + T_2 y$$

$$\hat{y} = \hat{x}_2,$$

(3.1)

where $z \in R^n$ is the state vector of the SMO, $\hat{x} = [\hat{x}_1^T \ \hat{x}_2^T]^T$ with $\hat{x}_1 \in R^{n-p}$ and $\hat{x}_2 \in R^p$ is the estimation of the state vector x, $G_n = [0 \ I]^T$, T_1 and T_2 are defined by

$$T_1 = \begin{bmatrix} Z_1 & I_{n-p} \\ Z_2 & 0 \end{bmatrix}$$

(3.2)

$$T_2 = \begin{bmatrix} 0 \\ I_p \end{bmatrix} - T_1 \begin{bmatrix} E_{12} \\ E_{22} \end{bmatrix}$$

(3.3)

$$Z_1 = \begin{bmatrix} Z_{11} & 0 \end{bmatrix},$$

(3.4)

$Z_1 \in R^{(n-p) \times (m-n+p)}$, $Z_{11} \in R^{(n-p) \times (m-n+p-q)}$, $Z_2 \in R^{p \times (m-n+p)}$ is full rank, υ is the discontinuous output error injection vector defined by:

$$\upsilon = \begin{cases} -\eta \dfrac{P_2 e_2}{\|P_2 e_2\|} & e_2 \neq 0 \\ 0 & \text{other,} \end{cases}$$

(3.5)

$e_2 = \hat{x}_2 - x_2$, $\eta > 0$, Z_{11}, Z_2, K_1, F, and P_2 are parameters to be designed.

In [33], it is shown that for an appropriate choice of observer parameters an ideal sliding motion takes place on $S = \{(e_1, e_2) \mid e_2 = 0\}$ in finite time.

Define $e = \hat{x} - x$ as the state estimation error, the following estimation error dynamic is obtained:

$$\dot{e}_1 = (A_{21} + Z_1 A_{11})e_1 + (Z_1 A_{12} + A_{22} - K_{11})e_2 + D_{22} f_2$$

$$\dot{e}_2 = Z_2 A_{11} e_1 + (Z_2 A_{12} - K_{12})e_2 - Z_2 D_{11} f_1 + \upsilon,$$

(3.6)

where $e = [e_1^T \ e_2^T]^T$, $e_1 \in R^{n-p}$, $K_1 = [K_{11}^T \ K_{12}^T]^T$, $K_{11} \in R^{(n-p) \times p}$, and $K_{12} \in R^{p \times p}$.

Assuming the primary sliding mode observer has been designed, and that a sliding motion has been achieved, then $e_2 = \dot{e}_2 = 0$, and the error equation becomes

$$\dot{e}_1 = (A_{21} + Z_1 A_{11})e_1 + D_{22} f_2$$

$$Z_2^+ \upsilon_{eq} = -A_{11} e_1 + D_{11} f_1,$$

(3.7)

where Z_2^+ is the generalized inverse matrix of Z_2, v_{eq} is the equivalent output error injection term that can be approximated to any degree of accuracy by replacing (3.8) with

$$v_{eq} = -\eta \frac{P_2 e_2}{\|P_2 e_2\| + \delta}, \tag{3.8}$$

where δ is a small positive constant.

The remaining system freedom can be used to estimate the state x_1 and reconstruct the fault f_2. Equation (3.7) can be rewritten as

$$\dot{e}_1 = (A_{21} + Z_1 A_{11})e_1 + D_{22}f_2 \tag{3.9}$$

$$D_{11}^{\perp} Z_2^+ v_{eq} = -A_{111}e_1, \tag{3.10}$$

where $D_{11}^{\perp} = \begin{bmatrix} I_{m+p-n-l} & 0 \end{bmatrix}$.

For any A_{111}, there exists a nonsingular matrix W so that $W A_{111} = \begin{bmatrix} \hat{A}_{111}^T & 0 \end{bmatrix}^T$, where $\hat{A}_{111} \in R^{\hat{p} \times (n-p)}$ is full row rank. We have

$$\begin{bmatrix} I_{\hat{p}} & 0 \end{bmatrix} W D_{11}^{\perp} Z_2^+ v_{eq} = -\hat{A}_{111}e_1. \tag{3.11}$$

The system (3.9) and (3.11) may be considered as the linear system with the $q-l$ faults, the $n-p$ states f_1 and the $\hat{p}$ outputs. Using the sliding mode observer design method for the linear system in [17], we can design a secondary sliding mode observer to estimate e_1 and f_2 if the following conditions hold:

$$\text{rank}\left(\hat{A}_{111} D_{22}\right) = \text{rank}(D_{22})$$

$$\text{rank}\begin{bmatrix} sI - (A_{21} + Z_1 A_{11}) & D_{22} \\ \hat{A}_{111} & 0 \end{bmatrix} = n - p + q - l, \quad \text{Re}(s) \geq 0. \tag{3.12}$$

Obviously, (3.12) are equivalent to

$$\text{rank}(A_{111} D_{22}) = \text{rank}(D_{22}) \tag{3.13}$$

$$\text{rank}\begin{bmatrix} sI - (A_{21} + Z_1 A_{11}) & D_{22} \\ A_{111} & 0 \end{bmatrix} = n - p + q - l, \quad \text{Re}(s) \geq 0. \tag{3.14}$$

Combined with (3.4), (3.14) is equivalent to

$$\text{rank}\begin{bmatrix} sI - A_{21} & D_{22} \\ A_{111} & 0 \end{bmatrix} = n - p + q - l, \quad \text{Re}(s) \geq 0. \tag{3.15}$$

From the above analysis, if the conditions (3.13) and (3.15) satisfy, there exists a cascaded sliding mode observer for the descriptor system (2.1).

Next, the fault reconstruction method based cascaded sliding mode observer is given.

Assuming that the secondary sliding mode observer has been designed and the $\widehat{e}_1$ and $\widehat{f}_2$ are the estimations of e_1 and f_2, respectively. Then, the reconstruction signal of the fault f_1 is described by

$$\widehat{f}_1 = A_{112}\widehat{e}_1 + \begin{bmatrix} 0 & I_l \end{bmatrix} Z_2^+ v_{\mathrm{eq}}, \tag{3.16}$$

and the estimation of the state x_1 is described by

$$\widehat{x}_1 - \widehat{e}_1 \longrightarrow x_1. \tag{3.17}$$

The reconstruction of fault is described by

$$\widehat{f} = U^{-1}\begin{bmatrix} \widehat{f}_1^T & \widehat{f}_2^T \end{bmatrix}^T \tag{3.18}$$

Equations (3.13) and (3.15) are the sufficient conditions for the existence of the cascaded sliding mode observer, but these cannot be checked using the parameters of the original system (2.1). Now, for system (2.1), sufficient conditions for the existence of the cascaded sliding mode observer can be given by Theorem 3.1.

Theorem 3.1. *There exists a cascaded sliding mode observer for system* (2.1) *if the following conditions hold:*

$$\mathrm{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n \tag{3.19}$$

$$\mathrm{rank}\begin{bmatrix} E & A & D & 0 \\ 0 & E & 0 & D \\ C & 0 & 0 & 0 \\ 0 & C & 0 & 0 \end{bmatrix} = n + q + \mathrm{rank}\begin{bmatrix} E & D \\ C & 0 \end{bmatrix}, \tag{3.20}$$

$$\mathrm{rank}\begin{bmatrix} sE - A & D \\ C & 0 \end{bmatrix} = n + q, \quad \mathrm{Re}(s) \geq 0. \tag{3.21}$$

Proof. If l is equal to q, the conclusion is obviously true. So, the following is to prove the case that l is less than q.

Substituting (2.8) and (2.25) into (3.20), we have

$$
\begin{aligned}
\operatorname{rank} &\begin{bmatrix} E & A & D & 0 \\ 0 & E & 0 & D \\ C & 0 & 0 & 0 \\ 0 & C & 0 & 0 \end{bmatrix} \\
&= \operatorname{rank} \begin{bmatrix} 0 & E_{12} & A_{11} & A_{12} & D_{11} & 0 & 0 & 0 \\ I_{n-p} & E_{22} & A_{21} & A_{22} & 0 & D_{22} & 0 & 0 \\ 0 & 0 & 0 & E_{12} & 0 & 0 & D_{11} & 0 \\ 0 & 0 & I_{n-p} & E_{12} & 0 & 0 & 0 & D_{22} \\ 0 & I_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p & 0 & 0 & 0 & 0 \end{bmatrix} \\
&= n + p + l + \operatorname{rank} \begin{bmatrix} A_{11} & D_{11} & 0 \\ I_{n-p} & 0 & D_{22} \end{bmatrix} \\
&= n + p + l + \operatorname{rank} \begin{bmatrix} A_{111} & 0 & 0 \\ A_{112} & I_l & 0 \\ I_{n-p} & 0 & D_{22} \end{bmatrix} \\
&= n + p + 2l + \operatorname{rank} \begin{bmatrix} I_{n-p} & D_{22} \\ A_{111} & 0 \end{bmatrix} \\
&= 2n + q + l.
\end{aligned}
\tag{3.22}
$$

So we have

$$
\operatorname{rank} \begin{bmatrix} I_{n-p} & D_{22} \\ A_{111} & 0 \end{bmatrix} = n - p + q - l.
\tag{3.23}
$$

By Lemma 1 in [32], (3.13) holds.
Substituting (2.18), (2.19), and (2.20) into (3.21), we can obtain (3.15). □

4. Cascaded Sliding Mode Observer Design and Fault Reconstruction for General Descriptor Systems

In Section 2, it is assumed that E is full row rank. In the following, it is discussed that E is rank deficient. Let $r := \operatorname{rank}(E) \le \min\{m, n\}$.

Now, since $\operatorname{rank}(E) = r$, there exists a regular matrix P^* such that (2.1) is restricted system equivalent to

$$
\begin{aligned}
E^* \dot{x} &= A^* x + B^* u + D^* f \\
y_1 &= -\overline{B}_1 u = \overline{A}_1 x + \overline{D}_1 f \\
y &= Cx,
\end{aligned}
\tag{4.1}
$$

where $P^*E = \begin{bmatrix} E^* \\ 0 \end{bmatrix}$, $P^*A = \begin{bmatrix} A^* \\ \overline{A}_1 \end{bmatrix}$, $P^*B = \begin{bmatrix} B^* \\ \overline{B}_1 \end{bmatrix}$, $P^*D = \begin{bmatrix} D^* \\ \overline{D}_1 \end{bmatrix}$, $E^* \in R^{r \times n}$, $A^* \in R^{r \times n}$, $B^* \in R^{r \times k}$, $D^* \in R^{r \times p}$, $\overline{A}_1 \in R^{(m-r) \times n}$, $\overline{B}_1 \in R^{(m-r) \times k}$, and $\overline{D}_1 \in R^{(m-r) \times q}$.

First passing the output y_1 through a nonsingular matrix P_* so that

$$P_* y_1 = \begin{cases} y_{11}(t) = A_{11}x \\ y_{12}(t) = A_{12}x + D_1 f, \end{cases} \tag{4.2}$$

where $D_1 \in R^{\overline{p} \times q}$ is row full rank.

Consider a new state $x_f \in R^{\overline{p}}$ which is a filtered version of y_{12} satisfying

$$\dot{x}_f = -A_f x_f + A_f y_{12}, \tag{4.3}$$

where $-A_f \in R^{\overline{p} \times \overline{p}}$ is a stable (filter) matrix.

Equations (4.1), (4.2), and (4.3) can be combined to form an augmented state-space system with order $\tilde{n}$ as follows:

$$E_a \dot{\tilde{x}} = A_a \tilde{x} + B_a u + D_a f$$
$$y_a = C_a \tilde{x}, \tag{4.4}$$

where $E_a = \begin{bmatrix} E^* & 0 \\ 0 & I \end{bmatrix}$, $A_a = \begin{bmatrix} A^* & 0 \\ A_f A_{12} & -A_f \end{bmatrix}$, $B_a = \begin{bmatrix} B^* \\ 0 \end{bmatrix}$, $D_a = \begin{bmatrix} D^* \\ A_f D_1 \end{bmatrix}$, $C_a = \begin{bmatrix} A_{11} & 0 \\ C & 0 \\ 0 & I \end{bmatrix}$, $\tilde{x} = [x^T \ x_f^T]^T$, $y_a = [y_{11}^T \ y^T \ x_f^T]^T$, $\tilde{n} = n + \overline{p}$, and $\tilde{p} = p + m - r$.

Obviously, the matrix E_a is full row rank so that the cascaded sliding mode observer can be designed using the method in Section 3.

In the following, the existence conditions of the cascaded sliding mode observer for general descriptor systems are given by Theorem 4.1.

Theorem 4.1. *There exists a cascaded sliding mode observer for system (2.1) with rank-deficient E if the following conditions hold:*

$$\text{rank} \begin{bmatrix} E & A & D \\ 0 & C & 0 \end{bmatrix} = m + p \tag{4.5}$$

$$\text{rank} \begin{bmatrix} E & A & D \\ 0 & E & 0 \\ 0 & C & 0 \end{bmatrix} = n + \text{rank}[E \ D] \tag{4.6}$$

$$\text{rank} \begin{bmatrix} E & A & 0 & D & 0 & 0 \\ 0 & E & A & 0 & D & 0 \\ 0 & 0 & E & 0 & 0 & D \\ 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 \end{bmatrix} = n + q + \text{rank} \begin{bmatrix} E & A & D & 0 \\ 0 & E & 0 & D \\ 0 & C & 0 & 0 \end{bmatrix}, \tag{4.7}$$

$$\text{rank} \begin{bmatrix} sE - A & D \\ C & 0 \end{bmatrix} = n + q, \quad \text{Re}(s) \geq 0. \tag{4.8}$$

Proof. Define a nonsingular matrix as follows:

$$P_1 = \text{diag}\left(\begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, \begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, I \right).$$

(4.9)

We have

$$\text{rank}\, P^* \begin{bmatrix} E & D \end{bmatrix} = \text{rank} \begin{bmatrix} E^* & D^* \\ 0 & D_1 \end{bmatrix} = r + \overline{p}$$

(4.10)

$$\text{rank} \begin{bmatrix} E & A & D \\ 0 & E & 0 \\ 0 & C & 0 \end{bmatrix} = \text{rank}\, P_1 \begin{bmatrix} E & A & D \\ 0 & E & 0 \\ 0 & C & 0 \end{bmatrix}$$

$$= \text{rank} \begin{bmatrix} A_{11} & 0 \\ 0 & I_{\overline{p}} \\ E^* & 0 \\ C & 0 \end{bmatrix} + r,$$

(4.11)

$$\text{rank} \begin{bmatrix} E_a \\ C_a \end{bmatrix} = \text{rank} \begin{bmatrix} E^* & 0 \\ 0 & I_{\overline{p}} \\ A_{11} & 0 \\ C & 0 \\ 0 & I_{\overline{p}} \end{bmatrix} = \text{rank} \begin{bmatrix} A_{11} & 0 \\ 0 & I_{\overline{p}} \\ E^* & 0 \\ C & 0 \end{bmatrix}.$$

(4.12)

Combining (4.6), (4.10), (4.11), and (4.12), we have

$$\text{rank} \begin{bmatrix} E_a \\ C_a \end{bmatrix} = \widetilde{n}.$$

(4.13)

Define a nonsingular matrix as follows:

$$P_2 = \text{diag}\left(\begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, \begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, \begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, I, I \right).$$

(4.14)

We have

$$
\operatorname{rank} P_2 \begin{bmatrix} E & A & 0 & D & 0 & 0 \\ 0 & E & A & 0 & D & 0 \\ 0 & 0 & E & 0 & 0 & D \\ 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 \end{bmatrix} = r + \overline{p} + \operatorname{rank} \begin{bmatrix} E^* & A^* & D^* & 0 \\ 0 & A_{12} & D_1 & 0 \\ 0 & E^* & 0 & D^* \\ 0 & 0 & 0 & D_1 \\ C & 0 & 0 & 0 \\ A_{11} & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & A_{11} & 0 & 0 \end{bmatrix} \tag{4.15}
$$

$$
\operatorname{rank} \begin{bmatrix} E_a & A_a & D_a & 0 \\ 0 & E_a & 0 & D_a \\ C_a & 0 & 0 & 0 \\ 0 & C_a & 0 & 0 \end{bmatrix} = 2\overline{p} + \operatorname{rank} \begin{bmatrix} E^* & A^* & D^* & 0 \\ 0 & A_{12} & D_1 & 0 \\ 0 & E^* & 0 & D^* \\ 0 & 0 & 0 & D_1 \\ A_{11} & 0 & 0 & 0 \\ C & 0 & 0 & 0 \\ 0 & A_{11} & 0 & 0 \\ 0 & C & 0 & 0 \end{bmatrix}, \tag{4.16}
$$

$$
\begin{aligned}
\operatorname{rank} P_1 \begin{bmatrix} E & A & D & 0 \\ 0 & E & 0 & D \\ 0 & C & 0 & 0 \end{bmatrix} &= \operatorname{rank} \begin{bmatrix} E & A & D & 0 \\ 0 & A_{11} & 0 & 0 \\ 0 & A_{12} & D_1 & 0 \\ 0 & E & 0 & D \\ 0 & 0 & 0 & D_1 \\ 0 & C^* & 0 & 0 \end{bmatrix} \\
&= \operatorname{rank} \begin{bmatrix} A_{12} & D_1 & 0 \\ E & 0 & D \\ 0 & 0 & D_1 \\ A_{11} & 0 & 0 \\ C^* & 0 & 0 \end{bmatrix} + r \\
&= \operatorname{rank} \begin{bmatrix} E_a & D_a \\ C_a & 0 \end{bmatrix} + r.
\end{aligned} \tag{4.17}
$$

Combining (4.7), (4.15), (4.16), and (4.17), we have

$$
\operatorname{rank} \begin{bmatrix} E_a & A_a & D_a & 0 \\ 0 & E_a & 0 & D_a \\ C_a & 0 & 0 & 0 \\ 0 & C_a & 0 & 0 \end{bmatrix} = \widetilde{n} + q + \operatorname{rank} \begin{bmatrix} E_a & D_a \\ C_a & 0 \end{bmatrix}. \tag{4.18}
$$

Define a nonsingular matrix as follows:

$$
P_3 = \operatorname{diag}\left(\begin{bmatrix} I & 0 \\ 0 & P_* \end{bmatrix} P^*, I \right). \tag{4.19}
$$

We have

$$
\operatorname{rank}\begin{bmatrix} sE^* - A^* & D^* \\ C^* & 0 \end{bmatrix} = \operatorname{rank} P_3 \begin{bmatrix} sE^* - A^* & D^* \\ C & 0 \end{bmatrix}
$$

$$
= \operatorname{rank}\begin{bmatrix} sE - A & D \\ -A_{11} & 0 \\ -A_{12} & D_1 \\ C^* & 0 \end{bmatrix} = n + q; \tag{4.20}
$$

thus,

$$
\operatorname{rank}\begin{bmatrix} sE^* - A^* & 0 & D^* \\ -A_{11} & 0 & 0 \\ -A_{12} & 0 & D_1 \\ C^* & 0 & 0 \\ 0 & I_{\bar{p}} & 0 \end{bmatrix} = n + q + \bar{p}. \tag{4.21}
$$

Define

$$
P_4 = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & A_f & 0 & s + A_f \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \tag{4.22}
$$

Then,

$$
\operatorname{rank}\begin{bmatrix} sE^* - A^* & 0 & D^* \\ -A_{11} & 0 & 0 \\ -A_{12} & 0 & D_1 \\ C^* & 0 & 0 \\ 0 & I_{\bar{p}} & 0 \end{bmatrix} = \operatorname{rank} P_4 \begin{bmatrix} sE^* - A^* & 0 & D^* \\ -A_{11} & 0 & 0 \\ -A_f A_{12} & 0 & A_f D_1 \\ C^* & 0 & 0 \\ 0 & I_{\bar{p}} & 0 \end{bmatrix}
$$

$$
= \operatorname{rank}\begin{bmatrix} sE^* - A^* & 0 & D^* \\ -A_f A_{12} & s + A_f & A_f D_1 \\ -A_{11} & 0 & 0 \\ C^* & 0 & 0 \\ 0 & I_{\bar{p}} & 0 \end{bmatrix} \tag{4.23}
$$

$$
= \operatorname{rank}\begin{bmatrix} sE_a - A_a & D_a \\ C_a & 0 \end{bmatrix}.
$$

Hence,

$$
\operatorname{rank}\begin{bmatrix} sE_a - A_a & D_a \\ C_a & 0 \end{bmatrix} = \tilde{n} + q, \quad \operatorname{Re}(s) \geq 0. \tag{4.24}
$$

Combining (4.12), (4.16), and (4.24), we get the conclusion by Theorem 3.1.   □

Corollary 4.2. *There exists a cascaded sliding mode observer for linear system if the following conditions hold:*

$$\text{rank}\begin{bmatrix} CAD & CD \\ CD & 0 \end{bmatrix} = \text{rank}(CD) + \text{rank}(D), \tag{4.25}$$

$$\text{rank}\begin{bmatrix} sI - A & D \\ C & 0 \end{bmatrix} = n + q, \quad \text{Re}(s) \geq 0. \tag{4.26}$$

Proof. Obviously, since $E = I$ for linear systems, (4.5), (4.6), and (4.8) hold. Hence, the following is to prove that (4.7) holds.

In [21], the canonical form of the linear system is given as follows:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad C = [0 \ T], \quad D = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}, \quad A_3 = \begin{bmatrix} A_{3a} & A_{3b} \\ A_{3c} & A_{3d} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} D_{11} \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ D_{22} \end{bmatrix}, \tag{4.27}$$

where $A_1 \in R^{(n-p)\times(n-p)}$, $A_{3a} \in R^{(p-l)\times(q-l)}$, $T \in R^{p\times p}$ is orthogonal, $D_1 \in R^{(n-p)\times l}$, $D_{11} \in R^{(q-l)\times(q-l)}$, and $D_{22} \in R^{l\times l}$ are invertible.

Substituting (4.27) into (4.25), we obtain

$$\text{rank}\begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}\begin{bmatrix} CAD & CD \\ CD & 0 \end{bmatrix} = \text{rank}(A_{3a}D_{11}) + 2l. \tag{4.28}$$

Combining (4.25) and (4.28), we obtain

$$\text{rank}(A_{3a}) = q - l. \tag{4.29}$$

Substituting (4.27) into (4.7), we obtain

$$\text{rank}\begin{bmatrix} E & A & 0 & D & 0 & 0 \\ 0 & E & A & 0 & D & 0 \\ 0 & 0 & E & 0 & 0 & D \\ 0 & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 \end{bmatrix} = n + \text{rank}\begin{bmatrix} I_{n-p} & 0 & A_1 & A_2 & D_1 & 0 & 0 & 0 \\ 0 & I_p & A_3 & A_4 & 0 & D_2 & 0 & 0 \\ 0 & 0 & I_{n-p} & 0 & 0 & 0 & D_1 & 0 \\ 0 & 0 & 0 & I_p & 0 & 0 & 0 & D_2 \\ 0 & I_p & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= 2n + p + \text{rank}\begin{bmatrix} A_3 & D_2 & 0 & 0 \\ I_{n-p} & 0 & D_1 & 0 \\ 0 & 0 & 0 & D_2 \end{bmatrix}$$

$$= 3n - q + l + 2\operatorname{rank}(D_{22}) + \operatorname{rank}(A_{3a}) + \operatorname{rank}(D_{11})$$

$$= 3n + l + q,$$

$$\operatorname{rank}\begin{bmatrix} E & A & D & 0 \\ 0 & E & 0 & D \\ 0 & C & 0 & 0 \end{bmatrix} = \operatorname{rank}\begin{bmatrix} I_{n-p} & 0 & D_1 & 0 \\ 0 & I_p & 0 & D_2 \\ 0 & I_p & 0 & 0 \end{bmatrix} + n$$

$$= 2n + l.$$

$$(4.30)$$

Obviously, (4.7) holds.   □

Remark 4.3. For the linear system, the rank conditions (4.25) and (4.26) are identical to the ones in [21], it is obvious that the conclusion of the paper is more general compared with [21].

5. Simulation

A machine infinite bus system linear model is described as follows [29]:

$$\dot{x}_1 = x_4 \qquad \dot{x}_2 = x_5 \qquad \dot{x}_3 = x_6$$

$$\dot{x}_4 = \frac{1}{M_1}(u_1 - Y_{12}V_1V_2(x_1 - x_2) - Y_{15}V_1V_5(x_1 - x_7) - D_2 x_4)$$

$$\dot{x}_5 = \frac{1}{M_2}(u_2 - Y_{21}V_1V_2(x_2 - x_1) - Y_{25}V_2V_5(x_2 - x_7) - D_2 x_5)$$

$$\dot{x}_6 = \frac{1}{M_3}(u_3 - Y_{34}V_3V_4x_3 - Y_{35}V_3V_5(x_3 - x_7) - D_3 x_6)$$

$$0 = P_{ch} - Y_{51}V_5V_1(x_7 - x_1) - Y_{52}V_5V_2(x_7 - x_2)$$

$$- Y_{53}V_5V_3(x_7 - x_3) - Y_{54}V_5V_4x_7,$$

$$(5.1)$$

where x_1, x_2, x_3, and x_7 are the generator angles, x_4, x_5, and x_6 are the generator speeds. u_1, u_2, and u_3 are the mechanical power, P_{ch} is unknown load, the nominal values of inertia M_1, M_2 and M_3, of damping D_1, D_2, and D_3, of admittance Y_{15}, Y_{25}, Y_{35}, Y_{51}, Y_{52}, Y_{53}, and Y_{54} and of potential V_1, V_2, V_3, V_4, and V_5 are shown in

$$
\begin{array}{c|c|c|c|c}
M_1 = 0.014 & M_2 = 0.026 & M_3 = 0.02 & D_1 = 0.057 & D_2 = 0.15 \\
D_3 = 0.11 & Y_{15} = 0.5 & Y_{25} = 1.2 & Y_{34} = 0.7 & Y_{45} = 1 \\
Y_{34} = 0.7 & Y_{12} = 1 & V_i = 1 \quad (i = 1\text{--}5). & &
\end{array}
\qquad (5.2)
$$

It is assumed that the available measurements are the generator angles x_1, x_2, x_3, and x_7. In order to illustrate the effectiveness of the design method, it is assumed that there exist faults on the actuator $u_1 - u_3$. It is easy to verify that the existence conditions of sliding mode
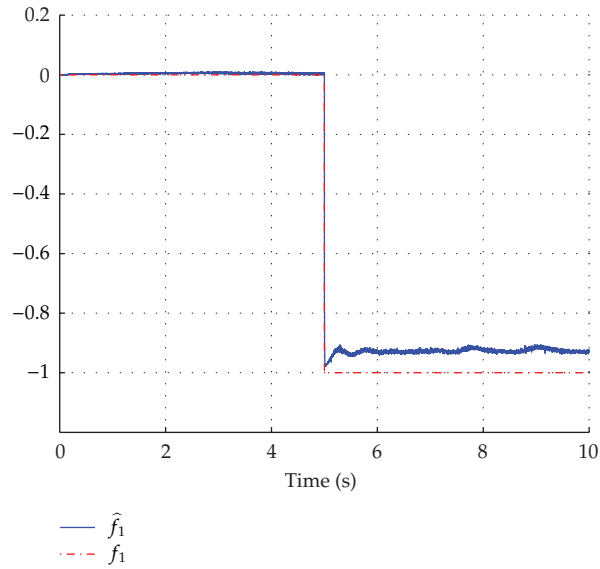
Figure 1: Fault signal f_1 and its reconstruction signal $\widehat{f}_1$.

observer in [32] do not hold, but the existence conditions of cascaded sliding mode observer hold.

In the following simulation, the cascaded sliding mode observer in Section 3 is designed to reconstruct the actuator faults.

Considering system (5.1) affected by the inputs $u_1 = 1$, $u_2 = 1$, and $u_3 = 2 + \sin(5t)$, the unknown load $P_{ch} = \sin(t)$ and an uncertain admittance

$$Y_{ij} = Y_{ij} + \Delta Y_{ij}, \tag{5.3}$$

where $\Delta Y_{ij} = \delta_{ij} \sin(\omega_{ij} t)$, $|\delta_{ij}| < 0.1$, $|\omega_{ij}| < 1 rd/s$, $i = 1, \ldots, 5$, $j = 1, \ldots, 5$.

Figures 1, 2, and 3 show faults and reconstruction signals. Although there exists unknown input and parameter uncertainty in the system, the cascaded sliding mode observer faithfully reconstructs the faults.

6. Conclusions and Future Works

This paper proposes a fault reconstruction method for a class of descriptor systems using cascaded sliding mode observer. The method can effectively relax the restrictions on the existence of a sliding mode observer, which allows the applicability of our proposed method to a wider range of systems. In our future work, the proposed actuator fault reconstruction schemes can be extended to some sensor fault reconstruction problems by using a suitable output filtering technique. Another interesting future research topic is to extend the current results to fault estimation of nonlinear systems based on T-S fuzzy models [34–36].
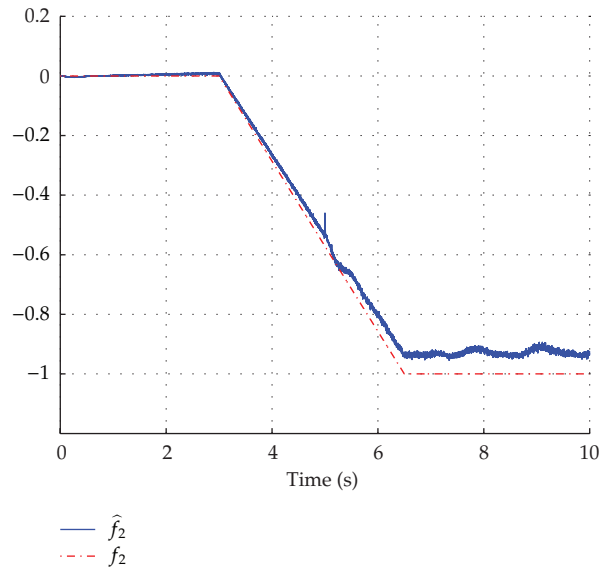
Figure 2: Fault signal f_2 and its reconstruction signal $\widehat{f}_2$.
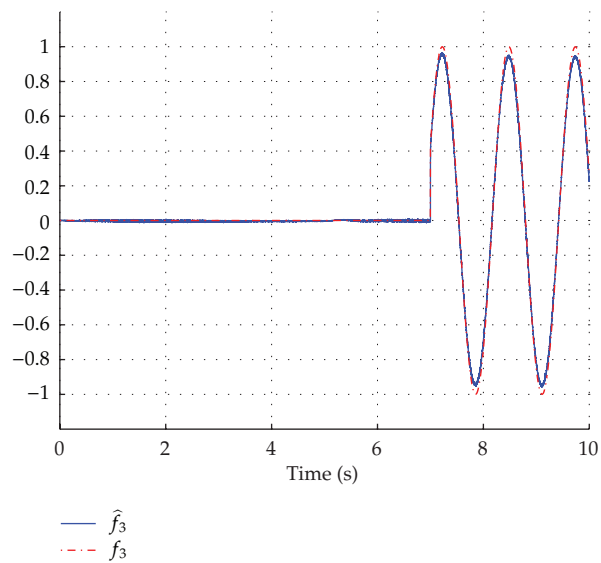

Figure 3: Fault signal f_3 and its reconstruction signal $\widehat{f}_3$.

Acknowledgment

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