Application of Wavelet Networks to Adaptive Control of Robotic Manipulators

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Abstract. In this paper, a wavelet-based adaptive control is proposed for a class of robotic manipulators, which consist of nonlinearities for friction effects and uncertain terms as disturbances. The controller is calculated by using a mixed of feedback linearization technique, supervisory control and H_∞ control. In addition, the parameter adaptive laws of the wavelet network are developed using a Lyapunov-based design. It is also shown that both system tracking stability and convergence of the error estimation can be guaranteed in the closed-loop system. Simulation results on a three-link robot manipulator show the satisfactory performance of the proposed control schemes even in the presence of large modeling uncertainties and external disturbances.

Keywords: Wavelet networks, robotic manipulators, adaptive control.

1 Introduction

Wavelet theory has a profound impact on signal processing as it offers a rigorous mathematical approach to the treatment of multiresolution. The combination of soft computing and wavelet theory has lead to a number of new techniques: wavelet networks and fuzzy wavelet [2, 7, 25]. It has been applied in a wide range of engineering disciplines such as signal processing, control engineering, pattern recognition and computer graphics. In the literature, some of the attempts are made in solving surface integral equations, improving the finite difference time domain method, solving linear differential equations and nonlinear partial differential equations and modelling nonlinear semiconductor devices [3, 6, 10-15, 21, 23, 24]. It has been shown that by employing the technique of feedback linearization and the theory of wavelet network, the robust adaptive control is designed based on Lyapunov method. The combination of wavelet theory and neural networks has lead to the development of wavelet networks. Wavelet networks are feed forward neural networks using wavelets as activation function. Wavelet networks have been used in classification and identification problems with some success. The strength of wavelet networks lies in their capabilities of catching essential features in “frequency-rich” signals. The origin of wavelet networks can be traced back to the work by [5] in which Gabor wavelets were used for image classification. Wavelet networks have become popular after the work by works

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[22, 26]. Recently, application of wavelet networks in identification and control design for a class of nonlinear dynamical systems has been investigated in [13].

On the other hand, robotic manipulators are non-linear models, if we take into account static frictions and dead zone will have non-linear models with unknown parameters. For systems that move at opposite directions and low speed (about zero), if we need high accuracy and performance, we have to consider static frictions and dead zone and if not, we will encounter a reduction in performance of system and even instability.

In the literature, there are some appreciable works related to utilizing different control techniques to the nonlinear robotic manipulators. These approaches often combine feedback linearization and optimal control techniques. It has been shown that how optimal control and adaptive control of robot motion may act in concert in the case of unknown or uncertain system parameters. Until now, often techniques that have been expressed need to exact model and exact value of model's parameters. However, in actual situations, the robot dynamics is rarely completely known, and it is thus difficult to express real robot dynamics in exact mathematical equations or to linearize the dynamics with respect to the operating point, see [4, 8, 9, 16, 17] and the references therein.

In this paper, a wavelet-based adaptive control is designed for a class of robotic manipulators. Model of robotic manipulators consists of some nonlinearity for friction effects and uncertain terms as disturbances. The controller is found by using the technique of feedback linearization, supervisory control and H_∞ control and the parameter adaptive laws of the wavelet network are developed using a Lyapunov-based design. It is also shown that both system tracking stability and error convergence of the estimation for nonlinear function can be guaranteed in the closed-loop system. Simulation results on a three-link robot manipulator show the satisfactory performance of the proposed control schemes even in the presence of large modeling uncertainties and external disturbances.

The paper is organized as follows. In Section 2 we will review some fundamentals of wavelet networks and mathematical notations. In Section 3 we give a wavelet-based adaptive control design for rigid robot systems. In Section 4, in order to demonstrate the validity of the proposed control method, a three-link robot controller is designed and simulated in the face of large uncertainties and external disturbances.

2 Wavelet Networks

The original objective of the wavelet theory is to construct orthogonal bases of $L_2(\Re)$. These bases are constituted by translation and dilation of the same function $\psi(.)$, namely wavelet function. It is preferable to take $\psi(.)$ localized and regular. The principles of wavelet construction are as follows [13, 26]:

1. $\phi(.)$ is a scaling function and the family $\phi(2^j x - k)$ for $0 \le k < 2^j$ constitutes an orthogonal basis of V_j,
2. the function $\phi(x-k)$ are mutually orthogonal for k ranging over Z ,

3. the family $\psi(2^j x - k)$ for $0 \le k < 2^j$ constitutes an orthogonal basis of W_j.
4. the family $\{\phi(2^{j0} x - k), \psi(2^j x - k) \text{ for } j \ge j_0\}$ forms an orthogonal basis of $L_2(\Re)$.

The wavelet subspaces W_j are defined as

$$W_j = \{\psi(2^j x - k), \quad 0 \le k < 2^j\} \tag{1}$$

which satisfy $W_j \cap W_i = 0$ for $\forall\, j \ne i$. For each $j \in Z$, let us consider the closed subspaces $V_j = \cdots \oplus W_{j-2} \oplus W_{j-1}$ of $L_2(\Re)$, where $\oplus$ denotes the direct sum, these nested subspaces have the following properties [13, 26]:

i. $\cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots$;

ii. $\text{close}_{L_2}(\bigcup_{j\in Z} V_j) = L_2(\Re)$;

iii. $\bigcap_{j\in Z} V_j = 0$;

iv. $V_{j+1} = V_j \oplus W_j \quad j \in Z$;

v. $f(x) \in V_j \leftrightarrow f(2x) \in V_{j+1} \quad j \in Z$.

If $\phi(.)$ and $\psi(.)$ are compactly supported, they give a local description, at different scales j, of the considered function. The wavelet series representation of the one-dimensional function $f(x)$ is given by

$$f(x) = \sum_{k\in Z} a_{j0k}\phi_{j0k} + \sum_{j\ge j0}\sum_{k\in Z} b_{jk}\psi_{jk}(x) \tag{2}$$

where $\phi_{j0k}(x) = 2^{j0/2}\phi(2^{j0}x - k)$, $\psi_{jk}(x) = 2^{j/2}\psi(2^j x - k)$, and using the inner product property $<.,.>$, the wavelet coefficients a_{j0k} and b_{jk} are obtained as

$$a_{j0k} = < f(x), \phi_{j0k}(x) >, \tag{3}$$

$$b_{jk} = < f(x), \psi_{jk}(x) >. \tag{4}$$

While the function $f(x)$ is unknown, the wavelet coefficients a_{j0K} and b_{jk} cannot be calculated simply by (3) and (4), respectively. Since, it is not realistic to use an infinite number of wavelets to represent the function $f(x)$, we consider the following wavelet representation form of the function $f(x)$

$$\hat{f}(x) = \sum_{j=-M_1}^{M_2}\sum_{k=-N_1}^{N_2} b_{jk}\psi_{jk}(x) = \underline{\theta}^T \underline{\psi}(x) \tag{5}$$

for some positive integers M_1, N_1, M_2, N_2, vector $\underline{\theta} = (b_{M_1N_1}, \ldots, b_{M_1N_2}, \ldots, b_{M_2N_1}, \ldots, b_{M_2N_2})^T$ and vector $\underline{\psi}(x) = (\psi_{M_1N_1}(x), \ldots, \psi_{M_1N_2}(x), \ldots, \psi_{M_2N_1}(x), \ldots, \psi_{M_2N_2}(x))^T$.

If $\Xi_f(M_1,M_2,N_1,N_2) = f(x) - \hat{f}(x)$ is the Network Error (or approximation error), then it is easy to show that for arbitrary constant $\eta \geq 0$, there exist some constants M_1, N_1, M_2, N_2 such that $\|\Xi_f(M_1,M_2,N_1,N_2)\|_2 \leq \eta$ for all $x \in \Re$ [1]. This means that $f(x)$ can be approximated to any desired accuracy by a wavelet network $\hat{f}(x)$ with large enough M_1, N_1, M_2, N_2. The variable wavelet networks were introduced to achieve desired estimation accuracy and a suitable size network, and to adapt to variations of the characteristics and operating points in nonlinear systems [13, 26].

The wavelet series representation can be easily generalized to any dimension ***n***. For the ***n***-dimension case $\underline{x} = [x_1, x_2, \cdots, x_n]^T$, we introduce the wavelet function

$$\psi(\underline{x}) = \psi(x_1, x_2, \ldots, x_n) = \psi(x_1)\,\psi(x_2)\cdots\psi(x_n)\,. \tag{6}$$

Now, we make a modification to replace the wavelet bases in (5) with (6). Then the modified wavelet network becomes

$$\hat{f}(\underline{x}) = \sum_{j=M_1}^{M_2} \sum_{k=N_1}^{N_2} b_{jk}\psi_{jk}(x) = \sum_{j=-M_1}^{M_2} \sum_{k=-N_1}^{N_2} b_{jk} \prod_{l=1}^{n} \psi_{jk}(x_l) = \underline{\theta}^T \underline{\psi}(\underline{x}) \tag{7}$$

where $\underline{\theta} = (b_{M_1N_1}, \ldots, b_{M_1N_2}, \ldots, b_{M_2N_1}, \ldots, b_{M_2N_2})^T$ and $\underline{\psi}(\underline{x}) = (\psi_{M_1N_1}(\underline{x}), \ldots, \psi_{M_1N_2}(\underline{x}), \ldots, \psi_{M_2N_1}(\underline{x}), \ldots, \psi_{M_2N_2}(\underline{x}))^T$.

3 Robot Manipulators Dynamics

The dynamics of an n-link robot manipulator may be expressed in the Lagrange form [18]:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_V\dot{q} + f_c(\dot{q}) + g(q) + \tau_d(t) = \tau(t) \tag{8}$$

with $q(t) \in \Re^n$ joint variable, $M(q) \in \Re^{n\times n}$ inertia, $V_m(q,\dot{q}) \in \Re^{m\times n}$ Coriolis/centripetal forces, $g(q) \in \Re^n$ gravitational forces, $F_v \in \Re^{n\times n}$ diagonal matrix of viscous friction coefficients, $f_c(q) \in \Re^n$ Coulomb friction coefficients, and $\tau_d(t) \in \Re^n$ external disturbances. The bounded values of the external disturbances are given by $\|\tau_d(t)\| < b_d$. The external control torques to each joints are $\tau(t) \in \Re^n$. Given a desired trajectory $q_d(t) \in \Re^n$, the tracking error is

$$e(t) = q_d(t) - q(t) \tag{9}$$

and the instantaneous performance measure is defined as

$$r(t) = \dot{e}(t) + \Lambda e(t) \tag{10}$$

where Λ is the constant gain matrix or critic (not necessarily symmetric). The robot dynamics (8) may be written as

$$M(q)\dot{r}(t) = -V_m(q,\dot{q})r(t) + h(x) - \tau(t) + \tau_d(t) \tag{11}$$

where the robot nonlinear function is

$$h(x) = M(q)(\ddot{q}_d + \Lambda\dot{e}) + V_m(q,\dot{q})(\dot{q}_d + \Lambda e) + g(q) + F_v\dot{q} + f_c(\dot{q}) \tag{12}$$

where $\underline{x} = [e^T, \dot{e}^T, q_d^T, \dot{q}_d^T, \ddot{q}_d^T]^T$. This key function h(x) captures all the unknown dynamics of the robot arm. We employ an adaptive wavelet networks

$$\hat{h}(\underline{x}, \underline{\theta}_f) = \underline{\theta}_f^T \, \underline{\psi}_f(\underline{x}) \tag{13}$$

to approximate (or model) the nonlinear function h(x). The optimal weight vector $\underline{\theta}_f^*$ is quantities required only for analytical purposes. Typically $\underline{\theta}_f^*$ are chosen as

$$\underline{\theta}_f^* = \arg \min_{\underline{\theta}_f} \{ \max_{\underline{x}} \left| h(\underline{x}) - \underline{\theta}_f^T \, \psi_f(\underline{x}) \right| \}, \tag{14}$$

and the function $h(.)$ which is valid for all $\underline{x} \in U_{\underline{x}}$ has the following representation

$$h(\underline{x}) = \hat{h}(\underline{x}, \underline{\theta}_f^*) + \Xi_f(\underline{x}) = \underline{\theta}_f^{*T} \, \underline{\psi}_f(\underline{x}) + \Xi_f(\underline{x}) \tag{15}$$

By using definitions of (12)-(15), we rewrite (11) as

$$M(q)\dot{r}(t) = -V_m(q,\dot{q})r(t) + \underline{\theta}_f^{*T} \, \underline{\psi}_f(\underline{x}) + \Xi_f(\underline{x}) - \tau(t) + \tau_d(t)$$

Define the position error dynamics as

$$\dot{e}(t) = -\Lambda e(t) + r(t) \tag{16}$$

The following augmented system is obtained:

$$\dot{z}(t) = A(q,\dot{q})z(t) + B(q)(\underline{\theta}_f^{*T} \, \underline{\psi}_f(\underline{x}) + \Xi_f(\underline{x}) - \tau(t) + \tau_d(t)) \tag{17}$$

with

$$z(t) = \begin{bmatrix} e \\ r \end{bmatrix}, \quad A(q,\dot{q}) = \begin{bmatrix} -\Lambda & I \\ 0 & -M^{-1}V_m \end{bmatrix}, \quad B(q) = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}.$$

Definition 1. The effect of $\tau_d(t)$, denoting the external disturbance, will be attenuated by the H_∞ control signal if the following H_∞ tracking performance holds

$$\int_0^T \underline{z}^T Q \, \underline{z} \, dt \le \underline{z}^T(0) P \underline{z}(0) + \gamma^2 \int_0^T \tau_d^T \tau_d \, dt \qquad \forall\, 0 \le T < \infty \tag{18}$$

where γ is a prescribed attenuation level, and P, Q are positive definite weighting matrixes.

4 Control Design

Consider the Lyapunov function

$$V = \underline{z}^T P \underline{z} + \frac{1}{2} tr(\underline{\tilde{\theta}}_f \, \underline{\tilde{\theta}}_f^T) \tag{19}$$

where matrix P is a positive definite matrix and $\underline{\tilde{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$. The first derivative of the Lyapunov function V with respect to time *t* is

$$\dot{V} = \underline{z}^T \left[\dot{P} + PA + A^T P\right] \underline{z} + 2\underline{z}^T P\underline{B}\,(\underline{\theta}_f^{*T}\, \underline{\psi}_f(\underline{x}) + \Xi_f(\underline{x}) - \tau(t) + \tau_d(t)) + \mathrm{tr}(\dot{\underline{\tilde{\theta}}}_f \underline{\tilde{\theta}}_f^T) \quad (20)$$

Substituting [13, 19]

$$\tau(t) = \underline{\theta}_f^T\, \underline{\psi}_f(\underline{x}) + \tau^a + \tau^s \quad (21)$$

with

$$\tau^a = \frac{-1}{\beta^2} \underline{B}^T P \underline{z} \quad (22)$$

$$\tau^s = \mu_s \,\mathrm{sgn}\,(\underline{z}^T P \underline{B})(h^U(\underline{x}) + \left|\hat{h}(\underline{x}, \underline{\theta}_f)\right|) \quad (23)$$

where

$$\mu_s = \begin{cases} 1 & \text{if} \quad \|\underline{z}(t)\| \geq E \\ 0 & \text{if} \quad \|\underline{z}(t)\| < E \end{cases}$$

into (20), we have:

$$\begin{aligned} \dot{V} = \underline{z}^T &\left[\dot{P} + PA + A^T P + \frac{2}{\beta^2} P\underline{B}\underline{B}^T P\right] \underline{z} \\ &+ 2\underline{z}^T P\underline{B}\,(\underline{\theta}_f^{*T}\, \underline{\psi}_f(\underline{x}) + \Xi_f(\underline{x}) - \underline{\theta}_f^T\, \underline{\psi}_f(\underline{x}) - \tau^s + \tau_d(t)) + \mathrm{tr}(\dot{\underline{\tilde{\theta}}}_f \underline{\tilde{\theta}}_f^T) \end{aligned} \quad (24)$$

By using (28), (35) and inequality $X^T Y + Y^T X \leq \gamma X^T X + \frac{1}{\gamma} Y^T Y$ for any matrices X and Y with appropriate dimensions and for any constant $\gamma > 0$, and the fact that $\dot{\underline{\tilde{\theta}}}_f = \dot{\underline{\theta}}_f$, we conclude:

$$\begin{aligned} \dot{V} \leq \underline{z}^T &\left[\dot{P} + PA + A^T P + \frac{2}{\beta^2} P\underline{B}\underline{B}^T P + \frac{1}{\gamma^2} P\underline{B}\underline{B}^T P\right] \underline{z} \\ &+ 2\underline{z}^T P\underline{B}(\Xi_f(\underline{x}) - \underline{\tilde{\theta}}_f^T\, \underline{\psi}_f(\underline{x}) - \tau^s) + \mathrm{tr}(\dot{\underline{\theta}}_f \underline{\tilde{\theta}}_f^T) + \gamma^2\, \tau_d^T(t)\tau_d(t) \end{aligned} \quad (25)$$

Considering

$$\dot{P} + PA + A^T P + P\underline{B}(\frac{2}{\beta^2} + \frac{1}{\gamma^2})\underline{B}^T P + Q = 0 \quad (26)$$

and the adaptation law

$$\dot{\underline{\theta}}_f = \mathrm{Proj}(\underline{\theta}_f, \underline{\Pi}_f) = \underline{\Pi}_f \quad (27)$$

with $\underline{\Pi}_f = 2\underline{z}^T P\underline{B}\underline{\psi}_f(\underline{x})$, we find:

$$\dot{V} \leq \begin{cases} -\underline{z}^T Q\, \underline{z} + \gamma^2\, \tau_d^T(t)\tau_d(t) & \text{if} \quad \|\underline{z}(t)\| \geq E \\ -\underline{z}^T Q\, \underline{z} + \gamma^2\, \tau_d^T(t)\tau_d(t) + 2\left|\underline{z}^T P\underline{B}\right|(\left|h(\underline{x})\right| + \left|\hat{h}(\underline{x}, \underline{\theta}_f^*)\right|) & \text{if} \quad \|\underline{z}(t)\| < E \end{cases} \quad (28)$$

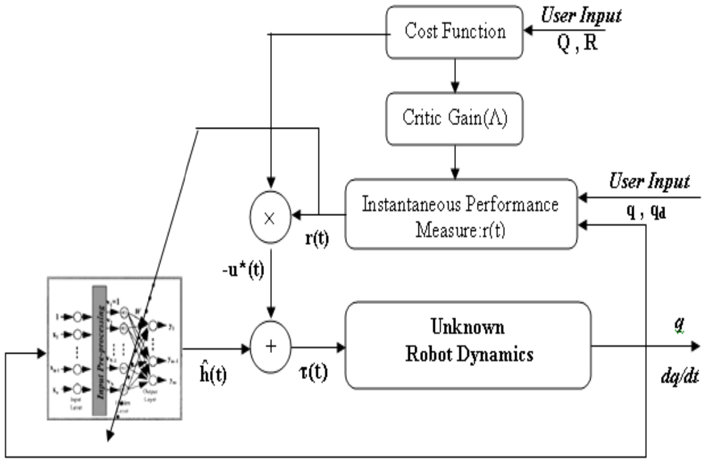

Fig. 1. Adaptive control design based on wavelet networks

By integrating both sides of (28) from 0 to T, we have:

$$V(T)+\int_0^T \underline{z}^T Q\,\underline{z}\,dt \le$$

$$\begin{cases} V(0)+\gamma^2\int_0^T \tau_d^T(t)\tau_d(t)\,dt & \text{if} \quad \|\underline{z}(t)\|\ge E \\ V(0)+\gamma^2\int_0^T \tau_d^T(t)\tau_d(t)\,dt+2\int_0^T\left|\underline{z}^T P\underline{B}\right|\left(\left|h(\underline{x})\right|+\left|\hat{h}(\underline{x},\underline{\theta}_f^*)\right|\right)\,dt & \text{if} \quad \|\underline{z}(t)\|< E \end{cases}$$

and after some simple manipulations on the inequality above, we conclude:

$$\lambda_{\min}(Q)\,\|\underline{e}(t)\|^2 \le \begin{cases} V(0)+\gamma^2\, B_d^2 & \text{if} \quad \|\underline{z}(t)\|\ge E \\ V(0)+\gamma^2\, B_d^2+2\rho\|\underline{z}(t)\| & \text{if} \quad \|\underline{z}(t)\|< E \end{cases} \tag{29}$$

where $\rho := \left\| \left|P\underline{B}\right|\left(\left|h(\underline{x})\right|+\left|\hat{h}(\underline{x},\underline{\theta}_f^*)\right|\right) \right\|$. This demonstrates all states and signals involved of the closed loop system are bounded, furthermore, the H_∞ tracking performance can be achieved from the results above.

In summary, the block diagram in Fig. 1 shows the major components that embody the wavelet-based adaptive controller (21) with τ^a given by (22) and τ^s given by (23). Finally, we can guarantee that $\|\underline{z}(t)\|< E$ and the criteria of H_∞ tracking performance (18) will be satisfied.

5 Simulation Results

The dynamic equations for an n-link manipulator can be found in [20]. In this study we have simulated a three-link robot manipulator.

An external disturbance and frictions are

$$\tau(t) = [3+\sin(2t)\ \ 5+\cos(t)\ \ 2+\sin(t)]^{T} \tag{30}$$

$$F_V\,\dot{q} + f_c(\dot{q}) = \mathrm{diag}\begin{bmatrix}2 & 2\end{bmatrix}\dot{q} + 1.5\,\mathrm{sgn}(\dot{q}) \tag{31}$$

where sgn(x) is a signum function. The weighting matrices in (18) are as follows:

$$Q_{11}=10\ I\ ,\ Q_{12}=-10\ I\ ,\ Q_{21}=Q_{12}^{T}\ ,\ Q_{22}=30\ I\ .$$

Our target is that the manipulator moves in a predetermined path without error. With determinate path, by solving inverse kinematics, we obtain the desired joints trajectory. In this study, we want that robot moves on a crescent path wobbly, as $q(t) = [0.5\mathrm{Sin}(0.1t), 1, 1.5]^{T}$.

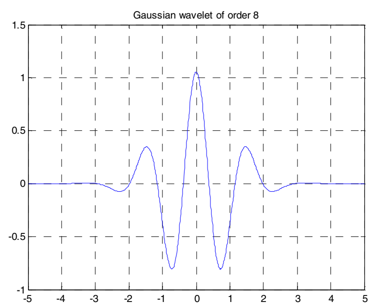

Fig. 2. Gaussian wavelet of order 8

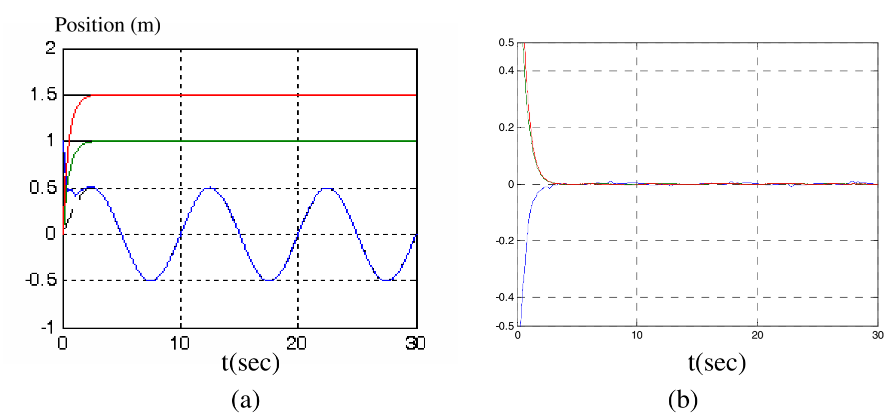

Fig. 3. Performance of adaptive control based on wavelet networks for system with uncertainties: (a) Tracking curve, (b) Tracking error

Fundamentally, the wavelet network can be characterized by

- Number of hidden neurons: 7*n+1, where ‘n’ is number of link of manipulator
- Hidden neuron activation functions: Gaussian wavelet of order 8 (Fig. 2)
-Output neuron activation functions: $\delta(x) = x$ *;*
- Learning rate in the weight tuning law: F= diag [70 70], K=0.01;
- Network input: $x = \begin{bmatrix} 1 & q^T & \dot{q}^T & e^T & \dot{e}^T & \dot{r}^T & q_d^T & \dot{q}_d^T \end{bmatrix}$;
- Inputs to hidden neurons: $p = x$.

The simulation results for wavelet-based neural network controllers with considering varieties of uncertainties for plant are shown in Fig. 3. The simulation results with assumption change system’s parameters for wavelet-based controller are shown in Fig. 4. The results in Fig. 3 and Fig. 4 show the capability of the wavelet network for designing an adaptive controller for overcoming uncertainties; both structured and unstructured.

6 Conclusion

An adaptive control scheme was developed for a robotic manipulator using wavelet networks. It was shown that the entire closed-loop system behavior depends on the user specified performance indexes. The weighting matrices automatically generate the Lyapunov function for the stability of the overall system. In derivation of the computed torque controller, it has been assumed that nonlinearities in the robotic manipulator are completely known. However even with the knowledge about the nonlinearities, it is difficult to achieve the control objective in the presence of modeling uncertainties and frictional forces. Due to use of Gaussian wavelet function in neural network, an estimation of non-linear functions with presence uncertainties and friction forces have more accuracy and will improve performance of control system considerably because wavelet functions keep time and frequency domain properties. The proposed neural adaptive learning shows both robustness and adaptation to changing system dynamics.

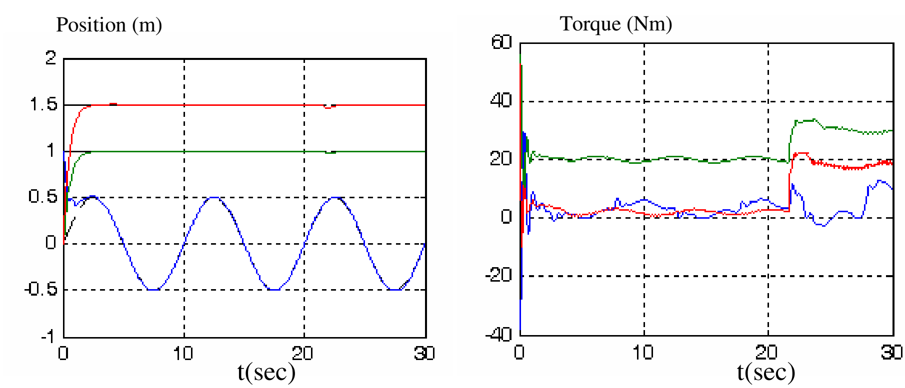

Fig. 4. Performance of adaptive control based on wavelet networks for system with inconstant parameters: (a) Tracking curve, (b) Controller output-$\tau(t)$

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