LMI-Based H_∞ Synchronization of Second-Order Neutral Master-Slave Systems Using Delayed Output Feedback Control

Hamid Reza Karimi, Huijun Gao

Abstract: The H_∞ synchronization problem of the master and slave structure of a second-order neutral master-slave systems with time-varying delays is presented in this paper. Delay-dependent sufficient conditions for the design of a delayed output-feedback control are given by Lyapunov-Krasovskii method in terms of a linear matrix inequality (LMI). A controller, which guarantees H_∞ synchronization of the master and slave structure using some free weighting matrices, is then developed. A numerical example has been given to show the effectiveness of the method. The simulation results illustrate the effectiveness of the proposed methodology.

Keywords: Second-order systems; H_∞ synchronization; output-feedback control; time- delay; LMI.

1. INTRODUCTION

In the last few years, synchronization in chaotic dynamical systems has received a great deal of interest among scientists from various fields [1, 2]. The results of chaos synchronization are utilized in biology, chemistry, secret communication and cryptography, nonlinear oscillation synchronization and some other nonlinear fields. The first idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll [3], and the method was realized in electronic circuits. The methods for synchronization of the chaotic systems have been widely studied in recent years, and many different methods have been applied theoretically and experimentally to synchronize chaotic systems, such as feedback control [4-10], adaptive control [11-15], backstepping [16] and sliding mode control [17-21].

One of the most attractive dynamical systems is the second-order systems which capture the dynamic behaviour of many natural phenomena, and have found applications in many fields, such as vibration and structural analysis, spacecraft control, electrical networks, robotics control and, hence, have attracted much attention (see, for instance, [22-32]). It has been proved that in special situations a second-order system may show chaotic dynamics. A second-order linear plant containing a relay with hysteresis is analyzed in [33], showing the chaotic nature of its dynamical behavior. Complex dynamical behavior of second-order linear plants controlled with conventional controllers is investigated in [34, 35]. On the other hand, in view of the time-delay phenomenon, which is frequently encountered in practical situations, this delay may induce complex behaviors (oscillation, instability, bad performances) for the systems concerned (see for instance the references [36-41] and the references therein). Up to now, to the best of the authors' knowledge, no results about the synchronization of second-order master-slave systems with time-varying delays using delayed output-feedback control are available in the literature, which remains to be important and challenging. This motivates the present study.

In this paper, we make an attempt to develop an efficient approach for H_∞ synchronization problem of second-order neutral master-slave systems with time-varying state delays. The main merit of the proposed method lies in the fact that it provides a convex problem via introduction of additional decision variables such that the control gains can be found from the LMI formulations without reformulating the system equations into a standard form of a first-order neutral system. By using a Lyapunov-Krasovskii method and some free weighting matrices, new sufficient conditions are established in terms of a delay-dependent LMI for the existence of desired delayed output-feedback control such that the resulting closed-loop system is asymptotically stable and satisfies a prescribed H_∞ performance. A significant advantage of our result is that the desired control is designed directly instead of coupling the model to a first-order neutral system and then designing the control law in a higher dimensional space. Therefore, our result can be

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implemented in a numerically stable and efficient way for large-scale second-order neutral systems. Furthermore, as pointed out in [25], retaining the model in matrix second-order form has many advantages such as preserving physical insight of the original problem, preserving system matrix sparsity and structure, preserving uncertainty structure and entailing easier implementation (feedback control can be used directly). Finally, the simulation results are given to illustrate the usefulness of our results.

The notations used throughout the paper are fairly standard. I_n and 0_n represent, respectively, n by n identity matrix and n by n zero matrix; the superscript $'T'$ stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{n\times m}$ is the set of all real m by n matrices. The matrices $\hat{I}$ and $\tilde{I}$ are defined, respectively, as $\hat{I} := [I \quad 0]$ and $\tilde{I} := [0 \quad I]$. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm and $diag\{\cdots\}$ represents a block diagonal matrix. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote, respectively, the smallest and largest eigenvalue of the square matrix A. The operator $sym\{A\}$ denotes $A + A^T$. The notation $P > 0$ means that P is real symmetric and positive definite and the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. In addition, $L_2[0, \infty)$ is the space of square-integrable vector functions over $[0, \infty)$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

I. Problem description

Consider a model of second-order neutral master-slave systems in the form of

$$\begin{cases} M\,\ddot{x}_m(t) + M_1\,\ddot{x}_m(t-d(t)) + A\,\dot{x}_m(t) + A_1\,\dot{x}_m(t-r(t)) \\ \quad + B\,x_m(t) + B_1\,x_m(t-r(t)) + N_1 f(x_m(t)) \\ \quad + N_2\, g(x_m(t-r(t))) = 0, \\ x_m(t) = \phi(t), \qquad t \in [-\max\{d_M, r_M\},\, 0], \\ \dot{x}_m(t) = \dot{\phi}(t), \qquad t \in [-\max\{d_M, r_M\},\, 0], \\ z_m(t) = C_1\, x_m(t) + C_2\, x_m(t-r(t)), \\ y_m(t) = C_3\, x_m(t), \end{cases} \tag{1}$$

$$\begin{cases} M\,\ddot{x}_s(t) + M_1\,\ddot{x}_s(t-d(t)) + A\,\dot{x}_s(t) + A_1\,\dot{x}_s(t-r(t)) \\ \quad + B\,x_s(t) + B_1\,x_s(t-r(t)) + N_1 f(x_s(t)) \\ \quad + N_2\, g(x_s(t-r(t))) = B_2\, u(t) + D\, w(t), \\ x_s(t) = \varphi(t), \qquad t \in [-\max\{d_M, r_M\},\, 0], \\ \dot{x}_s(t) = \dot{\varphi}(t), \qquad t \in [-\max\{d_M, r_M\},\, 0], \\ z_s(t) = C_1\, x_s(t) + C_2\, x_s(t-r(t)), \\ y_s(t) = C_3\, x_s(t), \end{cases} \tag{2}$$

where $x_m(t), x_s(t)$ are the $n\times 1$ state vector of the master and slave systems, respectively; $u(t)$ is the $r\times 1$ control input; $w(t)$ is the $q\times 1$ external excitation (disturbance), $z_m(t), z_s(t)$ are the $s\times 1$ controlled output and $y_m(t), y_s(t)$ is the $l\times 1$ measured output. The coefficient matrices M, M_1, A, A_1, B and B_1 are square and real matrices, and the matrices N_1, N_2, B_2, C_1, C_2, C_3 and D are real matrices with appropriate dimensions. The time-varying vector valued initial functions $\phi(t)$ and $\varphi(t)$ are continuously differentiable functionals, and the time-varying delays $d(t)$ and $r(t)$ are functions satisfying, respectively,

$$\begin{cases} 0 < d(t) \le d_M\,, \qquad \dot{d}(t) \le d_D < 1, \\ 0 < r(t) \le r_M\,, \qquad \dot{r}(t) \le r_D\,. \end{cases} \tag{3}$$

Assumption 1: The nonlinear functions $f, g : \Re^n \to \Re^n$ are continuous and satisfy $f(0) = g(0) = 0$ and the Lipschitz condition, i.e., $\|\underline{f}(x_0 - y_0)\| \le \|f(x_0) - f(y_0)\| \le \|\bar{f}(x_0 - y_0)\|$ and $\|\underline{g}(x_0 - y_0)\| \le \|g(x_0) - g(y_0)\| \le \|\bar{g}(x_0 - y_0)\|$ for all $x_0, y_0 \in \Re^n$ and $\bar{f}, \underline{f}, \bar{g}, \underline{g}$ are known constants.

Remark 1: The dynamical system (1) arises naturally in a wide range of applications, including: teleoperator systems, mechanical multi-body systems and robotics control (see for instance [20, 27-30] and the many references therein). In mechanical systems the matrices $(M, M_1), (A, A_1)$ and (B, B_1) , respectively, correspond to the mass, damping (viscous friction coefficient), and stiffness matrices. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$, respectively, are position, velocity and acceleration vectors. The matrix B_2 distributes the force input to the correct degrees of freedom (see [22-24]).

Now, the synchronization error of the master and slave systems (1) and (2) is defined as $e(t) = x_s(t) - x_m(t)$, then the error dynamics between (1) and (2), namely synchronization error system, can be expressed by

$$\begin{cases} M\,\ddot{e}(t)+M_1\,\ddot{e}(t-d(t))+A\,\dot{e}(t)+A_1\,\dot{e}(t-r(t)) \\ \quad +B\,e(t)+B_1\,e(t-r(t))+N_1\hat{f}(e(t)) \\ \quad +N_2\hat{g}(e(t-r(t)))=B_2\,u(t)+D\,w(t), \\ z_e(t)=C_1\,e(t)+C_2\,e(t-r(t)), \\ y_e(t)=C_3\,e(t), \end{cases} \tag{4}$$

where $z_e(t)=z_s(t)-z_m(t)$, $\hat{f}(e(t)):=f(x_s(t))-f(x_m(t))$ and $\hat{g}(e(t-r(t))):=g(x_s(t-r(t)))-g(x_m(t-r(t)))$.

The problem to be addressed in this paper is formulated as follows: given the second-order neutral master-slave systems (1) and (2) with any time-varying delays satisfying (3) and a prescribed level of disturbance attenuation $\gamma>0$, find a delayed output-feedback control $u(t)$ of the form

$$\begin{aligned} u(t)&=K_1\,y_e(t)+K_2\,\dot{y}_e(t)+K_3\,y_e(t-r(t)) \\ &\quad +K_4\,\dot{y}_e(t-r(t)):=K\,C\,\xi(t)+K_r\,C\,\xi(t-r(t)) \end{aligned} \tag{5}$$

where $K:=[K_1 \quad K_2]$, $K_r:=[K_3 \quad K_4]$, $C:=diag\{C_3,C_3\}$, $\xi(t)=col\{e(t),\dot{e}(t)\}$ and the matrices $\{K_i\}_{i=1}^{4}$ are the control gains to be determined such that

1) the synchronization error system (4) is asymptotically stable for any time delays satisfying (3);
2) under zero initial conditions and for all non-zero $w(t)\in L_2[0,\infty]$, the H_∞ performance measure, i.e., $J_\infty=\int_0^\infty z_e^T(t)\,z_e(t)-\gamma^2 w^T(t)\,w(t) \quad dt$, satisfies $J_\infty<0$ (or the induced L_2–norm of the operator form $w(t)$ to the controlled outputs $z(t)$ is less than γ);

in this case, the second-order neutral master-slave systems (1) and (2) are said to be robustly asymptotically stable with H_∞ performance measures.

III. Main results

In this section, sufficient conditions for the solvability of the delayed output-feedback control design problem are proposed using the Lyapunov method and an LMI approach. Before proceeding further, we give two technical lemmas, which are useful in the proof our main results.

Lemma 1 ([42]): For any arbitrary positive definite matrix H and a matrix W the following inequality holds:

$$-2\int_{t-r(t)}^{t} b(s)^T a(s)\,ds \le$$
$$\int_{t-r(t)}^{t}\begin{bmatrix} a(s) \\ b(s)\end{bmatrix}^T \begin{bmatrix} H & HW \\ * & (HW+I)^T H^{-1}(HW+I)\end{bmatrix}\begin{bmatrix} a(s) \\ b(s)\end{bmatrix} ds$$

Lemma 2 ([43]): For a given $\mathrm{M}\in\Re^{p\times n}$ with $rank\,(\mathrm{M})=p<n$, assume that $Z\in\Re^{n\times n}$ is a symmetric matrix, then there exists a matrix $\hat{Z}\in\Re^{p\times p}$ such that $\mathrm{M}Z=\hat{Z}\mathrm{M}$ if and only if

$$Z=V\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2\end{bmatrix}V^T,\quad \hat{Z}=U\,\hat{\mathrm{M}}\,Z_1\,\hat{\mathrm{M}}^{-1}U^T,$$

where $Z_1\in\Re^{p\times p}$, $Z_2\in\Re^{(n-p)\times(n-p)}$ and the singular value decomposition of the matrix M is represented as $\mathrm{M}=U[\hat{\mathrm{M}} \quad 0]V^T$ with the unitary matrices $U\in\Re^{p\times p}$, $V\in\Re^{n\times n}$ and a diagonal matrix $\hat{\mathrm{M}}\in\Re^{p\times p}$ with positive diagonal elements in decreasing order.

We firstly present a delay-dependent condition for the stability and H_∞ performance of the synchronization error system (4) for any time-varying delays satisfying (3) in the following theorem.

Theorem 1: For given scalars $d_M, r_M>0$, $d_D<1$, r_D and $\gamma>0$, the second-order neutral master-slave systems (1) and (2) with any time-varying delays satisfying (3) is robustly stabilizable by (5) and satisfies the H_∞ performance measure, if there exist some matrices P_2, P_3, W, F_1, F_2, positive-definite matrices P_1, Q_1, Q_2, H and positive-definite diagonal matrices $\Lambda_1,\Lambda_2,\Lambda_3$, such that the following matrix inequality is feasible,

$$\Pi=\begin{bmatrix} \Pi_{11}+\begin{bmatrix}(C_1\hat{I})^T \\ 0\end{bmatrix}[C_1\hat{I} \quad 0] & \Pi_{12}+\begin{bmatrix}(C_1\hat{I})^T C_2\hat{I} \\ 0\end{bmatrix} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22}++\hat{I}^T C_2^T C_2\hat{I} & 0 & \Pi_{24} \\ * & * & \Pi_{33} & 0 \\ * & * & * & \Pi_{44}\end{bmatrix}<0 \tag{6}$$

with

$$\Pi_{11} := sym\{P^T \begin{bmatrix} \hat{I}^T \tilde{I} & \tilde{I}^T \\ \hat{A}_1 & M \end{bmatrix}\} - sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{f}^T \Lambda_1 \underline{f}\, \hat{I}\}$$

$$- sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{g}^T \Lambda_2 \underline{g}\, \hat{I}\} + sym\{P^T W^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} [\tilde{I} \quad 0]\}$$

$$+ r_M P^T (W^T H + I) H^{-1} (HW + I) P$$

$$+ \begin{bmatrix} Q_1 + F_1 + F_1^T & 0 \\ 0 & Q_2 \end{bmatrix} + r_M \begin{bmatrix} \tilde{I} & 0 \\ 0 & I \end{bmatrix}^T P_1 \begin{bmatrix} \tilde{I} & 0 \\ 0 & I \end{bmatrix}$$

$$+ \frac{r_M}{1 - r_D} \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} 0 \\ A_1 \end{bmatrix}^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} [0 \quad I]$$

$$\Pi_{12} = P^T (\begin{bmatrix} 0 \\ \hat{A}_2 \end{bmatrix} - W^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \tilde{I}) + \begin{bmatrix} F_2^T - F_1 \\ 0 \end{bmatrix}$$

$$\Pi_{22} = -(1 - r_D) Q_1 - sym\{ F_2 \} - sym\{ (\bar{g}\ \hat{I})^T \Lambda_3 \underline{g}\ \hat{I} \}$$

$$\Pi_{13} = \begin{bmatrix} P^T \begin{bmatrix} 0 \\ M_1 \end{bmatrix} & P^T \begin{bmatrix} 0 \\ N_1 \end{bmatrix} + \begin{bmatrix} \hat{I}^T (\bar{f} + \underline{f})^T \Lambda_1 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\Pi_{33} = diag\{ -(1 - d_D) Q_2, -sym\{\Lambda_1\} \}$$

$$\Pi_{14} = \begin{bmatrix} \begin{bmatrix} \hat{I}^T (\bar{g} + \underline{g})^T \Lambda_2 \\ 0 \end{bmatrix} & P^T \begin{bmatrix} 0 \\ N_2 \end{bmatrix} & -P^T \begin{bmatrix} 0 \\ D \end{bmatrix} & \begin{bmatrix} F_1 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\Pi_{24} = \begin{bmatrix} 0 & \hat{I}^T (\bar{g} + \underline{g})^T \Lambda_3 & 0 & F_2 \end{bmatrix}$$

$$\Pi_{44} = diag\{ -sym\{\Lambda_2\}, -sym\{\Lambda_3\}, -\gamma^2 I, -r_M P_1 \}$$

where $\hat{A}_1 := B \hat{I} + (A + A_1) \tilde{I} - B_2 K C$ and $\hat{A}_2 := [B_1 \quad 0] - B_2 K_r C$.

Proof: Firstly, we represent the synchronization error system (4) in an equivalent descriptor model form as

$$\begin{cases} \ddot{e}(t) = \eta(t), \\ 0 = M \eta(t) + M_1 \eta(t - d(t)) + \hat{A}_1 \xi(t) + \hat{A}_2\, \xi(t - r(t)) \\ \quad - A_1 \int_{t-r(t)}^{t} \eta(s) \quad ds + N_1 \hat{f}(e(t)) + N_2 \hat{g}(e(t - r(t))) \\ \quad - D\, w(t), \end{cases} \tag{7}$$

Define the Lyapunov-Krasovskii functional

$$V(t) = \sum_{i=1}^{5} V_i(t), \tag{8}$$

where

$$V_1(t) = \xi(t)^T P_1\, \xi(t) := [\xi(t)^T \quad \eta(t)^T]\, T\, P \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix},$$

$$V_2(t) = \int_{t-r(t)}^{t} \xi(s)^T Q_1\, \xi(s) \ \ ds,$$

$$V_3(t) = \int_{t-d(t)}^{t} \eta(s)^T Q_2\, \eta(s) \ \ ds,$$

$$V_4(t) = \int_{-r(t)}^{0} \int_{t+\theta}^{t} \dot{\xi}(s)^T P_1\ \dot{\xi}(s) \ \ ds\, d\theta,$$

and

$$V_5(t) = \frac{1}{1 - r_D} \int_{t-r(t)}^{t} (s - t + r(t)) \eta(s)^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix}^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \eta(s) \ \ ds,$$

with $T = diag\{I,\ 0\}$ and $P = \begin{bmatrix} P_1 & 0 \\ P_3 & P_2 \end{bmatrix}$, where $0 < P_1 = P_1^T$.

Differentiating $V_1(t)$ along the system trajectory becomes

$$\dot{V}_1(t) = 2\xi(t)^T P_1\, \dot{\xi}(t) = 2[\xi(t)^T \quad \eta(t)^T]\ P^T \begin{bmatrix} \dot{\xi}(t) \\ 0 \end{bmatrix}$$

$$= 2[\xi(t)^T \quad \eta(t)^T]\ P^T \begin{bmatrix} \dot{\xi}(t) \\ (.) \end{bmatrix}$$

$$= 2[\xi(t)^T \quad \eta(t)^T]\ P^T \{ \begin{bmatrix} \hat{I}^T \tilde{I} & \tilde{I}^T \\ \hat{A}_1 & M \end{bmatrix} \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ M_1 \end{bmatrix} \eta(t - d(t)) + \begin{bmatrix} 0 \\ \hat{A}_2 \end{bmatrix} \xi(t - r(t)) + \begin{bmatrix} 0 \\ N_1 \end{bmatrix} \hat{f}(e(t))$$

$$+ \begin{bmatrix} 0 \\ N_2 \end{bmatrix} \hat{g}(e(t - r(t))) - \begin{bmatrix} 0 \\ D \end{bmatrix} w(t) \} + \beta(t) \tag{9}$$

where

$$(.) := M\, \eta(t) + M_1\, \eta(t - d(t)) + \hat{A}_1 \xi(t) + \hat{A}_2\, \xi(t - r(t))$$

$$- A_1 \int_{t-r(t)}^{t} \eta(s) \quad ds + N_1 \hat{f}(e(t)) + N_2 \hat{g}(e(t - r(t)))$$

$$- D\, w(t)$$

and

$$\beta(t) = -2 \int_{t-r(t)}^{t} [\xi(t)^T \quad \eta(t)^T]\ P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \eta(s)\, ds \quad .$$

Using Lemma 1 for $a(s) = col\{0, A_1\} \eta(s)$ and $b = P\, col\{\xi(t), \eta(t)\}$ we obtain

$$\beta(t) \le r_M [\xi(t)^T \quad \eta(t)^T] P^T (W^T H + I) H^{-1}$$

$$\times (HW + I) P \begin{bmatrix} \xi(t) \\ \eta(t) \end{bmatrix} + 2[\xi(t)^T \quad \eta(t)^T] P^T$$

$$\times W^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \tilde{I} (\xi(t) - \xi(t - r(t)))$$

$$+ \int_{t-r(t)}^{t} \eta(s)^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix}^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \eta(s) \ \ ds$$

$$(10)$$

Also, differentiating the second to forth Lyapunov terms in (8) give

$$\begin{aligned}\dot{V}_2(t) &= \xi(t)^T Q_1 \xi(t) - (1-\dot{r}(t))\xi^T(t-r(t))Q_1\xi(t-r(t)) \\ &\le \xi(t)^T Q_1 \xi(t) - (1-r_D)\xi^T(t-r(t))Q_1\xi(t-r(t)),\end{aligned} \tag{11}$$

and similarly,

$$\begin{aligned}\dot{V}_3(t) &= \eta(t)^T Q_2 \eta(t) - (1-\dot{d}(t))\eta^T(t-d(t))Q_2\eta(t-d(t)) \\ &\le \eta(t)^T Q_2 \eta(t) - (1-d_D)\eta^T(t-d(t))Q_2\eta(t-d(t)),\end{aligned} \tag{12}$$

and

$$\begin{aligned}\dot{V}_4(t) &= r(t)\ \dot{\xi}(t)^T P_1\ \dot{\xi}(t) - \int_{t-r(t)}^{t} \dot{\xi}(s)^T P_1\ \dot{\xi}(s)\,ds \\ &\le r_M\ \dot{\xi}(t)^T P_1\ \dot{\xi}(t) - \int_{t-r(t)}^{t} \dot{\xi}(s)^T P_1\ \dot{\xi}(s)\,ds\end{aligned} \tag{13}$$

and the time derivative of the last term of $V(t)$ in (8) is

$$\begin{aligned}\dot{V}_5(t) &= \frac{r}{1-r_D}\eta(t)^T \begin{bmatrix}0\\A_1\end{bmatrix}^T H \begin{bmatrix}0\\A_1\end{bmatrix}\eta(t) \\ &\quad - \frac{1-\dot{r}(t)}{1-r_D}\int_{t-r(t)}^{t} \eta(s)^T \begin{bmatrix}0\\A_1\end{bmatrix}^T H \begin{bmatrix}0\\A_1\end{bmatrix}\eta(s)\ ds \\ &\le \frac{r_M}{1-r_D}\eta(t)^T \begin{bmatrix}0\\A_1\end{bmatrix}^T H \begin{bmatrix}0\\A_1\end{bmatrix}\eta(t) \\ &\quad - \int_{t-r(t)}^{t} \eta(s)^T \begin{bmatrix}0\\A_1\end{bmatrix}^T H \begin{bmatrix}0\\A_1\end{bmatrix}\eta(s)\ ds\end{aligned} \tag{14}$$

Moreover, from the Leibniz-Newton formula ($\xi(t) = \xi(t-r(t)) + \int_{t-r(t)}^{t}\dot{\xi}(s)\,ds$), the following equation holds for any matrices F_1, F_2 with appropriate dimensions:

$$\begin{aligned}&2(\xi^T(t)F_1 + \xi^T(t-r(t))F_2) \\ &\quad \times(\xi(t) - \xi(t-r(t)) - \int_{t-r(t)}^{t}\dot{\xi}(s)\,ds) = 0\end{aligned} \tag{15}$$

On the other hand, using Assumption 1 for any positive scalars $\{\Lambda_i\}_{i=1}^3$, we have:

$$\begin{aligned}&0 < -2(\hat{f}(e(t)) - \bar{f}\,e(t))^T \Lambda_1 (\hat{f}(e(t)) - \underline{f}\,e(t)) \\ &0 < -2(\hat{g}(e(t)) - \bar{g}\,e(t))^T \Lambda_2 (\hat{g}(e(t)) - \underline{g}\,e(t)) \\ &0 < -2(\hat{g}(e(t-r(t))) - \bar{g}\,e(t-r(t)))^T \Lambda_3 \\ &\quad \times(\hat{g}(e(t-r(t))) - \underline{g}\,e(t-r(t)))\end{aligned} \tag{16}$$

Using the obtained derivative terms (9)-(15) and adding the right-hand sides of equation (16) into, we obtain the following result for $\dot{V}(t)$,

$$\begin{aligned}\dot{V}(t) &= \sum_{i=1}^{5}\dot{V}_i(t) \\ &\le 2[\xi(t)^T \quad \eta(t)^T]\,P^T\{\begin{bmatrix}\hat{I}^T\tilde{I} & \tilde{I}^T\\ \hat{A}_1 & M\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} + \begin{bmatrix}0\\M_1\end{bmatrix} \\ &\times\eta(t-d(t)) + \begin{bmatrix}0\\ \hat{A}_2\end{bmatrix}\xi(t-r(t)) + \begin{bmatrix}0\\N_1\end{bmatrix}\hat{f}(e(t)) \\ &+ \begin{bmatrix}0\\N_2\end{bmatrix}\hat{g}(e(t-r(t))) - \begin{bmatrix}0\\D\end{bmatrix}w(t)\} + r_M[\xi(t)^T \quad \eta(t)^T] \\ &\times P^T(W^T H + I)H^{-1}(HW + I)P\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix} \\ &+ 2[\xi(t)^T \quad \eta(t)^T]P^T W^T H\begin{bmatrix}0\\A_1\end{bmatrix}\tilde{I}(\xi(t) - \xi(t-r(t))) \\ &+ \frac{r_M}{1-r_D}\eta(t)^T\begin{bmatrix}0\\A_1\end{bmatrix}^T H\begin{bmatrix}0\\A_1\end{bmatrix}\eta(t) + \xi(t)^T(Q_1 \\ &+ sym\{F_1\})\xi(t) - \xi^T(t-r(t))((1-r_D)Q_1 + sym\{F_2\}) \\ &\times\xi(t-r(t)) + 2\xi(t)^T(F_2^T - F_1)\xi(t-r(t)) + r_M\ \dot{\xi}(t)^T \\ &\times P_1\ \dot{\xi}(t) + \eta(t)^T Q_2\,\eta(t) - (1-d_D)\eta^T(t-d(t))Q_2 \\ &\times\eta(t-d(t)) + r_M\vartheta^T(t)F P_1^{-1}F^T\vartheta(t) - 2(\hat{f}(e(t)) \\ &- \bar{f}\,\hat{I}\,\xi(t))^T\Lambda_1\times(\hat{f}(e(t)) - \underline{f}\ \hat{I}\,\xi(t)) - 2(\hat{g}(e(t)) \\ &- \bar{g}\ \hat{I}\,\xi(t))^T\Lambda_2(\hat{g}(e(t)) - \underline{g}\ \hat{I}\,\xi(t)) - 2(\hat{g}(e(t-r(t))) \\ &- \bar{g}\ \hat{I}\,\xi(t-r(t)))^T\Lambda_3(\hat{g}(e(t-r(t))) - \underline{g}\ \hat{I}\,\xi(t-r(t))) \\ &- \int_{t-r(t)}^{t}(\vartheta^T(t)F + \dot{\xi}^T(s)P_1)P_1^{-1}(\vartheta^T(t)F + \dot{\xi}^T(s)P_1)^T\ ds\end{aligned} \tag{17}$$

where the vectors $\vartheta(t)$ and N are, respectively,

$$\vartheta(t) := col\{\xi(t), \eta(t), \xi(t-r(t)), \eta(t-d(t)),\ w(t)\}, \tag{18}$$

$$F := col\{F_1, 0, F_2, 0,\ 0\}. \tag{19}$$

The H_∞ performance measure can be rewritten as

$$J_\infty = \int_0^\infty [(z_s(t) - z_m(t))^T(z_s(t) - z_m(t)) - \gamma^2 w(t)^T w(t) + \dot{V}(t)]\,dt \tag{20}$$

Substituting the terms of

$$z_s(t) - z_m(t) = C_1\hat{I}\ \xi(t) + C_2\,\hat{I}\,\xi(t-r(t)),$$

$$\dot{\xi}(t) = \begin{bmatrix}\hat{I}^T\tilde{I} & \tilde{I}^T\end{bmatrix}\begin{bmatrix}\xi(t)\\ \eta(t)\end{bmatrix},$$

and upper bound of $\dot{V}(t)$ in (17) results in (20) being less than the integrand $\vartheta(t)^T \, \Pi \, \vartheta(t)$ where the matrix Π, by Schur complement, is given in (6). Now, if $\Pi < 0$, then $J_\infty < 0$ which means that the L_2 –gain from the disturbance $w(t)$ to the controlled output $z(t)$ is less than γ. This completes the proof. ■

Remark 2: It is easy to see that the inequality (6) imply $\Pi_{11} < 0$. Hence by Proposition 4.2 in the reference [44], the matrix P is nonsingular. Then, according to the structure of the matrix P, the matrix $X := P^{-1}$ has the form

$$X = \begin{bmatrix} X_1 & 0 \\ X_3 & X_2 \end{bmatrix},$$

where $X_i = P_i^{-1} \, (i = 1, 2)$ and $X_3 = -X_2 P_3 X_1$.

Remark 3: According to structure of matrix C, i.e., $C := diag\{C_3, C_3\}$, with $rank\,(C_3) = l < n$, Lemma 2 proposes that an equivalent condition on matrix equation $CX_1 = \hat{X}_1 C$ is

$$X_1 = V \begin{bmatrix} X_{11} & 0 \\ 0 & X_{22} \end{bmatrix} V^T,$$

$$\hat{X}_1 = U \, \hat{C} \, X_{11} \hat{C}^{-1} U^T,$$

where $X_{11} \in \Re^{2l \times 2l}$, $X_{22} \in \Re^{2(n-l) \times 2(n-l)}$ and $C = U[\hat{C} \quad 0]V^T$ (the singular value decomposition of the matrix C), with $rank\,(C) = 2l$, $U \in \Re^{2l \times 2l}$, $V \in \Re^{2n \times 2n}$ and $\hat{C} \in \Re^{2l \times 2l}$.

Theorem 2: Consider the second-order neutral master-slave systems (1) and (2) with any time-varying delays satisfying (3). For given scalars $d_M, r_M > 0, d_D < 1, r_D$ and $\gamma > 0$, there exits an output-feedback control in the form of (5) such that the resulting closed-loop system is robustly asymptotically stable and satisfies H_∞ performance measure in Definition 1, if there exist a scalar α, matrices $\hat{F}_1, \hat{F}_2, \tilde{X}_1, \tilde{X}_2, X_2, X_3$, positive-definite matrices $X_{11}, X_{22}, \hat{Q}_1, \hat{Q}_2, \overline{H}$ and positive-definite diagonal matrices $\overline{\Lambda}_1, \overline{\Lambda}_2, \overline{\Lambda}_3$, satisfying the following LMI

$$\begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & \hat{\Pi}_{13} & \hat{\Pi}_{14} & \hat{\Pi}_{15} & \hat{\Pi}_{16} & \hat{\Pi}_{17} \\ * & sym\{M X_2\} & 0 & 0 & \hat{\Pi}_{25} & 0 & \hat{\Pi}_{27} \\ * & * & \hat{\Pi}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Pi}_{44} & 0 & 0 & 0 \\ * & * & * & * & \hat{\Pi}_{55} & 0 & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & 0 \\ * & * & * & * & * & * & \hat{\Pi}_{77} \end{bmatrix} < 0 \tag{21}$$

where

$$\hat{\Pi}_{11} = sym\{\hat{I}^T \tilde{I} \, X_1 + \hat{F}_1 + \tilde{I}^T X_3\} + \hat{Q}_1$$

$$\hat{\Pi}_{12} = X_1 (B \hat{I} + (A + (1+\alpha) A_1) \tilde{I})^T - C^T \tilde{X}_1^T B_2^T + \tilde{I}^T X_2 + X_3^T M^T$$

$$\hat{\Pi}_{13} = \left[\begin{bmatrix} \hat{F}_2^T - \hat{F}_1 \\ (B_1 \hat{I} - \alpha A_1 \tilde{I}) X_1 - B_2 \tilde{X}_2 C \end{bmatrix} \quad \begin{bmatrix} 0 \\ M_1 \end{bmatrix} X_1 \right]$$

$$\hat{\Pi}_{33} = diag\{-(1 - r_D) \hat{Q}_1 - sym\{\hat{F}_2\}, -(1 - d_D) \hat{Q}_2\}$$

$$\hat{\Pi}_{14} = \left[\begin{bmatrix} 0 \\ N_1 \end{bmatrix} \overline{\Lambda}_1^T + X^T \begin{bmatrix} \hat{I}^T (\bar{f} + \underline{f})^T \\ 0 \end{bmatrix} \quad X^T \begin{bmatrix} \hat{I}^T (\bar{g} + \underline{g})^T \\ 0 \end{bmatrix} \right]$$

$$\hat{\Pi}_{44} = diag\{-sym\{\overline{\Lambda}_1\}, -sym\{\overline{\Lambda}_2\}\}$$

$$\hat{\Pi}_{15} = \left[\begin{bmatrix} 0 \\ N_2 \end{bmatrix} \overline{\Lambda}_3^T \quad -\begin{bmatrix} 0 \\ D \end{bmatrix} \quad \begin{bmatrix} \hat{F}_1 \\ 0 \end{bmatrix} \quad r_M \begin{bmatrix} X_1 \tilde{I}^T \hat{I} + X_3^T \tilde{I} \\ X_2^T \tilde{I} \end{bmatrix} \right]$$

$$\hat{\Pi}_{25} = \left[X_1 \hat{I}^T (\bar{g} + \underline{g})^T \quad 0 \quad 0 \quad 0 \right]$$

$$\hat{\Pi}_{55} = diag\{-sym\{\overline{\Lambda}_3\}, -\gamma^2 I, -r_M X_1, -r_M X_1\}$$

$$\hat{\Pi}_{16} = \left[\begin{bmatrix} (C_1 \hat{I})^T \\ 0 \end{bmatrix} \quad r_M (\alpha + 1) \overline{H} \quad \begin{bmatrix} X_3^T \\ X_2^T \end{bmatrix} \quad \begin{bmatrix} 0 & X_3^T A_1^T \\ 0 & X_2^T A_1^T \end{bmatrix} \right]$$

$$\hat{\Pi}_{66} = diag\{-I, -r_M \overline{H}, -\overline{Q}_2, -\frac{1 - r_D}{r_M} \overline{H}\}$$

$$\hat{\Pi}_{17} = \left[\left[\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{f} \quad \hat{I}^T \underline{f} \right] \quad \left[\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{g} \quad \hat{I}^T \underline{g} \right] \quad 0 \right]$$

$$\hat{\Pi}_{27} = \left[0 \quad 0 \quad \left[\hat{I}^T \bar{g} \quad \hat{I}^T \underline{g} \right] \right]$$

$$\hat{\Pi}_{77} = diag\{-\overline{\Lambda}_1, -\overline{\Lambda}_1, -\overline{\Lambda}_2, -\overline{\Lambda}_2, -\overline{\Lambda}_3, -\overline{\Lambda}_3\}$$

The desired control gain in (5) is given by

$$K = \tilde{X}_1 \hat{X}_1^{-1}, K_r = \tilde{X}_2 \hat{X}_1^{-1} \text{ from LMI (21),} \tag{22}$$

where the matrices X_1 and $\hat{X}_1$ follow from Remark 3.

Proof: By introducing $T := HWP$ as a new decision variable and applying the Schur complement to the matrix inequality (6) in Theorem 1, we obtain

$$\begin{bmatrix} \tilde{\Pi}_{11} & \Pi_{12} & \Pi_{13} & \tilde{\Pi}_{14} & \tilde{\Pi}_{15} \\ * & \Pi_{22} & 0 & \tilde{\Pi}_{24} & \tilde{\Pi}_{25} \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & \tilde{\Pi}_{44} & 0 \\ * & * & * & * & \tilde{\Pi}_{55} \end{bmatrix} < 0 \qquad (23)$$

where

$$\tilde{\Pi}_{11} := sym\{P^T \begin{bmatrix} \hat{I}^T \tilde{I} & \tilde{I}^T \\ \hat{A}_1 & M \end{bmatrix}\} - sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{f}^T \Lambda_1 \underline{f}\, \hat{I}\}$$

$$- sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{g}^T \Lambda_2\, \underline{g}\, \hat{I}\} + sym\{T^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} [\tilde{I} \quad 0]\}$$

$$+ \frac{r_M}{1 - r_D} \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} 0 \\ A_1 \end{bmatrix}^T H \begin{bmatrix} 0 \\ A_1 \end{bmatrix} [0 \quad I]$$

$$+ \begin{bmatrix} Q_1 + F_1 + F_1^T & 0 \\ 0 & Q_2 \end{bmatrix}$$

$$\tilde{\Pi}_{14} = \begin{bmatrix} \begin{bmatrix} \hat{I}^T (\bar{g} + \underline{g})^T \Lambda_2 \\ 0 \end{bmatrix} & P^T \begin{bmatrix} 0 \\ N_2 \end{bmatrix} & -P^T \begin{bmatrix} 0 \\ D \end{bmatrix} \end{bmatrix}$$

$$\tilde{\Pi}_{24} = \begin{bmatrix} 0 & \hat{I}^T (\bar{g} + \underline{g})^T \Lambda_3 & 0 \end{bmatrix}$$

$$\tilde{\Pi}_{44} = diag\{-sym\{\Lambda_2\}, -sym\{\Lambda_3\}, -\gamma^2 I\}$$

$$\tilde{\Pi}_{15} = \begin{bmatrix} r_M \begin{bmatrix} \tilde{I}^T \hat{I} \\ \tilde{I} \end{bmatrix} P_1 & \begin{bmatrix} F_1 \\ 0 \end{bmatrix} & \begin{bmatrix} (C_1 \hat{I})^T \\ 0 \end{bmatrix} & r_M (T + P)^T \end{bmatrix}$$

$$\tilde{\Pi}_{25} = \begin{bmatrix} 0 & 0 & \hat{I}^T C_2^T & 0 \end{bmatrix}$$

$$\tilde{\Pi}_{55} = diag\{-r_M P_1, -r_M P_1, -I, -r_M H\}$$

Let

$$\zeta = diag\{X^T, X_1, X_1, \overline{\Lambda}_1, \overline{\Lambda}_2, \overline{\Lambda}_3, I, X_1, X_1, I, \overline{H}\} \qquad (24)$$

where $\overline{\Lambda}_i := \Lambda_i^{-1}$ and $\overline{H} = H^{-1}$. Premultiplying ζ and postmultiplying ζ^T to (23) and using the inequalities

$$-sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{f} \Lambda_1 \underline{f}\, \hat{I}\} \le \begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{f} \Lambda_1 \bar{f}\, \hat{I} \begin{bmatrix} I \\ 0 \end{bmatrix}^T + \hat{I}^T \underline{f}\, \Lambda_1 \underline{f}\, \hat{I}$$

$$-sym\{\begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{g} \Lambda_2\, \underline{g}\, \hat{I}\} \le \begin{bmatrix} I \\ 0 \end{bmatrix} \hat{I}^T \bar{g} \Lambda_2 \bar{g}\, \hat{I} \begin{bmatrix} I \\ 0 \end{bmatrix}^T + \hat{I}^T \underline{g}\, \Lambda_2\, \underline{g}\, \hat{I}$$

$$-sym\{\hat{I}^T \bar{g} \Lambda_3\, \underline{g}\, \hat{I}\} \le \hat{I}^T \bar{g} \Lambda_3\, \bar{g}\, \hat{I} + \hat{I}^T \underline{g}\, \Lambda_3\, \underline{g}\, \hat{I} \qquad (25)$$

and considering $T X = \alpha I$ to eliminate the nonlinearities in the matrix inequality with $\tilde{X}_1 := K \hat{X}_1$, $\hat{Q}_i = X_1^T Q_i X_1$ and $\hat{F}_i = X_1^T F_i X_1$, we obtain (by Schur complement) the LMI (21). This completes the proof. ■

Remark 4: If $rank(C_3) = l = n$, the matrix C is non-singular, it is clear that the matrix equation $C X_1 = \hat{X}_1 C$ is solvable on $\hat{X}_1$, i.e., $\hat{X}_1 = C X_1 C^{-1}$. In this case, the results of Theorem 2 are true for a full (non-diagonal) matrix X_1, i.e., $X_1 = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix}$, and the desired control gains in (5) are given by $K = \tilde{X}_1 C X_1^{-1} C^{-1}$ and $K_r = \tilde{X}_2 C X_1^{-1} C^{-1}$.

IV. Numerical example

Consider the second-order neutral master-slave systems (1) and (2), where the system matrices are given by

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0.5 \\ 0.3 & 0.5 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.25 & 0.125 \\ 0.075 & 0.125 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 & 0.025 \\ 0.05 & 0.075 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.8 & 0.5 \\ 0.4 & 0.5 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.2 & 0.125 \\ 0.1 & 0.125 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T,$$

$$C_3 = \begin{bmatrix} 1 & 0.5 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

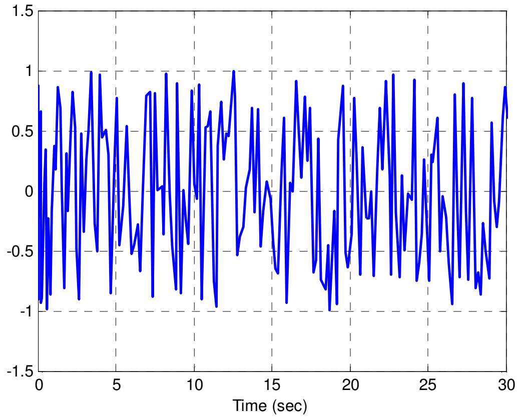
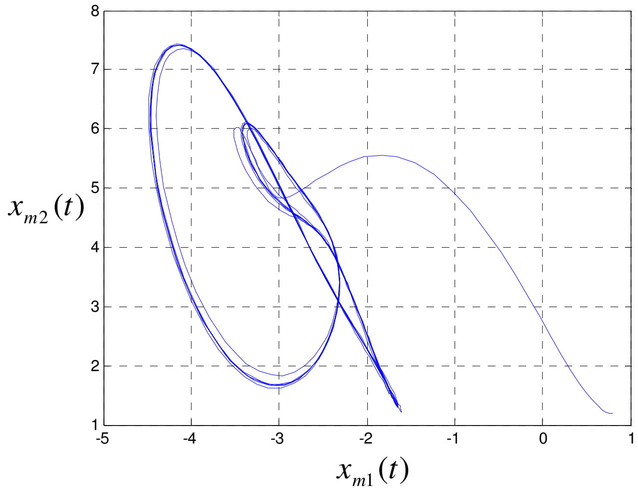
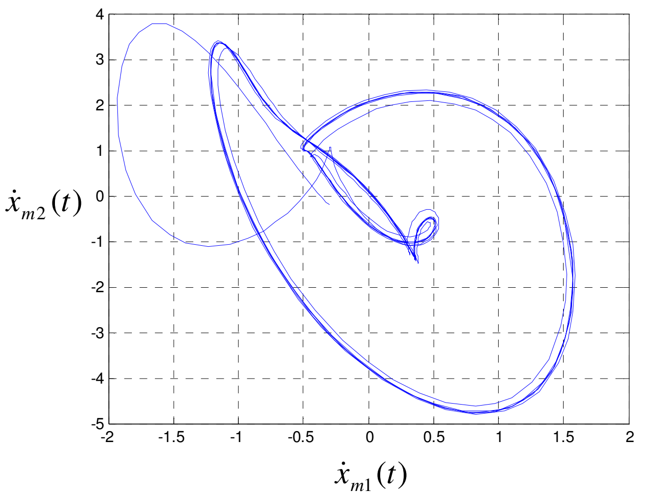


Figure 1. The disturbance signal.

The delays $r(t) = d(t) = (1 - e^{-t})/(1 + e^{-t})$ are time-varying and satisfy $0 \le r(t) = d(t) \le 1$ and $\dot{r}(t) = \dot{d}(t) \le 0.5$. For simulation purpose, a uniformly distributed random signal, shown in Fig. 1, with minimum and maximum -1 and 1, respectively, as the disturbance is imposed on the response system. With the above parameters, the neutral master-slave systems (1) and (2) exhibit chaotic behaviours such the $x_{m1}-x_{m2}$ and $\dot{x}_{m1}-\dot{x}_{m2}$ planes with initial conditions $\xi(0)=col\{0.4,0.6,-0.3,-0.2\}$ and $\zeta(0)=col\{0.8,-0.7,0.1,0.1\}$, respectively, are shown in Fig. 2.



(a)



(b)

Figure 2. The phase trajectories: a) $x_{m1}-x_{m2}$ plot and b) $\dot{x}_{m1}-\dot{x}_{m2}$ plot.

It is required to design the control law (5) such that the closed-loop system is asymptotically stable and satisfies the H_∞ performance measure. To this end, in light of Theorem 2, we solved LMI (21) with the disturbance attenuation $\gamma=0.2$ and obtained the following control gains by using Matlab LMI Control Toolbox [45]

$$K=[8.9681\quad -9.0207\quad 37.1101\quad -30.0309],$$

$$K_r=[-0.0250\quad 0.1896\quad 0.5152\quad -2.1808].$$

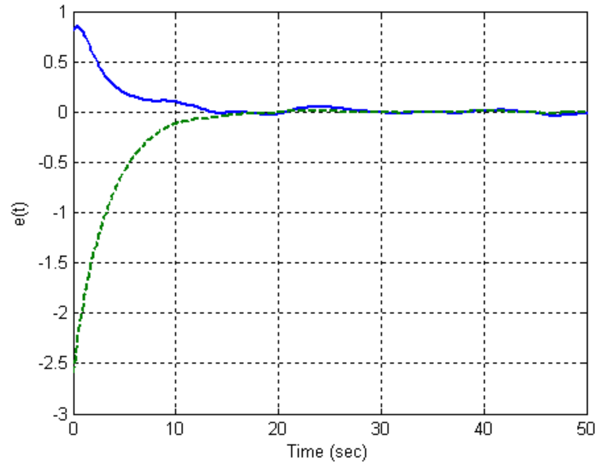


Figure 3. The synchronization errors.

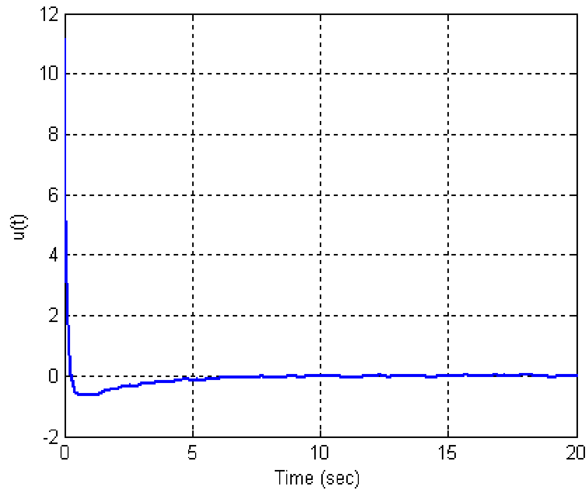


Figure 4. Control law for system.

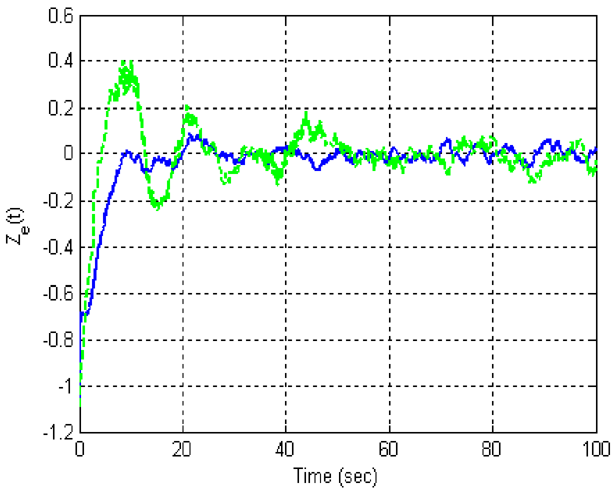


Figure 5. Comparison of the controlled outputs: a) closed-loop system (solid line) and b) open-loop system (dashed line).

Now, by applying the delayed state feedback controller (5) with the parameters above, the synchronization error between the drive system and response system is shown in Fig. 3. It shows that the synchronization error converges to zero. The curve of output-feedback control is also shown in Fig. 4. To observe the H_∞ performance, the response of the controlled output, i.e., $z_e(t)$, is depicted and compared with the output signal in the open-loop system under the disturbance in Fig. 5, which shows the delayed output-feedback controller (5) reduces the effect of the disturbance input $w(t)$ on the controlled output error.

V. Conclusion

This paper presented the H_∞ synchronization problem of the master and slave structure of a second-order neutral chaotic system with time-varying delays. Delay-dependent sufficient conditions for the design of a delayed output-feedback control were given by Lyapunov-Krasovskii method in terms of an LMI. A controller guaranteeing asymptotic stability, and H_∞ synchronization of the master and slave structure using some free weighting matrices was developed directly instead of coupling the model to a first-order neutral chaotic system. A numerical example has been given to show the effectiveness of the method.

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