THESIS FOR THE DEGREE OF LICENTIATE OF ARCHITECTURE

Prestress in Nature and Technics

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Cover:
Prestressed torus with the thicker bars in compression and the thinner in tension. Image credits Chris Williams.

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Abstract

To direct the forces of nature is a central task in the creation of spaces and load-carrying structures for architecture. This research investigates how prestress can be used as a design tool for the creation of material efficient and well-functioning structures, and in early design stages contribute to sustainable, functional and beautiful architecture.

The thesis begins with a discussion about central concepts such as stress and stiffness. Stiffness can be understood as the sum of elastic stiffness and geometric stiffness and the latter is differently influenced by the presence of tensile or compressive stresses. Only structures that are statically indeterminate are possible to prestress so that the stress pattern is affected. The terms externally-equilibrated and auto-equilibrated prestressed structures are introduced.

The design of load-bearing structures for architecture requires a collaboration between architects and engineers and the conditions for a successful collaboration is reflected upon. Prominent design cultures are highlighted and the one this research is linked to is described.

A collection of historic and contemporary examples of prestressed structures is presented. The focus is architectural applications but examples from other realms are also included. From this collection, a framework for prestressed structures is proposed and discussed which considers five perspectives. The first explores the historical knowledge development. The second is devoted to structural mechanical modes of actions where material behaviour, member actions and structural systems are discussed. The third highlights computational strategies and those appropriate for early stage design are distinguished from those suitable for late stage verification. The fourth perspective seeks to establish objectives for why prestress is used. The fifth perspective leads to suggestions for strategies for how the prestress is achieved.

Three papers are included. Paper A presents a numerical method for the form finding of prestressed gridshells consisting of both compressed and tensioned members. Paper B describes a structural design process where methods usually applied by architects are used by structural engineers. The work resulted in the construction of a temporary pavilion consisting of a post-tensioned wooden gridshell called the Wood Fusion Pavilion. Paper C explores under what conditions an unloaded shell formed of a closed surface unattached to any supports can contain a state of membrane stress which can be induced by prestressing. It is concluded that a torus can be prestressed, but there must almost certainly be more to explore.

Keywords: Prestress, Geometrical stiffness, Stress pattern, Conceptual design, Structural design, Form finding, Architecture, Engineering

Sammanfattning

Att samspela med naturens krafter är centralt vid utformningen av arkitekturens bärande konstruktioner. Denna forskning undersöker hur förspänning kan användas som ett designverktyg för skapandet av materialeffektiva och välfungerande strukturer, och i tidiga designskeden bidra till hållbar, funktionell och vacker arkitektur.

Avhandlingen inleds med en diskussion kring centrala begrepp så som spänning och styvhet. Styvhet kan förstås som summan av elastisk och geometrisk styvhet och den senare påverkas olika av drag- och tryckspänningar i strukturen. Endast statiskt obestämda strukturer är möjliga att förspänna så att spänningsmönstret förändras. Termerna externt balanserad och självbalanserad förspänd struktur introduceras.

Utformning av bärande konstruktioner inom arkitektur förutsätter ett samarbete mellan arkitekt och ingenjör och en diskussion förs kring förutsättningar för lyckade samarbeten. Förebildliga designmiljöer lyfts fram och den designkultur som denna forskning är kopplad till beskrivs.

En samling historiska och samtida exempel på förspända strukturer presenteras. Fokus är arkitektur men exempel från andra områden är också inkluderade. Med utgångspunkt i de studerade exemplen diskuteras för förspänning utifrån fem perspektiv. Det första undersöker den historiska kunskapsutvecklingen. Det andra ägnas åt strukturmekaniskt verkningssätt och beaktar de tre nivåerna material, element och struktur. Det tredje redogör för beräkningsmetoder och de som är lämpade för tidiga designskeden särskiljs från de som är lämpade för verifiering i sena skeden. Det fjärde perspektivet söker motiv till varför förspänning kan förbättra konstruktioner i olika avseenden. Det femte formulerar förslag på generella strategier för hur förspänning kan åstadkommas.

Tre undersökningar har därtill genomförts vilka presenteras i varsin artikel. Artikel A föreslår en numerisk metod för formsökning av förspända gitterskal bestående av både tryckta och dragna strukturelement. Artikel B beskriver processen med att utforma en konstruktion där designmetoder som vanligen används av arkitekter tillämpades av ingenjörer. Arbetet ledde till bygget av en skalformad och efterspänd paviljong av björkplywood kallad Wood Fusion Pavilion. Artikel C utforskar under vilka förutsättningar ett obelastat skal bestående av en sluten yta utan stöd kan innehålla membranspänningar som kan induceras genom förspänning. Artikeln visar att en torus uppfyller villkoren, men det måste med största sannolikhet finnas fler geometrier att upptäcka.

To Andreas

Preface

Though I always have been curious and had an urge to understand things, the topic of this thesis was not self-evident. In my application to become a PhD student I imagined I would study concrete shell structures. Six months later, when I began my studies in October 2016, this had changed to timber shell structures. During the first months of my studies I was suggested to study the Grand Louvre Pyramid which consists of a cable supported steel frame supporting the glass panes. From these studies the interest in prestress as a design tool emerged, which eventually became the topic of my licentiate thesis. It appears to me that the concept has a large potential in making structures more efficient and contribute to sustainable, functional and beautiful architecture, but little literature is devoted to the concept.

Reading about the Grand Louvre Pyramid also introduced me to the structural engineer Peter Rice (1935-1992). His book *An Engineer Imagines* (Rice 1996) fascinated me and showcased an approach towards engineering that I believe more engineers could apply in their work to make the world a better place. In the book, Rice discusses his collaboration with architects from the very early stages to the completion of the buildings. Rice was involved in many notable buildings, for example the Sydney Opera House (1957) and Centre Pompidou (1971), and it is evident that his collaborations were fruitful.

Another structural engineer who embraced collaboration was Frei Otto (1925-2015), who is known for his many tensile membrane structures. Throughout his entire career, Otto worked extensively with physical models (Vrachliotis et al. 2017). Using soap film, hanging chain models and stretched fabric, he explored the geometry and behaviour of prestressed minimal surfaces of a given boundary whilst architectural qualities such as space, light and proportions could be understood. At the many institutes he chaired, Otto surrounded himself with architects, engineers, biologists, philosophers, historians, naturalists and environmentalists (The Hyatt Foundation 2015). All these professions, or even individuals, had their own way of thinking of the same problem; Otto thought through modelling, engineers through algebra, computer scientists through programming, biologist through evolution and so on. Together they published their findings, and the book 'Pneu in nature and technics' (*IL9: Pneus in Natur und Technik* 1977) has inspired the title of this licentiate thesis: 'Prestress in nature and technics'. In addition, the creative environment Otto setup around himself lead to many notable projects such as the German pavilion at Expo 67, Olympiastadion in Munich (1972, see fig. 1.1 on page 4), Mannheim Multihalle (1975), Umbrellas for Pink Floyd's 1977 concert tour and the Japan Pavilion at Expo 2000 (see fig. 2.18c on page 26).

But the environments surrounding both Rice and Otto are rare exceptions. Instead there seem to be a general lack of communication between architects and engineers and the design process is fragmented into specialities. Architect Eugene-Emmanuel Viollet-le-Duc, who designed the recently burnt down spire at Notre-Dame de Paris, dwelled on the matter in the late 1800s concluding that 'the interests of the two professions will be best saved by their union' (Viollet-le-Duc 1881, p. 72). To confront this issue, a symposium entitled 'Bridging the Gap' was held in 1989 in New York (Gans 1991). Many renowned practitioners and academics participated and contributed to a three-day long exploration of the history of the split between our professions, their current relationship, and discussed models for future interaction. Perhaps can the existence be explained by the different perceptions of the same reality that architects and engineers tend to have (Charleson and Pirie 2009). It follows that the gap can be closed, or at least made smaller, if the professions understand one another's perception better.

It is my hope that, through the exploration of prestress as a design tool, I can contribute to the quest of bridging the gap. It has indeed affected the way my research has been conducted and I have put an effort in trying to make the thesis interesting for both practising architects and engineers whilst still appealing to my academic peers.

Göteborg, August 2019
Alexander Sehlström

Acknowledgements

First of all, I would like to thank my examiner, Prof. Dr. Karl-Gunnar Olsson, for helping me to see the big picture and being an amazing mentor. I strongly value his endless assistance, input and confidence in me and my studies. Karl-Gunnar wants his students to excel and he do the utmost to create an environment to support his students. It is a privilege and a true honour to be one of his students.

I also thank my supervisors, Dr. Chris Williams, Dr. Mats Ander and Joel Gustafsson, for their continuous support, constructive feedback and insightful comments, which have pushed this thesis that one step further. I want to thank Chris especially for pushing me into the fascinating world of prestress, his endless new perspectives and references from outside the world of architecture, and for being such an inspiration. I am grateful for Mats' ability to simplify and explain new concepts and topics, his enthusiasm and ability to give confidence that you will cope with the tasks at hand. Finally, I am thankful for Joel's help with prioritising among the many exiting endeavours I have encountered during my studies. Without his help, I would not have made it in one piece.

I am forever grateful for the support I have gotten from my industrial partner WSP. I would like to especially thank Mårten Sandell for providing the opportunity to perform my PhD studies. In addition, I would like to thank all my fantastic colleagues at WSP, especially Linda Leijonberg, Karin Stagemyr, Pär Gustafsson and Docent Dan Engström, for their interest in what I am doing whilst away from the office.

I want to thank several people for their feedback, comments, inspiring discussions or help, including Morten Lund, Peter Christensson, Jonas Carlsson, Peter Lindblom, Tabita Nilsson, Maja Kovacs, Dr. Paul Shepherd, Dr. Caitlin Mueller, Paul Mayencourt, Pierre Cuvilliers, Ian Liddell, Jürg Conzett, Andrew Weir, Prof. Dario Gasparini and last, but certainly not least, my fellow PhD colleagues at Chalmers, Erica Henrysson, Jens Olsson, Emil Adiels, Lic. Elke Miedema, Saga Karlson, Anita Ollár, Sofie Anderson and Giliam Dokter.

During spring 2018, I had the honour to act as master's thesis supervisor for Johanna Isaksson and Mattias Skeppstedt. Their project made the Wood Fusion Pavilion possible, and I am forever grateful for their fine work. Furthermore, the project was dependent on the trust and efforts of Stefan Sundqvist and his colleagues at the Swedish Exhibition & Congress Centre and for that I thank them.

I have the many brilliant people of the Swedish Federation of Young Scientists to thank for planting the very concept of research and PhD studies in my mind. A special thanks to Daniel Langkilde, Anders Lundberg, Joni Lindgren, Theresia Silander Hagström, Johan Pacamonti and Anna-Maria Wiberg. I am looking forward to meating you all soon again ☺

I could never have done this without the unconditional love and support from my family. I am especially thankful to my fantastic husband Andreas Tengström. I love you all.

Thesis

This thesis consists of an extended summary and the following appended papers:

Paper A	M. Ander et al. (2017). "Prestressed gridshell structures". *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2017*
Paper B	A. Sehlström, J. Isaksson, et al. (To be submitted). Embracing design methods from architects for conceptual design of structures
Paper C	A. Sehlström and C. J. K. Williams (Submitted and accepted; awaits publication). "Unloaded prestressed shell formed from a closed surface unattached to any supports". *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2019*

Other publications related to this thesis:

Publication I	A. Sehlström, H. Johansson, and M. Ander (2016). "On multi objective topology optimization and tracing of Pareto optimal structures". *Proceedings of 29th Nordic Seminar on Computational Mechanics – NSCM29*. Ed. by R. Larsson. Forskningsrapporter (Tillämpad mekanik). 3. Göteborg: Chalmers University of Technology
Publication II	E. Adiels et al. (2018). "The use of virtual work for the formfinding of fabric, shell and gridshell structures". *Proceedings of the Advances in Architectural Geometry conference 2018*. Ed. by L. Hesselgren et al. Klein Publishing, pp. 286–315. ISBN: 978-3-903015-13-5
Publication III	J. Larsson et al. (2017). "Moving Mesh and Image Registration in FEniCS". *Proceedings of 30th Nordic Seminar on Computational Mechanics – NSCM30*, pp. 180–183

My, the author of this licentiate thesis, contribution to the appended papers and publications are as follows:

Paper A was written in cooperation with the other authors. I helped with the literature review, execution of numerical examples and the writing and editing of the paper, which I also presented at the IASS 2017 symposium.

Paper B was written by me and I also acted as master's thesis supervisor for Johanna Isaksson and Mattias Skeppstedt, which thesis project the paper discusses. Morten Lund, Dr. Mats Ander, Joel Gustafsson, Dr. Chris Williams and Prof. Dr. Karl-Gunnar Olsson provided input during the project and later during the writing of the paper. The appended paper is a preprint (author's original manuscript) and will be sent to a journal soon.

Paper C was mainly written by Dr. Chris Williams and I supported with the final editing of the paper. I will present the paper at the IASS 2019 symposium in Barcelona, Spain in October 2019.

Publication I was written by me with input from Dr. Mats Ander and Docent Håkan Johansson. I presented the paper at the NSCM 29 symposium in 2016.

Publication II was written by Dr. Chris Williams with input from the rest of the authors. I provided help with some of the figures and the final editing of the paper.

Publication III was written in collaboration with the other authors on my initiative. The theoretical basis was provided by Docent Klas Modin. Me and Jenny Larsson wrote an implementation in FEniCS and conducted the numerical experiments. Prof. Dr. Anders Logg provided input on the FEniCS software.

Contents

Part I
Extended Summary

1 Introduction

Prestressing is simply the introduction of stresses in a structure to improve its performance during service. Since prehistoric times humans have employed the concept in structures and everyday objects, for example in tents and bows. Today it is used in string instruments, sports equipment, sailing boats, bicycle wheels et cetera, but the concept is perhaps mostly associated with prestressed concrete. In nature prestress is ubiquitous and can for example be found in cells and proteins (Edwards, Wagner, and Gräter 2012; Krieg, Dunn, and Goodman 2014) but also in structures made by animals, such as birds nests and spiders webs (Kullmann, Nachtigall, and Schurig 1975; Mortimer et al. 2016). In all of these examples, prestress is used to make the structure more efficient in different aspects. For many examples, prestress commonly lead to a weight reduction and to an efficient material usage.

Frei Otto devoted much of his work to lightweight structures where prestress was a central concept (e.g. Burkhardt et al. 1978; Hennicke and Schauer 1974; *IL12: Convertible Pneus* 1975; *IL15: Lufthallenhandbuch* 1983; *IL9: Pneus in Natur und Technik* 1977; Kullmann, Nachtigall, and Schurig 1975; Otto 1954, 1963, 1995; Otto and Rasch 1995; Otto and Schleyer 1966). Otto took inspiration from nature and argued that *pneu*, a lightweight structural membrane filled with air or fluid, was tied to the origin of life — 'Am Anfang war der Pneu' ('in the beginning was the pneu') he wrote, and it was 'the essential basis of the world of forms of living nature' (Fabricius 2016, p. 1264; *IL9: Pneus in Natur und Technik* 1977, p. 5). A living cell is an example of a pneu in which the turgor pressure, which can be as small as 0.1–0.4 MPa yet can also exceed 2–3 MPa, plays an important role in key processes such as growth, development, mechanical support, signalling, organ movement, flowering and responses to stress (Beauzamy, Nakayama, and Boudaoud 2014; Luchsinger, Pedretti, and Reinhard 2004). The effect can be substantial, such as in trees where the prestress, generally referred to as growth stresses, can be large enough to cause significant problems in the conversion of felled trees to timber (Wilkins 1986).

Much of the prestress terminology has its etymological roots in the development of prestressed concrete during late 19^{th} and early 20^{th} century, highlighting the importance of prestressed concrete for our society. One of the key player in the early development of prestressed concrete was Eugène Freyssinet (Menn 1990), who, on the matter of prestress as a concept, summarised in the foreword of (Guyon 1965): 'This idea is of an extreme simplicity in its foundation, even if it is not in the execution'. There are those who argue that prestress 'has only been completely understood [*sic!*] and implemented in the past century' (Sanabra-Loewe and Capellà-Llovera 2014, p. 93). Perhaps this is true for concrete structures, even though there is still research conducted on the topic providing more knowledge. However in general, this is not true even though there are those who have a deep understanding of the concept. For example, structural engineer Jörg Schlaich has designed many prestressed structures including the Munich Olympic stadium (fig. 1.1; Kullmann, Nachtigall, and Schurig 1975; Tomlow 2016), cable net façades (J. Schlaich, Schober, and Moschner 2005), bridges, towers, and solar power collectors (J. Schlaich 1999). But most architects and engineers lack enough conceptual understanding and tools for mathematical modelling of the structural behaviour to be able to with ease design and analyse prestressed structures. One has to understand the interplay between form and forces, how materials behave, how to formulate and use suitable analysis methods, careful detailing and precise construction and assessments methods to successfully design prestressed structures.

Figure 1.1: *Olympiastadion in Munich (1972) by Frei Otto, Jörg Schlaich, Rudolf Bergermann, et al.*

Source: *Taxiarchos228, Munich: Olympic Stadium, 2016-08-01.* `https://de.wikipedia.org/`. *Copyleft: This is a free work, you can copy, distribute, and modify it under the terms of the Free Art License* `http://artlibre.org/licence/lal/en/`

1.1 Preliminary concepts of prestress

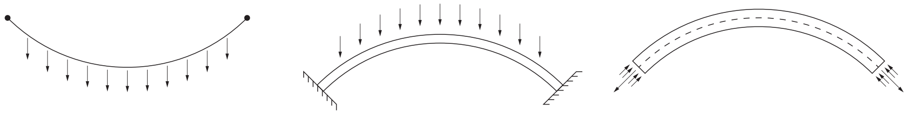
A key to understanding and be able to design with prestress is to understand how the internal stress distribution, or stress pattern, and the stiffness of an object is modified when it is prestressed.

Stress is a physical quantity measured as the resistive force per unit area in a material object that arises due to applied loading. The stress will either be a tensile stress or a compressive stress. *Tensile stresses* are related to material extensions whereas *compressive stresses* are related to material shortenings. All materials respond differently to stresses. The *strength* of the material is the limit under which the stress is considered to be safe not causing (local) failure or plastic deformation. Materials that fails without undergoing any plastic deformation are called brittle, whereas those which do are called ductile; the latter failure is considered more safe than the former in construction since it gives some warnings before collapse.

Stiffness is the extent to which a material or structural object resists deformation in response to an applied force. It is usually measured in terms as of how much force has to be applied to the object to deform it a unit length. For structural objects, stiffness can be understood as the sum of *elastic stiffness*, which depends on material properties, geometry (shape, topology, cross sections) and boundary conditions of the object, and *geometric stiffness*, which depends on the internal stress state of the object. Compressive stresses result in a negative geometric stiffness contribution (weakening) whereas tensile stresses result in a positive geometric stiffness contribution (stiffening) (K.-G. Olsson and Dahlblom 2016). It must be ensured that the geometric stiffness does not weaken the elastic stiffness to such an extent that stiffness is completely lost, or instability phenomenons may occur leading to partial or full collapse of the structure.

A structure has to be *statically indeterminate* to be possible to prestress so that the stress pattern is affected (F. W. Maxwell and Benson 1937). This means that the static equilibrium equations are insufficient for determining all the unknown inner forces and/or reaction forces; there is at least one more unknown reaction force than there are equations of equilibrium. This gives the possibility to 'choose' the stress state by prestressing, which introduces more equations so that the reaction forces can be determined. To illustrate this, one can compare the effect of shortening a leg of a tripod (statically determinant) and a quadpod (statically indeterminate), both composed of inextensible members pinned to the ground (Pellegrino and Calladine 1986). In the tripod, the legs will be stress-free and the top joint will move to compensate for the shortening of one leg. In the quadpod, on the other hand, there will be stress (prestress) in all four legs and the top joint will stay in its original position as the length of one leg is altered.

Figure 1.2 depicts three prestressed structures which are similarly shaped but have different resulting properties. Both the cable in fig. 1.2a and arch in fig. 1.2b are prestressed with their own weight. The cable is in tension, resulting in a positive geometric stiffness contribution (stiffening), and the arch is in compression, resulting in a negative geometric stiffness contribution (weakening); there is a risk of buckling if the compressive stresses are two high. At the same time, if the thrust line is well adapted to the geometry, the compressive stresses in the arch reduces the sensitivity for live loads causing stability problems and collapse. The cable and the arch are *externally-equilibrated* structures meaning that an increase in prestress results in an increase in the reaction forces at the supports. Structures may also be *auto-equilibrated* and such do not depend at all on any external support to maintain the prestress. Figure 1.2c depicts such an example consisting of an arched member compressed by an internally symmetrically arranged tendon put in tension. The negative geometric stiffness in the arch is counteracted by the positive geometric stiffness in the tendon. While the geometric stiffness of the individual members may be largely influenced by the prestress, the net effect on the geometric stiffness contribution is relatively low.



(a) *Cable* (b) *Arch* (c) *Arch + Tendon*

Figure 1.2: *Prestressed arches.*

Properties including load bearing capacity, deformation pattern and dynamic response of a structure is controlled through the choice of material, geometry and boundary conditions. In addition, prestress can be used to obtain the desired properties. A key to understand how is to understand how (1) the stress pattern and (2) the stiffness is modified by the addition of prestress. And though the two are inseparable usually one or the other will be used as the primary control parameter in order to achieve higher objectives:

- **Material efficiency (1):** Bicycle wheel, cable roofs, growth stresses
- **Stability (2):** Remove internal mechanisms, e.g. tensegrities
- **Form stability (1/2):** Tension in cable or textile, pneumatic structures
- **Efficient joinery (1):** Masonry, traditional timber joints, birds nests
- **Ductile behaviour (1/2):** Masonry, concrete, frames and arches under earthquake loading
- **Energy storage (2):** Bows and racket
- **Frequency (2):** String instruments

The examples will be discussed in detail in chapter 2 and the objectives in section 3.4.

1.2 Design culture

To successfully use prestress as a tool to develop or improve structures in architecture—and design in general—requires more than mere an understanding of central structural phenomenons. Vitruvius proposed two thousand years ago that architecture should exhibit the three qualities of *utilitas*, *firmitas*, *venustas* — that is, utility/functionality, stability/sustainability, beauty (Vitruvius, Morgan, and Warren 1914). Architecture that possesses these qualities are characterised by holistic solutions of which the structure is just one of many contributing parts. For a structural engineer, responsible for the load bearing capacity, it is thus essential to be able to talk in conceptual terms about prestress to meet and interact with the architect and contribute to the whole. Renowned structural engineer Peter Rice talked about humanity, tactility and scale (Rice 1996), which is all about what the structure 'communicates', not at all about physical phenomenons. For example, a tensegrity (section 2.2.13) is often a highly material efficient type of prestressed structure, but tend to have a messy and confusing appearance which might not at all fit with the overall architectural idea — perhaps an airy, lightweight prestressed cable net is a better solution if the need for a sturdy rim can be accepted. Such reasoning and discussions takes place if the design culture allows creative ideas to nourish and develop.

This in turn requires a collaboration between architects and engineers which, however, tend to have different perceptions of the same reality making successful collaboration a challenge (Charleson and Pirie 2009). This has since long been recognised and the power of good collaboration was stressed already in the late 1800s by the architect Eugene-Emmanuel Viollet-le-Duc who concluded that 'the interests of the two professions will be best saved by their union' (Viollet-le-Duc 1881, p. 72). Architect Renzo Piano, unlike many architects, includes engineers and other relevant professionals as part of his process from the start (Tusa et al. 2018). For many years Piano worked with Peter

Rice who noted that 'engineers are associated with unimaginative dull solutions' and argued that exploration and innovation are the keys for engineers to contribute to the work of architects (Rice 1996, p. 71). His view is confirmed by (Uihlein 2015) who, based on interviews with architects and engineers, claims that, in general, architects prefer to work with structural engineers who can engage with the architectural design and design process. Further studies suggests that, in order to increase the opportunities for collaboration, consideration must be 'given to the exclusion of architecture in the training ... of structural engineers' (Uihlein 2017, p. 6). Similar ideas has been put forward by others as well, for example at a conference which proceedings are entitled 'Bridging the Gap' (Gans 1991). The importance of the conceptual design development, as a key to develop integrated qualitative architecture and sound engineering solutions, is stressed by for instance (Corres-Peiretti 2013; Larsen and Tyas 2003), and evident in the way structural engineer Jörg Schlaich works (Holgate 1997).

The 'Matter Space Structure' architecture studio at Chalmers University of Technology teaches such a design culture. Rooted in the matter, phenomenons are investigated and explored with the attention of people and the space we inhabit (M. Lund 2018). The structure (methodology) is iterative, where matter and space are explored in continuous loops covering four phases: concepts, prototypes, proposals and narratives. The transition between each phase is supported by a seminar—a *crit* (Doidge, Sara, and Parnell 2016)—where qualities and weaknesses of the design is highlighted and the next steps discussed and prioritised.

1.3 Aims, limitations and research approaches

This research aim to investigate how prestress can be used as a design tool for the creation of material effective and well-functioning structures, and in early design stages contribute to sustainable, functional and beautiful architecture. More specifically, the objective is to answer the following questions:

1. What can be learnt from historic and contemporary examples about how prestress has been and can be used?
2. Can, from these examples, general objectives with prestressing be established?
3. Can, from these examples, basic strategies for prestressing be established?
4. How can this knowledge be utilised and applied in a design process?

To answer these questions, mixed research approaches have been used and some limitations have been introduced.

The first question requires a collection of examples to be answered and these were compiled using literature studies and presented in chapter 2. No limitations were introduced in terms of applications, resulting in examples from both nature and technics. However, with architectural applications in mind, there is an emphasis on examples found in the built environment. All types of references were accepted (journal articles, conference articles, books, magazines, electronic resources et cetera) and key words such as 'prestress', ‘pre-stress’, ’pre-tension’, ’pre-compression’ and ‘tensegrity’ were used in the search of examples. The work of some notable architects and engineers was also studied in the search for examples.

Based on the examples, a framework for prestress is established, which is discussed in chapter 3. The framework is derived by classifying the examples in various ways. The historical knowledge development is briefly explored (section 3.1). Structural mechanical modes of actions relevant for prestressed structures are presented (section 3.2), ranging from material behaviour to member actions and structural systems. Strategies for the analysis of prestressed structures are discussed (section 3.3), and those suitable for early stage design are distinguished from those suitable for verification.

The second and third question are implicitly limited to the collection of examples. Objectives and strategies are identified, again using classifications and sortings of the examples, and presented (sections 3.4 and 3.5).

The framework is put in context of a project through a thought experiment reasoning about the design process of a prestressed stone bridge called Wasserfallbrücke designed by engineer Jürg Conzett in 2013 (section 3.6).

The fourth question, which is briefly touched upon in the thought experiment, is primarily investigated through numerical investigations (Paper A and Paper C) and a physical experiment (Paper B). The numerical investigations considers structures with a high degree of structural efficiency. More specific, structures composed of members carrying loads primarily through tension or compression have been studied, so that bending moments in members are avoided as much as possible. The physical experiment is conducted within the established design culture at Chalmers (section 1.2). The papers are summarised in chapter 4.

2 Examples of prestress usage

This chapter presents examples of prestressed structures found in both nature and in technics, the part of human activity wherein, by an energetic organisation of the process of work, man controls and directs the forces of nature for own purposes (Mumford 2000). The collection of examples is compiled in order to answer what historic and contemporary examples can tell about how prestress have been used. This overview serve as a basis for the framework for prestress presented in chapter 3.

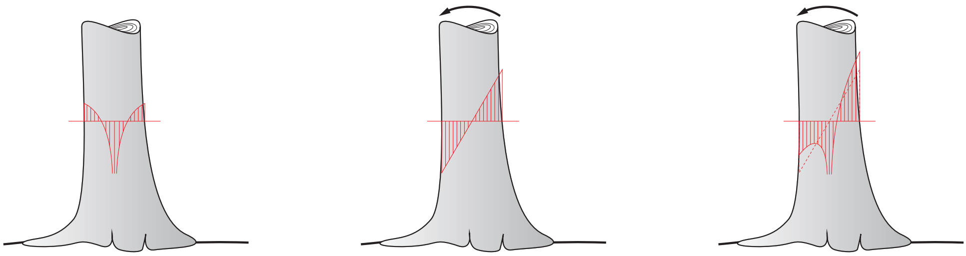
2.1 Prestress in nature

Prestress is ubiquitous in nature and plays a central role in many vital functions in nature.

A simple, yet essential application of prestress is the turgor pressure in living cells which, similarly as a balloon filled with air, gains its structural rigidity when the cell wall is stretched by the pressurised fluid it contains. The turgor pressure, which can be as small as 0.1–0.4 MPa yet can also exceed 2–3 MPa (a bike tire is around 0.2–0.6 MPa), plays an important role in key processes such as growth, development, mechanical support, signalling, organ movement, flowering and responses to stress (Beauzamy, Nakayama, and Boudaoud 2014; Luchsinger, Pedretti, and Reinhard 2004).

A special case of turgor pressure causing substantial effects can be seen in trees. The phenomenon, generally referred to as growth stresses, can be large enough to cause significant problems in the conversion of felled trees to timber (Wilkins 1986). Understanding these stresses provides not only a knowledge of how prestress can be used to overcome material weaknesses, but also why timber deform after it has been sawn (bow, crock, cup, twist).

The stresses arise as a response to effectively resist external forces, primarily from wind. Due to the high risk fibre buckling in wood (Boyd 1950), the longitudinal compressive strength is only about half of the tensile strength. When the growth stresses (fig. 2.1a) and temporary external stresses (fig. 2.1b) are acting simultaneous, a more even utilisation of the wood strength (fig. 2.1c) is obtained and thus a much higher overall load bearing capacity than if there were no prestress present (Mattheck and Kubler 1995).



(a) *Growth stresses* (b) *Bending stresses* (c) *Resulting stresses*

Figure 2.1: *Longitudinal stresses in tree trunk*

The growth stresses, which are orthotropically distributed (see e.g. Mattheck and Tesari 2004), originate when wood cells in the outer part of the trunk contract in the longitudinal direction and expand in the transverse direction (Münch 1938). The longitudinal contraction is restrained by older cells, putting the new cells in tension (Kubler 1987). This causes a compression of the adjacent interior layers that reduces the tension of older cells (Jacobs 1938) leading to severe compression near the pith (fig. 2.1a) (Cassens and Serrano 2004).

A complex application of prestress can also be found in some proteins. The protein *ubiquitin* has a 'tensegrity-like pattern of prestress' and that such knowledge could be used to create tailor made proteins with special mechanical properties for applications in medicine, material design and nanotechnology (Edwards, Wagner, and Gräter 2012, p. 4). The sensation of mechanical forces depends on a continuous, prestressed spectrin protein inside neurons (Krieg, Dunn, and Goodman 2014).

Animals use prestress when building structures. For instance, birds bend grass and branches as they build their nests, effectively inducing stresses in the members that, with the help of friction, is restrained against one another and thus kept in place. Spiders prestress their webs (fig. 2.2) in order to make them stiff enough to support the weight of themselves and their prey without substantial deflection using a minimum of material (Kullmann, Nachtigall, and Schurig 1975). The induced prestress also effects the sonic properties of the web which transmits vibratory information to the spider; by alternating the tension the spider can tune its web (Mortimer et al. 2016).

Figure 2.2: *A typical orb web constructed by an Araneus spider.*

Source: *Image distributed under CC BY-SA 3.0 licence;* `https://commons.wikimedia.org/wiki/File:Typical-orb-web-photo.jpg`

2.2 Prestress in technics

Humans have used prestress to construct objects since prehistoric time, and some of the applications are presented in this section. The examples are grouped according to common structural characteristics and what the prestress has made possible. The groups are presented in a somewhat chronological order and with an ambition to showcase a progress in terms of how advanced the applications are.

2.2.1 Early development: to ensure basic needs

Prestress has since prehistoric time been used to ensure basic needs such as shelter and food. The design concepts were simple, yet clever enough to make robust joinery and achieve a high degree of material efficiency, resulting in lightweight objects.

Though impossible to date due to its simplicity, tents are one of the oldest examples where prestress has been used. It is plausible that hide was supported by slender branches restrained against one-another by the prestress induced as the branches were bent into place (*active-bending*), much like in a bird's nest. With the development of ropes and textiles, which also require prestress for their making, the tents developed further into portable lightweight structures such as the yurt and tipi.

In parallel, boats for fishing and transportation utilising prestress were developed. Boats used in the Arctic and Subarctic zones, dating back at least 10,000 years, had a skin membrane wrapped and stretched around a timber frame (Evguenia 2016).

2.2.2 Ancient vessels: to secure the shape

One of the earliest records of an advanced usage of prestress dates back to c. 2,700 BCE in ancient Egypt (Leonhardt 1964). Reliefs depicts boats and barges in which a system of struts and ropes from stern to bow prevents the vessel from hogging. The ropes were entwined and by twisting the ropes the level of prestress could be adjusted and, thus, keep the deck in level (Casson 1971; Torr 1895). The mightiest is perhaps the barge depicted on the wall of Queen Hatshepsut's temple at Deir el-Bahri. Loaded with two obelisks, each weighing around 375 t, it was towed on the river Nile 213 km from the quarry in Aswan to the temple in Karnak (fig. 2.3).

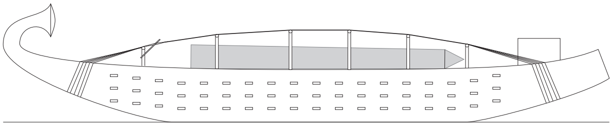


Figure 2.3: *c. 1470 BCE Hatshepsut's barge.*

2.2.3 Stretched strings: to store energy and control vibration

A simple strategy to induce prestress is to stretch a string and attach it to an anvil. The spider web and the Egyptian barges are some of the examples all ready discussed where this strategy is used. The objective is for these structures primarily related to structural integrity, that is, the ability to hold together under a load, including its own weight, without breaking or deforming excessively. But the same strategy can be used to achieve other objectives as well.

A stretched string will loose its prestress as soon as it is released from its anvil in a search to minimise the stored energy within it. This phenomenon is utilised in bows, initially developed for hunting but later warfare. By bending the bowstave and attach the string, strain (deformation) energy is stored in the bow which is further increased as the archer pulls the string backwards. On release, the bow seek to minimise the energy leading to a rapid acceleration of the arrow which shoots of the bow forward. As the bow were developed, more and more elaborate designs of the bowstave were developed eventually leading to the powerful medieval English longbow with a range of up to 315 meters (Oakeshott 1960).

The very same thing happens in sports rackets (Kullmann, Nachtigall, and Schurig 1975), with two main differences. First, multiples strings are used in a net instead of a single string. This provides a large area making it easier to hit the ball with the racket. Secondly, the strings are not deformed by an active pull exerted from the player, but by the impact energy released upon the collision between the moving ball and racket. This causes more strain energy to be induced in the racket, which is quickly minimised by the conversion into kinetic (movement) energy in the ball which springs off the net.

Similar energy conversion processes takes place in a string instrument such as the violin, piano or guitar, but the deformations of the strings are smaller. And while the strings in the bow and racket are tensioned to provide as much energy to the arrow and ball, respectively, string instruments are tensioned to be tuned to a specific frequency, or tone. The first natural frequency f_n of a stretched string with length L may, assuming small displacements, be computed from

$$f_n = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}, \tag{2.1}$$

where T is the (constant) tension in the string, ρ the density of the string and A its cross-sectional area. As string instruments are played, for instance by plucking or bowing, the musician influences the pitch by pressing the fingertips on the strings, effectively shortening the length of the string while the tension in the string is retained. From eq. (2.1) it is thus evident that a shortening of L to, say, half its length results in a higher pitch, in this case it is doubled. Equation (2.1) is derived from the partial differential equation describing the vibration mode at time t of the string

$$T\frac{\partial^2 y}{\partial x^2} - EI\frac{\partial^4 y}{\partial x^4} = \rho A\frac{\partial^2 y}{\partial t^2},$$

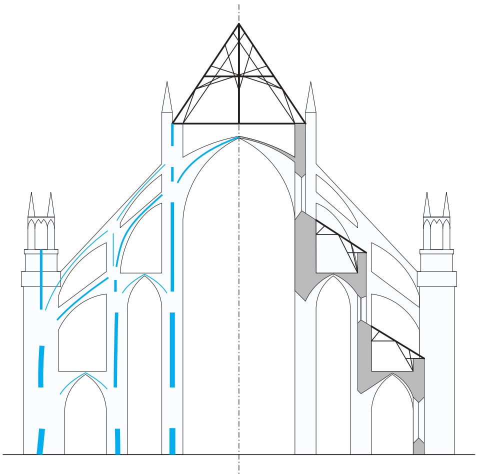
where y is the lateral displacement of the string at longitudinal position x and EI the elastic bending stiffness—which in this case has been neglected. The equation can be used to derive the Euler buckling formula by setting the right hand side to 0 and solving for T, which would then correspond to the critical buckling load usually denoted P_{cr}.

2.2.4 Compressing with weight: to increase stability

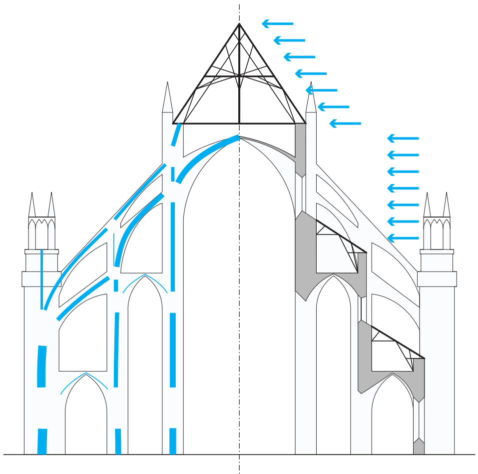
The tensile strength of masonry is mainly related to the strength of the joints; while dry-mortar joints have none, mortar joints have some tensile strength. Regardless of joint type, its low strength constitutes a challenge, especially if the structure is tall and light. The bending moment due to wind may cause tensile stresses to arise surpassing the tensile capacity of the masonry. This must of course be avoided, so that the structure doesn't fall over. According to (Heyman 1966), who introduced a rigorous framework of limit state analysis applied to masonry structures, the masonry is safe if the *thrust line* lie within the cross section for all possible load cases. The thrust line represents the path of the compressive resultants of the stresses acting within the structure.

Though the Romans didn't know about Heyman's theory, they effectively controlled the the thrust line by prestressing their masonry structures (Todisco 2016). For example, the attic of Colosseum in Rome, Italy (70–80 CE) adds extra weight to the lower part of the wall to counteract wind load causing tensile stresses. At Pantheon in Rome, Italy (118–128 CE), varying density of the concrete together with step-rings were used to fine-tune the thrust line. The Mausoleum of Centcelles in Tarragona, Spain relies on the back fill for its rigidity.

The technique relies on the elaboration of density and weight distributions to control the stress state. Later, during the medieval, the technique was refined to enabled the construction of Gothic cathedrals, where pinnacles adds weight to steer outward thrusts in the flying buttresses down into the buttresses (fig. 2.4a). Though the loads on heavy masonry buildings can be considered constant, this is not the case in the flying buttresses (Addis 2007). The upper tier of flying buttresses carries primarily wind loads exerted on



(a) *Load paths of thrusts from main vault and buttresses, due to gravity.*



(b) *Load paths of thrusts due to gravity and wind on the roof and the walls.*

Figure 2.4: *Cross section of the Boruges Cathedral (c. 1230); geometry derived from (Bork 2014) and load paths adopted from (Addis 2007).*

the roof, and their size and weight need be only the minimum necessary to perform this function (fig. 2.4b). For structural analysis of flying buttresses based on the strength of materials concepts, see (Quintas 2016).

Similarly, the grass and stone roofs of traditional timber houses, such as the Swedish fäbod, contributes with additional weight preventing the logs from separating at the bed joints when the building is subjected to overturning wind loads.

A more recent example, designed by structural engineer Peter Rice, is the Pavilion of the Future (fig. 2.5). Rice used a series of tie-rods to lift up the weight of an adjacent roof and apply it radially to the stone arches of the façade (Addis 1994; Lenczner 1994; Rice 1996). The originally catenary shaped thrust line was in this way transformed into a more semi-circular trajectory that can be contained within the structural depth. By adjusting the level of post-tensioning force, the intensity of the thrust can be controlled to compensate for the low self-weight of the arch and reducing the sensitivity for varying live loads imposed by wind and earthquake.

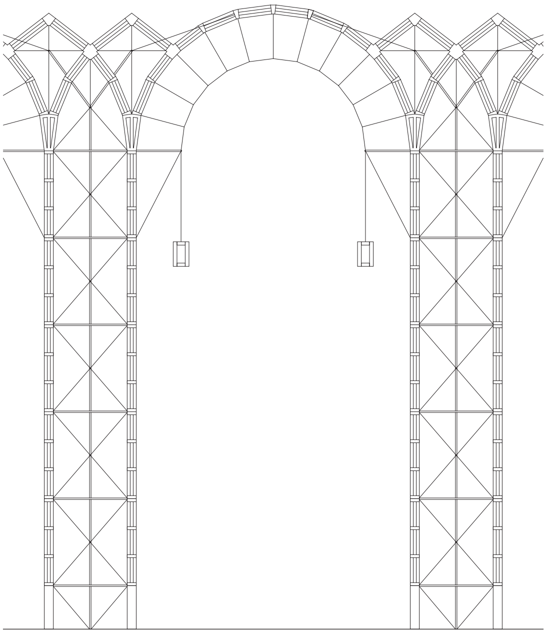


Figure 2.5: *Elevation of parts of the façade of Pavilion of the Future, Seville (1992).*

2.2.5 Joining timber members: to increase span width

Joints under tension are more troublesome to design than joints under compression since they risk to separate. In bio-mechanical structures, for example where bone and tendons or tendons and muscles are joined, this is solved by an intergrown transition between the parts (Benjamin et al. 2006). But such solutions are hard to obtain in technics and other solutions have been developed and many can be found in timber structures.

Timber has been used for thousands of years and a vast range of methods along with suitable tools have been developed to join timber members together (Zwerger 2000). For a long time, mechanical joints dominated, but today adhesives are widely used in finger joints, glulam beams, cross laminate timber panels and for glue-in metals rods and plates. Most mechanical joints requires some kind of weakening in the timber, for example notches to fit wedges or holes for dowels, bolts and screws. Adhesive joints requires large enough surface areas for the transmission of forces. Without proper care, these joints may lead to local stress concentration (G. Larsson, Gustafsson, and Crocetti 2017) easily exceeding the strength limit of the timber causing failure.

A common traditional joinery technique developed before metal fasteners were readily available is the usage of wedges, for example as in joint *f* in fig. 2.6 (Krauth and Meyer 1893). As the wedge is driven in between the members, it pushes the members apart and locks the connection by means of friction and a prestressing normal force. The technique enabled the construction of large span timber structures (James 1982) of which some of the most notable designs were made by members of the Grubenmann family in Switzerland (Brunner 1921, 1924; James 1982; Killer 1942; Weinand 2016).

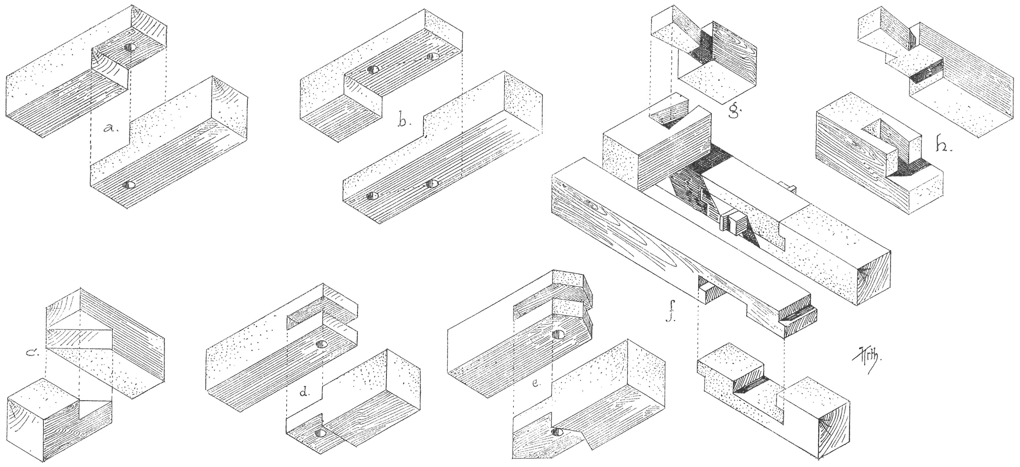


Figure 2.6: *Various types of lapped joint designed to transfer tension between two members. Published in (Krauth and Meyer 1893).*

The Grubenmanns engaged in large span roof trusses, but are perhaps most famous for their bridges, especially one erected by the Grubenmann brothers Johannes (1707-1771) and Hans Ulrich (1709-1783) at Wettingen in 1765 (fig. 2.7). The bridge aroused admiration of their contemporaries almost immediately and were, partly due to the explosion of architectural research, travel and publication starting in the 1750s (Bergdoll 2000), already widely known in 1770 (Angelo and Maggi 2003). The novelty of the bridge was the use of timber arches as the primary load bearing structure (S. Samuelsson 2015); prior to the Wettingen bridge only polygon shaped arches had been used. The arches, one at each side of the bridge deck raising 7.5 meters, were made of a lamination of several layers of heavy oak timber members wrapped by iron straps. Depending on source, six, seven or eight layers were used (James 1982). The timber members had notches and sloping surfaces along their lengths, and wooden wedges were put at each notch (Killer 1942). As the wedges were installed, the members slipped against one another causing a prestress both locally and at the iron straps which were tightened (fig. 2.8), altogether ensuring a high degree of composite action of the arch. Recent studies has provided some insights into the design and analysis of straight beams laminated using wedges (Miller 2009).

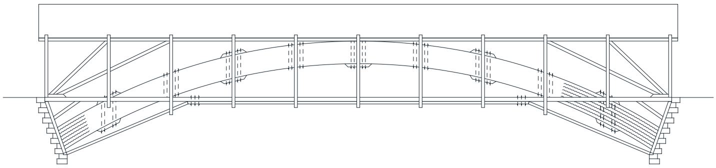


Figure 2.7: *Wettingen brücke (1765) by J. and H. U. Grubenmann.*

(a) *Before installation of wedges.* (b) *After installation of wedges.*

Figure 2.8: *Laminated timber beams using wedges and metal wraps.*

Little more than half a century later, the timber building technology took a new leap forward when Stephen Harriman Long (1784-1864) in 1829 completed the 'Jackson Bridge' in the U.S. The railway timber truss bridge was patented (fig. 2.9; Long 1830) and had shape similarities with one depicted in (Navier 1826). The patent contained an important claim about the 'counterbraces', which normally would have been in tension, but by the use of wedges were prestressed into compression, thus avoiding tension connections (Gasparini and Provost 1989). The patent also introduced mathematical principles of engineering to American bridge building that prior to this had relied upon empirical methods (Christianson and Marston 2015). Long continued to file for patents for variations of his bridge design. In the (Long 1839) patent, he had altered the connection detail putting the wedge between the chord and the vertical rendering compression in the verticals and tension in the diagonals (Gasparini and Provost 1989).

Long's 1830 and 1839 patents defined the two basic designs for parallel-chord trusses. Even though the contemporary German Carl Culmann devoted many pages to Long's bridges in a paper (Culmann 1851), Long's contribution was not widely recognised because of the rapid changes that occurred in material technology between 1840 and 1870. By substituting vertical members of timber with iron, William Howe's 1840 patent (fig. 2.10; Howe 1840) facilitated prestressing and eliminated the main weakness in Long's trusses, quickly making Long's 1830 design obsolete (Christianson and Marston 2015). Thomas and Caleb Pratt's 1844 patent (fig. 2.11; T. W. Pratt and C. Pratt 1844) made Long's first 1839 design practical through the use of pre-tensioned iron diagonals and counters (Sutherland 1997).

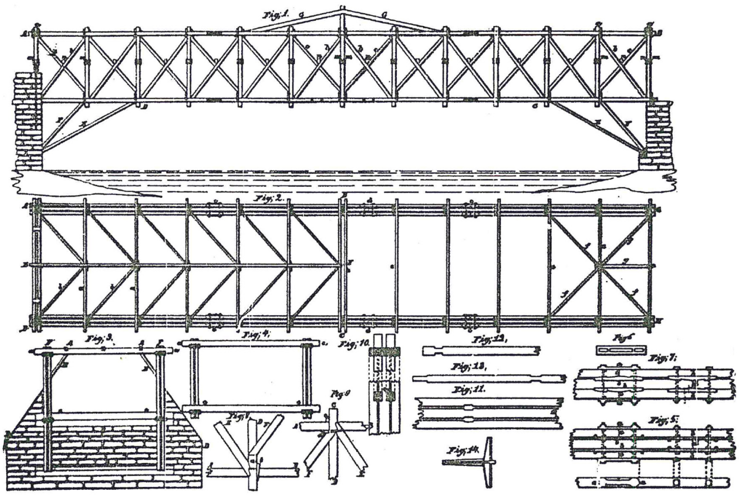


Figure 2.9: *Longs's 1830 patent (Long 1830).*

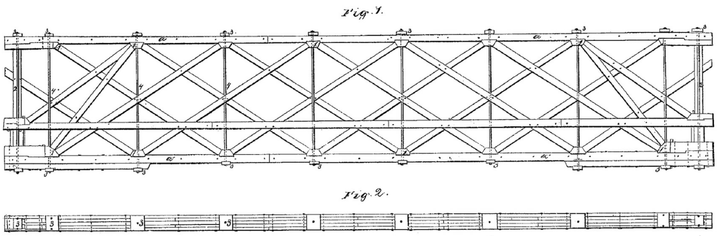


Figure 2.10: *Howe's 1840 patent (Howe 1840).*

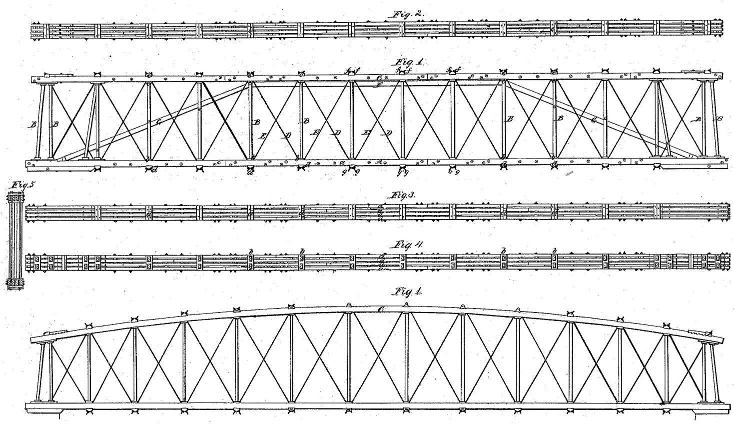


Figure 2.11: *Thomas and Caleb Pratt's 1844 patent (T. W. Pratt and C. Pratt 1844).*

2.2.6 Post tensioned timber frames: for improved ductility

The prestressed timber trusses discussed in section 2.2.5 were indeed post-tensioned, but such timber and timber-iron structures are nowadays superseded by better performing steel and concrete structures. However, there has recently been a renewed interest in post-tensioned timber (e.g. beams (D'Aveni and D'Agata 2017; McConnell, McPolin, and Taylor 2014), stress-laminated decks (Ekholm 2013; Oliva et al. 1990)) with post-tensioned frames perhaps being the most promising.

An early example, completed in 1995, is the post-tensioned bridge in Mursteg, Austria (fig. 2.12) by Jürg Conzett (S. Samuelsson 2015). The bridge has a 47 meters span, which connects a variety of routs in an almost urban situation, and is made of glue laminated timber beams and built-up timber panels. The decision to create a Vierendeel like truss

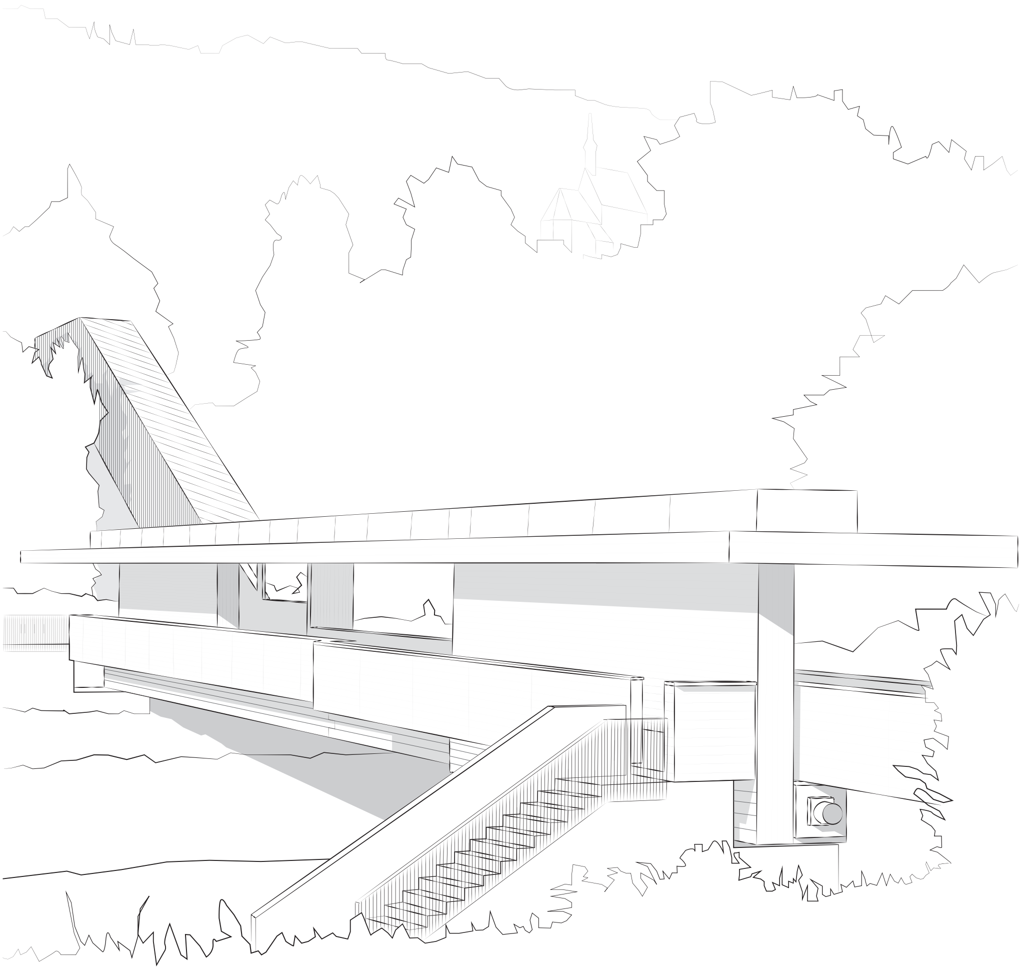


Figure 2.12: *Bridge in Mursteg (1995).*

with displaced shear panels from the central longitudinal axis, offered not only spatial possibilities to create multiple routs across the bridge, but also made it possible to fix these elements to the chords using simple connections with screw rods and ductile dowels (Conzett, Mostafavi, and Reichllin 2006; Fédération internationale du béton 2005). At mid-span, where a 23.8 meters wide 'window' sits disabling any shear connection between the upper and lower chord, the bridge is very flexible. To compensate for the lack of stiffness inherent in the structure and thus avoiding large deformations, Conzett has placed a post-tensioned steel tendon in the centre of the lower glulam chord which counteracts the tensile forces in the timber .

Since 2005, the concept of post-tensioned timber frames have been investigated further (Palermo et al. 2005), taking advantage of un-bonded post-tensioned steel tendons passing through internal ducts through timber members to create moment-resisting connections (Granello et al. 2018). In New Zealand a system know as Pres-Lam, where the tendon passes through timber box beams, frames, or walls, have been developed at the University of Canterbury (Buchanan, Deam, et al. 2008; Newcombe 2011) and built (Buchanan, Palermo, et al. 2011; Curtain et al. 2012). The system usually requires steel fasteners in the beam-column joint to protect the timber that is loaded perpendicular to grain (Granello et al. 2018). To overcome this, a system called Flexframe using hardwood reinforcement (ash wood) of glulam beams with internal tendons has been developed in Switzerland and a prototype building, named the ETH House of Natural Resources, was erected in 2014 (Granello et al. 2018; Wanninger and Frangi 2014, 2016; Wanninger 2015). Regardless of system, the post-tensioned timber frames have shown favourable seismic behaviour, being able to prevent residual deformations after earthquakes (Wanninger and Frangi 2014).

2.2.7 Prestressed wheels: to make them light and stiff

The concept of the wheel is simple, but making a rigid yet lightweight wheel, that can withstand the forces and wear exerted as they are used, is a challenge. A solid disk would work as a wheel, but is heavy. By dissolving the disk into discrete elements, weight can be reduced but then joinery becomes an issue as well as the potential risk of buckling any of the elements if they are compressed too much.

Traditional wooden wagon wheels are made of a hub, spokes and rim segments that are bound together on the exterior of the rim. Early constructions relied on wet rawhide for the binding which would shrink whilst drying, compressing and binding the woodwork together. Later the wheels where either fitted with an iron hoop or straked with iron, compressing the woodwork and protecting against wear from the ground (fig. 2.13). Using hoops for the binding of wheels has similarities with barrels consisting of wooden staves bound by wooden or metal hoops.

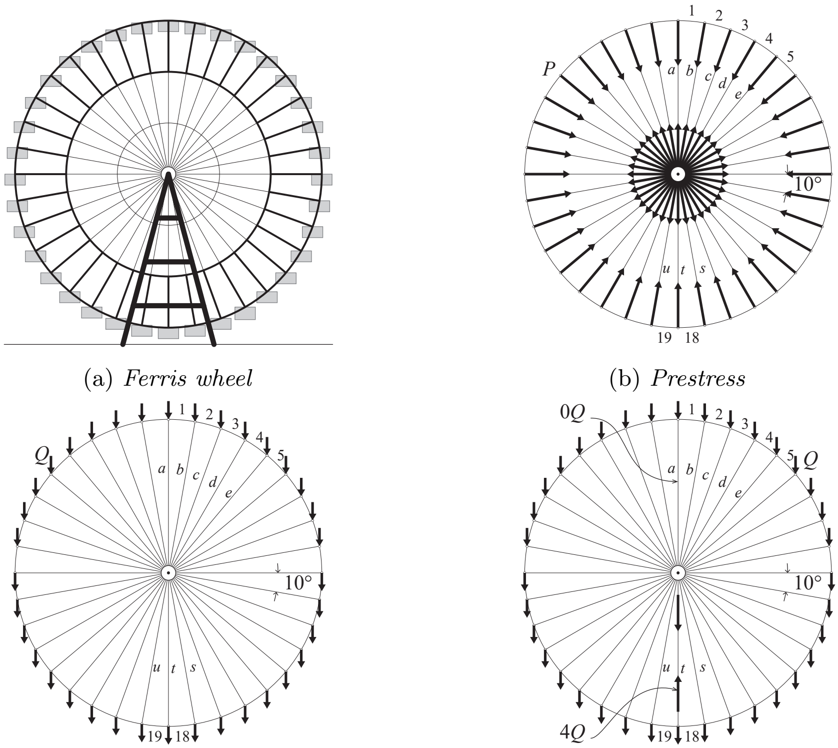
Another kind of wheel is the Ferris wheel. In the shadows of Eiffel's tower for the 1889 Paris Exposition, George Washington Gale Ferris Jr., a 33-year-old engineer from Pittsburgh, U.S., suggested to build a huge revolving iron wheel for the 1893 World Columbian Exposition in Chicago. U.S. After thorough design work and testing, much paid for by Ferris himself, the 76 meter wheel with 36 cars, each designed for sixty persons on a two-revolution twenty-minute ride, was approved and built. Ferris' wheel

Figure 2.13: *Plate depicting two methods of shoeing a wheel. In the centre the labourers are using hammers and "devils" to fit a hoop onto the rim, and on the right they're hammering strakes into place. Published in a volume of Encyclopédie in 1769 Diderot and Rond d'Alembert 1769.*

had an inner layer of cable spokes connected to an outer layer of bars and was prestressed by post-tensioning the cable spokes (fig. 2.14a). Ferris's wheel has not survived until today, but the Wiener Riesenrad in Vienna, Austria, built just a few years later in 1897 (Kullmann, Nachtigall, and Schurig 1975), has a similar structure and is still operating.

Until this point, designers of prestressed structures understood the concept sufficiently and applied it 'effectively and safely, albeit without analyses based on structural mechanics' (Gasparini, Bruckner, and Porto 2006, p. 418). Johnson, Turneaure, and Bryan 1894 presented, in their third edition of the book *The theory and practice of modern framed structures – Designed for the use of schools and for engineers in professional practice*, an early and realistic mathematical model for the analysis of the effects of prestressing, live load and the sum of all in the context of a Ferris wheel. Assuming linear relations they showed, by superposition of load cases and a symmetry argument, that the prestress force P in each cable (fig. 2.14b) has to be twice the weight of each car Q (fig. 2.14c), i.e. $P = 2Q$. Then there will be 0 force in cable a and the maximum tensile force $4Q$ will occur in cable t (fig. 2.14d). This can be compared to the case without prestressing where the cables are replaced with bars that can take compression. Then a maximum compression force of $5.68Q$ occurs in bar a and a maximum tensile force of $11.48Q$ in bar t.

Since the original Ferris' Wheel, a large number of successors have been constructed, one taller than the other. The High Roller in Las Vegas, U.S. is since 2014 the tallest reaching a height of 167.6 meters. With few exceptions, all Ferris wheels rely on prestress to carry the load from the rim to the nave effectively. Many Ferris wheels, such as the



(a) *Ferris wheel* (b) *Prestress*

(c) *Loading* (d) *Result*

Figure 2.14: *Illustration of Ferris' wheel (1893) and load analysis according to Johnson, Turneaure, and Bryan 1894.*

High Roller and London Eye (D. Engström et al. 2004), have designs reminding of the design of a modern bicycle wheel with spokes aligned along a narrow rim diverging out of the plane of the wheel towards a wider nave. The inclination provides some elastic stiffness and by the addition of prestress a substantial amount of geometrical stiffness is further provided to withstand lateral forces in addition to the radial (Brandt 1993).

2.2.8 Compressed concrete: to master creep

Reinforced concrete is the most widely used construction material today. Concrete is a moldable material with high compressive strength, but comes with virtually no tensile strength and has a brittle failuremode in tension. In addition, it shrinks and creeps over time altering its material properties. To master these issues, investigations were triggered that led to the development of prestressed concrete during the late 19th and early 20th century.

The general idea is to compress the concrete by means of tensioning steel tendons in order to reduce or completely avoid tensile stresses within the concrete. The tendons used to compress the concrete are usually embedded—bonded or un-bonded—within the concrete section, but there are some rare cases where externally located tendons have successfully been used as well. The effect of prestressing is un-cracked concrete sections, with higher flexural capacity, greater resistance to corrosion and better liquid retaining properties compared to cracked reinforced concrete sections. Furthermore, it improves the shear resistance and makes it possible to reduce the cross sections with savings in materials and weight as a result (Kaylor 1961). But most importantly, prestressed concrete enables longer spans and offers a way to control creep deformations.

A short summary about the development of prestressed concrete follows, primarily based on (Leonhardt 1964; Menn 1990; Sanabra-Loewe and Capellà-Llovera 2014) and to some extent (B. Engström 2011; Haegermann, Huberti, and Möll 1964; Hellström, Granholm, and Wästerlund 1958).

The concept of prestressed concrete started to be investigated in the late nineteenth century. Peter H. Jackson received in 1886, preceded by at least three patents for systems of applying prestress to building construction, the first patent on prestressed concrete in which tie-rods were used to compress the concrete (Jackson 1886). It was followed by a number of patents by others over the next coming four decades, for example one by the Norwegian Jens Lund (J. G. F. Lund 1912), however few of these systems had any practical application. It was only when the french engineer Eugène Freyssinet (1879-1962) recognised the full potential that the concept became applicable at large scale.

In 1907 Freyssinet built a reinforced concrete bridge across the Allier river in France. Jacks were used at the crown of the three-hinge arches for easy removal of the formwork. The jacking also helped to avoid initial deflections due to elastic deformations. However, soon after completion, creep, at that time a little-known phenomenon, caused a 130 mm deflection. Again jacks were used to restore the original profile and the gap was concreted turning it into a two-hinged arch. By using the jacks, Freyssinet effectively induced a prestress in terms of a compressive thrust acting within the concrete in the bridge. The same method was later used for the 1934 Traneberg bridge (fig. 2.15) in Stockholm (Kasarnowsky 1936).

Figure 2.15: *Traneberg bridge, Stockholm (1934)*

Source: *Holger Ellgaard (2008); image distributed under CC BY-SA 3.0 licence; https://commons.wikimedia.org/wiki/File:Tranebergsbron_panorama_2008.jpg*

The deflection phenomenon observed at the Allier bridge made Freyssinet, beginning in 1911, study the subject of creep in concrete. It eventually lead to a number of patents in 1928 where he stressed the importance of having *full prestressing* in the steel, that is, prestressing to such a level that all tensile stresses in the concrete are removed under service load, to prevent creep. His design philosophy dominated the prestressed concrete industry for many years. It eventually became clear, however, that full prestressing was too restrictive and uneconomical. In 1946, P. W. Abels made an argument about the advantages of reducing the prestress in the tendons and to combine prestressed tendons with unstressed tendons. Ables showed that you could achieve approximately the same load-carrying capacity for fully prestressed and partially prestressed beams. Since the late 1960s, *partial prestressing* is the dominating design philosophy where resistance at ultimate limit state is determined considering both prestressing steel and mild reinforcing steel.

During the late 1940s, the Swedish company Strängbetong AB successfully implemented and developed a system based on a patent by German E. Hoyer. By pre-stressing thin high-strength steel wires, cast the concrete, let it harden and then release the jacks, the concrete is set into permanent compression (Hellström, Granholm, and Wästerlund 1958).

Prestress is however not only applied to new concrete structures. With ageing concrete structures comes a need to enhance the performance of existing buildings and bridges to prolong their technical lifespan. Since the early 1990s investigations have been made regarding post-tensioned externally bonded carbon fibre reinforced polymers to improve the performance of existing concrete structures. The increased performance can be seen in terms of increased elastic bending stiffness, smaller crack openings and improved ultimate capacity (Yang 2019).

2.2.9 Pneumatic structures: for form stability

Nature provides rigidity, or form stability, to biological structures by the use of fluid pressure (Luchsinger, Pedretti, and Reinhard 2004) and air can be used in the same way for man-made structures. The balloon is a simple example, where the gas pressure pushes the enclosing membrane outwards. As the membrane is stretched, it gains geometric stiffness making it stiffer. The sports ball can be seen as a special type of balloon. It is designed to, when bounced, transform its kinetic energy into strain energy which, due to the geometric stiffness, again is transformed into kinetic energy when it springs off from the ground or racket.

Much work in the exploration of the potential of pneumatic structures, also beyond architecture, was done by Frei Otto beginning in the 1960s leading to several IL publications on the theme (*IL12: Convertible Pneus* 1975; *IL15: Lufthallenhandbuch* 1983; *IL9: Pneus in Natur und Technik* 1977; Otto 1995). At the Expo' 70 in Osaka, many pioneering pneumatic buildings were shown, but since then no substantial development has been made other than the use of the airhouse to cover tennis courts and large sport arenas (Luchsinger, Pedretti, and Reinhard 2004). Pneumatic structures are however often used as components of building envelopes, such as the ETFE foil cushions used at the Eden Project (2001) in the UK, the Beijing Olympic Aquatics Centre (2007) and Roof Annex Lutherhaus (2010) in Germany (fig. 2.16; Liu, Zwingmann, and M. Schlaich 2015). In the latter, the cushions are supported by slender circular steel beams which in turn are cable supported both below and above themselves to withstand gravity and wind uplift.

There are also examples of where balloons, or pneus as Frei Otto called them, have

Figure 2.16: *Lutherhaus, Germany (2010). Pressurised transparent foil cushions supported by cable supported steel beams.*

Source: *Copyright schlaich bergermann partner. Reproduced with permission.*

been used as load bearing components (Otto 1995). For instance, the patented *Tensairity* system in which a long slender balloon is used to provide stiffness to plates used as beams or bridges (Luchsinger, Pedretti, and Reinhard 2004; Pedretti and Luscher 2007). FIDU – Freie Innen Druck Umformung, or internal pressure-forming, have been developed at CAAD, ETH Zürich leading to the construction of a 6 meters long steel-skin balloon bridge weighing 170 kg with a capacity to hold a load of up to 1,800 kg (Zieta, Dohmen, and Teutsch 2008).

2.2.10 Prestressed cable nets: for form stability and transparency

The most material efficient way to carry a load is by tension. But a single cable loosely hung between two supports has virtually no stiffness and easily changes shape for only the smallest pertubation. If the cable is stretched, it gains geometric stiffness and withstands pertubations with very little change in its form. A network of several such cables has even higher form stability, and the spiderweb is an example from nature of such. If given an anticlastic shape (curved in two opposite ways; saddle-shaped), the cable net becomes even stiffer.

A typical application of prestressed cable nets are roofs covering cover large areas. There are, for instance, evidence suggesting that ropes supported textiles covering the stands at the Colosseum in Rome, Italy (70–80 CE). But it took until the 1950s until any major advancements in the constructions of cable roofs took place (Krishna and Godbole 2013) and a collection of cable roof structures built until 1975 is presented in (Kullmann, Nachtigall, and Schurig 1975). The Dorton arena (1953) in Raleigh, U.S. is often, even though there are a some earlier examples, seen as the turning point after which substantial progress have taken place in the advancement of cable roof structures (fig. 2.17).

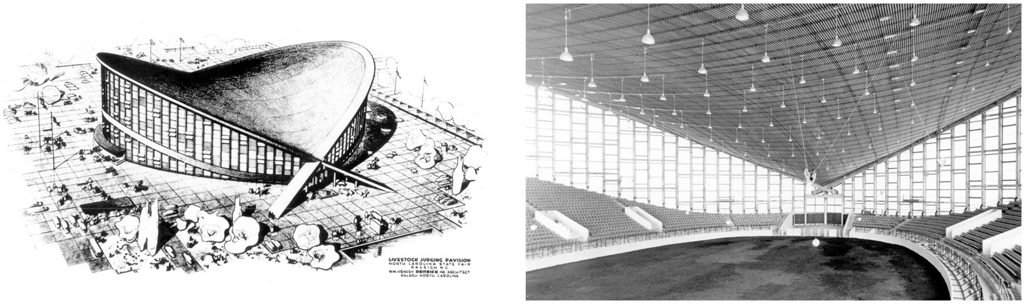


Figure 2.17: *Dorton arena (1953). Drawing (left) and view from inside (right).*

Source: *Photos are courtesy of the N.C. State Fairgrounds.*

Since then, similar roofs have been built, for example the arena Scandinavium (1971) in Göteborg, Sweden (Kärrholm and A. Samuelsson 1972) and the London 2012 Velodrome (Arnold et al. 2011). In these examples, the cable net is post-tensioned against a ring situated along the perimeter of the roof which together forms an auto-equilibrated system. And while the cable net gains geometric stiffness due to the pre-tension, the ring looses geometric stiffness due to pre-compression and has to be designed to resist buckling.

Cable roof structures can not be discussed without mentioning Frei Otto. He devoted his doctoral dissertation — 'Das hängende Dach' ('The hanging Roof') — to such structures (Otto 1954) and founded the Institute for Lightweight Structures (IL) at the University of Stuttgart which he directed from 1964 to 1994 (Aldinger 2016). At IL, extensive work was carried out regarding hanging roof structures (Kullmann, Nachtigall, and Schurig 1975). The work was closely linked to the form finding of compression shells, which can be seen as an inverted hanging chain (Hennicke and Schauer 1974; I. Liddell 2015), eventually leading to the design of the 1975 Mannheim Multihalle (Burkhardt et al. 1978; Happold and W. I. Liddell 1975; Vrachliotis 2017). One of Otto's major contributions is the work on the design of the 1972 Olympiastadion in Munich (fig. 1.1; Tomlow 2016). On the design team was also, among others, Jörg Schlaich and Rudolf Bergermann, who since then at their own practice *schlaich bergermann partner (SBP)* have contributed to the development of prestressed cable net structures (M. Schlaich 2018).

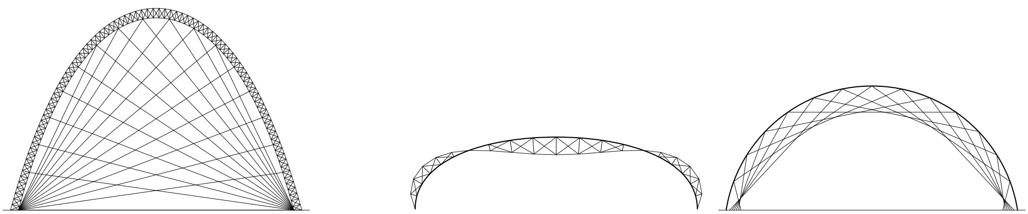
Due to the efficiency of the nets, almost transparent structures can be constructed which still can withstand large forces. The spiderweb is once again a good example from nature, and from techniques there is the 25 $\times$ 40 meters glass façade for the Hilton Hotel (1993) at Munich Airport (Schober and Schneider 2004). The façade, which is designed by SBP, consists of horizontal and vertical cables spanning between the building's two wings as well as between the ground and roof trusses, forming a planar net with a mesh size of 1.5 × 1.5 meters. The glass panels are hung at the intersections of the net using clamp plates which also secures the cables to one another. The cable net is prestressed, similarly as a tennis racket, providing geometric stiffness that reduces the out-of-plane deformations of the net, which under wind can be up to 90 centimetres at the centre of the façade (Barkhofen and Bögle 2010).

2.2.11 Restrained arches: to avoid buckling

The cable roofs are often provided with a sturdy rim which provides a stiff anvil for the tensile forces acting within the roof surface. But one can also do the opposite, letting the cables provide stiffness for the rim. By connecting cables to an arch or a ring, the structure's mode of action is transformed from in-plane bending to a combination of bending and truss action. This is an effective approach where the reduction of weight is of big importance, as in transverse bracing of Zeppelines compensating for the lifting forces at the upper ring half and the load forces at the lower ring half (Kullmann, Nachtigall, and Schurig 1975). There are similarities with how the bicycle wheel the and Ferris wheel work (section 2.2.7) enabling the use of very slender lightweight load bearing components. Therefore the approach is well suited for the construction of glass roofs.

A pioneer on lightweight shell structures was Vladimir Shukhov (M. Wells 2010), who stiffened the glass roof of the Moscow GUM department store (1890-1893) and the Pushkin Museum (1898–1912) using cables springing from the ends connecting once to the arch (Graefe and Tomlow 1990). Eugène Freyssinet used the same topology for the cables stiffening a 7.5 meters wide mobile timber frame on which slipform plywood moulds could slide used for the construction of the Orly airship hangars outside Paris in 1923 (fig. 2.18a; Frampton 2007; Frampton and Futagawa 1983). More recently, Peter Rice let, for the glass roof of the bus terminal in Chur, Switzerland (1988), all cables radiate towards the roof arch from a central nave located just above the focal point of the arch (Addis 1994; Rice 1996). Jörg Schlaich used the same principle for the glass roof at the 1989 Museum of Hamburg History (Barkhofen and Bögle 2010) and later reused the idea for the glass roof at the 1998 DZ-Bank in Berlin (J. Schlaich, Schober, and Helbig 2001).

These cable typologies are effective in restraining the arch, however, the cables interfere



(a) *Orly Hangar* (b) *Berlin Hauptbahnhof* (c) *Japan Pavilion*

Figure 2.18: *Examples of topology of cable systems for the stiffening of arches (not to scale).*

with the space below. For the design of the shallow glass roof covering the upper train tracks and platforms of Berlin Hauptbahnhof (2006) another cable topology was sought. Here Jürg Schlaich let a post-tensioned cable trace the tension side of the moment diagram of the supporting arches leaving enough space below (fig. 2.18b; "Berlin Hauptbahnhof – Lehrter Bahnhof" 2005). The cable and arch are connected using compression struts and cable bracing (Falk and Buelow 2009). Another example is the Japan Pavilion at Expo 2000 where Shigeru Ban, who collaborated with Frei Otto on the project, let the cables spring from the ends and connect multiple times to the arches (fig. 2.18c; Ban 2000).

The degree of prestress in these systems is rather low, just enough tensile stress is induced to make sure that the cables never run slack which would cause them to loose their stiffness completely. Such a loss would lead to an increase of the buckling length of the arch, making it more susceptible to instability phenomenons.

2.2.12 Principles for equilibrium: stressed for stability

While the examples in section 2.2.4 are examples of extenrally-equilibrated structures, Wasserfallbrücke (2013) by Swiss engineer Jürg Conzett is an example of a auto-equilibrated-strucutre (fig. 2.19). The bridge sits along a trail that at times takes hikers through narrow gorge landscapes just along the river Flem and at others offers panoramic views of the alps. It consists of a shallow semi-circular arch made from locally quarried gneiss, has a span of 18 meters, a rise of 1 meter and a cross sectional depth of 0.2 meters. Two steel plates are placed on top of the arch to which the handrail is welded. The plates are post-tensioned against the stone arch with initially a force of 400 kN each, but now it is expected to have decreased to 85 % of the initial value due to relaxation[1]. The effect of post-tensioning the steel plates is the exertion of a uniformly distributed radial acting load on the stone arch in a similar manner as in Pavilion of the Future.

Figure 2.19: *Wasserfallbrücke, Flims (2013).*

Source: *Photo courtesy of Linnea Jansson.*

[1]E-Mail correspondence with Jürg Conzett, 7-8 November 2018.

Another example is the stone arches supporting the roof of Padre Pio Pilgrimage Church (2004) in San Giovanni Rotondo, Italy. Here the arches are not semi-circular, but catenary shaped with deeper cross-sections at the base than at the crown. The arches are stable under normal service conditions (Rice 1996), yet they are prestressed by means of two internally placed steel tendons similarly as in fig. 1.2c. Their purpose is to ensure the stability of the structure during earthquakes which, due to large dynamic forces, otherwise would impose tension at the joints in-between the stones forming mechanism that could lead to collapse.

So far upright standing masonry arches have been discussed, where gravity in combination with prestress sets the arch in compression. It is, however, with the use of prestress possible to construct upside-down masonry arches. By providing enough pre-compression to the stone arch it can be ensured that the tension stress caused by gravity and live loads dose not put the joints under tension. One such example is Conzett's bridge Pùnt da Suransuns (1999) in Thusis, Switzerland (Conzett 2012). The pedestrian bridge is suspended 40 meters between rock anchored foundations and is thus externally-equilibrated. Prestress was applied by post-tensioning two steel plates placed under the stone arch upon which you walk, effectively raising the pathway and compressing the stones together (Conzett, Mostafavi, and Reichllin 2006). Over time, steel relaxes and (M. Wells 2010) argue that the loss in this case is substantial so that the stiffening effect of the prestress is more or less lost.

2.2.13 Tensegrity: for material efficiency

Tensegrity structures offers perhaps the most sophisticated use of prestress. Tensegrity structures are mechanical pin-jointed structures which consist of simple dedicated members, that is compressed struts and tensioned ties (Wroldsen 2007). Through the clear distinction between compression and tension, tensegrities gain a high mechanical efficiency (Ashwear 2016). The prestress induced is necessary to remove mechanisms and thus ensuring the rigidity of the structure. The word *tensegrity*, which is a contraction of *tens*ile and int*egrity*, was coined by Buckminster Fuller in a patent (Fuller 1962). There is, however, a dispute regarding the true inventor and the origins of the idea (Snelson 2012), but the artist Kenneth Snelson's *X-Piece* structure from 1948 (fig. 2.20) is generally regarded as the birth of the tensegrity concept (Tibert 2002). A more extensive exploration of the origins of the concept is provided through a series of articles and responses in a special issue of *International Journal of Space Structures* (Lalvani 1996).

Definitions

There is no consensus on the definition of tensegrity structures. Fuller defined in his patent tensegrity as 'an assemblage of tension and compression components arranged in a discontinuous compression system...' (Fuller 1962). In line with Fuller, (Pugh 1976) extends the definition stating that 'a tensegrity system is established when a set of discontinuous compressive components interacts with a set of continuous tensile components to define a stable volume in space.' (Hanaor 1994) highlights the need for prestress by defining tensegrity as 'internally pre-stressed, free standing pin-jointed networks, in which the cables [...] are tensioned against a system of bars [...].' (Skelton

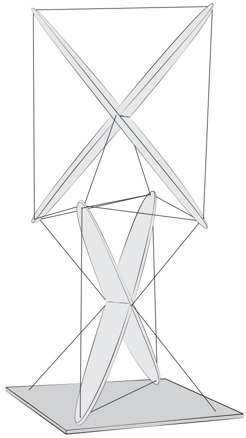


Figure 2.20: *Illustration of Early X-Peice (1948) by Kenneth Snelson*

et al. 2001) defines a class k tensegrity structure as 'a stable equilibrium of axially-loaded elements, with a maximum of k compressive members connected at the node(s).' (Motro and Raducanu 2003) employ a rather strict definition with for example limitations on element size: 'Tensegrity systems are spatial reticulate systems in a state of self-stress. All their elements have a straight middle fibre and are of equivalent size. Tensional elements have no rigidity in compression and constitute a continuous set. Compressed elements constitute a discontinuous set. Each node receives one and only one compressed elements.' Miura and Pellegrino interprets a tensegrity structure as 'any structure realised from cables and struts, to which *a state of prestress is imposed that imparts tension to all cables*' and adds that 'as well as imparting tension to all cables, the state of prestress serves the purpose of *stabilising the structure*, thus providing first-order stiffness to its infinitesimal mechanisms' (Tibert 2002; Tibert and Pellegrino 2003a).

Many definitions limit the possible element types to use to networks of struts and ties. However, relying on Fuller's more poetic definition of tensegrity 'islands of compression in a sea of tension' (Safaei 2012), one could argue that other kinds of element are possible to use in a tensegrity structure (Motro and Raducanu 2003), such as continuous fabric stretched against discontinuous struts.

Some examples

Kenneth Snelson devoted much of his life to tensegrity, leading to an amazing collection of artwork exhibited in different parks and museums, many depicted in his book *Art and ideas* (Snelson 2013). Cecil Balmond has also made use of the concept in 2006 for his *H_Edge* structure (fig. 2.21; Balmond 2007). In 2003, Snelsson's record of highest tensegrity structure was ousted by the 62.3 m tensegrity tower in Rostock, Germany designed by SBP relying on pre-tension forces up to 1,100 kN for its rigidity (M. Schlaich 2004).

The tensegrity concept has shown hard to implement in its pure form — if there is

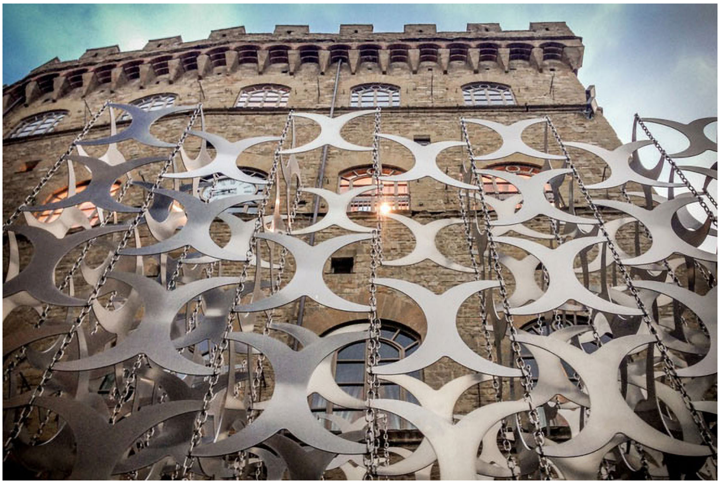


Figure 2.21: *H_Edge (2006) by Cecil Balmond.*

Source: *Image reproduced by permission of BALMOND STUDIO;* `http://www.balmondstudio.com/news/history-of-h_edge.php`

such a thing — within architecture and civil engineering (M. Schlaich 2004). However, there are structures that have tensegrity-like features, such as large domes, temporary structures, tents (Safaei 2012) and stadium roofs but also glass façades and roofs (cf. section 2.2.10). Both the Olympiastadion in Munich (fig. 1.1) and the Millennium Dome (1999) by Richard Rogers and Buro Happold (fig. 2.22; I. Liddell and Westbury 1999) are such examples with their tensile membranes, cables and so-called flying masts (Wroldsen 2007). The concept has also inspired bridge designs, such as the Royal Victoria Dock Bridge (1998) in London, UK by Techniker and Lifschutz Davidson, and the Kurilpa Bridge (2009) in Brisbane, Australia by Ove Arup & Partners.

Examples of façades with tensegrity features were designed by Rice Francis Ritchie (RFR) (Rice 1996), who during the 1980s were considered the best engineering firm in glassed tensed structures worldwide (S. Samuelsson 2015). With the Glass Walls (Les Serres) at the Parc de la Villette, Paris (1982-86) in their portfolio (Patterson 2011), RFR were consulted for the design of the Grand Pyramid (1989) and the Inverted Pyramid (1993) at the Louvre in Paris (fig. 2.23). Architect Ieoh Ming Pei asked the engineers to create a 'structure as transparent as technology could reach' (Knoll 1989; NCK n.d.). The Grand Pyramid consists of steel beams stiffened by post-tensioned steel cables and compression struts. These are placed in two directions parallel to the pyramid edges to support the glass panels and handle the wind pressure. To handle wind suction, the pyramid has been equipped with several horizontal cable rings redistributing suction forces from one side of the pyramid to the opposite (D. Engström et al. 2004). The Inverted Pyramid takes advantage of structural glass and post-tensioned rods rendering the need for a supporting frame unnecessary.

More successfully, the tensegrity concept has been applied within art and furniture design, leading to lightweight new designs. It can also be used to understand biomechanics

Figure 2.22: *Millennium Dome, London, UK (1999) by Richard Rogers and Buro Happold.*

Source: *James Jin (2004); image distributed under CC BY-SA 2.0 licence; cropped and black and white;* *https://www.flickr.com/photos/44768990@N00/58712717*

Figure 2.23: *Grand Pyramid (1989; left) and Inverted Pyramid (1993; right) at the Louvre, Paris.*

Source: *Babyaimeesmom (2018); image distributed under CC BY-SA 4.0 licence; cropped and converted to black and white;* *https://commons.wikimedia.org/wiki/File:Louvre_Palace.jpg*. *Lucas Lima (2017); image distributed under CC BY-SA 2.0 licence; cropped;* *https://www.flickr.com/photos/lucasnave/34167423466/*

and biology inspiring developments in robotics (Ashwear 2016; Safaei 2012). It is, due to the lightweight nature of the structures and the possibility to unfold them by adjusting the length of cables, applied in the design of deployable space structures (Tibert 2002).

3 Framework for prestress

Building on the collection of examples presented in chapter 2, this chapter proposes a framework for prestress in a search for answers to the first three research questions (section 1.3). This is done using various ways of classifying the examples, so that similarities can be identified and differences be distinguished. In the "1970 Key speech", Ove Arup noted that though classification is 'arbitrary and rough', it 'may nevertheless be useful as a help for understanding and discussion, if its imperfections and incompleteness are borne in mind' (Arup 1985, p. 34).

The first three classifications are related to the first question about what historic and contemporary examples tell about how prestress have been used. The first classification sorts all examples chronologically to provide an understanding of how the concept of prestress has been used over time. The time line is in addition divided into four ages enabling the exploration of the overall knowledge development and related challenges at each given time. The second classification takes its departure in structural mechanical modes of action to provide an understanding of how prestressed structures work. Three levels are considered: material behaviour, member action and the assembled structure. The third classification concerns strategies for the analysis and design of prestressed structures. Strategies suitable for early stage design are distinguished from those suitable for verification at the end of the design process.

The fourth classification seek to derive general objectives with prestressing, pursuing to clarify what it means to *improve the performance* and make a structure more *efficient*.

The fifth classification is made to be able to propose design strategies for how prestress is achieved.

The chapter is ended with a thought experiment about the design of the Wasserfallbrücke (recall fig. 2.19) where some of the aspects discussed in the chapter are highlighted in the context of the bridge.

3.1 Historical knowledge development

Regardless of the objective for using prestress the usage requires knowledge about materials, structures, design, analysis and construction. The state of the art is the sum of a historical development of knowledge and (Sanabra-Loewe and Capellà-Llovera 2014) suggest that the historical evolution of prestressed structures can be understood through four main design ages, or phases (table 3.1): *I. Intuition*, *II. Optimistic engineering*, *III. Struggling to minimise losses*, and *IV. Effective prestressing*. This approach gives an understanding of the level of knowledge and related unsolved challenges at a given age.

It also gives an insight of how the knowledge has been used and it is possible to distinguish between qualitative and quantitative approaches. During all ages, qualitative approaches have been used, which is characterised by a conceptual understanding of the physical phenomenons. For example, one may know that stretching a rope will make it stiffer, but can not predict how much stiffer it will get. As time evolved, quantitative approaches was developed, which are characterised by mathematical models targeted to describe aspects of the physical phenomena. This is done by combining

Table 3.1: Four ages of prestress according to (Sanabra-Loewe and Capellà-Llovera 2014) with related examples.

Age	Examples
I. Intuition *Benefits of prestress understood intuitive by designer*	Tents (section 2.2.1), bows (section 2.2.3), Gothic cathedrals (section 2.2.4)
II. Optimistic engineering *Main principle rationally understood and implemented: preloads are designed to act in opposition to service loads*	Timber truss bridges (section 2.2.5), Ferris Wheel (section 2.2.7), Moscow GUM department store (section 2.2.11)
III. Struggling to minimise losses *Recognition that losses exist but are not easily quantifiable or effectively controlled*	Bridge across Allier river and Traneberg bridge (section 2.2.8)
IV. Effective prestressing *Complete understanding of the long-term behaviour of materials and of engineering solutions able to overcome the effect of losses*	Pres-lam/Flexframe (section 2.2.6), prestressed concrete (section 2.2.8), London 2012 Velodrome (section 2.2.10)

assumptions regarding equilibrium (forces), constitutive (material) and kinematic relations (deformations). Quantitative methods, such as those based on geometric nonlinear elastic bar theory, enables the prediction of how much stiffer the rope used by a rope-dancer will get when it is stretched.

Empirical approaches have at all time been used and, depending on how it is used, can be both classified as a qualitative approach providing an understanding and a quantitative approach providing measurements and verification of analytic models.

3.2 Structural mechanical modes of actions

In an article called 'Fundamental concepts of structures' (Otto 1963), Frei Otto wrote 'structures are means of transmitting forces and moments' (Roland 1972). In the same article, Otto published a diagram in which he attempts to sort out how structures gain their ability to do so. In addition, Otto's purpose was to sort existing structures, but also make opportunities for new structures visible. In the diagram (fig. 3.1), which may seem complex at first, three primary levels may be distinguished.

The first level concerns material properties (solids, non-solids) and immaterial properties (magnetism). The second level is about member action where, on one hand, the type of loading is considered (material: tension, bending, compression; immaterial: attraction, rotation, repulsion) and, on the other, the direction of the load is considered (mono-, bi- and triaxial). The third level describes structural systems, that is, assemblies of strucutral members, where three main characteristics are distinguished: (1) the dimension (one-, two- or three-dimensional) and if they consist (2) of only one type of member action (basic

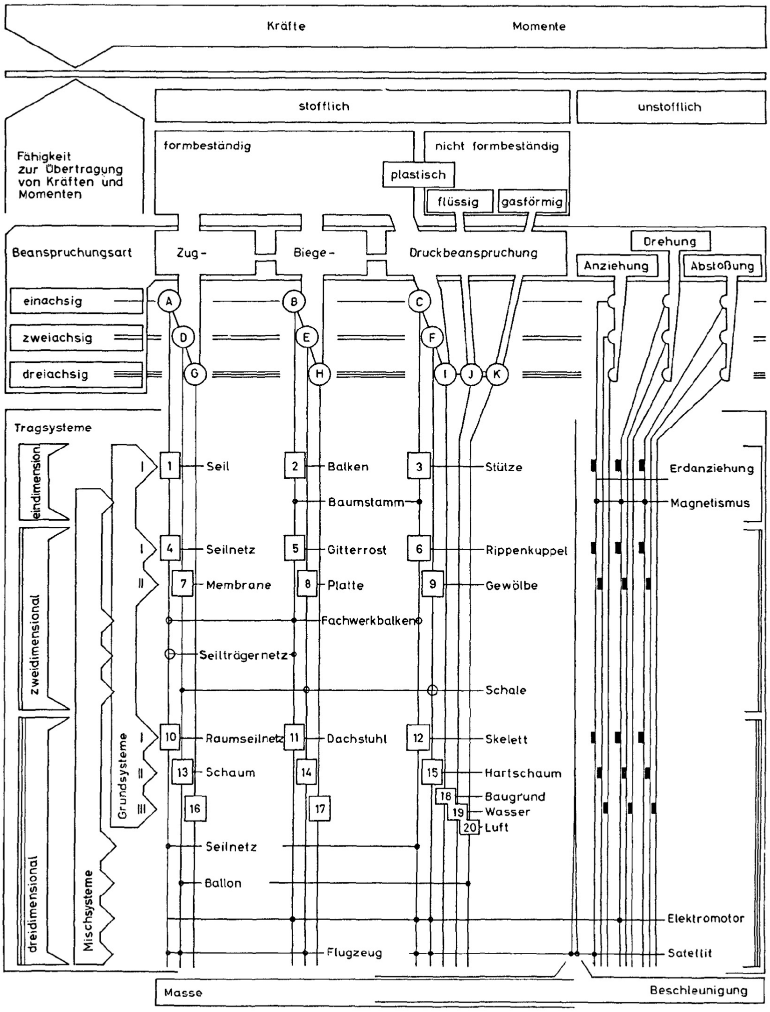


Figure 3.1: *Otto's diagram relating forces and moments to structures.*

Source: *Diagram from (Otto 1963); an English version was first published in (Roland 1972) but is also available in (K.-G. Olsson 2005, p. 49).*

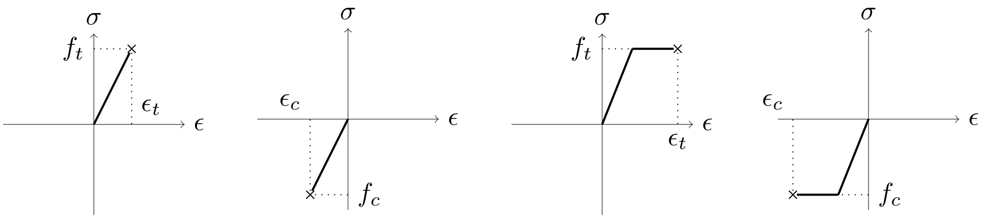
systems – 'grundsysteme') or (3) of a mix of several types of member actions (composite systems – 'mischsystems').

In the following, material behaviour, member action and structural behaviour will be discussed in detail, leaving the interesting, but within architecture less applicable, immaterial part of the diagram out.

3.2.1 Material behaviour

The examples in chapter 2 make it clear that a prestressed object consists not of one, but usually several materials. Materials are combined in such a way that they contribute with their best qualities, while their weaknesses are tried to be avoided. To understand how this is done in the best way, an understanding of material behaviour is essential. Stress-strain relations, material strength and failure modes are, along with the density, key factors for the understanding.

In general the stress-strain relation of a material is non-linear and may in addition be effected by temperature, moisture content and time. Idealised stress-strain relations, known as constitutive models, are used to model the behaviour of materials (see fig. 3.2). Usually only stresses that are within the elastic range of the stress-strain relation are allowed and the strength of the material is thus defined as where that range ends. Furthermore, two primary fracture modes of materials can be identified: brittle failure, where the fracture is sudden (figs. 3.2a and 3.2b), and ductile failure, where the fracture is preceded by extensive plastic deformations (figs. 3.2c and 3.2d). In general brittle failure is avoided and considered unsafe; no warning signs are given before failure thus not allowing safety precautions to take place. This is especially true for material under tension where a brittle failure may cause structures to collapse, whereas compressed brittle material that fails, i.e. is crushed, usually has some reduced capacity even after failure. Materials often have different failure modes in compression and in tension.



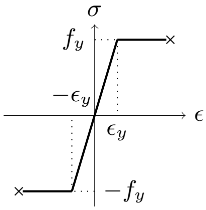
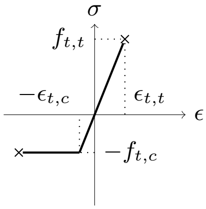
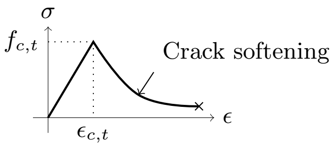
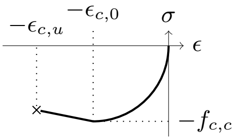
(a) *Brittle tension* (b) *Brittle compression* (c) *Ductile tension* (d) *Ductile compression*

Figure 3.2: *Idealised characteristic constitutive models*

Within the built environment the most common load bearing materials are masonry, concrete, timber and steel. Masonry and concrete have a brittle failure mode at very low tensile stresses, whereas they have a ductile failure at high compressive stresses. Timber is about twice as strong in tension than in compression if loaded parallel to the grain direction and has a brittle failure in tension and a ductile in compression. Steel is an exception and has a ductile failure mode in both compression and tension. Their idealised constitutive models are shown in table 3.2.

With only a few exceptions dating long back, the materials used in the examples in chapter 2 are used in such a way that they will undergo ductile failure if stresses exceed their strength. In addition, a combination of materials is more common than using only one material for the entire structure. When materials that have brittle failure in tension are used, they are always put into compression by the prestress so that they will have ductile failure instead.

Table 3.2: Typical constitutive models for metal, timber and concrete.

Material	**Constitutive model**	
Metal		f_y: yield strength ϵ_y: yield strain
Timber parallel to grain		$f_{t,t}$: tension strength $f_{t,c}$: compression strength $\epsilon_{t,t}$: tension strain $\epsilon_{t,c}$: compression strain
Concrete in tension		$f_{c,t}$: tension strength $\epsilon_{c,t}$: cracking strain
Concrete in compression		$f_{c,c}$: compressive strength $\epsilon_{c,0} = 0.002$ $\epsilon_{c,u} = 0.0035$

Most materials have properties that are stress and time dependent. Much of the development of prestressed concrete is related to the creep phenomenon of concrete, which reduces the elastic stiffness of the material over time. Eventually the creep effect declines and the elastic stiffness reaches a steady state. By prestressing the concrete, the creep process can be accelerated and the steady state can be reached faster and long-term creep deformations can be avoided. Steel and timber have similar time-dependent behaviour, though by different underlying reasons. For steel, the phenomena is called relaxation, and will cause a steel tendon under tension to loose some of its prestress over time which has to be accounted for in the design. Steel is also susceptible to fatigue failure, which is easily demonstrated by bending a metal paper clip at the same point many times so that it finally breaks. The failure is related to a cyclic change between compression stresses and tension stresses. By prestressing the steel, the fluctuation point can be adjusted so that the steel is always in tension and the risk of fatigue failure reduced.

In some early examples, like the Colosseum and Gothic cathedrals, weight was added to prestress parts of the structure. High density materials, such as stone, masonry, soil and gravel, were used so that small quantities of additional materials were needed. With the invention of steel, the same prestressing effect could be achieved with virtually no additional weight, despite the fact that steel has a very high density. Steel has a high strength and high modulus of elasticity making it possible to strain the material to a high degree without failure. Specific strength, which is the quota between strength and density, can be used to compare material efficiency in terms of how much material is needed to support a given load. Wood and timber products obtain high specific strength, even if the effect of buckling is considered, while concrete has a comparatively low specific strength. Various material properties are listed in table 3.3 for wood, steel and concrete, but also for carbon fibre reinforced polymers (CFRP). Due to its high cost, CFRP is rarely used today within the built environment, but may very well be a more commonly used material in the future since it has both high strength end elasticity and unchallenged specific strength.

Table 3.3: Typical material properties for common construction materials.

Material	Strength f [MPa]	Density ρ [kg/m^3]	Elasticity modulus E [GPa]	Specific strength[a] f/ρ	Specific strength w.r.t. buckling $(f/\rho)^{1/3}$
Softwood	20–30	350–450	9–13	44–85	3–3.3
Hardwood	30–50	450–750	7–12	56–78	3.8–4.3
Carbon steel	235–355	7800	210	30–45	3
Concrete	30–50	2500	30	1–2[b]	2.3
CFRP[c]	500–1400	1600	70–300	300–900[d]	5–6

[a] In case of compression, members are assumed to be restrained against buckling.
[b] Applies only for member in compression.
[c] Carbon Fibre Reinforced Polymer
[d] Applies only for member in tension.

3.2.2 Member action

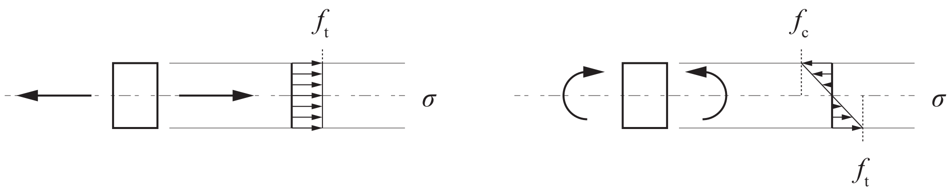
At member level, forces and moments are resisted by tension/compression, bending and twisting.

For one-dimensional members, which are characterised by being long in one principal direction and comparatively short in the other two, there is *bar action*, *beam action* and *twisting action*. Bar action transmits forces acting in the axial direction (e.g. cable in tension, strut in compression), beam action transmits loads acting laterally to the axis (e.g. beam, bow stave, flagpole), and twisting action resists loads that twists the axis (e.g. crankshaft, stairwell).

For two-dimensional members, which are characterised by being long in two principal directions and comparatively short in the other one, there is *membrane action* and *plate action*. Membrane action transmits forces acting in the normal plane of the surface (e.g. textile in tension, wall and vault in compression) whereas plate action transmits forces acting perpendicular to the surface (e.g. slab).

For three-dimensional members, where all principal direction have about the same dimension, *solid action* describes the transmission of loads (e.g. masonry; contact regions between members). Concepts such as normal force, bending moment, shear force and torque, which are used to quantify the actions in one- and two-dimensional members, can no longer be used. Instead the action within the solid is quantified in terms of a (discretized) strain field with an associated stress field. Worth noting is that one- and two-dimensional members may be considered with solid action as well with accurate results. However, such an approach removes the possibility to distinguish the underlying mode of action dominating the stress field, reducing the opportunity to make wise design decisions, especially in early stage design.

Among the examples in chapter 2, bar action (1D) and membrane action (2D) are the most common member actions, while beam action (1D) and plate action (2D) are less common. For bar and membrane action, an even utilisation of the material strength across the cross-section is obtained (fig. 3.3a), possibly utilising the full strength. For beams and plates, however, the material strength can, due to the un-even stress distribution, only be fully utilised in the outer parts of the cross-section (fig. 3.3b).



(a) *Normal stresses in a bar.* (b) *Bending stresses in a beam.*

Figure 3.3: *Stresses σ due to external loading and material strengths f in a bar and beam*

3.2.3 Structural systems

For the structural systems (i.e. assemblies of one or several structural members), Otto sorts structures according to their geometry (one-, two-, three-dimensional). Linking members together along one direction gives a one-dimensional structure, along two directions a two-dimensional structure, and along all three directions a three-dimensional structure.

Structural systems consisting of members of one type of action is called basic systems in Otto's diagram. Assemblies of bar member action can thus form systems such as cable (1D), cable net (2D), cable net (3D), strut (1D), ribbed dome (2D) and skeleton (3D). Beam action gives beam (1D), beam grillage (2D) and roof frame (3D).

Structural systems consisting of members with different types of action are called composite systems. Combining bar and beam action gives systems such as tree trunk (1D), lattice girder (2D) and cable pole (3D), while the combination of membrane and plate action results in shells (2D).

So far, Otto's diagram has been used to discuss how structures transmits forces and moments, but three are other ways as well which could be used in parallel. One such system is provided by (Engel 1997), who published his first version of the book *Structure Systems* just a few years after Otto's was published. Engel's system, which just a Otto's diagram takes its departure in how the transmission of forces and moments are dealt with by the structure, sorts structures into to 'families' and 'types', see table 3.4. For each of these, there are distinct characteristics regarding the internal flow of forces but also the geometry of the structure. By classifying the examples in chapter 2 according to Engel's system, it is evident that the *form active* and *vector active* families are the most common families for prestress structures, while *section active* and *surface active* families seldom are prestressed. Furthermore, *hybrid action* is common, in which a combination of two or more families or their types are used (e.g. prestressed concrete beam: form/vector active + section active).

Structure family	Structure type
1 Form active *Systems of flexible, non-ridgid matter, in which the redirection of forces is effected by particular FORM design and characteristic FORM stabilisation.*	1.1 Cable structures 1.2 Tent structures 1.3 Pneumatic structures 1.4 Arch structures
2 Vector active *Systems of short, solid, straight linear members (bars), in which the redirection of forces is effected by VECTOR partition, i.e. by multi-directional splitting of single forces (compressive or tensile bars).*	2.1 Flat trusses 2.2 Transmitted flat trusses 2.3 Curved trusses 2.4 Space trusses
3 Section active *Systems of rigid, solid linear elements – including their compacted form as slab –, in which the redirection of forces is effected by mobilisation of SECTIONAL (inner) forces.*	3.1 Beam structures 3.2 Frame structures 3.3 Beam grid structures 3.4 Slab structures
4 Surface active *Systems of flexible, but otherwise rigid planes (=resistant to compression, tension, shear), in which the redirection of forces is effected by SURFACE resistance and particular SURFACE form.*	4.1 Plate structures 4.2 Folded plate structures 4.3 Shell structures
Hybrid action *Systems consisting of a combination of families.*	

Table 3.4: Structure families and types according to (Engel 1997).

3.3 Computation strategies

In order to design the structure, various computation strategies can be used. Some are more suitable in the early stage design, were understanding of structural behaviour and structural optimisation is needed, whereas others are are suitable in verification stages, where numerical results for code checks are required.

3.3.1 Early stages

There exists many tools suitable for early stage design. Some are targeted to provide an understanding of the structural behaviour providing qualitative measures for design decisions. Others are targeted for structural optimisation especially applicable when seeking to reduce bending moments, as in vector active structures and form active structures.

Graphic statics

Graphic statics is a graphical method for determining the forces in two-dimensional axially loaded members such as trusses, cables and arches. It was was developed during 18^{th}- and 19^{th}-century through the works of (Varignon 1725; Culmann 1866; Cremona 1872; J. C. Maxwell 1864b, 1870; Rankine 1858) and others. The chronology is described by (Kurrer 2008) and *Form and forces: designing efficient, expressive structures* (Allen and Zalewski 2009) is a popular book introducing the method.

The method relies on the reciprocal relation between a form diagram representing the geometry of the structure and a force diagram representing the external and internal forces using vectors. To find the internal force of a member, one simply measures the length of the corresponding, parallel vector, in the force diagram, which has to be drawn to scale. Any change in either the form or force diagram is reflected in the other, providing a visual understanding of form and forces.

Recently graphic statics has been extended to three dimensions (Akbarzadeh, Van Mele, and Block 2013, 2015; Block and Ochsendorf 2007) and applied to the design of structural masonry (Fraternali 2010; Rippmann, Van Mele, et al. 2016). Methods to apply graphic statics on post-tensioned funiculars are presented in (Todisco 2016) and graphic statics has been combined with structural optimisation in (Beghini et al. 2014). Computer implementations have been developed making the drawing of the diagrams faster (Greenwold and Allen 2003; Rippmann, Lachauer, and Block 2012; Van Mele, Brunier-Ernst, and Block 2009). A general algebraic implementation of graphic statics is presented in (Van Mele and Block 2014) which given a form diagram allows the direct generation of a force diagram. Alic and D. Åkesson extends algebraic statics to be bi-directional allowing changes in the force diagram to generate the form diagram (Alic and D. Åkesson 2017); an example of a prestress funicular arch is provided, much similar to the Wasserfallbrücke, where alterations of the prestressing force generates changes in the form diagrams.

Canonical stiffness

The concept of canonical stiffness, which is a scalar representation of the global stiffness of a structure, was introduced by (K.-G. Olsson 2005). By solving a static eigenvalue problem, canonical stiffnesses λ (eigenvalues) and associated deformation patterns $\mathbf{x}$ (eigenmodes) can be found from

$$(\mathbf{K} - \lambda\mathbf{I})\,\mathbf{x} = \mathbf{0} \tag{3.1}$$

where **I** is the identity matrix. Effects of prestress can be considered using geometric nonlinearity by including the geometric stiffness in the stiffness matrix **K**. The canonical stiffness can be interpreted as being indicative of the inherent global stiffness of the structure. The deformation pattern indicates weaknesses of the structure providing an understanding of how it may respond to external forces. The concept is implemented using first order theory in the computer program 'pointSketch' (P. Olsson 2006).

Mechanics of bar frameworks and Maxwell's rule

A bar framework is a theoretical model of bars connected with friction less joints which can be used to model many structures, for example trusses, tensegrities, gridhsells, cable nets. There exists a simple condition for the rigidity of such frameworks, initially presented in (Möbius 1837): a general plane framework consisting of j frictionless joints, has to have at least $2j-3$ bars in order to be rigid, while a space framework needs $3j-6$. Möbius were aware of exceptions to this rule, and observed that this happens when the determinant of the equilibrium equations of the nodes vanishes. Möbius also notes that if you remove a bar from a framework that has the minimum number of bars, according to the rule, the framework in general transforms from a rigid structure into a finite mechanism. Furthermore he points out that the removal of a bar does not introduce any further degree of internal mobility, if the bar length is either minimum or maximum. However, Möbius's findings were to general and its presentation to abstract, and his work remained unknown to engineers for a long time (Pellegrino 1986).

About 30 years later, Möbius' rule was rediscovered (J. C. Maxwell 1864a) and the rule is nowadays widely known as *Maxwell's rule* for the construction of rigid three-dimensional frameworks,

$$3j - b - c = 0, \tag{3.2}$$

where j is the number of joints, b the number of bars and c the number of kinematic constraints ($c \geq 6$ in three dimensions, $c \geq 3$ in two dimensions). Just as Möbius, Maxwell anticipated exceptions to the rule (Calladine 1978; J. C. Maxwell 1864a): (i) 'In those cases which stiffness can be produced with a smaller number of lines, certain conditions must be fulfilled, rendering the case one of a maximum or minimum value of one or more of its lines.' and (ii) 'The stiffness of such frames (i) is of inferior order, as a small disturbing force may produce a displacement infinite in comparison with itself.'

In 1978, Calladine went back to the paper by Maxwell in order to explain Buckminster Fuller's tensegrity structures (Calladine 1978). Calladine's paper presents rigorous derivations of a tensegrity structure with 12 joints and 24 bars which should be loose with 6 degrees of freedom, according to Maxwell's rule; yet, it is stiff (Tibert 2002). Calladine's key finding is an extended version of Maxwell's rule, dealing with all possible special cases:

$$3j - b - c = m - s, \tag{3.3}$$

where m is the number of internal mechanisms and s the number of states of self-stress. Equation (3.3) dose not by itself solve m and s of a general bar framework, but it introduces a clear explanation of the fundamental mechanics of bar frameworks. The values of m and s depend not only on the number of bars and joints, nor even on the topology of the connections, but on the complete specification of the framework (Pellegrino and Calladine 1986).

Making use of the linear-algebraic relationships of equilibrium and kinematic as well as the principle of virtual work gives, together with the rank r of the equilibrium matrix and its transpose—the kinematic matrix—, the following expressions (Calladine 1978; Pellegrino and Calladine 1986)

$$s = b - r, \qquad m = 3j - c - r. \tag{3.4}$$

Topology optimisation

Topology optimisation (Bendsøe and Sigmund 2004; Sigmund and Maute 2013) is used for the optimisation of material layouts allowing the design to attain any shape within a given design space. It is a numerical method that minimises some performance objective under a set of loads, boundary conditions and constraints. Topology optimisation problems can be solved in several ways, for example in a discrete sense by discretizing the design domain into finite elements. Though targeted to the design of solids, the methods may be applied to find the topology of a dissolved vector active structures such as space frames and trusses.

Form finding

Form finding, or shape optimisation as it is called in a more general sense outside the domain of architecture, is used to determine the global form of a structure and is especially suitable for form active structures. Form finding can either be done using a physical model or a numerical model. In both cases, the equilibrium equations are solved by adjusting the shape – the form – of the structure. The model used for the form finding has to be a mechanism in order to be able to deform from some initial state to the form found state. As the real structure can not be a mechanism, the form found structure has to be 'frozen' when constructed either by adding bending stiffness or bracing.

Most numerical methods for form finding simulate a physical model which has to be stable in order to achieve equilibrium. The physical model might involve hanging chains which will be inverted to form a compression structure as explained by (Hooke 1675), a technique used by for example Antoni Gaudí (Huerta 2006). The model may also involve a combination of soap film and cotton threads in tension as well as masts in compression described by (Otto and Rasch 1995), in which the tension elements stabilises the masts.

Two of the more commonly used numerical methods are the force density method (Linkwitz and Schek 1971; Schek 1974) and the dynamic relaxation method (Day 1965). Recent development has also lead to the thrust network analysis method targeted towards the design of compression shells (Block and Lachauer 2014; Block and Ochsendorf 2007). The book *Shell structures for architecture: form finding and optimization* (S. Adriaenssens et al. 2014) provides a comprehensive introduction to these and other form finding methods within the context of designing shell structures. (Tibert 2002) classifies form finding methods into two groups, kinematic and static methods, where dynamic relaxation is an example of the former class and force density method the latter. A review of these methods, within the context of designing tensegrity structures, can be found in (Tibert and Pellegrino 2003b). A compact summary of tensegrity form finding methods is provided by (Safaei 2012), who concludes that 'most of the proposed methods for form-finding do not consider self-weight and external forces'.

3.3.2 Verification

To verify that a design complies with regulations and other demands, mathematical analysis is primarily used, even though physical testing occasionally occurs. The mathematical analysis may be done analytically with simplified models for early stage design, however, for complex problems, final design and code checks, numerical approaches dominates.

Mathematical analysis approaches are based on either rigid body theory or theory of elasticity. In the former, deformations are neglected while they are accounted for in the latter by considering the constitutive relation between stress and strain. In rigid body theory and theory of elasticity of first order, the equilibrium equation is written for the undeformed state of the structure. Though there are methods for analysing prestressed structures using rigid body theory, for example (Alic and D. Åkesson 2017; Todisco 2016), as well as first order theory, these do not include the influence prestress has on the stiffness of the structure. This is, however, accounted for by higher order theory of elasticity (geometric nonlinear theory). Both second and third order theory are nonlinear and the equilibrium equations are written for the deformed state of the structure. In third order theory, the geometry is also updated in each computation step. Second order theory is in most case sufficient for the analysis of prestressed structures. Figure 3.4 maps the relation between established theories.

For the numerical computation matrix methods such as the finite element method is often used even though there are examples where dynamic methods are used. Dynamic relaxation was for example used to considered dynamic earthquake loading of the Pavilion of the Future (Rice 1996), but also for stress analysis for the London 2012 Velodrome roof[1].

3.4 Objectives for prestressing

It has already been noted that prestress is used to improve the performance of a structure during service and offers a vast range of possibilities of making structures more efficient. But what does *improve the performance* and more *efficient* means? Among the examples in chapter 2, some general objectives for prestressing has been identified which are listed in order of commonness in table 3.5 and will be discussed more in detail in the following.

In most examples material efficiency is sought. For instance, in the Ferries Wheel and bicycle wheel, prestress ensures that buckling in spokes could be avoided by putting them all in tension. The growth stresses in living trees reduces the compressive stresses due to external forces (primarily from wind) so that the stresses better comply with the strength of the wood fibres. In these examples, the more efficient use of the material results in the possibility to use more slender cross sections to withstand the same amount of force.

[1]E-Mail correspondence with Andrew Weir, Director Expedition Engineering, 7 March 2019

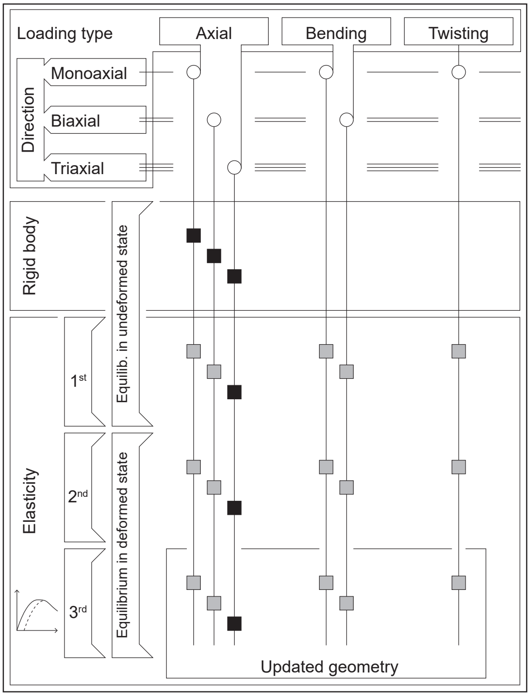


Figure 3.4: *Diagram of how consideration of material properties (rigid body theory, theory of elasticity), type of loading (axial, bending, twisting) and loading direction (mono-, bi- and triaxial) maps to established theories for analysis. Black boxes represents theories where the actions within the members are quantified at material level whereas grey boxes represents theories where the actions are quantified at cross-sectional level.*

Prestress is also commonly used to ensure the stability of structures. Tensegrity structures and pneumatic structures are perhaps the most elaborate examples of such, which both gain most of their stiffness and thus stability from the prestress. Efficient joinery is also often sought, where prestressing is added so that joints are kept in compression rather than tension as in masonry structures and many timber joints. Prestress is also to obtain ductility in terms of overall structural behaviour as in Padre Pio Pilgrimage Church, but also in terms of material ductility reversing forces in materials so that brittle failure is avoided. The latter may be classified as material efficiency, to use the material so that it operates in a ductile manner.

Other, less common, reasons for prestressing includes storing energy, as in the bow or racket, obtaining a specific frequency, as in string instruments, and having adjustable geometry, as in the Orly hangar.

Worth noting is that material efficiency is not an end in itself, but should in turn be linked to higher purposes. Using less material is in most cases cheaper and more sustainable, as for prestressed concrete compared to slack reinforced concrete. It may also allow the architectural ambition to bloom which the Louvre Grand Pyramid and Hilton

Table 3.5: Objectives for prestressing.

Objective	Examples
1. Material efficiency *Prestress to avoid compression/buckling*	Ferries wheel (section 2.2.7), cable roofs (section 2.2.10), growth stresses (section 2.1)
2. Ensure stability *Prestress to provide positive geometric stiffness and remove internal mechanisms*	Tensegrities (section 2.2.13)
3. Provide form stability *Prestress to maintain geometry*	Tensile membranes, pneumatic structures (section 2.2.9), Ferris Wheel & bicycle wheel (section 2.2.7), spiderweb (section 2.1), Egyptian barges (section 2.2.2), cable roofs (section 2.2.10)
4. Efficient joinery *Prestress to secure connections (avoid tension in joints)*	Masonry structures (section 2.2.4), traditional timber joints (section 2.2.5), birds nest (section 2.1)
5. Ductile behaviour	
Prestress to use material ductile stress-strain behaviour	Masonry (section 2.2.4), concrete (section 2.2.8)
Prestress to achieve global ductility	Pres-lam/Flexframe (section 2.2.6), Padre Pio Pilgrimage Church (section 2.2.12)
6. Storing energy *Prestress to store strain energy*	Bow & racket (section 2.2.3)
7. Obtain frequency *Prestress to tune (often slender) members to a specific frequency*	String instruments (section 2.2.3), bicycle spokes (during truing of wheel; section 2.2.7)
8. Adjustable geometry *Prestress adjusted frequently during operation to adjust the geometry*	Egyptian barges (section 2.2.2), Cables at Orly hangar (section 2.2.11)
9. Easy operation *Prestress to reduce weight*	Tents, boats & weaving (section 2.2.1), sports equipment (section 2.2.3), wheels (section 2.2.7)

hotel at Munich Airport are examples of where the vision was to create invisible structures. In everyday objects such as the bow, tent, boat, and sports racket, the material efficiency is linked to an urge of making the object lightweight and thus easier to carry around and use.

3.5 Design strategies for how to achieve prestress

In section 3.4 reasons for why prestress is used were discussed. As soon as the reason for prestressing is clarified in a design process, the question about how the prestressing can be achieved arises. Once again the examples in chapter 2 are examined and this time in the search for general strategies for how to acheive the prestress.

Two main strategies are found, linking back to the discussion in section 1.1 about how the prestressing forces in the structure are equilibrated: either (1) *externally-equilibrated* or (2) *auto-equilibrated*. The main strategies are further divided in a number of sub strategies. In the first case the type of reaction forces obtained are considered whereas in the second case the division is base on how the prestress is balanced within the structure. The strategies and some associated examples are presented in table 3.6.

Table 3.6: Design strategies for how to achieve prestress.

How	**Examples**
1. Externally-equilibrated system *Inner forces balanced by reaction forces at boundary; prestress increases reaction forces*	
1a. Tensile reaction forces *Inner stresses balanced by tensile reactions*	Spiderweb (section 2.1) and Hilton hotel at Munich Airport (section 2.2.10)
1b. Compressive reaction fores *Inner stresses balanced by compressive reactions*	Gothic Cathedrals & timber buildings with heavy roofs (section 2.2.4)
1c. Mixed reaction fores *Inner stresses balanced by* both *tensile and compressive reactions*	Suspension bridges, cable-stayed radio masts
2. Auto-equilibrated system *Tension and compression internally in balance; prestress do not affect reaction forces*	
2a. Inflation *Membrane in tension enclosing compressed fluid*	Pneu & turgor pressure (section 2.1), airhouse & ETFE cushions (section 2.2.9)
2b. Active bending *Prestress induced by active bending, usually restrained by string or membrane in tension*	Birds nest (section 2.1) bow, tents & skin on frame boat (section 2.2.1), hull of Egyptian barges (section 2.2.2)
2c. Aligned tension/compression *Tension and compression member along a mutual line of action*	Prestressed concrete beam (section 2.2.8), Pres-lam/Flexframe (section 2.2.6), Wasserfallbrücke & Padre Pio Pilgrimage Church (section 2.2.12)
2d. Distributed tension/compression *Tension and compression members along individual lines of actions*	Egyptian barges (section 2.2.2), string instruments & racket (section 2.2.3), cable roofs & nets (section 2.2.10), Restrained arches & wheels (section 2.2.7), tensegrities (section 2.2.13)
2e. Local prestress *Pushing parts away/together*	Timber joinery with wedges (section 2.2.5)

All strategies have their strengths, but also their weaknesses, and some of these will be discussed briefly.

The two main strategies results in, for the same level of prestress force, different support reaction magnitudes. In externally-equilibrated systems (1) the reaction forces are modified with a change in prestress, while they are unchanged for auto-equilibrated systems (2). If there are good ground conditions, like solid bedrock, an increase in support reactions is perhaps not an issue and can be accepted. But increases in reaction forces may be challenging for less good conditions, for example clay, especially if there are tensile reaction forces. The construction of auto-equilibrated systems offers the possibility for complete off-site manufacturing and pre-stressing whereas externally-equilibrated systems requires on-site prestressing. Among the examples, it is evident that most auto-equilibrated systems consists of a combination of several materials, for example steel in tension and masonry, concrete and timber in compression. Externally-equilibrated are often constructed from a single material.

Inflated structures (2a) rely on a constant high-enough internal pressure to make sure the membrane is under tension and do not wrinkle. The internal pressure needs, due to leakage and change of surrounding air pressure, temperature and loading, to be constantly monitored and adjusted throughout the life of the structure, for instance by using pumps or fans. This becomes an issue of redundancy, the structure should for instance not collapse due to a power failure. However, for temporary structures, such as inflatable children playgrounds, this offers a convenient way to quickly erect and take down the structure—just plug in or out the fan and wait.

Active bending (2b) is also convenient for temporary structures. Light-weight flexible elements can easily be designed for the bending stresses induced when bending and additional live loads for small structures such as a camping tent. But for larger span structures, the forces in each member increases resulting in larger elements which are more challenging to bend, both in terms of the required force and in terms of practicalities. Recently Quinn investigated inflation as a method to erect active bending gridshells (Quinn 2018), effectively combining strategy (2a) and (2b).

Aligned (2c) and distributed (2d) tension/compression are similar to one another. Both rely on equilibrium at joints where tension and compression elements meet. What differs is the geometrical configuration. These two strategies are perhaps the most common ones and are usually prestressed by post-tensioning one or several members, which then influences the stress pattern of the entire system.

Local prestress (2e) is local in the sense that one seeks to achieve some action at a specific part of the structure, like at a timber joint. However, it should not be forgotten that a locally induced prestress will affect the entire system, if only to a small degree.

Finally, it should be observed that it may be hard to make a clear distinction between the various strategies. Consider for example cable net structures, which both can be classified as externally-equilibrated or auto-equilibrated depending on where the system boundary is set. In table 3.6 the façade of the Hilton hotel at Munich Airport is listed as strategy (1a), whereas cable roofs and nets are listed as strategy (2d) while not much distinguishes the systems. In the former case, the large roof truss, adjacent buildings and the foundation are not considered to be part of the cable net structure while the compression ring of the roofs is, which explains why they may be classified differently.

3.6 Theoretical context of the Wasserfallbrücke

The Wasserfallbrücke in Flims (2013) (recall section 2.2.12 and fig. 2.19 on 27) is remotely situated and spans across a gorge. Locally quarried stone was to be used as the main load bearing material to root the bridge in its surroundings. With the site and chosen material in mind, the design could evolve. How is however not known, but the following description is a plausible illustration of the design process.

Masonry has virtually no tensile strength, so the shape of the bridge has to follow the flow of the compressive forces thus the arch shape. At member level, forces are transmitted primarily through bar action, tough it is possible to consider it as one-dimensional solid action as well.

The remoteness of the site makes transportation of materials and equipment a challenge. Reducing the weight would thus not only be beneficial in terms of transportation of materials, but would also require less sturdy false work. Setting the width of the bridge to a minimum, one meter, leaves the cross-sectional depth of the bridge as the next parameter to minimise in order to cut weight. However, with a reduced self weight, live loads will dominate the flow of forces, just as wind loads dominates the force flow in the upper tier of the flying buttresses of Gothic cathedrals (section 2.2.4, fig. 2.4b), with stability issues as a result.

To be able to reduce the weight to a minimum and obtain a light weight, slender expression of the bridge and yet secure the stability, prestress was added. The prestress was induced by placing post-tensioned steel plates on top of the arch which pushed the arch downwards whilst providing restraints at its ends, resulting in the stone arch being compressed. Together it forms an auto-equilibrated system. With the added prestress, equivalent to 3-4 times as much stone as was used, the permanent stresses dominates the flow of forces instead of stresses caused by live loads. The bridge was built recently, in the current era of efficient prestressing (Sanabra-Loewe and Capellà-Llovera 2014), and expected relaxation of the steel was accounted for during design[2].

During the design, different analysis approaches may have been used at different stages. As a very first approach, rigid body mechanics in terms of graphic statics, for example adopting the method presented by (Alic and D. Åkesson 2017), may have been used to find the (walk friendly) height of the arch and a suitable initial guess for the prestress force. During this stage, the Heyman stability condition for masonry (Heyman 1966) could have been used to position the thrust line.

Using graphic statics does, however, not provide any insights on the deflections of the bridge. These could be captured using by modelling the stone arch in 2D using first order linear elasticity bar members. The influence on the internal forces caused by prestressing can in such a model be accounted for by adding the equivalent external forces acting perpendicular to the arch. Such a model is easy to construct and analyse using for example a finite element analysis software.

As the stone arch deflects, the prestress force will be reduced since the steel plates will get less stretched. First order linear elasticity is unable to capture this, but second order nonlinear geometric elasticity is. An analysis where the steel plates are modelled as

[2]E-Mail correspondence with Jürg Conzett, 7-8 November 2018.

bar members and coupled to the stone arch bar members with constraints can be used.

The deflection also leads to a rotation of the arch members which influences the geometry, so loading directions should be updated accordingly using third order theory. Such deformations are however most likely too small to cause any significant influence of the bridge.

During construction, the prestress was applied using jacks at one side of the bridge, adjusting the length of the steel plates by stretching them.

4 Summary of papers and publications

Table 4.1 contains an overview of the appended papers and publications produced during the work with this thesis. Three of these considers the topic of prestress and are thus included as papers part of this licentiate thesis. The others are seen as related publications.

Table 4.1: Overview of appended papers and publications

	Prestress	**Shell & Gridshell Structures**	**Form Finding**	**Design Process**
Paper A	●	●	●	
Paper B	●	●	○	●
Paper C	●	●	●	
Publication I			●	
Publication II		●	●	
Publication III			○	

All appended work touches upon form finding, directly or indirectly. Tools for form finding are suitable to apply in early stage design to find a shape of the structures which is in equilibrium with its loading. Most rely on dynamic relaxation (Day 1965), but topology optimisation and image registration is also covered. In addition has unpublished work resulted in a demonstrator program using dynamic relaxation with spline elements simulating bending elements (S. M. L. Adriaenssens and Barnes 2001) to understand creep buckling of shell structures, an issue currently acutely addressed at the Multihalle Mannheim[1].

Shell and gridshell structures are investigated in most of the appended works.

Each paper and publications is summarised in the following.

4.1 Paper A

Prestressed gridshell structures

The aim of the study presented in Paper A was to develop a numerical form finding method which produces only compressive and tensile axial forces in members which may lie in one or several layers. Such structures can be prestressed and auto-equilibrated. Two algorithms based on dynamic relaxation are provided which allows *negative fictitious mass* if part of the structure is unstable due to compression forces. In such case, *nodes will be moved in the opposite direction to the out of balance force* towards equilibrium and instability phenomenons due to mechanisms avoided. In both algorithms force densities are used as parameter for the internal member force. The first algorithm form finds for the case of constant force densities, which results in no control of the obtained member

[1] "Expertengespräch über die zukünftige Sanierung der Multihalle Mannheim" (Expert discussion about the future refurbishment of the Multihalle Mannheim), 23 October 2017, Restaurang Multihalle Mannheim.

lengths. The second algorithm provides such control by prescribed required length for some members. The length requirement is met by adjusting the force density during form finding, again using dynamic relaxation in an inner loop. Finally, case studies are presented where the applied load and the prestress is used to govern the form found shape.

4.2 Paper B

Embracing design methods from architects for conceptual design of structures

Paper B describes and discusses the development and outcome of a structural engineering project which design process were enriched by a conscious use of working methods applied by architects. The project is rooted in an environment that aims to bridge a gap between architects and engineers which makes successful collaboration a challenge. Two MSc students in Structural Engineering designed and built a pavilion enclosing a seminar space at a wood technology fair as their master's thesis project (Isaksson and Skeppstedt 2018). The design process, which is taught in an architectural design studio (recall section 1.2), was iterative allowing numerous viable solutions to be explored and successively refined into a design proposal. The early stages resemble work usually performed by architects, whereas the later are such work undertaken by engineers daily. The research by design project (Megahed 2017) led to the construction of the *The Wood Fusion Pavilion*; an active-bending geodesic gridshell prestressed by means of an external post-tension cable system. This project has proven that, given suitable processes and tools, engineers can be creative and come up with imaginative solutions. With the gained experience, the students will most likely be able to help bridging the gap and be active co-creative participants in their future careers.

The appended paper is the Author's Original Manuscript (AOM)/Preprint version of the paper, which soon will be submitted to a journal.

4.3 Paper C

Unloaded prestressed shell formed from a closed surface unattached to any supports

The aim of the study presented in Paper C was to begin to answer the question, 'under what conditions can an unloaded shell formed of a closed surface unattached to any supports contain a state of membrane stress which can be induced by prestressing?' The study is limited to unloaded shell surfaces with inextensional deformation and membrane action. Using Maxwell's rule, it is shown that a pin-jointed representation of a sphere cannot be prestressed, but a torus can be. A fine triangulated pin-jointed framework behaves very much like a continuous shell, and a particular state of stress is for a torus is examined. However there must almost certainly be more, and this is the topic of future research.

Paper C is a conference paper which will be presented at the IASS 60th Anniversary Symposium (IASS Symposium 2019) in October, 2019 and thereafter published in its proceedings.

4.4 Publication I

On multi objective topology optimization and tracing of Pareto optimal structures

The publication explores multi-objective topology optimisation as a way to find topological configurations of matter yielding high elastic stiffness under mass and eigenfrequency constraints. The principles of topology optimisation may be applicable on prestressed structures prior to any form finding, but has not been explored further.

4.5 Publication II

The use of virtual work for the formfinding of fabric, shell and gridshell structures

With differential geometry and statics as the theoretical basis, just as in Paper C, the principle of virtual work for form finding of fabric, shell and gridshell structures is investigated.

4.6 Publication III

Moving Mesh and Image Registration in FEniCS

The potentials of image registrations is investigated, where a transformation that warps a source image into a target image is sought matching gross features. An implementation was done in the Finite Element Method solver FEniCS (Logg, Mardal, G. N. Wells, et al. 2012) using a moving mesh to solve the governing elliptic, vector valued partial differential equation. Ideas of how to use the method for form finding applied in architectural and structural design is presented, but has not been explored further.

5 Discussion and future research

This licentiate thesis reports research that investigates how prestress can be used as a design tool for the creation of material efficient and well-functioning structures, and in early design stages contribute to sustainable, functional and beautiful architecture. This has been done through the search for answers to four main questions.

The first question is about what can be learnt from historic and contemporary examples about how prestress have been and can be used. For this, a collection of examples is presented in chapter 2. This has been compiled searching broadly for historic and contemporary applications and showcases a diverse range of applications of prestress. In nature the pressurised cell—the pneu—is interesting and the effect of the prestress can be substantial as with the growth-stresses in trees. The spiderweb has most likely inspired the creation of tensile cable net structures used in architecture. In technics, history learn about sophisticated applications such as the skin-on-frame boat. It is also possible to track the development from historic prestressed spoked wooden wheels to contemporary bicycle wheels and modern Ferris wheels such as the Millennium wheel in London, UK. When looking at the collection, it is clear that the common notion that prestressing has only to do with concrete is wrong. Contrary, prestress is ubiquitous in nature and commonly applied within technics. Furthermore, it is evident that the stress level needed for prestress to be effective can be very small, as in living cells, but also large, as in the high strength steel tendons used for prestressed concrete.

Based on the examples, a framework for prestress is derived and presented in chapter 3. The historical development is considered in section 3.1, shedding some light on design challenges that has been overcome. Structural mechanical modes of actions that can be used to understand the effect of prestress are discussed in section 3.2 considering three levels: material, member and structure. Computational strategies suitable for early stage design and verification in late stages are exemplified in section 3.3. These three perspectives then support the search for answers to the next two research questions, which also are included in the framework.

The second question seeks for general objectives with prestressing, and suggestions for such are presented in table 3.5 and discussed in section 3.4. The objectives answer *what* is sought after with prestress pursuing to clarify what it means to *improve the performance* and make a structure more *efficient.*

The third question is if it is possible to establish design strategies for how to achieve the prestress. Two main strategies are discussed in section 3.5, externally-equilibrated and auto-equilibrated, and further divided into sub-strategies. The strategies are presented in table 3.6 and discussed, and answers *how* the prestress is achieved.

The fourth question explores how the gained knowledge about prestress can be utilised and applied in a design process. To answer the question, three studies has been performed from which the following can be observed:

1. In Paper A an effort is made to provide numerical form finding tools for prestressed pin jointed frameworks. The tools are suitable to use in early stage design. Using dynamic relaxation, force densities and allowing negative fictitious masses, an algorithm is provided allowing equilibrium configurations to be found, even though

the structure is a mechanism and unstable. The real structure must of course be stable, which can be achieved by providing moment stiff joints or add bracing to the form found structure.

2. Paper B describes a case study focusing on the design process and to some extent the construction of a post tensioned timber shell pavilion. It was found that iterative design processes used by architects, which successively increases the complexity of associated investigations, are suitable also for conceptual structural design. Such processes are strengthened by oscillating between creative explorations and critical evaluations. Preferably, the critical evaluations are performed inviting peers and others not directly involved in the project who can provide new perspectives on the suggestions at hand. In addition, the process offers the possibility to perform research by design in practice by successfully search for and develop tools that can enforce qualitative and quantitative evaluations. Such an approach is especially helpful for concepts with few precedents to learn from. Iterative design process of this kind is not limited to the design of prestressed structures, but can be applied on any conceptual design development, even outside the field of architecture. It allows for a successful development of skills, knowledge and tools and is likely to ease the application of challenging concepts such as prestress.

3. In Paper C work has begun exploring what geometric requirements there are to be able to prestress shells. It has been concluded that a sphere can not be prestressed, but a tours can. Further work may result in constraints which may be used for form finding of prestressed shells using virtual work, similarly as in Publication II.

While literature exists with the aim to explain concepts of structures, for example by Freio Otto (Otto 1963) and Heino Engel (Engel 1997), no literature has been found specifically targeted to prestress. The findings presented in chapter 3 and especially in sections 3.4 and 3.5 has the potential to be developed further and complement the literature.

Some of the research presented herein relies on calculations based on existing as well as in the research group newly developed computational models.

The work presented herein opens up for some possible future work.

To begin with, the examples presented in chapter 2 and the linked framework in chapter 3 could be completed and improved to make it qualitative enough to publish it as a journal article. Such work would strengthen the discussion related to the first three questions of this thesis.

Also, further work could be done related to the fourth question and two subsequent questions arises relevant to seek answers to:

1. How much do prestress actually help? The question could be answered through comparative studies where designs without prestress are compared with designs with prestress given the same design scope. There is, for instance, ongoing initiatives to replace a large flat roof in a shopping mall in Göteborg with a glass roof and a ‘conventional’ glass shell could be compared with a prestressed one. Such investigations could be a suitable continuation of the work presented in Paper C.

2. When you have your design, how do you apply the prestress during construction? This question became relevant during the construction of the Wood Fusion Pavilion discussed in Paper B. Due to the many applications points of the prestress, it was challenging to find the correct order of prestressing and level of prestress not causing the geometry to deviate to much.

Both of these questions could possibly be answered applying existing computational methods and models in new ways. It may also be so that new computational methods needs to be developed to better answer such questions.

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