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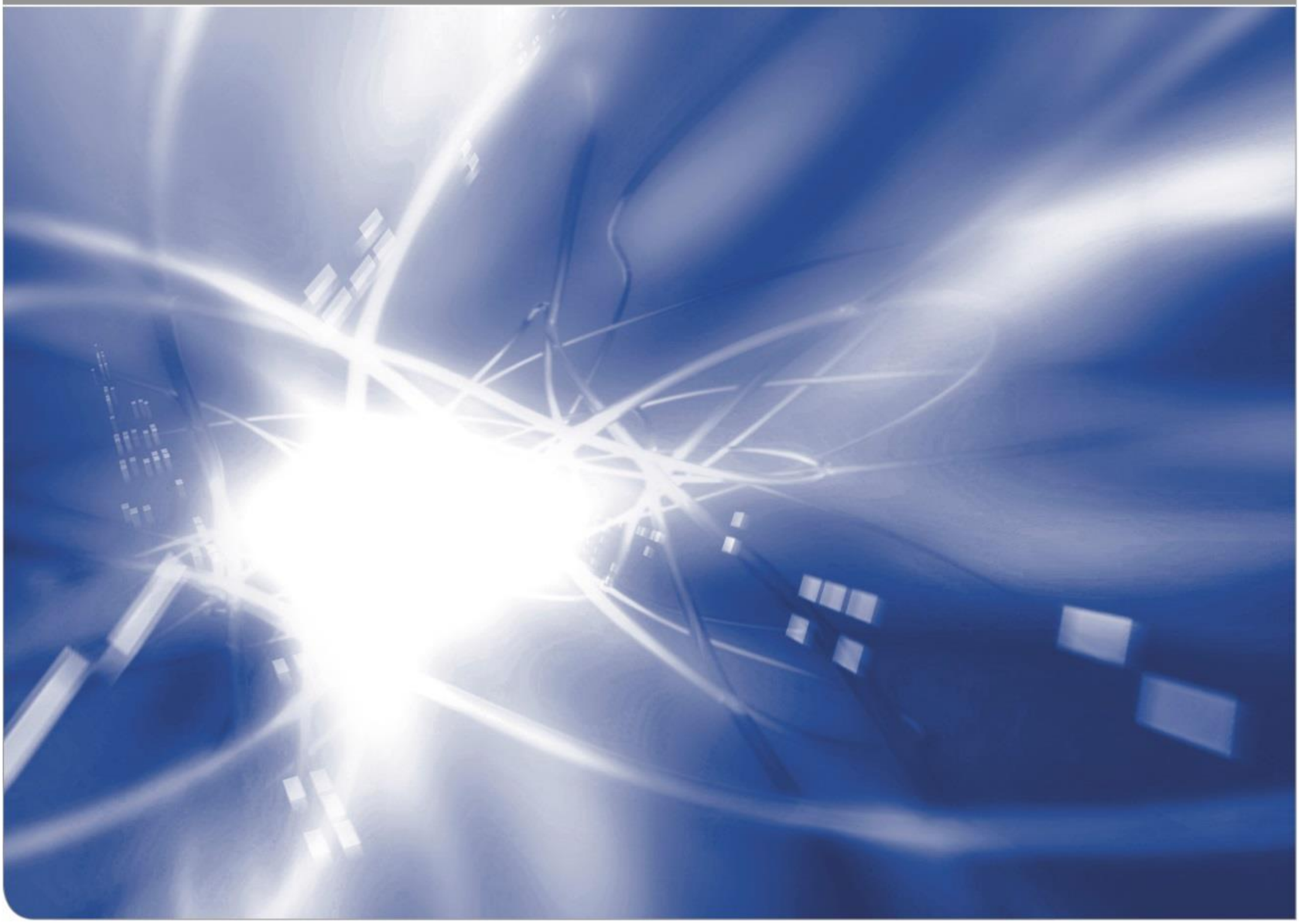

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AFRgeo_v1.0: A Geoid Model for Africa

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Abstract

The paper presents an attempt to compute a geoid model for Africa in the framework of the IAG African Geoid Project. The available gravity data set consists of land point gravity data as well as shipborne and altimetry-derived gravity anomaly data, having a lot of significant gaps allover the continent. The reduced gravity anomalies are gridded on a $5' \times 5'$ grid using an iterative process employing a tailored reference model, to fill in the data gaps, and a weighted least-squares prediction technique. The tailored reference model, up to degree and order 2160, has been used to compute a geoid model for Africa within the window remove-restore technique employing the Stokes integral in frequency domain by the 1-D FFT technique. For the sake of comparison, another geoid model for Africa has been computed using a different approach. This approach renounces the use of the topographic-isostatic reduction and uses the recent global combined geopotential model EIGEN-6C4, complete to degree and order 2190, serving as the reference model. The computed geoids are scaled using the GO_CONS_GCF_2_DIR_R5 GOCE satellite-only model, which represents the best available global geopotential model approximating the African gravity field. An extensive comparison between the geoids computed within the current investigation and the former geoid model for Africa has been carried out.

Keywords Africa · geopotential theory · inverse theory · numerical modelling · window technique

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1 Introduction

This paper faces the great challenge of determining the geoid model for Africa. The challenge consists in the available data set, which suffers from significantly large gaps, especially on land.

The geoid model for Africa has been determined within the current investigation employing the Stokes integral in the frequency domain by 1-D FFT technique (Haagmans et al., 1993; Sideris and Li, 1993). This requires the gravity data to be interpolated on a sufficiently dense regular grid. Since the data gaps are significantly large (attaining hundreds of kilometers in some areas), and in order to avoid the random freedom of the interpolation solution at those areas, an underlying grid has been used to fill in these gaps. This underlying grid is computed using a tailored geopotential model for Africa, because none of the existing global models proved to fit the African gravity field with the desired accuracy (Abd-Elmotaal, 2015). The main idea of the used interpolation process is based on the fact that the interpolation errors are directly proportional to the degree of smoothness of the field, i.e., the smoother the field, the smaller are the interpolation errors. Accordingly, the paper attempts to get a residual field as smooth as possible. This has been achieved by using the window remove-restore technique (Abd-Elmotaal and Kühtreiber, 1999, 2003).

The tailored geopotential model for Africa is established within an iterative process. This iterative process works in such a way that in each iteration step the data gaps are filled with a $30' \times 30'$ grid computed by the tailored geopotential model generated at the previous iteration step. The weighted least-squares prediction technique (Moritz, 1980; Kraiger, 1988) is thus applied to estimate gridded gravity anomalies, which are used to compute a new tailored geopotential model. The tailored model has been computed using two harmonic analysis techniques; the FFT technique (Abd-Elmotaal, 2004) and the least-squares technique (Heck and Seitz, 1991). The iterative process is terminated when the solution stabilizes, i.e., when no further improvement in the residual field is achieved. The latest tailored geopotential model has thus been used to generate the final underlying grid. Accordingly, a weighted least-squares prediction technique using all data types took place to interpolate the gravity anomalies on a regular grid to be used for the geoid determination. The geoid model for Africa determined through this developed somewhat lengthy approach is called **AFRgeo_v1.0**.

The available data sets are described and the method used to interpolate the reduced gravity data on a denser $5' \times 5'$ grid is explained. The geoid model for Africa has been determined using 1-D FFT technique. For comparison purposes, another geoid model for Africa (AFRgeo-EIGEN-6C4) has been computed using the EIGEN-6C4 geopotential model (Förste et al., 2014) without topographic-isostatic reduction. The geoids computed within the current investigation are scaled and compared.

It is worth mentioning that the first attempt to determine a geoid model for Africa has been made by Merry et al. (2005). For that geoid computation project, a $5' \times 5'$ mean gravity anomaly grid developed at Leeds University was used. Unfortunately, that data set has never become available again. For that first geoid solution for Africa, the remove-restore method, based on the EGM96 geopotential model (Lemoine et al., 1998), was employed. The AFRgeo_v1.0 geoid developed within the current investigation is also compared to the geoid model computed by Merry et al. (2005).

Finally, it may be useful to mention that the topic of computing a tailored geopotential model for a particular part of the earth has been tackled by many researchers. The reader may refer, e.g., to Weber and Zomorrodian (1988), Wenzel (1998), Abd-Elmotaal (2007, 2014), and Abd-Elmotaal et al. (2015a).

2 Used Data

2.1 Gravity Data

The available gravity data set for the current investigation comprises point gravity data on land as well as shipborne and altimetry-derived gravity anomalies in sea regions. These data sets are described below.

2.1.1 Land Data

The available gravity data set on land in Africa for the current investigation consists of 96,954 gravity data points. These gravity data have been collected over the past decade from different sources by the first author. A gross-error detection scheme has been carried out on the land data set using a smart gross-error detection technique (Abd-Elmotaal and Kühtreiber, 2014). That gross-error detection scheme uses the least-squares prediction technique (Moritz, 1980; Kraiger, 1988). The gross-error detection technique works first to estimate the gravity anomaly value at the computational point using the values of the surrounding stations excluding the computational point. Hence, a comparison between the estimated and observed data values is used to define a possible gross-error. Accordingly, the effect of the computational point on the surrounding stations is examined. Data points which show a real gross-error behaviour are removed from the database. The number of land gravity stations after removing the gross-errors is 96,472 stations. Figure 2.1 shows their distribution, which contains very large data gaps. The free-air gravity anomalies on land range between -624.5 mGal and 452.8 mGal with an average of about 1.9 mGal and a standard deviation of 60.7 mGal.

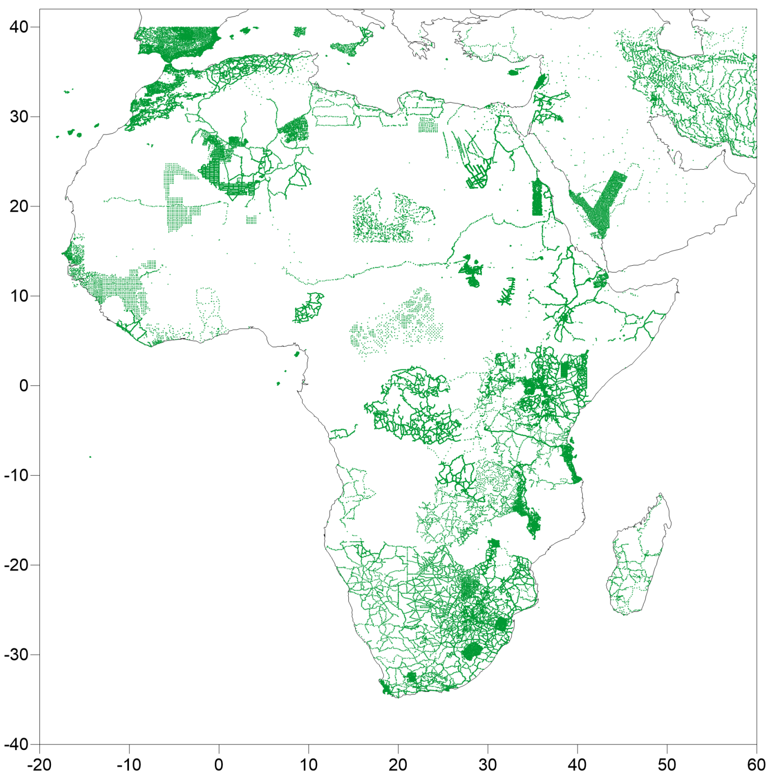


Figure 2.1: The distribution of the used gravity data set on land for Africa.

2.1.2 Shipborne Data

The available shipborne gravity data set for the current investigation consists of 1,091,351 gravity data points. A gross-error detection scheme has been carried out on the shipborne data set using an approach described in Abd-Elmotaal and Makhloof (2013) based on the least-squares prediction technique (Moritz, 1980). It estimates the gravity anomaly value at the computational point using the values of the surrounding stations excluding the computational point. Hence, a comparison between the estimated and observed data values is used to define a possible blunder. The gross-error technique works in an iterative scheme till the standard deviation of the residuals (data minus estimated) becomes below 1.5 mGal. The number of the shipborne data points after the gross-error removal is 971,945. Figure 2.2 illustrates their distribution, which shows a better distribution than that of the land data. The remaining gaps of the shipborne data are partially filled with the altimetry-derived gravity anomalies. The shipborne free-air gravity anomalies range between −238.3 mGal and 364.8 mGal with an average of about −6.5 mGal and a standard deviation of 40.4 mGal.

2.1.3 Altimetry Data

The available altimetry-derived gravity anomaly data set for the current investigation consists of 161,735 gravity data points. A gross-error detection technique, similar to that employed on the shipborne data, was carried out. A combination between the shipborne and altimetry-derived data took place (Abd-Elmotaal and Makhloof, 2014). This combination has resulted in some gaps along altimetry tracks when they do not match with the shipborne data (cf. Fig. 2.3). The final number of the used altimetry-derived free-air anomalies is 119,249. Figure 2.3 illustrates their distribution, which shows, more or less, a

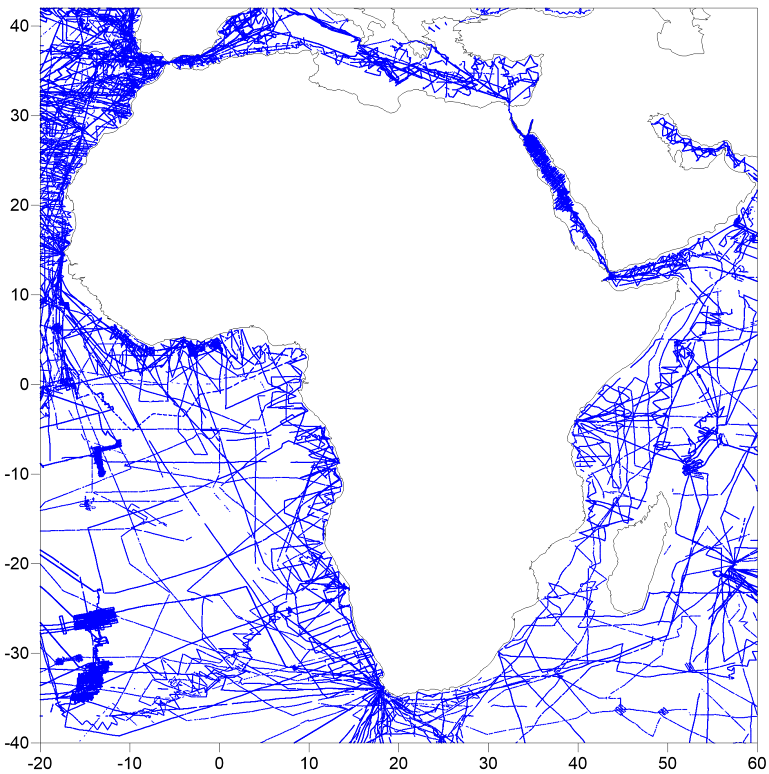


Figure 2.2: The distribution of the used shipborne gravity data set for Africa.

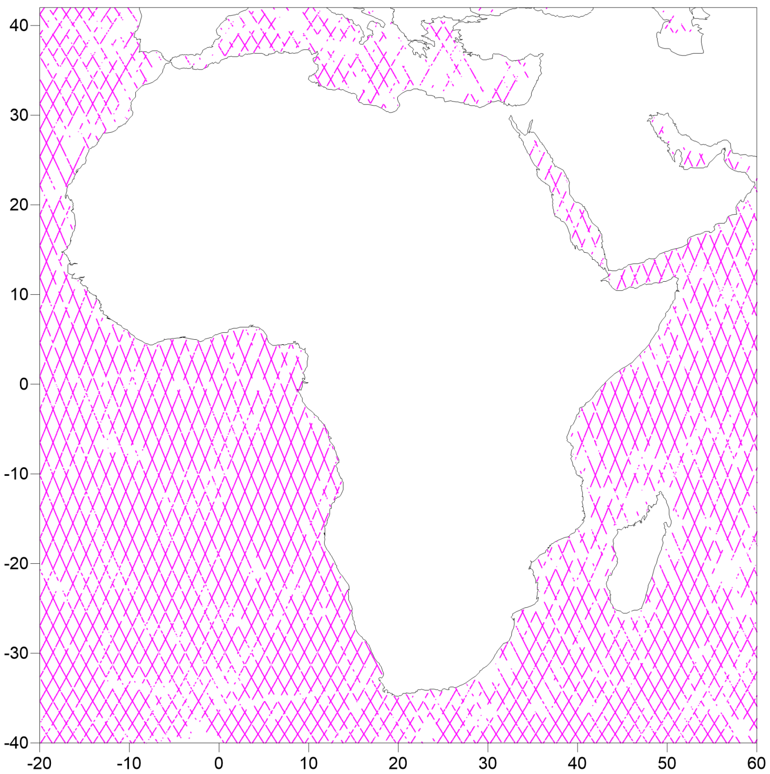


Figure 2.3: The distribution of the used altimetry-derived gravity anomaly data set for Africa.

regular distribution. The altimetry-derived free-air gravity anomalies range between -172.2 mGal and 172.7 mGal with an average of 4.0 mGal and a standard deviation of 18.6 mGal.

2.2 Digital Height Models

For the terrain reduction computation, a set of fine and coarse Digital Height Models (DHMs) is required. The $30'' \times 30''$ AFH13S30 DHM (Abd-Elmotaal, personal communication, 2013), which is based mainly on the SRTM30+ (Shuttle Radar Topography Mission) (Farr et al., 2007), is employed as the fine DHM. Figure 2.4 illustrates the $30'' \times 30''$ AFH13S30 fine DHM. The heights of the AFH13S30 DHM range between -8344 m and 5778 m with an average of -1622.2 m and a standard deviation of 2407.8 m. The $3' \times 3'$ AFH13M03 is used as the coarse DHM.

3 Gravity Reduction

As stated above, two geoid models for Africa are determined in the current investigation, they are:

- AFRgeo_v1.0 geoid model: the geoid model for Africa using the developed approach employing the window remove-restore technique with the EGM2008 geopotential model, up to degree and order 2160, and a tailored reference model (developed in an iterative scheme), up to degree and order 2160, to fill in the data gaps.

- AFRgeo-EIGEN-6C4 geoid model: the geoid model for Africa which avoids the use of the topographic-isostatic reduction employing the EIGEN-6C4, up to degree and order 2190. The data gaps are filled with the resulting tailored model determined in the developed approach.

The reduction steps for both geoid models are described in the following subsections.

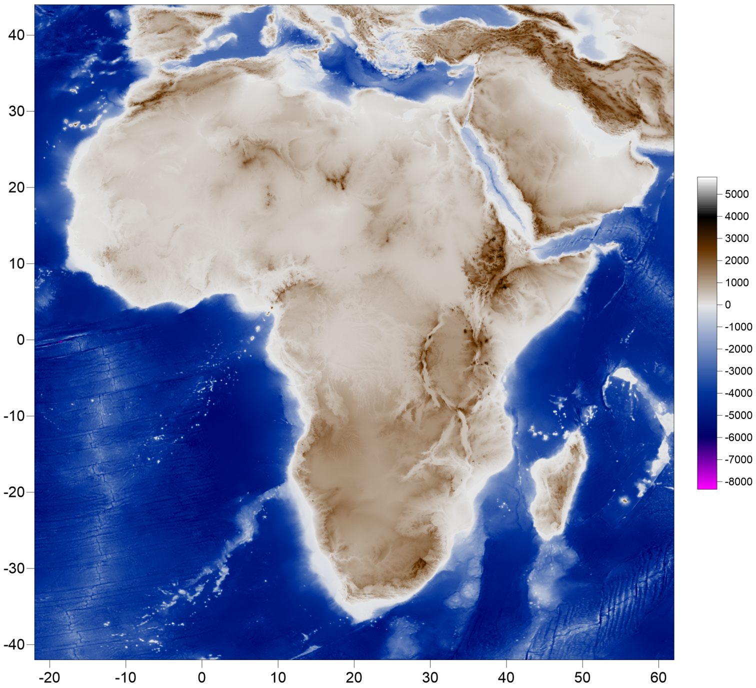


Figure 2.4: The fine $30'' \times 30''$ AFH13S30 Digital Height Model. Units in [m].

3.1 Gravity Reduction for AFRgeo_v1.0 Geoid Model

The gravity reduction for the AFRgeo_v1.0 geoid model follows the window remove-restore technique (Abd-Elmotaal and Kühtreiber, 1999, 2003). The window reduction step can be described by

$$\Delta g_{win\text{-}red} = \Delta g_F - \Delta g_{TI\,win} - \Delta g_{GM} + \Delta g_{wincof}\,, \tag{3.1}$$

where $\Delta g_{win\text{-}red}$ refers to the window-reduced gravity anomalies, Δg_F refers to the measured free-air gravity anomalies, Δg_{GM} stands for the contribution of the global reference geopotential model, $\Delta g_{TI\,win}$ is the contribution of the topographic-isostatic masses for a fixed data window computed by forward modelling and Δg_{wincof} stands for the contribution of the dimensionless harmonic coefficients of the topographic-isostatic masses of the same data window.

It is worth mentioning that the dimensionless harmonic coefficients of the topographic-isostatic masses of the data window can be computed using the approach developed by Abd-Elmotaal and Kühtreiber (2015). For the computation of the contribution of the involved geopotential models, one may use one of the software introduced in, e.g., Rapp (1982), Tscherning et al. (1983, 1994), and Abd-Elmotaal (1998). The terrain reduction computations are done using the TC-program (Forsberg, 1984) with the modifications given by Abd-Elmotaal and Kühtreiber (2003).

As stated above, the developed approach for generating the AFRgeo_v1.0 geoid model uses a tailored global reference model for better fitting to the African gravity field to be used for filling-in the large gravity data gaps. In order to create such a tailored model, Eq. (3.1) is re-written in the form

$$\Delta g_{win\text{-}red} = \Delta g_F - \Delta g_{TI\,win} - \Delta g_{GM}\Big|_{n=2}^{N} - \Delta g_{GM}\Big|_{n=N+1}^{n_{max}} + \Delta g_{wincof}\Big|_{n=2}^{N}\,, \tag{3.2}$$

where n_{max} is the upper used degree of the global reference model and N is the upper degree of the generated tailored model. The EGM2008 (Pavlis et al., 2012) has been used as the reference field in the current investigation with an upper degree $n_{max} = 2160$. The upper degree of the tailored model has been set to $N = 360$. This means that only the lower harmonic coefficients till degree and order N will be tailored, while the other coefficients from $N+1$ till n_{max} will keep their values as the EGM2008 model. Accordingly, in order to get theoretically zero reduced anomalies, the left hand side of Eq. (3.2) is set to zero, and the contribution of the tailored reference model to the gravity anomalies Δg_{GM_T} is thus computed from the following expression

$$\Delta g_{GM_T}\Big|_{n=2}^{N} = \Delta g_F - \Delta g_{TI\,win} - \Delta g_{EGM2008}\Big|_{n=N+1}^{n_{max}} + \Delta g_{wincof}\Big|_{n=2}^{N} . \tag{3.3}$$

The large data gaps are filled, as mentioned above, by a $30' \times 30'$ underlying grid using the tailored model, generated through an iterative scheme. For the first iteration step, the underlying grid is generated from the tailored model AFR2013 (Abd-Elmotaal et al., 2015a) available to degree and order $N = 360$. Hence the free-air anomalies for the underlying grid are computed by

$$\Delta g_{F_{underlying}} = \Delta g_{GM_{AFR2013}}\Big|_{n=2}^{N} . \tag{3.4}$$

Eq. (3.3) is used to generate the contribution of the underlying grid to the gravity anomalies used to estimate the tailored reference model. For the successive iteration steps, the free-air anomalies are computed from the previously estimated tailored model as follows

$$\Delta g_{F_{underlying}} = \Delta g_{GM_{T_{i-1}}}\Big|_{n=2}^{N} + \Delta g_{EGM2008}\Big|_{n=N+1}^{n_{max}} . \tag{3.5}$$

The reduced gravity anomalies are interpolated using an unequal weight least-squares interpolation technique (Moritz, 1980) on a $30' \times 30'$ grid with the following standard deviations: $\sigma_{land} = 1$ mGal, $\sigma_{shipborne} = 3$ mGal, $\sigma_{altimetry} = 5$ mGal, $\sigma_{underlying\,grid} = 10$ mGal.
The lower harmonic coefficients of the tailored geopotential model for Africa, till degree and order N, have been computed using two harmonic analysis techniques:

- Least-squares harmonic analysis technique (Heck and Seitz, 1991),
- FFT harmonic analysis technique (Abd-Elmotaal, 2004).

The higher harmonic coefficients ($N+1 \le n \le n_{max}$) are kept as their values of EGM2008 model, as mentioned above (cf. Eq. (3.2)).
The process of creating a tailored reference model for Africa, used to generate the underlying grid at the large data gaps, has been iteratively implemented. The iteration process has been terminated after the second iteration where no significant improvement (standard deviation below 0.7 mGal) in the reduced anomaly field has been achieved. The latest tailored model is then used to generate the underlying grid (see sec. 4). More details on the creation of the tailored model and its iterative approach is found in Abd-Elmotaal et al. (2015b).
Figure 3.1 shows the reduced gravity anomalies for the AFRgeo_v1.0 geoid model, computed by Eq. (3.2), after being gridded on a $5' \times 5'$ grid using the interpolation technique described in sec. 4. These anomalies range between -466.5 mGal and 297.8 mGal with an average of -1.1 mGal and a standard deviation of 20.3 mGal. The white pattern in Fig. 3.1 indicates anomalies below 10 mGal in magnitude.

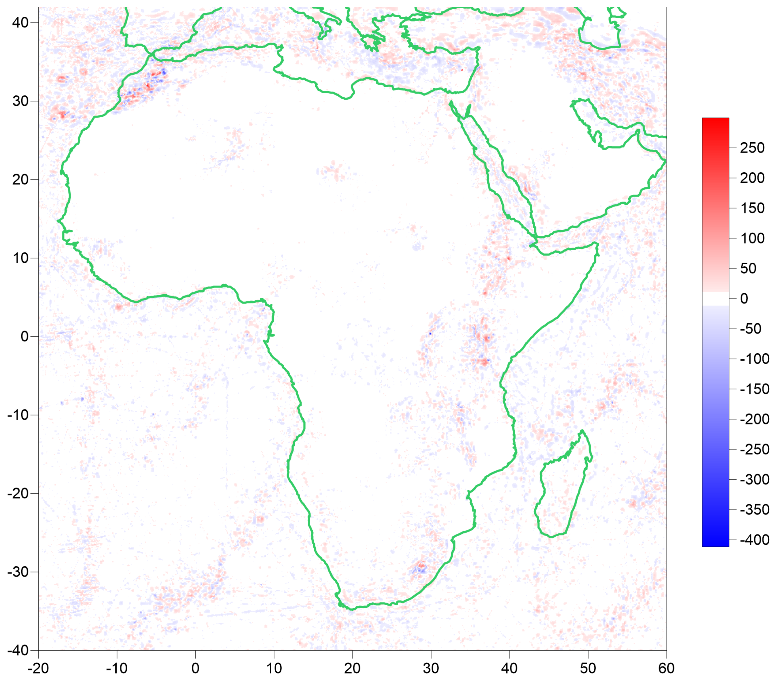


Figure 3.1: The $5' \times 5'$ reduced anomalies $\Delta g^{G}_{win\text{-}red}$ for the AFRgeo_v1.0 geoid model. Units in [mGal].

3.2 Gravity Reduction for AFRgeo-EIGEN-6C4 Geoid Model

The gravity reduction approach for the AFRgeo-EIGEN-6C4 geoid model avoids the use of the topographic-isostatic reduction and uses the EIGEN-6C4 global combined geopotential model, complete to degree and order 2190, as reference field. Accordingly, the reduction step is given by

$$\Delta g_{red} = \Delta g_F - \Delta g_{EIGEN\text{-}6C4} \Big|_{n=2}^{2190} , \quad (3.6)$$

where Δg_{red} refers to the reduced gravity anomalies, $\Delta g_{EIGEN\text{-}6C4}$ stands for the contribution of the EIGEN-6C4 global reference geopotential model, from degree 2 to degree 2190.
The free-air anomalies for the underlying grid $\Delta g_{F_{underlying}}$ are computed by

$$\Delta g_{F_{underlying}} = \Delta g_{GM_T} \Big|_{n=2}^{N} + \Delta g_{EGM2008} \Big|_{n=N+1}^{n_{max}} , \quad (3.7)$$

where Δg_{GM_T} is the contribution of the latest tailored reference model, computed in sec. 3.1, on a $30' \times 30'$ grid covering the African window ($40°S \leq \phi \leq 42°N$, $20°W \leq \lambda \leq 60°E$). Eq. (3.6) is then used to compute the reduced anomalies for the underlying grid for the AFRgeo-EIGEN-6C4 geoid model.
Figure 3.2 shows the reduced anomalies for the AFRgeo-EIGEN-6C4 geoid model, computed by Eq. (3.6), after being gridded on a $5' \times 5'$ grid using the interpolation technique described in sec. 4. These anomalies range between -779.3 mGal and 270.8 mGal with an average of -2.4 mGal and a standard deviation of 19.2 mGal. The white pattern in Fig. 3.2 indicates anomalies below 10 mGal in magnitude.
Table 3.1 illustrates the statistics of the free-air and reduced anomalies using the two different reduction techniques for each data region. Comparing Figs. 3.1 and 3.2 shows that the window-reduced anomalies for the AFRgeo_v1.0 geoid model are smaller in size than those for the AFRgeo-EIGEN-6C4 geoid model, especially at the continent region. Table 3.1 also confirms the above conclusion showing that

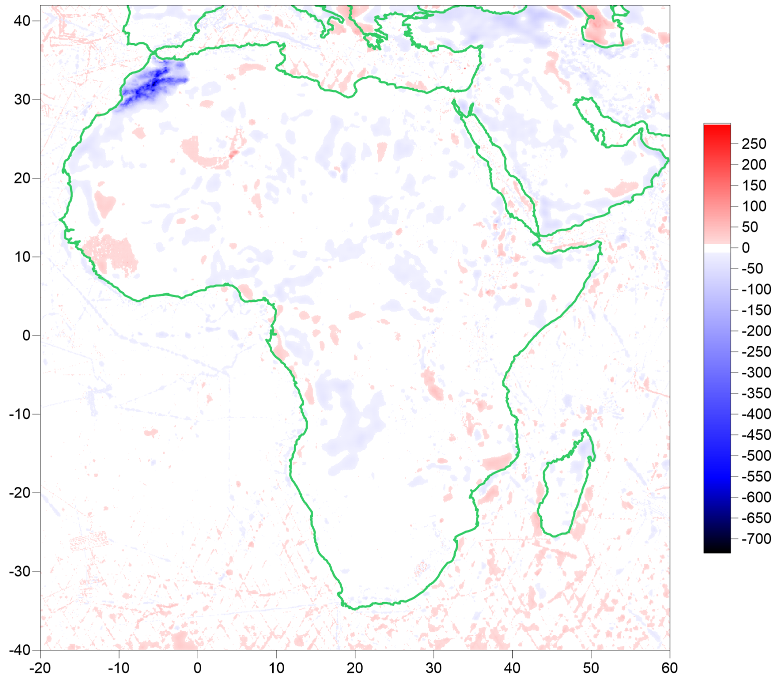


Figure 3.2: The $5' \times 5'$ reduced anomalies Δg^{G}_{red} for the AFRgeo-EIGEN-6C4 geoid model. Units in [mGal].

the range of the reduced anomalies for the AFRgeo_v1.0 geoid model is less than that for the AFRgeo-EIGEN-6C4 by 37%. Also the reduced anomalies for the AFRgeo_v1.0 are more centered than those for the AFRgeo-EIGEN-6C4. The standard deviation of the reduced anomalies for the AFRgeo_v1.0 geoid model is, however, only 1 mGal larger than that for the AFRgeo-EIGEN-6C4 geoid model.

Table 3.1: Statistics of the free-air and reduced gravity anomalies using the two different reduction techniques

Reduction technique and geoid model	Region	No. of data points	Statistical parameters			
			min mGal	max mGal	mean mGal	std mGal
Free-air	Land	96,472	−624.47	452.80	1.86	60.71
	Shipborne	971,945	−238.30	364.80	−6.48	40.43
	Altimetry	119,249	−172.23	172.73	3.98	18.59
	Total	1,187,666	−624.47	452.80	−4.76	41.05
window-reduced AFRgeo_v1.0	Land	96,472	−466.46	297.77	0.18	23.92
	Shipborne	971,945	−187.24	184.50	−1.57	20.64
	Altimetry	119,249	−158.73	137.51	1.90	12.66
	Total	1,187,666	−466.46	297.77	−1.08	20.31
no terrain reduction AFRgeo-EIGEN-6C4	Land	96,472	−779.34	270.80	−10.25	54.02
	Shipborne	971,945	−70.10	67.96	−2.53	11.56
	Altimetry	119,249	−161.79	97.45	5.37	10.03
	Total	1,187,666	−779.34	270.80	−2.36	19.17

4 Interpolation Technique

For both reduced anomalies types of the current investigation, an unequal weight least-squares interpolation technique (Moritz, 1980) on a $5' \times 5'$ grid covering the African window ($40°S \leq \phi \leq 42°N$, $20°W \leq \lambda \leq 60°E$) took place with the following standard deviations: $\sigma_{land} = 1$ mGal, $\sigma_{shipborne} = 3$ mGal, $\sigma_{altimetry} = 5$ mGal, $\sigma_{underlying\,grid} = 20$ mGal. The free-air anomalies for the underlying grid are computed using Eq. (3.7) on a $15' \times 15'$ unregistered grid with the final output grid for the data gaps *only* (i.e., the underlying grid is shifted by half of the grid cell of the output grid) employing the latest tailored reference model computed in sec. 3.1. The reduction strategies described in sec. 3 are then applied for both geoids under investigation. The interpolation process yields $5' \times 5'$ gridded reduced anomalies $\Delta g^G_{win\text{-}red}$ for the AFRgeo_v1.0 geoid model and $5' \times 5'$ gridded reduced anomalies Δg^G_{red} for the AFRgeo-EIGEN-6C4 geoid model.

It is worth mentioning that Figs. 3.1 and 3.2 showing the $5' \times 5'$ reduced anomalies $\Delta g^G_{win\text{-}red}$ and Δg^G_{red} for the AFRgeo_v1.0 and AFRgeo-EIGEN-6C4 geoid models, respectively, are generated from gridded anomalies using the unequal weight least-squares interpolation technique employing the relative standard deviations described above.

5 Geoid Determination

The contribution of the reduced gravity anomalies to the geoid $N^G_{\Delta g_{win\text{-}red}}$ and $N^G_{\Delta g_{red}}$ have been computed using the 1-D FFT technique (Haagmans et al., 1993; Sideris and Li, 1993) for the AFRgeo_v1.0 and AFRgeo-EIGEN-6C4 geoid models, respectively, on a $5' \times 5'$ grid covering the African window ($40°S \leq \phi \leq 42°N$, $20°W \leq \lambda \leq 60°E$). The different restore steps for the developed geoids within the current investigation are described in the following subsections.

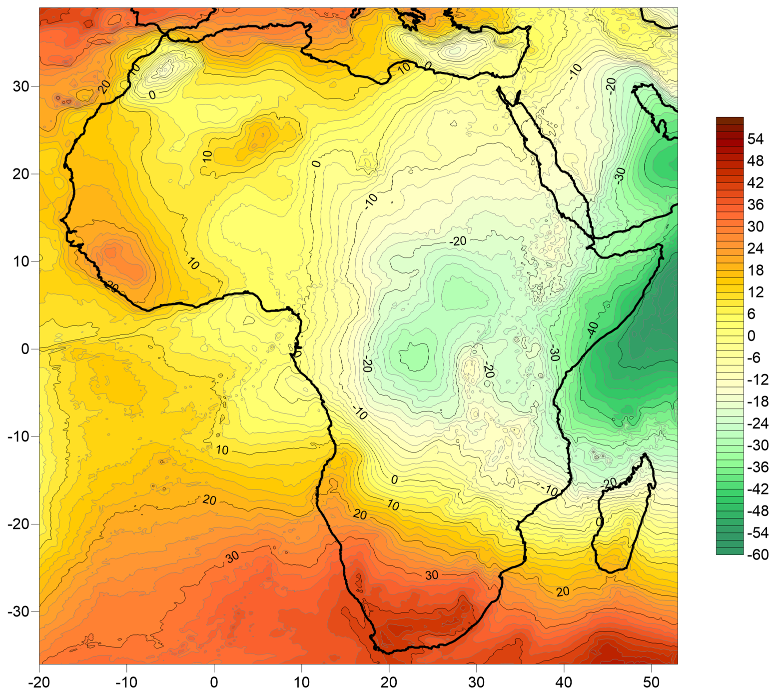


Figure 5.1: The AFRgeo_v1.0 geoid model before scaling. Contour interval: 2 m.

5.1 The AFRgeo_v1.0 Geoid Model

The restore step for the AFRgeo_v1.0 geoid model follows the window technique (Abd-Elmotaal and Kühtreiber, 1999, 2003). The geoid undulation on the computed $5' \times 5'$ grid $N^G_{AFRgeo\_v1.0}$ for the AFRgeo_v1.0 geoid model is mathematically given by

$$N^G_{AFRgeo\_v1.0} = N^G_{\Delta g_{win\text{-}red}} + N^G_{TI\,win} + N^G_{GM_T}\Big|_{n=2}^{N} + N^G_{EGM2008}\Big|_{n=N+1}^{n_{max}} - N^G_{wincof}\Big|_{n=2}^{N}, \tag{5.1}$$

where $N^G_{\Delta g_{win\text{-}red}}$ gives the contribution of the reduced gravity anomalies $\Delta g^G_{win\text{-}red}$ computed using the 1-D FFT technique, $N^G_{TI\,win}$ gives the contribution of the topography and its compensation (the indirect effect) for the same fixed data window as used for the remove step, N^G_{wincof} stands for the contribution of the dimensionless harmonic coefficients of the topographic-isostatic masses of the data window, $N^G_{EGM2008}$ gives the contribution of the EGM2008 geopotential model (from degree $N+1$ to degree n_{max}) and $N^G_{GM_T}$ is the contribution of the tailored geopotential model (from degree 2 to degree N).

Figure 5.1 illustrates the AFRgeo_v1.0 geoid model before scaling. The values of the AFRgeo_v1.0 before scaling range between -58.87 m and 49.12 m with an average of 3.83 m and a standard deviation of 21.70 m.

5.2 The AFRgeo-EIGEN-6C4 Geoid Model

The restore step for the case of the AFRgeo-EIGEN-6C4 geoid model is more simple. The geoid undulation on the computed $5' \times 5'$ grid $N^G_{AFRgeo\text{-}EIGEN\text{-}6C4}$ for the AFRgeo-EIGEN-6C4 geoid model can easily be given by

$$N^G_{AFRgeo\text{-}EIGEN\text{-}6C4} = N^G_{\Delta g_{red}} + N^G_{EIGEN\text{-}6C4}\Big|_{n=2}^{2190}, \tag{5.2}$$

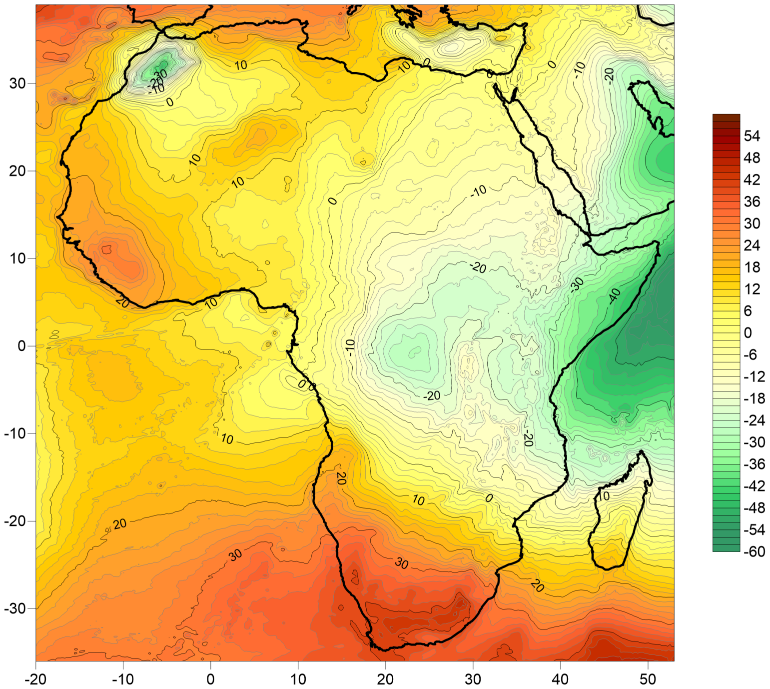


Figure 5.2: The AFRgeo-EIGEN-6C4 geoid model before scaling. Contour interval: 2 m.

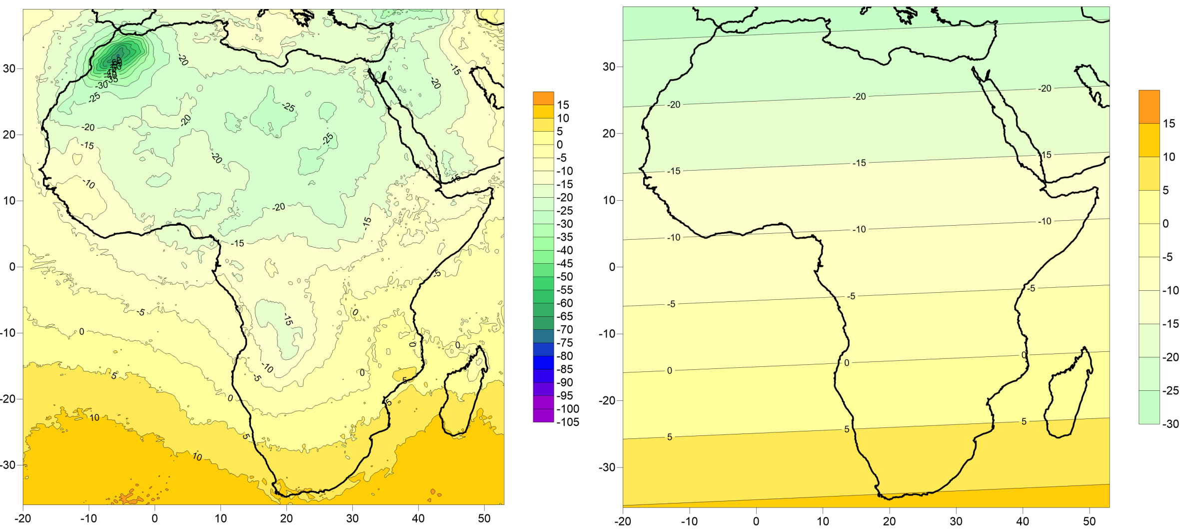
where $N^G_{\Delta g_{red}}$ stands for the contribution of the reduced gravity anomalies Δg^G_{red} computed using 1-D FFT technique and $N^G_{EIGEN\text{-}6C4}$ gives the contribution of the EIGEN-6C4 geopotential model (from degree 2 to degree 2190).

Figure 5.2 illustrates the AFRgeo-EIGEN-6C4 geoid model before scaling. The values of the AFR-geo-EIGEN-6C4 before scaling range between −58.24 m and 47.24 m with an average of 4.90 m and a standard deviation of 20.85 m.

6 Geoid Scaling

Unfortunately, there are no GNSS data available with known orthometric height covering the continent of Africa. Accordingly, the computed geoids are scaled using the GOCE satellite-only model GO_CONS_GCF_2_DIR_R5 (shortly in the sequel "DIR_R5 model"), complete to degree and order 280, which represents the best available global geopotential model approximating the African gravity field (Abd-Elmotaal, 2015). A planar trend surface, computed through a least-squares regression technique, has been used to scale the computed geoids within the current investigation.

Figure 6.1 (a) shows the difference between AFRgeo_v1.0 and DIR_R5 geoids. These differences range between −71.26 m and 16.93 m with an average of −7.90 m and a standard deviation of 12.41 m. Figure 6.1 (b) shows the planar trend surface computed by least-squares regression of the difference between AFRgeo_v1.0 and DIR_R5 geoid.



(a) (b)

Figure 6.1: (a) Difference between AFRgeo_v1.0 and DIR_R5 geoids, (b) plane trend surface computed by least-squares regression of the difference between AFRgeo_v1.0 and DIR_R5 geoid. Contour interval: 5 m.

Figure 6.2 shows the AFRgeo_v1.0 scaled geoid after removing the trend surface illustrated in Fig. 6.1 (b). The values of the AFRgeo_v1.0 geoid range between −49.86 m and 71.88 m with an average of 11.73 m and a standard deviation of 20.94 m.

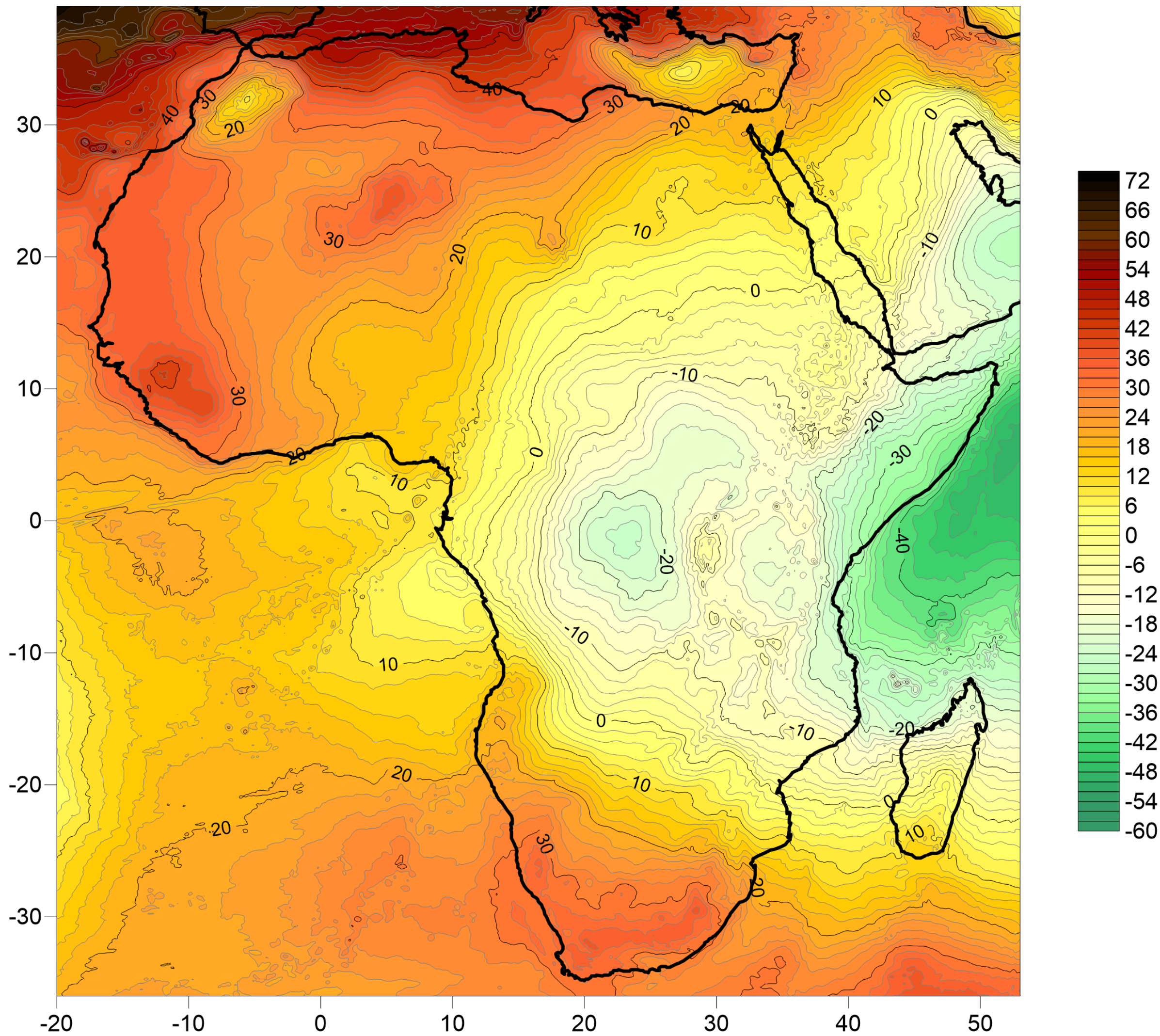
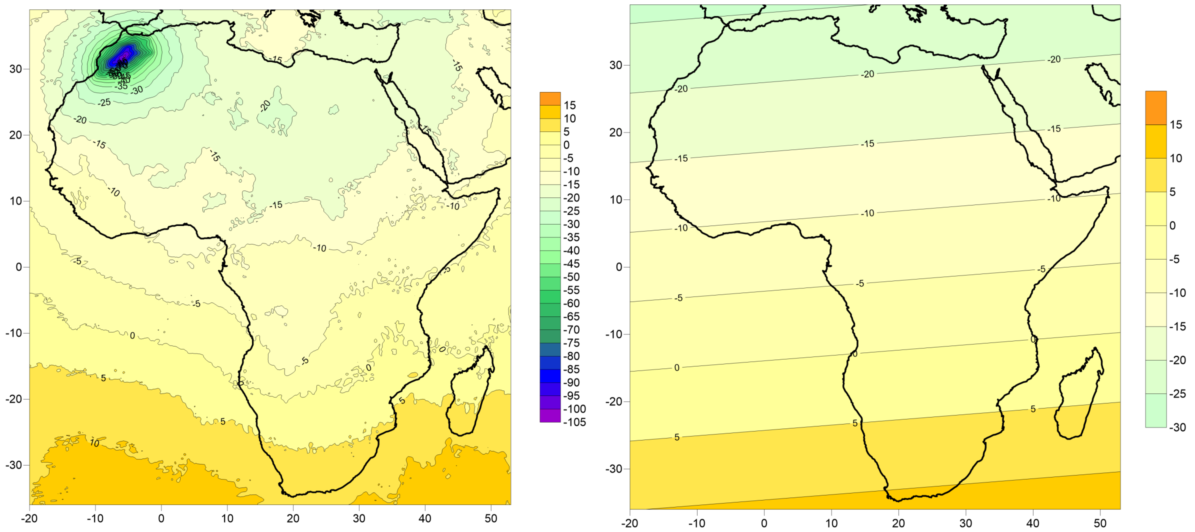


Figure 6.2: The AFRgeo_v1.0 scaled geoid model. Contour interval: 2 m.

Figure 6.3 (a) shows the difference between AFRgeo-EIGEN-6C4 and DIR_R5 geoids. These differences range between −100.95 m and 14.13 m with an average of −6.83 m and a standard deviation of 11.94 m. Figure 6.3 (b) shows the planar trend surface computed by least-squares regression of the difference between AFRgeo-EIGEN-6C4 and DIR_R5 geoid.



(a) (b)

Figure 6.3: (a) Difference between AFRgeo-EIGEN-6C4 and DIR_R5 geoids, (b) plane trend surface computed by least-squares regression of the difference between AFRgeo-EIGEN-6C4 and DIR_R5 geoid. Contour interval: 5 m.

Figure 6.4 shows the AFRgeo-EIGEN-6C4 scaled geoid after removing the trend surface illustrated in Fig. 6.3 (b). The values of the AFRgeo-EIGEN-6C4 geoid range between −51.03 m and 69.03 m with an average of 11.73 m and a standard deviation of 20.41 m.

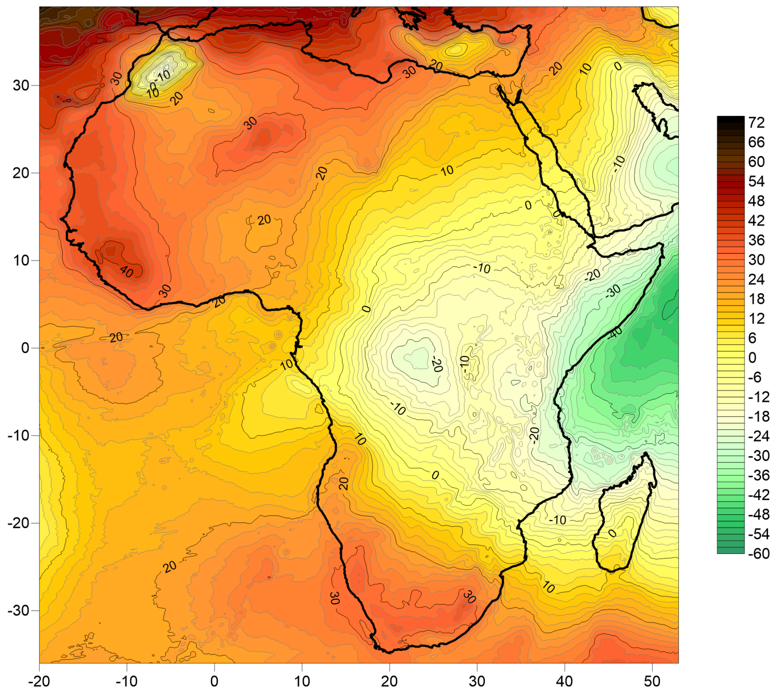


Figure 6.4: The AFRgeo-EIGEN-6C4 scaled geoid model. Contour interval: 2 m.

It should be noted that there is a strange anomaly behaviour in the high mountainous area of Morocco. This phenomenon is being under deep investigation.

7 Geoid Comparison

The first geoid model for Africa AGP2003 has been developed by Merry et al. (2005). Figure 7.1 illustrates the AGP2003 geoid model for Africa. The values of the AGP2003 geoid model range between −53.03 m and 56.13 m with an average of 11.97 m and a standard deviation of 20.26 m.
As the AGP2003 geoid model has used the EGM96 (Lemoine et al., 1998) as reference model, it is interesting to compute the difference between the AGP2003 and EGM96 geoid models. Figure 7.2 shows the difference between the AGP2003 geoid model and the geoid undulation computed by EGM96, complete to degree and order 360. These differences range between −10.96 m and 10.01 m with an average of −0.39 m and a standard deviation of 0.87 m. The light yellow pattern in Fig. 7.2 indicates differences below 2 m in magnitude. Figure 7.2 shows clearly that the AGP2003 is mainly the EGM96 geoid. This may come from the fact that the AGP2003 geoid model has been computed using $5' \times 5'$ mean gravity anomalies developed at Leeds University (cf. Merry et al., 2005).
It is rather more interesting to compare the AGP2003 geoid model with the AFRgeo_v1.0 geoid model developed in the current investigation. Figure 7.3 shows the difference between the scaled AFRgeo_v1.0 and the AGP2003 geoid models. These differences range between −12.26 m and 47.15 m with an average of 2.32 m and a standard deviation of 5.36 m. The light yellow pattern in Fig. 7.3 indicates differences below 5 m in magnitude. Figure 7.3 shows significant differences between the two geoids in the middle region of Africa. This is due to the fact that more point gravity data values are inherent in the solution of the AFRgeo_v1.0 geoid model. Accordingly, it is considered as a progress of the geoid model for Africa. Again, in the high mountainous part of Morocco there is a high anomaly difference, which is, as stated above, under deep investigation.

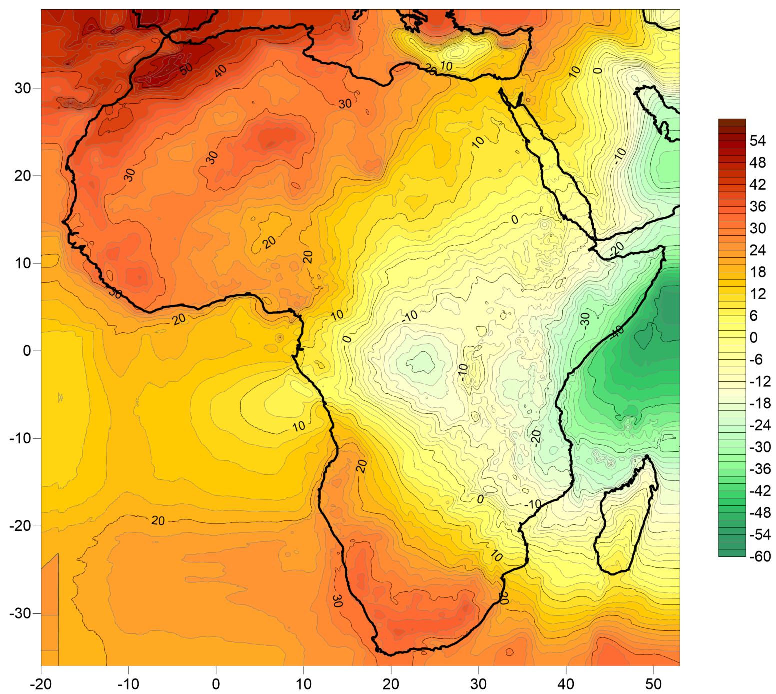


Figure 7.1: The AGP2003 geoid model, after Merry et al. (2005). Contour interval: 2 m.

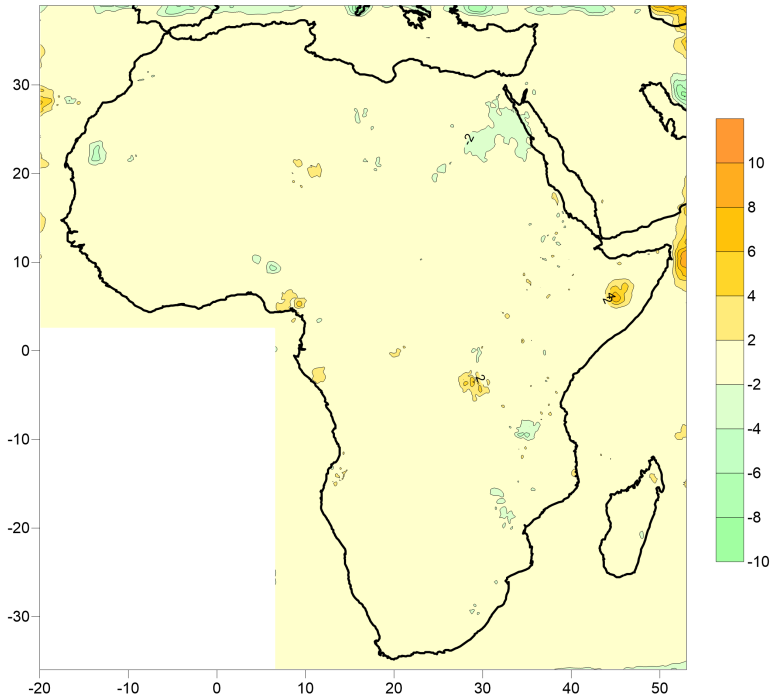


Figure 7.2: Difference between the AGP2003 geoid model and the geoid undulation computed by the EGM96 geopotential model (up to degree and order 360). Units in [m].

Let us compare the two geoids computed within the current investigation. Figure 7.4 shows the difference between the AFRgeo_v1.0 and AFRgeo-EIGEN-6C4 scaled geoid models. These differences range between −7.47 m and 31.66 m with an average of 3.75 m and a standard deviation of 3.24 m. The light yellow pattern in Fig. 7.4 indicates differences below 5 m in magnitude. Figure 7.4 illustrates that most

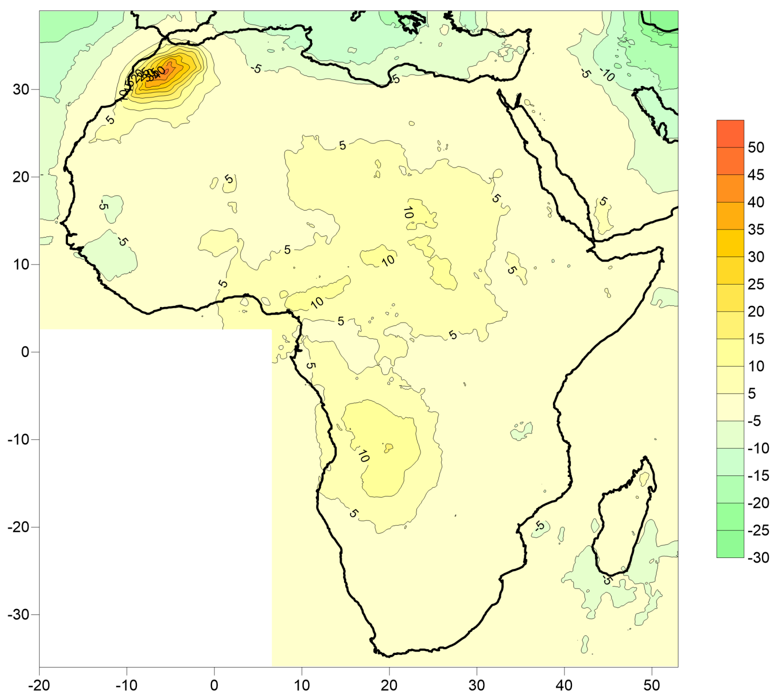


Figure 7.3: Difference between the scaled AFRgeo_v1.0 and the AGP2003 geoid models. Units in [m].

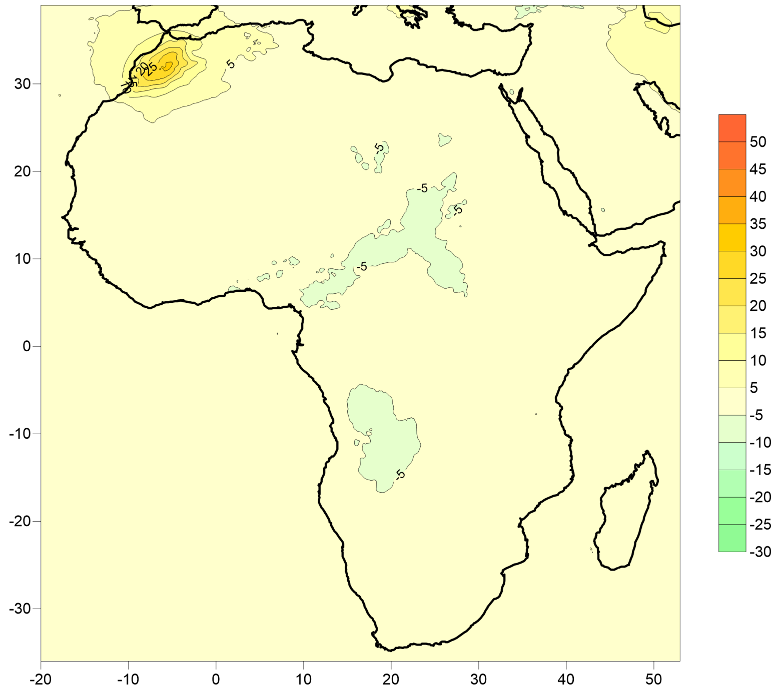


Figure 7.4: Difference between the AFRgeo_v1.0 and AFRgeo-EIGEN-6C4 scaled geoid models. Units in [m].

of the area, apart from Morocco and a small region in the middle Africa, has differences between the two geoids below 5 m.

8 Conclusions

In this paper, two geoid models for Africa have been determined. The first one, the AFRgeo_v1.0 geoid model, is based on the non-ambiguous window remove-restore technique employing a tailored geopotential model, complete to degree and order 2160, used in the reduction process as well as in filling the large data gaps by the so-called underlying grid. The process of computing the tailored model follows a lengthy iterative process. The second geoid model, AFRgeo-EIGEN-6C4, avoids the use of the topographic-isostatic reduction and uses the recent global combined geopotential model EIGEN-6C4, complete to degree and order 2190, as the reference field. It also used the same tailored model computed in the first approach to fill in the large data gaps.

The reduced anomalies for the AFRgeo_v1.0 geoid model are better (especially on land) because they are centered, smoother and have less range than those of the AFRgeo-EIGEN-6C4 geoid model (cf. Figs. 3.1 and 3.2). Hence, they give less interpolation errors, especially in the large gravity data gaps. The reduced gravity data for both geoid models were interpolated using an unequal least-squares interpolation technique, giving the land data the highest precision, the sea data a moderate precision and the underlying grid the lowest precision.

The geoids are computed using the 1-D FFT technique, and the proper restore steps have been carried out for both geoids at hand. The computed geoids have been scaled to the DIR_R5 GOCE model. The computed AFRgeo_v1.0 geoid model for Africa has been compared to the former geoid model AGP2003, where significant improvements are obtained. The two geoids computed in the current investigation are compared, where some intermediate differences are visible.

Finally we would like to introduce the AFRgeo_v1.0 scaled geoid model as the new geoid model for Africa, because its reduced gravity anomalies are smoother, which produce less interpolation errors as stated above. It is worth mentioning here that in order to have a better judge on the geoid quality of Africa, a reasonably well distributed GNSS network with known orthometric heights is essentially needed. The International Association of Geodesy (IAG) and its sister bodies are cordially invited to help in achieving this important goal.

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References

Abd-Elmotaal, H. A. (1998): An alternative capable technique for the evaluation of geopotential from spherical harmonic expansions. *Bollettino di Geodesia e Scienze Affini* 57(1):25–38.

Abd-Elmotaal, H. A. (2004): An Efficient Technique for Harmonic Analysis on a Spheroid (Ellipsoid and Sphere). *VGI–Österreichische Zeitschrift für Vermessung und Geoinformation* 92(3-4):126–135.

Abd-Elmotaal, H. A. (2007): Reference Geopotential Models Tailored to the Egyptian Gravity Field. *Bollettino di Geodesia e Scienze Affini* 66(3):129–144.

Abd-Elmotaal, H. A. (2014): Egyptian Geoid using Ultra High-Degree Tailored Geopotential Model. *Proceedings of the 25th International Federation of Surveyors FIG Congress, Kuala Lumpur, Malaysia, June 16–21, 2014*. URL: www.fig.net/pub/fig2014/papers/ts02a/TS02A_abd-elmotaal_6856.pdf.

Abd-Elmotaal, H. A. (2015): Validation of GOCE Models in Africa. *Newton's Bulletin* 5:149–162. URL: http://www.isgeoid.polimi.it/Newton/Newton_5/11_Hussein_149_162.pdf.

Abd-Elmotaal, H. A. and Kühtreiber, N. (1999): Improving the geoid accuracy by adapting the reference field. *Phys Chem Earth Pt A* 24(1):53–59. DOI: 10.1016/S1464-1895(98)00010-6.

Abd-Elmotaal, H. A. and Kühtreiber, N. (2003): Geoid Determination Using Adapted Reference Field, Seismic Moho Depths and Variable Density Contrast. *Journal of Geodesy* 77(1-2):77–85. DOI: 10.1007/s00190-002-0300-7.

Abd-Elmotaal, H. A. and Kühtreiber, N. (2014): Automated Gross Error Detection Technique Applied to the Gravity Database of Africa. *Geophysical Research Abstracts* 16, EGU General Assembly 2014:92. URL: http://meetingorganizer.copernicus.org/EGU2014/EGU2014-92.pdf.

Abd-Elmotaal, H. A. and Kühtreiber, N. (2015): On the Computation of the Ultra-High Harmonic Coefficients of the Topographic-Isostatic Masses within the Data Window. *Geophysical Research Abstracts* 17, EGU General Assembly 2015:355. URL: http://meetingorganizer.copernicus.org/EGU2015/EGU2015-355.pdf.

Abd-Elmotaal, H. A. and Makhloof, A. (2013): Gross-Errors Detection in the Shipborne Gravity Data Set for Africa. *Geodetic Week, Essen, Germany, October 8–10, 2013*. URL: http://geodaetische-Woche.de/2013/session02/Abd-Elmotaal-Makhloof.pdf.

Abd-Elmotaal, H. A. and Makhloof, A. (2014): Combination between Altimetry and Shipborne Gravity Data for Africa. *3rd International Gravity Field Service (IGFS) General Assembly, Shanghai, China, June 30 – July 6, 2014*.

Abd-Elmotaal, H. A., Seitz, K., Abd-Elbaky, M., and Heck, B. (2015a): Tailored Reference Geopotential Model for Africa. *International Association of Geodesy Symposia* 143:383–390. DOI: 10.1007/1345_2015_84.

Abd-Elmotaal, H. A., Seitz, K., Kühtreiber, N., and Heck, B. (2015b): Establishment of the Gravity Database AFRGDB_V1.0 for the African Geoid. *International Association of Geodesy Symposia* 144:131–138. DOI: 10.1007/1345_2015_51.

Farr, T., Rosen, P., Caro, E., Crippen, R., Duren, R., Hensley, S., Kobrick, M., Paller, M., Rodriguez, E., Roth, L., Seal, D., Shaffer, S., Shimada, J., Umland, J., Werner, M., Oskin, M., Burbank, D., and Alsdorf, D. (2007): The Shuttle Radar Topography Mission. *Reviews of Geophysics* 45(RG2004). DOI: 110.1029/2005RG000183.

Forsberg, R. (1984): A Study of Terrain Reductions, Density Anomalies and Geophysical Inversion Methods in Gravity Field Modelling. *Ohio State University, Department of Geodetic Science and Surveying.* Rep 355.

Förste, C., Bruinsma, S., Abrikosov, O., Lemoine, J.-M., Schaller, T., Götze, H.-J., Ebbing, J., Marty, J., Flechtner, F., Balmino, G., and Biancale, R. (2014): EIGEN-6C4 The Latest Combined Global Gravity Field Model Including GOCE Data Up to Degree and Order 2190 of GFZ Potsdam and GRGS Toulouse. *5th GOCE User Workshop, Paris, November 25–28, 2014*.

Haagmans, R., de Min, E., and van Gelderen, M. (1993): Fast Evaluation of Convolution Integrals on the Sphere Using 1D FFT, and a Comparison with Existing Methods for Stokes' Integral. *Manuscripta Geodaetica* 18(4):227–241.

Heck, B. and Seitz, K. (1991): Harmonische Analyse. *Geodetic Institute, University of Karlsruhe.* Technical Report.

Kraiger, G. (1988): Influence of the Curvature Parameter on Least-Squares Prediction. *Manuscripta Geodaetica* 13(3):164–171.

Lemoine, F., Kenyon, S., Factor, J., Trimmer, R., Pavlis, N., Chinn, D., Cox, C., Klosko, S., Luthcke, S., Torrence, M., Wang, Y., Williamson, R., Pavlis, E., Rapp, R., and Olson, T. (1998): The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96. *NASA/TP-1998-206861, NASA Goddard Space Flight Center, Maryland*.

Merry, C. L., Blitzkow, D., Abd-Elmotaal, H. A., Fashir, H., John, S., Podmore, F., and Fairhead, J. (2005): A preliminary geoid model for Africa. *A Window on the Future of Geodesy, International Association of Geodesy Symposia* 128:374–379. DOI: 10.1007/3-540-27432-4_64.

Moritz, H. (1980): Advanced Physical Geodesy. Wichmann, Karlsruhe. ISBN: 978-3879071951.

Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K. (2012): The development and evaluation of the Earth Gravitational Model 2008 (EGM2008). *J Geophys Res* 117(B04406). DOI: 10.1029/2011JB008916.

Rapp, R. H. (1982): A Fortran Program for the Computation of Gravimetric Quantities From High Degree Spherical Harmonic Expansions. *Ohio State University, Department of Geodetic Science and Surveying.* Rep 334.

Sideris, M. G. and Li, Y. C. (1993): Gravity field convolutions without windowing and edge effects. *Bulletin Géodésique* 67(2):107–118. DOI: 10.1007/BF01371374.

Tscherning, C. C., Knudsen, P., and Forsberg, R. (1994): Description of the GRAVSOFT Package. *Geophysical Institute, University of Copenhagen.* Technical Report.

Tscherning, C. C., Rapp, R. H., and Goad, C. (1983): A Comparison of Methods for Computing Gravimetric Quantities From High Degree Spherical Harmonic Expansions. *Manuscripta Geodaetica* 8:249–272.

Weber, G. and Zomorrodian, H. (1988): Regional geopotential model improvement for the Iranian geoid determination. *Bulletin Géodésique* 62:125–141. DOI: 10.1007/BF02519221.

Wenzel, H. (1998): Ultra High Degree Geopotential Models GPM98A, B and C to Degree 1800 Tailored to Europe. *Reports of the Finnish Geodetic Institute* 98(4):71–80.

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