

Stochastic programming models for designing logistic networks

Modelos de programación estocásticos para el diseño de redes logísticas

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ABSTRACT

In a supply chain, logistical management is the planning stage, in which, the physical distribution of products, scheduling and supporting manufacturing operations, planning and activities are related with the final products and certain materials are organized for getting the wished results. The problem of designing a logistical network, in many cases, is associated to the correct formulation of the model according to the context in which, the systems develops; from the incursion of all the possible variables that can affect the processes, will depends the success of the said model and truthful results. This paper shows the design of a logistical network and the introduction of stochastic constrains in its programming, the random variables that change through the time with the natural variability associated to the processes, this with the purpose of responding more precisely to the reality inherent to the systems inside the organizations.

Key words: Logistic networks, Stochastic constraints, Programming, Random Variables.

RESUMEN

En la cadena de Abastecimiento, la gestión logística es la etapa de planificación, en la cual, la distribución física de productos, programación y apoyo los procesos de fabricación, la planeación y las actividades se relacionan con los productos terminados y materiales determinados y organizados para obtener el mejor resultados. El problema de diseñar una red logística, en muchos casos, se asocia a la formulación correcta del modelo en función del contexto, desde el desarrollo de los sistemas hasta la inclusión de todas las posibles variables que pueden afectar los procesos. De todo esto, depende el éxito de su construcción y los resultados veraces. Este trabajo muestra el diseño de una red logística y la aplicación de restricciones estocásticas en su programación, las variables aleatorias cambian todo el tiempo e incluso la variabilidad natural asociada a los procesos, esto con el fin de responder efectivamente la realidad de los sistemas dentro de las organizaciones.

Palabras Clave: Redes logísticas, Restricciones estocásticas, Programación, Variables aleatorias.

1. Introduction

In the most basic definition, logistic consists in the efficient and effective planning, implementation and control of strategies for directing suitably the flow and storage of products in a system. Designing a logistic network is one of the big purposes of the logistical management. In this stage, experts determine and study all the possible scenarios that may appear, the results previously obtained and the future changes, with the purpose of making a model that adjusts faithfully to the reality. This way, the results of the implementation of the configuration given by the model, once this is programmed to find solutions feasible, must be consequent with the awaited results.

At the time of identifying the variables, restrictions, and all the elements of the system, it is necessary to visualize that most of the variables and restrictions respond to the natural variability of the processes and scenes. By simplicity, this variability is avoided, coverall in problems of great scale is that the experts consider appropriate to define the parameters of the problem like values exactly well-known or approximated, or to foretell with a small margin of error. This with the purpose of reducing the time of solution and for a problem easier to solve; nevertheless, the

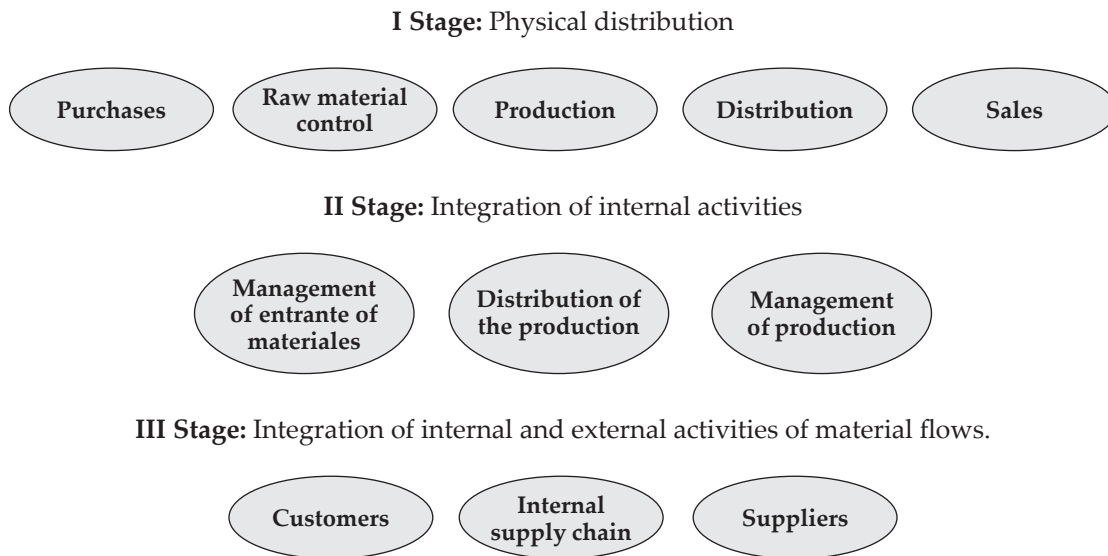
results are not very necessary. To add to the system a stochastic touch allows taking the parameters like elements that follow a probability distribution and therefore, must be modeled like varieties; this evidently makes difficult the problem, but it allows obtaining more realistic results.

It is as well as, like conclusion, for obtaining a good logistic network design is required: data, the construction of the associated model, computational tools for finding an optimal solution and a precise but exhaustive analysis of its processes and results.

2. Related Works

A very important element of the planning activities of a manufacturing firm is the efficient design and operation of its logistic network. Logistic network is a network of suppliers, manufacturing plants, warehouses, and distribution channels organized to acquire raw materials, convert these raw materials to finished products, and distribute these products to customers.

Stages of the conception of logistics networks inside the companies



Strategic level for logistic network planning involves deciding the configuration of the network, i.e. the number, location, capacity, and technology of the facilities. The tactical level planning of logistic network operations involves deciding the aggregate quantities and material flows for purchasing, processing, and distribution of products. The strategic configuration of the logistic network is a key factor influencing efficient tactical operations, and therefore

has a long lasting impact on the firm. Furthermore, the fact that the logistic network configuration involves the commitment of substantial capital resources over long periods of time makes the logistic network design problem an extremely important one.

A logistic network is a distribution options that functions to procure materials, transform these materials into inter-

mediate and finished products, and distribute these finished products to customers. Logistic network exist in both service and manufacturing organizations, although the complexity of the chain may vary greatly from industry to industry and firm to firm.

Traditionally, marketing, distribution, planning, manufacturing, and the purchasing of organizations along the logistic network operate independently. These organizations have their own objectives and they are often conflicting logistic network management is a strategy through which such an integration can be achieved. Supply chain management is typically viewed to lie between fully vertically integrated firms.

For a long time, the analysis of investigation of operations have created techniques for the study of such networks many problems have modeled themselves with excellent result and have managed themselves to optimize their better operation for the utilization of the resources.

Some of the studies that have been done about logistic network is the study made by Jean Francois Cordeau and Federico Pasin in their paper An Integrated Model for Logistics Network Design which introduce a new formulation of the logistics network design problem encountered in deterministic, single-country, single-period contexts. [1]

Mehmet Ferhat Candas and Erhan Kutanoglu approach design and inventory stocking problem as characterized by the interdependency of the design and stocking decisions in service parts logistics in their paper called how Benefits of Considering Inventory in Logistics Network Design Problems with Time-based Service Constraints. [2]

There exists an amount of problems in practices it in where no single it is possible to be assumed that the data associate are deterministic is to say that is, that the data pertaining to it such as flying times, refueling durations, maintenance time requirements, costs, etc., are known with certainty, but that there are times in where they work with stochastic data as it is the case in the following work.

Subjects like these have been studied by different authors, Tjendera Santoso, Shabbir Ahmed, Marc Goetschalckx, Alexander Shapiro in their paper A stochastic programming approach for supply chain network design under uncertain. They propose a stochastic programming model and solution algorithm for solving supply chain network design problems of a realistic scale. [3]

Julien Granger, Ananth Krishnamurthy, and Stephen M. Robinson describe a computational experiment directed at the problem of improving a stochastic network such as

those found in logistics planning in their paper Approximation and Optimization for Stochastic Networks. [4]

The doctor in psychology, industrial and system engineer Ming Dong proposes that Supply chain management has been recently introduced to address the integration of organizational functions ranging from the ordering and receipt of raw materials throughout the manufacturing processes, to the distribution and delivery of products to the customer to achieve higher quality products, better customer service, and lower inventory cost. [5]

Ovidiu Listes and Rommert Dekker in their paper a stochastic approach to a case study for product Recovery network design present a stochastic programming based approach by which a deterministic location model for product recovery network design may be extended to explicitly account for the uncertainties. [6]

S. H. Owen and M. S. Daskin developed a strategic level in their paper At the strategic level, there is a great deal of research in the facility location component of supply chain network design under uncertainty. [7]

G. J. Gutierrez, P. Kouvelis, and A. A. Kurawala. proposed a robust optimization framework for network design under uncertainty. This approach seeks network configurations that are good (nearly optimal) for a variety of scenarios of the design parameters at the expense of being suboptimal for any one scenario in their paper A robustness approach to uncapacitated network design problems. [8]

The authors of Partitioning procedures for solving mixed variables programming problems proposed a modification of the Benders decomposition algorithm and A. M. Geoffrion and G.W. Graves commonly used for deterministic network design problems, to generate robust designs. [9]

S. A. Mir Hassani, C. Lucas, G. Mitra, E. Messina, and C. A. Poojari. In their paper considered a two-stage model for multi-period capacity planning of supply chain networks. Here the first stage decisions, comprised of openings and closings of the plants and distribution centers and setting their capacity levels, are to be decided prior to the realization of future demands. Then, based upon the particular demand scenario realized, the production and distribution decisions are to be decided optimally. The overall objective is to minimize the cost of the first-stage strategic decisions and the expected production and distribution costs over the uncertain demand scenarios. [10]

To continuation we present the description, formulation y develop a stochastic modeling to logistic network with the objective to study and know to developed.

3. Definitions

The word stochastic is synonym of the word aleatory. A stochastic system is a system that is developed in the time while it goes at random by fluctuations. By mathematical definition:

A stochastic process is a collection of random variables. That is, for each t in the index set T , $X(t)$ is a random variable. We often interpret t as time and call $X(t)$ the state of the process at time t .

4. Formulation of the problem

PLASTIC HOUSE COMPANY, is a company specialized in the production and distribution of products made in plastic base. Actually offers a range of products that consists basically of 3 types of toys, A, B and C. Each one of that products pass by 3 stages: Assembly, testing and sale. There are two lines of production. In the line 1, the product A and B are assembled and tested. This line has a maximum capacity of 120 hours available by week. The line 2 is exclusive for assembling and testing the product C and has a maximum capacity of 48 hours available by week.

The total time of production has been considered and corresponds for A, B and C of this way: 10, 15, and 20 hour of work respectively.

The company has an initial inventory in warehouse of 22 units of product A, 42 units of product B and 36 units of product C. The cost of maintaining one unit in stock is: for A of 9 monetary units, for B of 10 monetary units and C of 18 monetary units. The cost of acquisition of raw material to produce A, B and C is 30, 30 and 70 monetary units respectively. The cost of transports of products to the final client is of 22, 22 and 30 for A, B and C respectively.

The capacity of the transporting truck cannot exceed the 250 units altogether (A, B and C without any distinction). The sale price of each product has been considered in: 350 monetary units for the product A, 470 for product B and 610 for product C.

The demand follows a probability distribution. For this case, the aleatority is included by means of stochastic constraints. Demand in the first week is known with certainty, but management is very uneasy about demand in the second week. In particular, it is unsure about the impact of a large advertising campaign on the quantities of A, B and C that the company will be able to sell in the second week. Analysis of previous advertising campaigns and intuitive estimates by marketing personnel has identified three radically different scenarios, as show as follows:

Table 1. Demand forecasts

WEEK1		WEEK2		
Product	Demand	Product	Low Demand	Probability
A	60	A	40	0.25
B	40	B	30	
C	50	C	30	
		Product	Medium Demand	
		A	80	0.50
		B	60	
		C	40	
		Product	High Demand	
		A	100	0.25
		B	120	
		C	100	

5. Mathematical description

Description of the variables:

i = Line, j =product, K =week. $i=\{1,2\}$, $j=\{1,2,3\}$, $k=\{1,2\}$, $e=\{1,2,3\}$

L_i = Capacity of the line i

L_1 = Capacity of the line 1

L_2 = Capacity of the line 2

P_j = Number or units of j , assembled, tested and sold during the weeks.

P_1 = Number or units of A, assembled, tested and sold during the weeks.

P_2 = Number or units of B, assembled, tested and sold during the weeks.

P_2 = Number or units of B, assembled, tested and sold during the weeks.

S_j = Sales Price of product j

S_1 = Sales Price of product 1

S_2 = Sales Price of product 2

S_3 = Sales Price of product 3

Q_{S_j} = Quantity of product j sold

Q_{S_1} = Quantity of product 1 sold

Q_{S_2} = Quantity of product 2 sold

Q_{S_3} = Quantity of product 3 sold

Q_{n_j} = Quantity of product j in beginning inventory

Q_{n_1} = Quantity of product 1 in beginning inventory

Q_{n_2} = Quantity of product 2 in beginning inventory

Q_{n_3} = Quantity of product 3 in beginning inventory

Q_{fj}= Quantity of product j in final inventory
 Q_{f1}= Quantity of product 1 in final inventory
 Q_{f2}= Quantity of product 2 in final inventory
 Q_{f3}= Quantity of product 3 in final inventory

IC_j= Inventory costs for product j
 IC₁= Inventory costs for product 1
 IC₂= Inventory costs for product 2
 IC₃= Inventory costs for product 3

FC_j= Fixed costs for product j
 FC₁= Fixed costs for product 1
 FC₂= Fixed costs for product 2
 FC₃= Fixed costs for product 3

NP_k= net profit for week k
 NP₁= net profit for week 1
 NP₂= net profit for week 2

LH_j= labor hours for product j
 LH₁= labor hours for the product 1
 LH₂= labor hours for the product 2
 LH₃= labor hours for the product 3

Labor= total quantity of hours available

D_{jke}=Demands for product j Week k scenario e
 D₁₁₁=Demands for product 1 Week 1 scenario 1
 D₁₁₂=Demands for product 1 Week 1 scenario 2
 D₁₂₂=Demands for product 1 Week 2 scenario 2
 D₁₃₂=Demands for product 1 Week 3 scenario 2
 D₂₁₁=Demands for product 2 Week 1 scenario 1
 D₂₁₂=Demands for product 2 Week 1 scenario 2
 D₂₂₂=Demands for product 2 Week 2 scenario 2
 D₂₃₂=Demands for product 2 Week 3 scenario 2
 D₃₁₁=Demands for product 3 Week 1 scenario 1
 D₃₁₂=Demands for product 3 Week 1 scenario 2
 D₃₂₂=Demands for product 3 Week 2 scenario 2
 D₃₃₂=Demands for product 3 Week 3 scenario 2

Objective Function

The mathematical formulation of the problem outlined responds to the following functions objective:

$$(1) \text{ Maximizing}(NP_1) = \sum_{j=1}^n P_j Qs_j - \sum_{j=1}^n IC_j Qf_j - \sum_{j=1}^n FC_j$$

This equation is for finding the net profit

$$(2) \text{ Maximizing}(NP_2) = NP_1 + \sum_{e=1}^n (\sum_{j=1}^n P_j Qs_j - \sum_{j=1}^n IC_j Qn_j - \sum_{j=1}^n FC_j) * P_e$$

Where e is the scenario, e{1,2,3}
 This equation is for finding the total net profit

Constraints

Line test capacity

$$(3) \sum_{j=1}^n P_j \leq L_i \quad i \in I$$

Labor capacity

$$(4) \sum_{j=1}^n P_j LH_j \leq \text{Labor}$$

Inventory

$$(5) \sum_{j=1}^n Qn_j + \sum_{j=1}^n P_j - Qs_j = Qn_j, \forall j \in J$$

Demand

$$(6) P_j \leq D_{jke}, \quad j \in J$$

To balance the network we first performed a single simulation run to collect data. Demand in the first week is known with certainty, but management is very uneasy about demand in the second week. The following table shows the results of that run.

Table 2. Results of the first week

Description	A	B	C
Quantity	38	82	20
Sales units	60	40	50
Ending inventory	0	84	6
Total profit	\$ 60.696		

The objective function is optimized in total net profit, which for the first week equals a \$ 69.361. As can be seen from this solution, quantities assembled during the first week and the quantities of sales of A, B, C and the quantities ending inventory.

For this model the quantities ending inventory for the first week are the Beginning inventory for each scenarios in the next week.

The following tables show the results of the week 2, scenarios 1, 2, 3:

Table 3. Results of scenarios 1

Description	A	B	C	OT
Quantity	56	64	24	
Overtime				0
Sales units	40	30	30	
Ending inventory	16	118	0	
Total profit	\$ 37.356			

Table 4. Results of scenarios 2

Description	A	B	C	OT
Quantity	96	24	34	
Overtime				0
Sales units	80	60	40	
Ending inventory	16	48	0	
Total profit	\$ 69.716			

Table 5. Results of scenarios 3

Description	A	B	C	OT
Quantity	100	20	48	
Overtime				260
Sales units	100	104	54	
Ending inventory	0	0	0	
Total profit	\$ 98.552			

For each of the three scenarios the model added a contingency decision option, labor time. This decision variable was used in the scenario 3 with 260 of over time.

The objective function is the first week net profit to the expected value of the second week net profit summed over the three scenarios. The total expected net profit is **\$129.531**

The value mentioned before is associated with the total net profit earned during the two weeks, taking into account that the profit of the second week has been estimated by assuming certain demand for it and using some probabilities in order to calculate it.

6. Conclusions

In this paper, we have developed a practical methodology for stochastic programming models for a logistic network. The proposed stochastic programming model may be extended to an arbitrary number of periods, although we must be concerned about controlling its size.

A stochastic programming model brings into clearer focus the need of identify and incorporate contingency options for different types of scenarios.

We have provided empirical results for the design realistic logistics networks. Furthermore, we have demonstrated that the solutions identified by the proposed method are not only superior to traditional mean-value problem solutions in an expectation sense.

About the linear programming, it has studied different models applied to deterministic scenarios its say, under given circumstances nevertheless, the principal disadvantage of this models is its limited capacity of reaction to determinate change of the variables that take part in the problem.

Nowadays, the decision maker it has much more tools to make decisions respect the future. The stochastic programming is one of those tools that allows analyze different scenarios where each one has assigned a determinate probability of occurrence, the idea, is design or build models that represent those possible scenarios and with base in that, determine contingency plans for each one.

The stochastic programming is a excellent option to design of a strategic plan because, allow predict and analyze all the possible causes of variation of the scenarios with determinate probability associated and establish according with the model a program of control of risks.

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