Causal nonseparability and the structure of spacetime

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1 Introduction

Quantum entanglement is a key ingredient of quantum mechanics, and might even be central for the very distinction between the quantum and classical theories. In a composite system made of several single sub-parts, this phenomenon refers to the situation for which it is impossible to attribute independent definite states to each of the sub-systems. Instead, the composite system has to be considered as a whole, and its sub-parts are said to be non-separable. Philosophically, quantum entanglement has important implications. First of all, it is at the core of the main conceptual puzzle arising from the development of non-classical physics, known as the measurement problem. Secondly, entanglement allows for the observation of very peculiar, non-classical correlations between spacelike-separated events, which cannot be explained by any local causal model and are thus said to be nonlocal. Overall, many interpretations of quantum mechanics have been developed in order to solve the measurement problem and provide an account for nonlocality. Within a realist framework, which postulates a direct link between a theory's ontology and that of the objective world, each interpretation commits to a particular set of assumptions regarding the properties of reality and space-time, by specifying the ontology of the theory and its dynamics. Which of those interpretations is the most successful is still an ongoing debate.

Yet, entanglement is not an exclusive property of standard quantum mechanics. Instead, it is also central in further theoretical developments of quantum physics, e.g. quantum field theory, and is expected to remain an important feature of quantum gravity. Investigating the philosophical implications of entanglement in a broader theoretical context could shed a new light on the conceptual problems in quantum theory. Indeed, since (i) the way entanglement and nonlocality are accounted for is partly conditioned by our conception of spacetime, and (ii) a radical shift in our conception of spacetime is expected to take place as quantum physics is developed into more general theories (where gravity would be ultimately taken into account), looking at entanglement in new theoretical frameworks generalizing standard quantum mechanics could help us understand the nature of entanglement and its connection to space and time in a radically new way. Identifying the relation between entanglement and spacetime is crucial for developing a consistent ontology of reality.

For these reasons, this work focuses on a particular theoretical development of quantum mechanics, called the process matrix formalism, in which no assumption is made about the global causal structure of the studied system. That broader theoretical context allows for the existence of quantum processes (a generalization of the concept of quantum state allowing to represent joint states over multiple parties without specifying *a priori* their spatio-temporal locations) that are causally non-separable, in direct analogy with the spatial non-separability involved in entangled systems. For a causally non-separable process, there is no definite causal order among its interacting elements. The corresponding causal structure is said to be indefinite. Therefore, the notion of indefinite causal structure somehow extends the notion of entanglement to the geometry of spacetime. In operational terms, the probability distribution encapsulating the results of measurements performed on certain causally nonseparable processes may violate the causal equivalent of Bell inequalities, called causal inequalities. Such a distribution is said to be non-causal.

The goal of this paper is to discuss the connection between the notions of quantum and causal nonseparability, and, in a realist framework, have a preliminary reflection regarding their potential implications for the world's ontology. Four main questions will be developed:

- A formal analogy with quantum nonseparability is at the basis of the articulation of the notion of causal nonseparability. What are the extent and the limits of this formal analogy?

- Is this analogy merely formal or does it have a deeper conceptual significance?
- Can the link between causal nonseparability and noncausal correlations help to understand better the link between quantum nonseparability and nonlocality (and *vice versa*)?
- What would be the possible implications of the above reflexions on the nature of space and time?

Before developing those questions (see section 4), the next two sections will present the notions of quantum nonseparability and nonlocal correlations on the one hand (see section 2), and causal nonseparability and noncausal correlations on the other hand (see section 3).

2 Quantum nonseparability and nonlocal correlations

2.1 Quantum nonseparability

This section will recall the formal definition of quantum nonseparability. Let be two subsystems, labelled 1 and 2, forming a composite system labelled 1-2. The quantum states of the subsystems A and B are *nonseparable*, or *entangled* if we cannot express the global quantum state as follows:

$$\rho_{AB} = \sum_i q_i \; \rho_A^i \otimes \rho_B^i \tag{1}$$

where the index i sums over classical probabilities (q_i) to have the subsystem (i.e. sub-event) x in the (pure or mixed) quantum state described by ρ_x^i.

2.2 Quantum nonlocality

Quantum nonseparability as defined above is shown to be a necessary (but not sufficient) condition for quantum nonlocality (Werner, 1989), which is the observation of nonlocal correlations (obtained from quantum measurements on quantum states) that cannot be explained by any local causal model. A local causal model is defined within the second version of Bell's theorem, where quantum nonlocality amounts to the negation of the *principle of local causality*. This principle states the following:

> "The direct causes (and effects) of events are near by, and even the indirect causes (and effects) are no further away than permitted by the velocity of light." (Bell, 1995)

Put in a more formal way, in relativistic terms, the principle of local causality requires that the statistical correlation between two spacelike-correlated events disappears upon conditioning on the intersections of their past lightcones (Maudlin, 2011).

2.3 Philosophical implications of quantum nonseparability and nonlocality

In our scientific realist framework, where our best scientific theories are believed to provide an approximately true description of the objective world, both quantum nonseparability and nonlocality have direct implications on the nature of reality.

As recalled in section 2.1, the quantum state of a composite system made of nonseparable (entangled) subsystems can not be expressed as a mixture of product states. There can be different reasons why a composite system's quantum state is not supervenient on the subparts' quantum states (Berkovitz, 2016):

1. **Dynamical origin**: The quantum states of the subsystems are well-defined, but they are not sufficient to determine in a non-ambiguous way the quantum state of the composite system.

2. **Ontological origin**: The quantum states of the subsystems themselves are not well-defined in the first place.

3. **Mixture of dynamical and ontological origins**: A combination of both previous options can be at stake in the phenomenon of quantum nonseparability.

At this stage, which of these three options is considered as correct remains an open question unless we adopt a (realist) account of quantum mechanics with a corresponding ontology that is developed enough to take a stance on the origin of quantum nonseparability (Maudlin, 2011). The most popular realist accounts of quantum mechanics are Bohmian mechanics, GRW theory and Everettian quantum mechanics. In each of those contexts, a clear view of what represents the quantum state and what is its dynamics is provided, so the notion of quantum nonseparability can be made more precise.

In the context of a well-developed ontological and dynamical account of quantum mechanics, where the link between the quantum state of the system and its physical state and properties is made clear, one can rely on metaphysical theses to articulate more explicitly the ontological content of the theory:

1. **Physical state nonseparability**: In case the quantum state coincides with the real physical state of the system, quantum state separability amounts to Einstein's notion of physical state separability: "Each region of space-time has its own intrinsic physical state, and the entire and complete physical state of the universe is specified once one has determined the intrinsic state of each small region" (Maudlin, 2011, p. 193).

2. **Property holism**: An ontological origin for quantum nonseparability seems to point towards a form of property holism, which states that some objects have properties that are not determined by physical properties of their basic physical parts.

3. **(Moderate) ontic structural realism**: Among the various forms that such a thesis can take, one of them states that fundamental object have no intrinsic properties and don't exist independently of the relations in which they stand (Ainsworth, 2010; Esfeld and Lam, 2010).

To sum up, quantum nonseparability can be explained by providing an adequate ontological and/or dynamical account of reality. This picture should ultimately account for quantum nonlocality, of which quantum nonseparability is a necessary but not sufficient condition[1]. Nonlocality is therefore expected to express stronger features of nature, and has been analyzed in dynamical terms by Maudlin (2011). Indeed, Maudlin investigated the kinds of physical events required by nonlocal correlations and argued that they require the existence of *superluminal causal connections*. He proposed a sufficient criterion for claiming the presence of such connections, defined as follows (Maudlin, 2011, p. 118):

[1]This is true when considering mixes quantum states. Yet, for pure quantum states, nonseparability is necessary **and** sufficient for nonlocality (Gisin, 1991).

> "Given a pair of space-like separated events A and B, if A would not have occurred had B not occurred even though everything in A's past light cone was the same, then there must be superluminal influences."

Such a statement invites us to revise our initial ideas about the notion of "past".

3 Causal nonseparability and noncausal correlations

3.1 Presentation of the process matrix formalism

The development of the process matrix formalism was motivated by the desire to provide a more general formalism for quantum mechanics in which no global predefined causal order is assumed for the system. Within such a formalism, one can investigate whether more general causal structures than the definite (yet possibly dynamical) ones are compatible with the formalism of quantum mechanics.

The formalism in which a theory is to be formulated without any reference to a global causal order needs to be *causally neutral*. This means that the formalism should express the relations among systems using the same mathematical objects, irrespectively of whether the systems are causally connected or disconnected. However, the standard formalism of quantum theory does not feature such a neutrality: while the correlations among sub-systems localized in different spatial regions are generated by a joint state pertaining to the tensor product of all the sub-systems' Hilbert spaces, the correlations of measurements' results obtained on a single system at different times are represented by linear maps transforming the initial states into final states in accordance with the specific outcome obtained (Brukner, 2014). A unified formulation for both spatial and temporal relations is achievable by using the Choi-Jamiolkowski (CJ) isomorphism (Jamiołkowski, 1972), which transforms a linear map between matrices acting on two Hilbert spaces (one for the system before and after the measurement) into a single matrix acting on the tensor product of these Hilbert spaces.

At this point, a mere causal neutrality is achieved. The process matrix formalism gives up on the assumption of a definite causal structure by postulating that the local systems on which each party performs an experiment obey the rules of quantum mechanics, but it makes no assumption regarding the spatio-temporal locations of these parties (Oreshkov et al., 2012).

In practice, the local quantum experiments of each party are described either operationally by joint probability distributions, or in a theory-dependent form in terms of a density matrix being acted on by a linear map between matrices in Hilbert spaces, describing how input states are transformed into final states upon a measurement. The correspondence between the operational and Hilbertian notions is thoroughly presented in Janotta and Hinrichsen (2014). The CJ isomorphism presented earlier is then applied to transform these linear maps into a matrix acting on the tensor product of the Hilbert spaces describing the system before and after the measurement. This provides the following expression for the joint probability distribution of having the outcomes labelled i for the party A, and j for the party B (the generalization to multiple parties is straightforward):

$$P(M_i^A, M_j^B) = Tr[W^{A_1A_2B_1B_2}(M_i^{A1A2} \otimes M_j^{B1B2})] \tag{2}$$

where M_i^A (M_j^B) is the completely positive trace non-increasing bilinear map transforming any input system of party A (B) into a given final state in agreement with the obtained measurement's outcome i (j). M_i^{A1A2} is the matrix acting on the tensor product of the Hilbert spaces in which the system is described before and after the measurement, $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2}$, obtained by applying the CJ isomorphism on M_i^A, and similarly for $M_j^{B_1 B_2}$. Tr[X] indicates the operation calculating the trace of a matrix X. W is a matrix acting on the tensor product $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2}$, with $\mathcal{H}^{X_m}$ being the Hilbert space of the system of party X before the measurement (m = 1) or after the measurement (m = 2). W satisfies a set of conditions ensuring its consistency with a probabilistic interpretation of Eq. (2). Eq. (2) is very similar to the one describing the link between the probability distribution of an experiment and the Hilbert space formalism (Janotta and Hinrichsen, 2014) in the context of a fixed causal structure:

$$P_a = Tr[E_a \rho] \tag{3}$$

where P_a is the probability to obtain the outcome labelled a, and E_a is the operator describing the corresponding measurement performed on the system. By comparing Eq. (2) with Eq. (3), it can be seen that W, called quantum process, is a generalization of the quantum state ρ allowing to represent joint states over multiple parties without mentioning their spatio-temporal locations.

3.2 Causal nonseparability

In direct analogy with the definition of *separable quantum states* (see section 2.1), the notion of *separable processes* can be defined. While the former notion relied on the spatial dimension by referring to subsystems possibly localized in different spatial regions but described at the same time, the latter notion relies on both spatial and temporal dimensions, as the concept of processes itself is defined across time and space. For those reasons, whereas we sometimes speak of *spatially* separable *quantum states*, we speak of *causally* (i.e. spatiotemporally) separable *processes*.

A bipartite process $W_c^{A,B}$ is said to be *causally separable* if it can be decomposed as a probabilistic mixture of one-way signalling causal processes (Oreshkov et al., 2012; Oreshkov and Giarmatzi, 2016):

$$W_c^{A,B} = qW^{A \preceq B} + (1-q)W^{B \preceq A} \tag{4}$$

where q is a number between 0 and 1 and $W^{X \preceq Y}$ represents a process for which signalling is only possible from X to Y. A generalization of Eq. (4) for multipartite processes has been developed in Oreshkov and Giarmatzi (2016); Wechs et al. (2018).

A process with a definite causal structure is necessarily a one-way signalling process, and *vice versa*. Indeed, if a process has a definite causal structure, either the events generated by the process are causally disconnected and no signalling occurs, or they are causally connected according to a definite time ordering, in such a way that signalling is in principle possible only from the temporally anterior events to the temporally posterior ones. Reciprocally, if a process is one-way signalling, this trivially means that its causal structure is definite. The definition formalized in Eq (4) therefore means that a causally separable process is a process for which a definite causal order exists. Even for an imperfect preparation procedure that yields a given causal order with a certain probability, it is still the case that the time ordering between the events in party A and party B is definite. This means that the events measured in party A are either preceding those measured in party B (this is is denoted A $\preceq$ B) or succeeding them (this is denoted A $\succeq$ B).

Chiribella et al. (2013) have found a way to implement a circuit, called quantum switch, of which the structure (hence the global causal order) is entangled with the state of a controlled qubit. Such a system has been proved to be causally nonseparable (Oreshkov and Giarmatzi, 2016;

Araújo et al., 2015). This confirms that the process matrix formalism indeed allows to describe more general systems existing in the world.

3.3 Noncausal correlations

A process as defined in section 3.1 generates specific correlations depending on the experiment that is performed. As a result, if one focuses on those correlations instead of on the process itself, it is possible to provide an operational characterization of the corresponding causal structures featured by the system. Let's consider a joint measurement performed by two observers, Alice and Bob, with a given set of inputs a and b corresponding to Alice's and Bob's input choices, respectively. The corresponding joint probability to obtain the outcomes x for Alice and y for Bob is noted $P^{AB}(a,b|x,y)$. A given correlation $P^{AB}(a,b|x,y)$ is causal if it satisfies a decomposition similar to Eq.(4):

$$P^{AB}(a,b|x,y) = qP^{A\preceq B}(a,b|x,y) + (1-q)P^{B\preceq A}(a,b|x,y) \tag{5}$$

where q $\in$ [0,1] and $P^{A\preceq B}(a,b|x,y)$ and $P^{B\preceq A}(a,b|x,y)$ are valid probability distributions (Oreshkov et al., 2012; Branciard et al., 2015).

Correlations $P^{AB}(a,b|x,y)$ can be geometrically represented as vectors in a multi-dimensional space, the number of dimensions depending on the number of parties, measurements settings and outcomes. It follows from Eq. (5) that any combination of causal correlations is still a causal correlation. The vectors corresponding to causal correlations form a polytope, and all the correlations in that causal polytope satisfy trivial constrains ensuring a probabilistic interpretation. They also satisfy non-trivial constrains originating from the definition of causal correlation expressed in Eq. (5). These constrains can be formulated as algebraic inequalities. The correlations satisfying all constrains but reaching the upper-bound value for a given inequality are still part of the causal polytope and constitute the various facets of the latter. Each facet corresponds to correlations reaching the upper-bound value of a specific inequality. As a result, it is said that non-trivial facets correspond to causal inequalities. Any valid correlation outside the causal polytope violates at least one causal inequality, and is therefore qualified as non-causal. Hence, by construction, such inequalities are used to test whether a correlation is causal.

A concrete example of causal inequalities and corresponding non-causal correlations violating them has been provided by Branciard et al. (2015) in the simplest bipartite configuration with two different measurement's settings and outcomes for each party. Further work needs to be done in order to establish the causal inequalities of more complex causal polytopes.

Previous work evidenced the fact that a causally non-separable process will not necessarily generate correlations that will violate a causal inequality (Oreshkov and Giarmatzi, 2016; Araújo et al., 2015). So far, no physical protocol has yet succeeded to generate non-causal correlations.

4 Discussion

We are now in a position to discuss the connection between the notions of quantum and causal nonseparability, and have a preliminary reflection regarding the possible metaphysical implications that it suggests.

4.1 Formal analogy with quantum nonseparability and causal nonseparability

As explained in section 3.1, the process matrix W can be seen as a generalization of the density matrix ρ. Yet, those two concepts are two distinct mathematical objects of a different nature.

First of all, these two objects describe different notions. The **density matrix** describes the *quantum state* of a given system (i.e. a physical event), localized at a given spacetime point (t,x,y,z). The **process matrix** describes the *process that causally (i.e. spatio-temporally) relates* the quantum states of different physical events at different spacetime points (t,x,y,z).

As a result, the density and process matrices are different objects acting on Hilbert spaces having different structures (see section 3.1): while the density matrix of a composite system (e.g. a bipartite system made of subsystems A and B) acts on the tensor product of the Hilbert spaces associated to each sub-system ($\mathcal{H}^A \otimes \mathcal{H}^B$, with $\mathcal{H}^X$ being the Hilbert space of the system of party X), the process matrix of a process (e.g. relating the quantum states of two systems A and B, each of them undergoing some linear evolution or transformation through time) acts on the Hilbert space $\mathcal{H}^{A_1} \otimes \mathcal{H}^{A_2} \otimes \mathcal{H}^{B_1} \otimes \mathcal{H}^{B_2}$, with $\mathcal{H}^{X_m}$ being the Hilbert space of the system of party X before the linear evolution or transformation (m = 1) or after the linear evolution or transformation (m = 2).

We see that in both cases, the composite nature of a system or process is described by the use of outer products. Yet, the structure of that outer product is different in each case, and a process matrix will never reduce to a density matrix. This last point is better illustrated by two particular process matrices relating the quantum states of two systems labelled system A and system B. For simplicity, we make the hypothesis that those systems are not entangled, neither among each other nor with their environment. We note t_0 the initial time at which the systems are simultaneously described, and t_1 a later time after the systems have possibly undergone some change. The process matrix has therefore four quantum states to relate, namely the quantum states of both systems at two different times. Let's consider two particular cases:

1. System A and system B are two distinct systems spatially distant and they don't undergo any change in time

   $$W_{A \preceq B} = \rho_{H_A(t_0)} \otimes \rho_{H_B(t_0)} \otimes \mathbb{1}_{H_A(t_1)} \otimes \mathbb{1}_{H_B(t_1)} \tag{6}$$

   where $H_x(t_0)$ is the associated Hilbert space of system x at the earlier time t_0 and $H_x(t_1)$ is the associated Hilbert space of system x at the later time t_1.

   This process matrix describes the process relating two subsystems described at the same time, i.e. a mere spatial relation among the two events. Even though there is no temporal evolution to describe, we see that the process matrix has still a different mathematical structure than the density matrix describing the composite system.

   Note: in that case, $W_{A \preceq B}$ is identical to $W_{A \succeq B}$.

2. System A and system B are one and the same system evolving across a certain time period $[t_0 - t_1]$

   $$W_{A \preceq B} = \mathbb{1}_{H_A(t_0)} \otimes C_{H_A(t_0) H_B(t_1)} \otimes \mathbb{1}_{H_B(t_1)} \tag{7}$$

where $H_A(t_0)$ is the associated Hilbert space of the system at time t_0 and $H_B(t_1)$ is the associated Hilbert space of the same system that has evolved from t_0 to t_1. $C_{H_B(t_1)H_A(t_0)}$ is the matrix that describes the effect on the quantum state of the system of the time evolution from t_0 to t_1, obtained using the C-J isomorphism to transform a linear map into a matrix (see section 3.1).

This process matrix describes the process relating two subsequent states of the same system, i.e. a (spatio)temporal relation among the two events. Such a description cannot be encoded within the density matrix alone. This fact illustrates in what sense the process matrix is a generalization of the density matrix.

As a conclusion of this comparison, we see clearly that the process matrix W and the density matrix ρ are distinct mathematical objects, with different inner structures. A process matrix W does not represent a quantum state, but a *process* that *causally relates* the quantum states of physical events. We can also see how the process matrix is generalizing the concept of density matrix in the sense that it allows to represent not only the quantum states of (possibly composite) systems, but also their evolution through time.

Since the process matrix and the density matrix are different objects, they will have different physical meanings within the context of a particular account of quantum mechanics. Therefore, the kind of nonseparability that corresponds to each of those notions has also a different physical meaning in each case. The next section will discuss that last point.

4.2 Conceptual significance of causal nonseparability

As seen in section 2.1 and 3.2, quantum and causal nonseparability are defined in a very similar way:

1. **Quantum nonseparability:** there is an impossibility to express the global *quantum state* as a probabilistic mixture of outer products that connects the quantum states of subsystems localized at possibly different spatial regions but described at the same time.

2. **Causal nonseparability:** there is an impossibility to express the global *process* as probabilistic mixture of outer products that connects the quantum states of the spatially and/or temporally distant events.

The physical interpretation of those two forms of nonseparability depends on that of the density and process matrices. Since a process matrix is built on density matrices, its meaning/interpretation depends on that of density matrices, i.e. on that of the quantum state. Therefore, the physical meaning of process matrices depends on the account of quantum mechanics under consideration.

The transposition of existing accounts of quantum mechanics to the process matrix formalism should not bring any particular technical difficulty, since the process matrix formalism does not introduce any new elements in addition to the formalism of standard quantum mechanics. Indeed, it only appeals to the C-J isomorphism, which is a purely mathematical operation that does not bring any substantial change to the content of the theory.

The task for future work will therefore to articulate, within a given realist interpretation of quantum mechanics, the meaning of a process matrix and the new idea that for a process relating causally nonseparable events, there is no well-defined causal structure among the events composing the process.

4.3 Link between causal nonseparability and noncausal correlations

As a recall from section 2.2, quantum nonseparability is necessary but non-sufficient for nonlocality, which expresses therefore stronger aspects of nature, namely the presence of superluminal causal connections.

Causal nonseparability is also necessary (Oreshkov et al., 2012; Wechs et al., 2018) but non-sufficient for causal correlations. Its non-sufficiency is demonstrated by the example of the quantum switch, which is causally nonseparable but does not lead to any noncausal correlations (see section 3.2). We are therefore in a similar situation as the one existing between quantum nonseparability and nonlocality. However, the important difference is that the existence or non-existence of noncausal correlation in practice has not yet been demonstrated. If they do exist, we should investigate about the kind of implications of such a feature of nature. If they don't exist, we should discover the reason why such correlations are forbidden, and which principle (if any) actually limits the process matrix formalism. In both cases, we expect to learn something about how causal structures behave, what they allow and why. In particular, such lessons are expected to concern the very notions of space and time.

4.4 Possible/suspected implications on the nature of space and time

Causal nonseparability seems to put into question the initial view of spacetime as a fixed, fundamental background for physical events. Indeed, an indefinite causal structure among two causally nonseparable events would imply that the spatiotemporal relation among these two events is indefinite. Hence, contrary to the case of quantum nonseparability, which was dealing with potential indefiniteness of the quantum state of systems/events, we are here dealing with the potential indefiniteness of causal (i.e. spatiotemporal) relations among events. This might indicate that those spatiotemporal relations are supervenient on some more fundamental elements. Yet, the role of spacetime in non-relativistic quantum mechanics is to provide a fixed background stage with a Galilean geometry for events to take place. Causal non-separability threatens to overthrow such a conception. Whether causal nonseparability indeed implies a non-fundamental nature of spacetime (as it is also suggested by others research fields in fundamental physics (Huggett and Wüthrich, 2013)) remains at this stage an open question that needs to be investigated more carefully.

5 Conclusion and future work

We presented a new notion of nonseparability affecting the spatiotemporal relations among events, which are described by a process within a new framework called process matrix formalism. This causal nonseparability is implemented by particular systems called quantum switches and constitutes therefore a real physical phenomenon. It is a necessary but non-sufficient condition for noncausal correlations, which are physical correlations incompatible with a definite causal structure. The extent and limits of the formal analogy existing between quantum nonseparability and nonlocal correlations on the one hand and causal nonseparability and noncausal correlations on the other hand were discussed in order to evidence how these notions differ in nature, and what would be the strategies to investigate further their conceptual significance.

Future work will aim first at reviewing and making more explicit the way the ontological developments of quantum mechanics account for quantum nonseparability and nonlocality (extending section 2.3). That basis will then be used to develop the discussion of section 4.2, which in turn will foster the development of some reflections presented in section 4.4.

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