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Preservice mathematics teachers' professional noticing of students' mathematical thinking with technology

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Research Highlights in Technology and Teacher Education 2018

TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK)

MATHEMATICS EDUCATION

APPLICATIONS

TEACHERS' PERCEPTIONS AND EXPERIENCES

K-12 STUDIES

ASSESSMENT AND EVALUATION

Preservice Mathematics Teachers' Professional Noticing of Students' Mathematical Thinking with Technology

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Abstract The purpose of this study is to examine the ways in which preservice secondary mathematics teachers (PSMT) professionally notice middle school students' mathematical thinking on a technology enhanced mathematical task. The middle school students' work was captured as a videocase for PSMTs to examine. Findings show that every PSMT included a discussion of the middle school students' interaction with the technology in their noticing prompts, demonstrating that PSMTs recognized that the middle school students' mathematical understanding was tied to their interactions with the technology. Additionally, results from PSMTs' justifications for their predictions of middle school students' responses to the task, incorporated the middle school students language and described how the middle school students would interact with the technology.

Introduction

Engaging students in meaningful mathematical tasks and capitalizing on available technological tools has been shown to improve attitudes towards mathematics and increase learning (Cheung & Slavin, 2013; Ellington, 2003). Whether or not the use of technology will enhance students' learning depends on teachers' decisions when using technology tools to design and implement meaningful tasks. These decisions are informed by teachers' knowledge of mathematics, technology, and pedagogy. To prepare teachers to implement lessons that draw on such knowledge, the Association of Mathematics Teacher Educators (AMTE) *Standards for Preparing Teachers of Mathematics* (AMTE, 2017) report discuss the need for PSMTs to engage with technology themselves as learners of mathematics, and to consider how technology can be leveraged to support students' mathematical learning. Further, they note the importance of opportunities to analyze student work samples with a goal of making sense of different students' mathematical thinking; this is consistent with the construct of professional noticing (Jacobs, Lamb & Philipp, 2010). Therefore the purpose of this study is to examine the ways in which PSMTs professionally notice students' mathematical thinking on a technology enhanced mathematical task.

Background and Framework

TPACK

Consider the following example that illustrates three components of teacher knowledge in a lesson on functions. A teacher who is teaching a lesson focusing on parameters' of function families needs to know how the parameters influence the behavior of the function (knowledge of content), use technology to investigate the influence of parameters (knowledge of technology specific to the content), and design activities that align with approaches students may take when asked to investigate the influence of parameters on a family of functions (knowledge of pedagogy specific to the content). The intersection of these forms of knowledge has been identified as technological pedagogical content knowledge (TPACK) (Koehler & Mishra, 2005; Mishra & Koehler, 2006; Niess, 2005), a type of knowledge several authors have characterized as necessary for teachers to understand how to use technology effectively to teach specific subject matter. Niess (2005) has articulated four components of TPACK: 1) an overarching conception of what it means to teach a particular subject integrating technology in the learning; 2) knowledge of instructional strategies and representations for teaching particular topics with technology; 3) knowledge of students' understandings, thinking, and learning with technology in a specific subject; and 4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area. It is the third component, knowledge of students' understandings, thinking, and learning with technology in a specific subject, that is the focus of this study.

Professionally Noticing

Jacobs, Lamb & Philipp (2010) developed the professional noticing of children's mathematical thinking framework which expounds that novices in any profession must learn to notice in ways unique to the profession. The three components of the professional noticing framework (Jacobs et al., 2010) are attending to students' strategies, interpreting of students' mathematical thinking, and deciding how to respond on the basis of students' understandings. With the goal of eliciting PSMTs' knowledge of students' mathematical thinking with technology, designing learning experiences by drawing on these professional noticing frameworks seems fruitful. Research on professional noticing has shown that knowledge influences what PSMTs notice (e.g., Dick, 2017; Hiebert, Morris, Berk & Janson, 2007; Jacobs et al., 2010; Wilson, Lee, & Hollebrands, 2011). Hiebert et al. (2007) explained that noticing student work "requires a set of competencies or skills that draw directly on subject matter knowledge combined with knowledge of student thinking" (p. 52). In the case of a technological mathematical task, this knowledge needed is TPACK.

In studying PSMTs' development of the skill of professional noticing, three different types of approaches have been employed: 1) noticing researcher selected artifacts from other teachers' classrooms; 2) retrospectively sharing noticing about one's own lesson/classroom; and 3) researchers observe teachers and infer what teachers notice (Sherin, Russ, & Colestock, 2011). Our study follows the first approach and most of the research employing this approach has focused on whole class video (e.g., Krupa, Huey, Lesseig, Casey & Monson, 2017; Leatham, Peterson, Stockero, & Van Zoest, 2015; McDuffie et. al, 2013) with less research on preservice teachers' noticing of student written work (e.g., Dick, 2017; Goldsmith & Seago, 2011) and even less on the act of professional noticing in the context of artifacts of students' technological mathematical work (e.g., Chandler, 2017; Wilson, Lee, & Hollebrands, 2011). Wilson et al. (2011) did not situate their work within the professional noticing framework, but they did discover four different categories of ways PSMTs made sense of students' work with technology: describing, comparing, inferring and restructuring and called on others to design tasks for PSMTs that provided opportunities to analyze students' technological work. To address the limited knowledge of PSMTs' professional noticing in technological contexts, we designed a lesson to engage PSMTs with professional noticing through analyzing middle school students' (MSS) work. Similar to the work of Wilson et al. (2011), the PSMTs examined the written work and a video-recording of MSS' mathematical technological work. Thus, we specifically address the following research questions:

How do PSMTs professionally notice middle school students' thinking with technology through analyzing middle school students' technological work?

Context of this Study

Given the importance of function in 6 – 12 mathematics and as a foundational topic in college-level mathematics we chose to focus on the concept of function for this lesson. Five "big ideas" related to function are outlined Cooney, Beckman, and Lloyd's (2010) *Developing Essential Understandings of Functions*, Grades 9-12 - the first of which is the function concept. The function concept includes the understanding of functions as single-valued mappings from one set to another (in which the domain and range do not have to be numbers) and as applicable to a wide range of situations. There is substantial evidence that students often have incorrect or incomplete views of the function concept. This includes a view of function that is limited to algebraic expressions and their associated graphs (e.g., Carlson 1998; Even, 1990) and that such understandings typically result in a "vertical line test" related definition of function (e.g., Breidenbach et al., 1992; Fernandez, 2005). To address this, we designed a lesson that used a preconstructed GeoGebra applet to introduce and problematize the function concept (McCulloch, Lovett, & Edgington, 2017).

Technology can be effective in helping students' develop the function concept and make connections between different representations of function (e.g., Dick & Hollebrands, 2011; Garofalo, Drier, Harper, Timmerman, & Shockey, 2000). As a result, both research and standards documents often suggest that technology tools be used to study functions. To this end we developed a set of activities in which PSMTs examined their own understanding of the function concept through interacting with an online applet, as well students' mathematical thinking through professionally noticing artifacts of students' engaging with a similar applet.

During the class session prior to the lesson of study, PSMTs were then given a homework assignment to engage with the Vending Machine applet (described below), complete a worksheet to record their answers, and screen record themselves following a talk aloud protocol while engaging with the applet. During the next class the PSMTs examined the MSS version Vending Machine applet and examined authentic MSS' definitions written following engagement with the applet. For homework, PSMTs engaged in a noticing assignment specifically focusing on the first two aspects of the Jacobs et al. (2010) noticing framework, attend and interpret. The decision was made to focus on these aspects and not the "decide" aspect as research has shown that PSMTs struggle making next-step decisions, especially when they are new to noticing (Gupta, Soto, Dick, Broderick & Appelgate, 2018). During the noticing assignment, PMSTs watched and analyzed video recordings of two different pairs of MSS' engagement with the applet. After analyzing the video recordings, the PSMTs completed a written reflection (see Figure 1) in which they were asked to attend to how the pair of MSS decided which machine was or was not a function and interpret the MSSs' understanding of function. Along with these two noticing components we included a third component of the reflection that asked the PSMTs to predict how the MSS would identify each of the other eight machines as functions or non-functions and to provide a justification for their predictions. This final component was included since PSMTs need to be able to predict and anticipate different strategies students might employ to solve mathematical tasks (Hiebert et al., 2007; Smith & Stein, 2011) prior to making decisions about what to do next (i.e., the third component of the noticing framework).

Watch the video of students engaging with Machines I and J, following along with the transcript. While watching the video, focus on the students' language. Based on their responses, predict the students' responses for the rest of the machines in the student version of the applet. Provide evidence from the video to justify your responses.

How did Group 1 decide which machine was or was not a function?

Explain Group 1's understanding of function. Use examples from the screencast as evidence to show how you know what they do or do not fully understand.

Predict (using language you believe the students will use) how Group 1 will answer each machine from the middle school student version of the applet and how Group 1 will engage with each machine. Your justification should include evidence from the video.

Figure 1. Noticing reflection assignment directions

The Vending Machine Applet

The Vending Machine applet (version 2.0) was designed to trigger a dilemma in PSMTs' understanding of function as it contains no numerical or algebraic expressions, but instead was built on the metaphor of a vending machine. Our Vending Machine applet (https://ggbm.at/qxQQQ7GP) is a GeoGebra book that asks the user to identify if each vending machines is a function or non-function. Each machine was designed to address misconceptions from the literature on distinguishing functions and non-functions. In the design of the applet, we are trying to disrupt are the notion of what represents an element in the range (Machines B, I, & J), students occasional use of the term "unique" when thinking about outputs (Machines B & I), and the notion that onto functions should be "predictable" (Machines A, C, I, & J) - meaning that if one knows the function rule and is given an input, it is possible to predict the output. A similar version of the applet was designed for MSS to develop a definition of the concept of function (https://ggbm.at/wcuPt43b). The MSS applet's directions on each page are slightly different than the PSMT version but all the machines in the MSS Vending Machine applet also appear in the PSMT version.

Method

For this study, we sought to develop PSMTs' knowledge of students' mathematical thinking with technology through engaging in a lesson (described above) that included opportunities for PSMTs to professionally notice MSS' thinking with technology. To do so, we conducted a study with eight PSMTs at a southeastern university. These PSMTs were juniors enrolled in a mathematics methods course. All were seeking a bachelor's degree.

Data Collection and Analysis

Even though the lesson had several components, for this paper we focus on PSMTs' written work on their noticing reflection assignment (Figure 1). Each PSMT was asked to explain how each group of MSS determined whether or not the machine was a function (attend) and to discuss the MSS' understanding of function (interpret). This resulted in a total of 16 attend and interpret statements, two for each PSMT that were recorded in a spreadsheet for analysis. Since each PSMT was asked to justify their predictions on how the MSS would classify the additional eight machines that were not shown in the video, there is a total of 14 predictions and justifications for each PSMT. (Note there are 14 and not 16 since the one MSS group did not finish the assignment). PSMTs' worksheets from completing the task themselves as learners, were also used to compare their language as learners to the language they used on the noticing reflection assignment. This allowed use to identify when PSMTs were drawing on their own language or MSS' language.

A team of three researchers began by analyzing individual PSMT's responses together to develop the codebook. Coding began by examining each reflection for evidence of attending to and/or interpreting the MSS' mathematical thinking. However, since the videocase of the MSS' work incorporated technology, an additional code for evidence of attending to and/or interpreting the MSS' interaction with technology emerged. To analyze PSMTs' justifications of their MSS predictions, we used content analysis through open coding with constant comparison (Creswell, 2007). Specifically we focused on the ways in which the PSMTs drew on their noticings (i.e., attention to and interpretation of student thinking) to anticipate what students might do next and make predictions. From this open coding, three overarching prediction themes emerged regarding PSMTs' language used in the prediction and manners in which they discussed the MSS' engagement with the machines (Figure 2).

Attend	**Interpret**	**Predictions**
Evidence of attend to interaction with technology	Evidence of interpretation of interaction with technology	Used MSS' language
Evidence of attend to MSS' mathematical thinking	Evidence of interpretation of MSS' mathematical thinking	Used own language
		Discussed engagement with machines

Figure 2. Codes for noticing reflection assignment

To establish reliability in our coding, responses were coded independently by all members of the research team (n=3) and the number of agreements were divided by the number of assigned codes. The team were in agreement over 90% of the time, so the codebook was considered reliable (Miles & Huberman, 1994). Differences in coding were reconciled through discussion. We examined these codes for themes relating to evidence of PSMTs' knowledge of students' mathematical thinking with technology. These themes are presented in the results section.

Results

To answer our research question, we first discuss how the PSMTs professionally noticed the MSS's technological work including how they attended to and interpreted the MSS' mathematical thinking and/or their interactions with the technology. Then, we discuss themes that emerged in the ways the PSMTs predicted and justified how they believed the MSS would engage with the applet.

PSMTs’ Professional Noticing of students’ mathematical thinking with technology

When asked to attend to how the MSS decided whether or not the machines were functions, three of the eight PSMTs discussed both how the MSS interacted with the machines as well as described the MSS’ ideas of function for at least one of the MSS groups. It was more common for the PSMTs to either describe how the MSS decided function or non-function in terms of interactions with the machines or in terms of how the MSS were thinking about functions. For example, with MSS Group 1, PSMT 6 stated, “They did one machine at a time; while working on each machine they selected each soda 3 or 4 times to see if the outputs were different.” This description was focused only on interactions with the technology. In contrast PSMT 3 explained, “Group 1 looks at each input and makes sure there is only 1 output and that it's the same one each time” which is focused on the MSS’ ideas of function.

For interpreting the MSS’ understanding of function, seven of the eight PSMTs discussed both how the MSS interacted with the machines as well as described the MSS’ ideas of function for at least one of the MSS groups. For example, PSMT 7’s interpretation of MSS Group 1’s thinking included both aspects,

> They fully understand that if an input is has same output each time then it is a function. They are thinking that if a function puts out 2 sodas with a different color each time and that colors are same every time then it is a function. I know this because they used word, ‘constant’, ‘random’, and ‘pattern’.

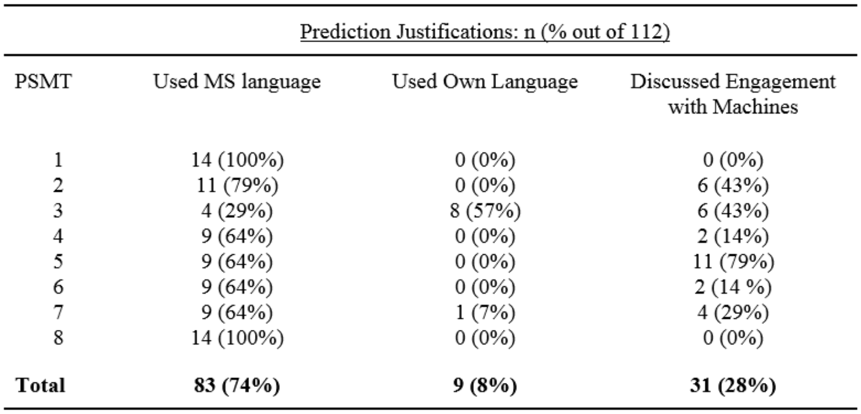
In general, the PSMTs’ interpretations of the MSS’ understanding of function were based both on how the MSS were using the technology and how their interactions with the technology influenced their understanding.

PSMTs’ knowledge of students’ mathematical thinking with technology seen through their Prediction Justifications

Through an examination of PSMTs’ predictions of MSS’ answers and justifications, two themes emerged relating to PSMTs’ noticing of students’ mathematical thinking with technology: incorporating MSS’ language; and considering MSS’ interactions with the applet.

Incorporating MSS’ language.

Overwhelmingly, PSMTs abandoned their own mathematical language and justified their predictions of the MSS’ answers using the MSS’ language (Table 1.).



	Prediction Justifications: n (% out of 112)		
PSMT	Used MS language	Used Own Language	Discussed Engagement with Machines
1	14 (100%)	0 (0%)	0 (0%)
2	11 (79%)	0 (0%)	6 (43%)
3	4 (29%)	8 (57%)	6 (43%)
4	9 (64%)	0 (0%)	2 (14%)
5	9 (64%)	0 (0%)	11 (79%)
6	9 (64%)	0 (0%)	2 (14 %)
7	9 (64%)	1 (7%)	4 (29%)
8	14 (100%)	0 (0%)	0 (0%)
Total	**83 (74%)**	**9 (8%)**	**31 (28%)**

Table 1. Occurrence of themes in PSMTs’ prediction justifications.

All eight PSMT incorporated the MSS’ language in at least 4 of their 14 justifications, with some PSMTs including the MSS’ language in all of their justifications. Overall, of 112 justifications, 74 percent of those incorporated the MSS’ language from the video and/or written artifacts. To illustrate, PSMT 1, justified all of his own answers to the vending machine task by explaining whether or not the machine would pass the vertical line test (i.e., he justified all responses by saying "passes the vertical line test" or "fails the vertical line test"). However, when predicting how MSS Group 1 would decide whether the machines were functions each of his justifications used the MSS' terms "constant" and "random." For example, PSMT 1 correctly predicted that MSS Group 1 would classify Machine L as a function and justified his response by stating "even though all the buttons put out green its not *random* but a *constant* output." There was no mention of the vertical line test; he abandoned his language and adopted that of the MSS. Even though all PSMTs adopted the MSS' language for some of the justification, there were two PSMTs who still relied heavily on their own mathematical language when justifying the MSS' answers. For example, instead of using the words "constant" and "random" as the MSS did, in his justifications of MSS' predictions, PSMT 3 discussed the inputs and number of outputs, which is the language he used in his own machine classifications from when he completed the task as a learner.

Considering MSS' interactions with the applet.

In considering the PSMT's justifications for their predictions of the MSS' responses, 6 of the 8 PSMTs included a description of how they predicted the MSS would interact with the machine applet. Their discussions of the MSS' interactions with the applet fell into two categories: assuming the MSS would continue to interact with the applet in the same manner as they progressed and incorrectly assuming the MSS would interact in ways they themselves did.

The first category was more prevalent; all six of these PSMTs assumed the MSS would continue to interact with the applet in the same manner as they did for Machines I and J. In MSS Group 1's video, the pair of students checked each button on the machine more than once. PSMT 5 used the evidence he saw in the video to justify his predictions for the other machines for 79% of his justifications. For example, PSMT 5 predicted that for Machine H, "The group will check each button multiple times to ensure the outputs are consistent for each input and conclude that it is a function." PSMT 5 also applied this in his justifications for Group 2. In Group 2's video, the pair of students did not check each button more than once before identifying the machine as a function or non-function which PSMT 5 used as part of his justification predictions. One example of this is below:

> After watching their approach for I and J, I can't be certain that they will check each button's output enough times to classify this machine correctly as not a function...Even if they check each button only once, they would still be able to correctly classify this as a function. Their answer would be correct even though their justification might be flawed.

PSMT 5's predictions for Group 2 referenced interactions with the applet for all but one justification.

For the second category, both PSMT 2 and 3 assumed that the MSS would pay close attention to the directions which stated that only one machine on each page is a function and would therefore figure out whether or not one of the machines was a function and effectively ignore the other. For example, for MSS Group 1 PSMT 2's predicted, "In comparison to G, H must be a function. And they will see that the output is consistent." Here PSMT 2 had already predicted that the MSS would say G is not a function, so based on the instructions H would have to be a function. While this type of predication of how the MSS would interact with the machines may have made sense to the PSMTs based on their own interactions, in reality neither group of MSS discussed the directions or made a decision identifying a machine as a function or non-function based on the fact that the only one on the page could be a function.

Overall, 5 of the 6 PSMTs who discussed MSS' interactions with the machines in their justification predictions did so for two to six machines. There was not a machine that seemed to elicit this type of justification more than others.

Discussion

To support PSMTs as they learn how to teach with technology, we designed a multi-part lesson to engage PSMTs with professionally noticing researcher-selected video and written artifacts of MSS' thinking with technology. Findings from our analysis of how the PSMTs professionally noticed the MSS' thinking showed every PSMT included a discussion of both the MSS' interactions with the applet and the MSS' understanding of the function concept for either the attend or interpret prompts. Thus, the PSMTs recognized that the MSS' understanding of function was tied to their interactions with the technology. Results from the analysis of the PSMTs' justifications for their predictions of the MSS' responses to each machine included a prevalence of justifications that incorporated the MSS' language and described how the MSS would interact with the technology either by making assumptions that the MSS would continue to interact with the applet in the same manner as they continued with the technological task or incorrectly assuming the MSS would interact with the technology in the same ways they themselves did.

Our findings closely mirror those of Wilson et al. (2011) in that PSMTs' professional noticing responses included instances of describing, comparing, inferring and restructuring. Wilson et al. (2011) explained describing as when PSMTs explicitly refer to or utilize "students' actions with the technology, words students have written or said, or mathematical terminology and symbols used by students" in their analysis of the students' work (p. 53). Comparing includes instances where the PSMTs compare the students' interactions with the technology to their own interactions with the technology either by implicitly or explicitly referring to a difference between the two. Inferring includes instances when PSMTs "use their technological, pedagogical, and/or mathematical knowledge to interpret students' work and make inferences about what students [are] thinking (p. 57). Finally restructuring includes instances where PSMTs expand their own understandings to include those of the students' whose work they analyzed.

When the PSMTs in this study were initially attending and interpreting how the MSS interacted with the technology, they tended to describe either the students interaction with the technology or to describe the MSS' mathematical understanding of function. Wilson et al. (2011) considered these in the same category, but we found that attention to and interpretation of the MSS' technological work to both to be necessary for a full evaluation of the MSS's thinking with technology. The PSMTs' attention to and interpretation of the MSS' thinking with technology did not include comparisons, inferences or restructuring. This is likely due to the manner that the professional noticing assignment asked the PSMTs to first explain how the MSS determined whether or not the machine was a function (attend) and then to discuss the MSS' understanding of function (interpret).

The final part of the professional noticing assignment in which PSMTs' predicted MSS' answers and interactions with the applet did elicit the other three ways (i.e., inferring, comparing, and restructuring) PSMTs make sense of students' work with technology as described by of Wilson et al. (2011). The task the PSMTs completed asked them to make predictions (i.e. infer) as to how the MSS would classify the remaining machines as function or non-function. The PSMTs had to use their TPACK to make these predictions. Their predictions often adopted the MSS' language (describe) and seemed to indicate an understanding of the ways the MSS were developing the concept of function. Because seven of the eight PSMTs included the MSS' language in the majority of their prediction justifications, we see that almost all of the PSMTs demonstrated knowledge of students' mathematical thinking with the applet to the extent that they understood their words, and used them to justify predictions about what the MSS would do with the other machines.

Additionally, the PSMTs sometimes assumed the MSS would interact with the technology in the same ways they interacted with it themselves (compare); for these instances it seemed that the PSMTs were not demonstrating an understanding of how the MSS would interact with the applet or the relationship of how interacting with the applet influenced the MSS' mathematical thinking. Finally, the PSMTs showed some evidence of altering their own understanding of function (e.g., PSMT 1 whose initial understanding was only based on the vertical line test) based on their attention to and interpretation of the MSS' work on the technological task (restructuring).

Wilson et al. (2010) claim that the fourth component, restructuring, "requires a reconciliation of PSTs' observations and inferences with their own understandings" (p. 61) and call for others to provide opportunities for PSMTs to reflect on their own understandings in light of their noticing of students' technological work. In this study, we did not have the PSMTs reflect on how their own understandings of function changed as a result in engaging in this professional noticing task, but will include a reflection component in the next iteration of the lesson in the hopes of seeing more explicit evidences of PSMTs restructuring their own understandings of function.

Overall, through attending, interpreting, and predicting the PSMTs showed evidence of making sense of both students' language and their actions with the technology itself. This is evidence of the PSMTs are able to engage in professional noticing of students' mathematical thinking with technology. In addition, this suggests that drawing on the noticing framework in technological contexts might be a powerful way to understand and develop particular aspects of PSMTs' TPACK. However, this study was focused on PSMTs' noticing of researcher-selected artifacts and thus we expect that these results would differ if the PSMTs were noticing in-the-moment.

The findings here (in addition to other studies that have utilized the vending machine applet - e.g., Sherman et al., (2018)) indicate that this particular lesson and its accompanying applet should be improved and further tested before it is widely used with the mathematics education community. The lesson has since been revised to provide PSMTs with an opportunity to design a middle school version of the applet prior to engaging with the version the instructor created for MSS. The purpose of this addition is to address another component of TPACK - knowledge of instructional strategies for teaching with technology. Additionally, since we did not specifically ask the PSMTs to include both a discussion of the MSS' interactions with the applet and the MSS' thinking for the attend and interpret prompts of the noticing assignment, we have changed the assignment to elicit both. This should provide a deeper understanding of how PSMTs draw on their knowledge of students' mathematical thinking with technology when noticing. Finally, we have added in a reflection component to capture how PSMTs feel their knowledge has been impacted through engaging in this lesson.

Implications for Teacher Educators

As technology tools are becoming more ubiquitous in our schools, using them to teach mathematics should be as well. Thus it is important that PSMTs have opportunities to engage with technology in their teacher education programs and learn how to design lessons using technology tools. But maybe even more importantly, PSMTs need to be able to make sense of students' mathematical understandings as they engage with technology-based mathematics tasks and how students' interactions with the technology influences their mathematical understandings. It has been shown that attending to student thinking is not trivial, this is especially true in technological contexts. The results of this study suggest that designing opportunities for PSMTs to engage in professional noticing tasks that include making predictions along with attending to and interpreting students' thinking will further support their development related to instructional decision making. Given the promise of these results, mathematics teacher educators need to provide PSMTs more opportunities to reason about students' understandings, thinking, and learning with technology.

References

Association of Mathematics Teacher Educators. (2017). *Standards for preparing teachers of mathematics*. Available online at amte.net/standards.

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics, 23*, 247–285.

Carlson, M. P. (1998). A cross-sectional investigation of the development of the function concept. *Research in collegiate mathematics education, III, Issues in mathematics education, 7*(1), 115-162.

Chandler, K. C. (2017). *Examining how prospective secondary mathematics teachers notice students' thinking on a paper and pencil task and a technological task*. Unpublished doctoral dissertation. North Carolina State University. Raleigh, NC.

Cheung, A. C. K., & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review, 9*, 88-113.

Cooney, T. J., Beckman, S., & Lloyd, G. M. (2010). *Developing essential understanding of functions for teaching mathematics in grades 9-12*. Reston, VA: National Council of Teachers of Mathematics.

Creswell, J. W. (2007). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Los Angeles, CA: Sage.

Dick, L. K. (2017). Investigating the relationship between professional noticing and specialized content knowledge. In E. O. Schack, M. H. Fisher & J. A. Wilhelm (Eds), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks* (pp. 339-358). Cham, Switzerland: Springer International Publishing.

Dick, T. P., & Hollebrands, K. F. (2011). *Focus in high school mathematics: Technology to support reasoning and sense making*. Reston, VA: National Council of Teachers of Mathematics.

Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students' achievement and attitude levels in precollege mathematics classes. *Journal for Research in Mathematics Education, 34*(5), 433-463.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. *Educational Studies in Mathematics, 21*, 521-544.

Fernandez, E. (2005). Understanding functions without using the vertical line test. *Mathematics Teacher, 99*(2), 96-100.

Garofalo, J., Drier, H. S., Harper, S., & Timmerman, M. A. (2000). Promoting appropriate uses of technology in mathematics teacher preparation. *Contemporary Issues in Technology and Teacher Education, 1*(1). Retrieved from http://www.citejournal.org/volume-1/issue-1-00/mathematics/promoting-appropriate-uses-of-technology-inmathematics-teacher-preparation

Goldsmith, L. T., & Seago, N. (2011). Using classroom artifacts to focus teachers' noticing. In M. G. Sherin, V. Jacobs, & R. A. Philipp (Eds.), *Mathematics Teacher Noticing: Seeing through teachers' eyes* (pp. 169–187). New York, NY: Routledge.

Gupta, D., Soto, M., Dick, L., Broderick, S. D., & Appelgate, M. (2018). Noticing and deciding the next steps for teaching: A cross-university study with elementary pre-service Teachers. In G. J. Stylianides & K. Hino (Eds), *Research Advances in the Mathematical Education of Pre-service Elementary Teachers* (pp. 261-275). Springer, Cham.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education*, *58*(1), 47–61.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research, 32*(2), 131-152.

Krupa, E. E., Huey, M., Lesseig, K., Casey, S. & Monson, D. (2017). Investigating secondary preservice teacher noticing of students' mathematical thinking. In Schack, E. O., Fisher, M. H. & Wilhelm, J. A. (Eds.). *Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks (*pp. 49-72). Cham, Switzerland, Springer.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*(1), 88-124.

McCulloch, A. W., Lovett, J. N., & Edgington, C. (2017). Developing preservice teachers' understanding of function using a vending machine metaphor applet. In E. Galindo & J. Newton (Eds.), *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1281-1288). Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, CA: SAGE.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*(6), 1017-1054.

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a pedagogical content knowledge. *Teaching and Teacher Education, 21*, 509-523.

Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011). Accessing mathematics teachers' in-the-moment noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), Mathematics teacher noticing: Seeing through teachers' eyes (pp. 79–94). New York, NY: Routledge.

Sherman, M. F., Lovett, J. N., McCulloch, A. W., Dick, L. K., Edgington, C., & Casey, S. A. (2018). Transforming students' definitions of function using a vending machine applet. In A. Weinberg, C. Rasmussen, J. Rabin, M. Wawro, & S. Brown (Eds.),s *Proceedings of the 21st Annual Conference on Research in Undergraduate Mathematics Education.* San Diego, CA: The Special Interest Group of the Mathematical Association of America (SIGMAA) for Research in Undergraduate Mathematics Education.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

Wilson, P. H., Lee, H. S., & Hollebrands, K. F. (2011). Understanding prospective mathematics teachers' processes for making sense of students' work with technology. *Journal for Research in Mathematics Education, 42*(1), 39-64.