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RESEARCH ARTICLE

Hydromagnetic boundary layer flow of a dusty fluid in a porous medium over a stretching sheet with slip effect

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Abstract

This paper investigated the problem of hydromagnetic boundary layer flow and heat transfer of a dusty fluid over a stretching sheet through a porous medium. The velocity slip was considered instead of the no-slip condition at the boundary. The governing partial equations were reduced into a set of non-linear ordinary differential equations by using the suitable similarity transformation. The transformed equations were numerically integrated using bvp4c in Matlab. The effects of various physical parameters on the velocity and temperature profiles of both phases, such as fluid-particle interaction parameter, magnetic parameter, mass concentration parameter, porosity parameter and Prandtl number were obtained and analyzed through several plots. Useful discussions were carried out with the help of plotted graphs and tables. Under the limiting cases, the obtained numerical results were compared and found to be in good agreement with previously published results.

Keywords: Boundary layer flow, dusty fluid, hydromagnetic flow

INTRODUCTION

Boundary layer flow and heat transfer of a fluid over a stretching surface has attracted many researchers in the last few decades. Since then, it has a wide range of applications in various fields such as polymer processing industries, extrusion, wire drawing, metal spinning, and hot rolling (Fischer, 1976). Crane (1970) pioneered the study of stretching sheet.. He presented an exact analytical solution for the steady two-dimensional stretching of a plate in a quiescent fluid. Then, Grubka and Bobba (1985) investigated heat transfer characteristics of a continuous stretching surface with variable temperature. Many authors have considered various aspects of this problem ever since, namely Fazlina *et al.* (2010)and Mansur and Ishak (2013).

Furthermore, the investigation on two-phase boundary layer flow in which solid spherical particles are distributed in fluid becomes extensive research in fluid dynamics due to its importance in engineering applications and technical problems such as flow through packed beds, sedimentation, environmental pollution, centrifugal separation of particles and blood rheology. The first research on boundary layer of dusty gas was carried out by Chakrabarti (1977). Then, Vajravelu and Nayfeh (1992) studied the boundary layer flow of dusty fluid over a stretching surface with the presence of suction. Gireesha *et al.* (2011) had studied the boundary layer flow of dusty fluid and heat transfer over a stretching sheet. Vajravelu *et al.* (2013) studied the hydromagnetic fluid flow of a dusty fluid and heat transfer at a stretching sheet with fluid-particle suspension. The analysis showed that the transverse velocity decreased with the increasing fluid-particle interaction parameter in the presence of magnetic field.

Recently,Sharena *et al.* (2016) investigated the magnetohydrodynamic flow of dusty fluid past a vertical stretching sheet with Hall effect.

Motivated by previous studies, the hydromagnetic boundary layer flow of a dusty fluid in a porous medium over a stretching sheet with slip effect is investigated. We consider the flow through porous medium since it plays an important role in many applications, such as geothermal operations, petroleum industries, thermal insulation, and many more. We also consider the slip condition since the velocity slip could possibly happen over stretching sheet in dusty fluid. For example, in emulsifier and polymer processing applications. The coupled nonlinear partial differential equations governing the problem are transformed into a coupled nonlinear ordinary differential equations by using appropriate similarity transformation. These resulting nonlinear ordinary differential equations are solved numerically by bvp4c function solver in Matlab software.
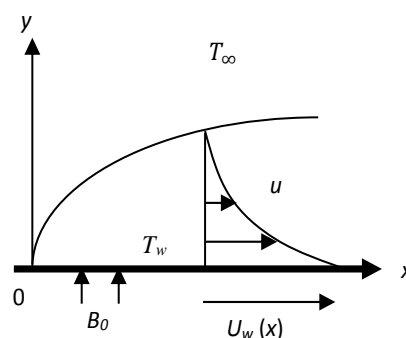

Fig. 1 Physical diagram for the problem

PROBLEM FORMULATION

Consider a steady two-dimensional laminar boundary layer flow and heat transfer of an incompressible, electrically conducting dusty fluid over a stretching sheet through a porous medium with the presence of applied strong magnetic field. The sheet is being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed.(see Fig. 1). The fluid and dust particle clouds are supposed to be static at the beginning. The non-reactive solid particles are assumed to be spherical and uniform in size. Furthermore, the density of the dust particle is taken as a constant throughout the flow.

Under these assumptions along with the boundary layer approximations, the governing boundary layer equations of both fluid and particle phase are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \frac{\rho_p}{\tau}(u_p - u) - \sigma_0 B_0^2 u + \frac{\mu}{k_p}u \tag{2}$$

$$\frac{\partial}{\partial x}(u_p) + \frac{\partial}{\partial y}(v_p) = 0 \tag{3}$$

$$u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{1}{\tau}(u - u_p) \tag{4}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \frac{c_s\rho_p}{\gamma_T}(T_p - T) \tag{5}$$

$$\rho_p c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = -\frac{c_s\rho_p}{\gamma_T}(T_p - T) \tag{6}$$

where (u, v) and (u_p, v_p) are the velocity components of the fluid and the dust particle phases along x and y directions respectively. μ, ρ_∞ and ρ_p are the dynamic viscosity of the fluid, density of the fluid, and mass of dust particles per unit volume of the fluid, T and T_p are the temperature of the fluid and temperature of the dust particle, k is the thermal conductivity, c_p and c_s are specific heat of fluid and dust particles, σ is the electrical conductivity and B_0 is the uniform magnetic field. γ_T is the temperature relaxation time whereas τ is the thermal conductivity. In deriving these equations, the drag force is considered for the interaction between the fluid and dust phases. The boundary conditions are

$$u = U_w(x) + L\frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w \text{ at } y = 0$$
$$u \to 0, \quad u_p \to 0, \quad v_p \to v, \quad T \to T_\infty, \quad T_p \to T_\infty,$$
$$\text{as } y \to \infty \tag{7}$$

where $U_w(x) = bx$ is the stretching sheet velocity, with prescribed surface temperature $T_w - T_\infty = A\left(\frac{x}{l}\right)$, with $b > 0$ is the stretching rate, l is the characteristic length and A are constants.

The governing equations (1) – (6) subjected to the boundary conditions (7) can be expressed in a simpler form by introducing the following dimensionless coordinates in term of similarity variable and similarity function as

$$u = bx f'(\eta), \quad v = -\sqrt{vb}f(\eta), \quad \eta = \sqrt{\frac{b}{v}}y$$

$$u_p = bxF'(\eta), \quad v_p = -\sqrt{vb}F(\eta), \tag{8}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}.$$

where v is the kinematic viscosity of the fluid.

By using the similarity equations from (8), we obtain the following nonlinear ordinary differential equations:

$$f'^2 - ff'' = f''' + \beta\rho_r(F' - f') - Mf' - kf' \tag{9}$$

$$F'^2 - FF'' = \beta(f' - F') \tag{10}$$

$$2f'\theta - f\theta' = \frac{1}{Pr}\theta'' + \frac{2}{3}\frac{\beta\rho_r}{Pr}(\theta_p - \theta) \tag{11}$$

$$F'\theta_p - F\theta_p' = -\frac{2}{3}\frac{\beta}{\gamma Pr}(\theta_p - \theta) \tag{12}$$

where $\rho_r = \rho_p/\rho_\infty$ is relative density, $\beta = 1/c\tau$ is the fluid particle interaction parameter, $M = \sigma B_0^2/\rho_\infty c$ is the magnetic parameter, k is the porosity parameter, $Pr = v/\alpha$ is Prandtl number and $\gamma = c_p/c_s$ is the ratio of specific heat capacity of the fluid dan dust phase.

The boundary condition in (7) becomes

$$f'(\eta) = 1 + Af''(\eta), \quad f(\eta) = 0, \quad \theta(\eta) = 0 \quad \text{at} \quad \eta = 0$$

$$f'(\eta) \to 0, \quad f(\eta) \to F(\eta), \quad F'(\eta) \to 0, \tag{13}$$

$$\theta(\eta) \to 0, \quad \theta_p(\eta) \to 0 \qquad \text{as} \quad \eta \to 0$$

where $A = L\left(\frac{b}{v}\right)^{1/2}$ is the slip parameter.

RESULTS AND DISCUSSION

The set of ordinary differential equations (9) – (12) subjected to boundary conditions (13) were solved numerically using the bvp4c function in MATLAB. The function is used to solve the equations due to its effectiveness in solving the boundary value problems (Mansur *et al.*, 2015). The numerical solutions were obtained to study the effects of various physical parameters such as fluid particle interaction parameter β, magnetic parameter M, porosity parameter k, Prandtl number Pr and mass concentration ρ_r on velocity, temperature as well as the gradients of both velocity and temperature. In order to verify the accuracy of this study, Table 1 is drawn to compare the results obtained by this study with previous studies done by Grubka and Bobba (1985) and Sharena *et al.* (2016). The table shows excellent agreement between the three sets of results.

Table 1 Results validation for values of $-\theta'(0)$ for existing solutions for $M = \beta = \rho_r = k = 0$.

Pr	Grubka & Bobba (1985)	Sharena *et al.* (2016)	Present study
0.72	1.0885	1.0886	1.0886
1	1.3333	1.3333	1.3333
3	2.5097	2.5097	2.5097
10	4.7969	4.7969	4.7969

Figs. 2 – 10 depict the distributions of velocity and temperature for different values of M, β, ρ_r, Pr and k. In Fig. 2, it is shown that the increasing M decreases the velocity for both fluid and dust. When the velocity of stretching sheet is high, the increase of M causes the main stream velocity to reduce the velocity of the fluid and dust. Hence, as M increases, the drag force which opposes the flow also increases and thus decreases the velocity of the flow.Fig. 3 shows the effect of magnetic parameter M on temperature, where the temperature increases as M increases.
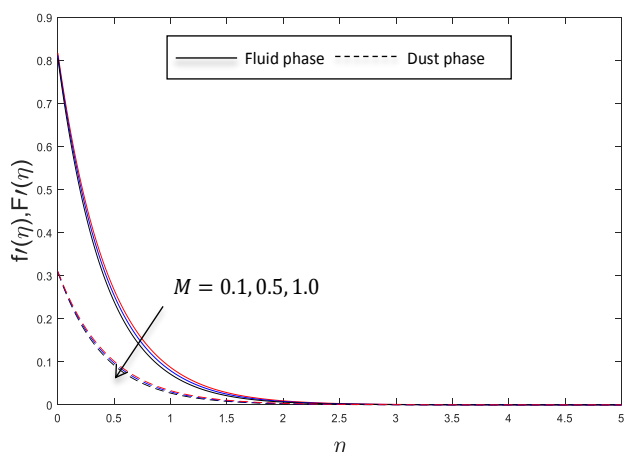
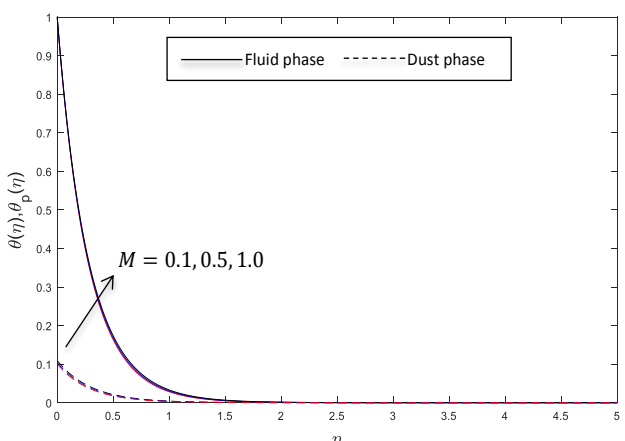**Fig. 2** Velocity profiles for fluid phase $f'(\eta)$ and dust phase $F'(\eta)$ for different values of M.

Fig. 3 Temperature profiles for fluid phase $\theta(\eta)$ and dust phase $\theta_p(\eta)$ for different values of M.

The influence of β on velocity and temperature are shown in Figs. 4 and 5, respectively. As β increases, velocity of dust also increases. However, the increment of β causes the velocity of fluid to decrease. It is expected that for a large value of β the velocities for both fluid and dust will be the same due to the decreasing relaxation time. On the other hand, the temperature for fluid decreases and increases for dust as β increases.
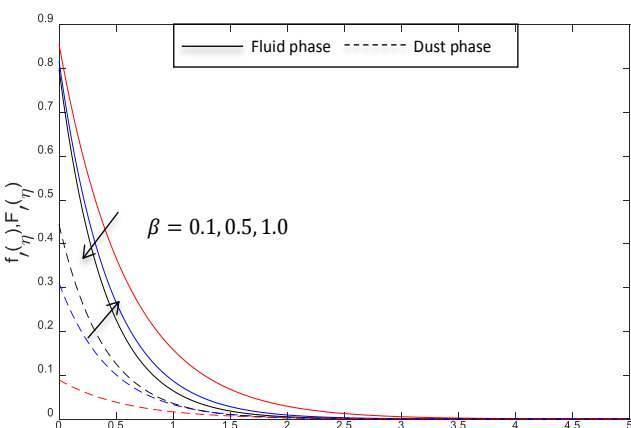

Fig. 4 Velocity profiles for fluid phase $f'(\eta)$ and dust phase $F'(\eta)$ for different values of β.
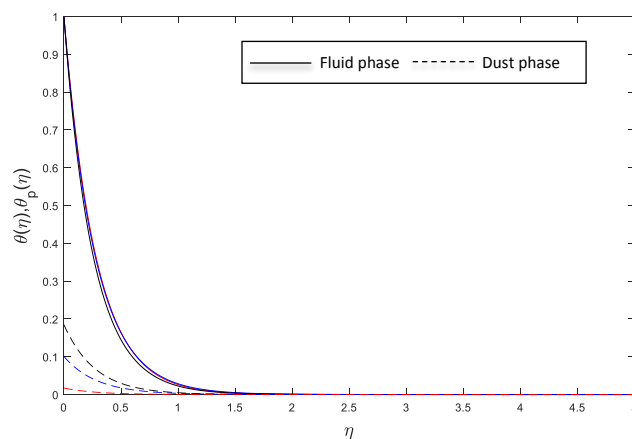

Fig. 5 Temperature profiles for fluid phase $\theta(\eta)$ and dust phase $\theta_p(\eta)$ for different values of β.

Through Figs. 6 and 7, we can see the effect of porosity k on velocity and temperature. Due to the presence of porous medium, we expect that it will increase the resistive force in fluid motion, as a result, we can observe that as k increases, velocity for fluid and dust decreases. However, the temperature for fluid and dust have a slight increase as k increases. This phenomenon occurs due to the thickening thermal boundary layer as porosity increases.
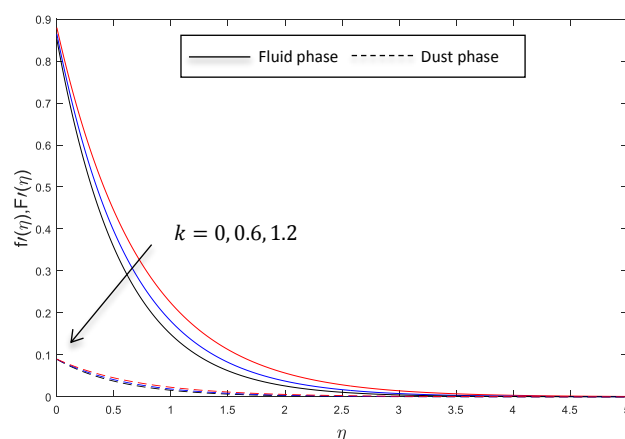

Fig. 6 Velocity profiles for fluid phase $f'(\eta)$ and dust phase $F'(\eta)$ for different values of k.
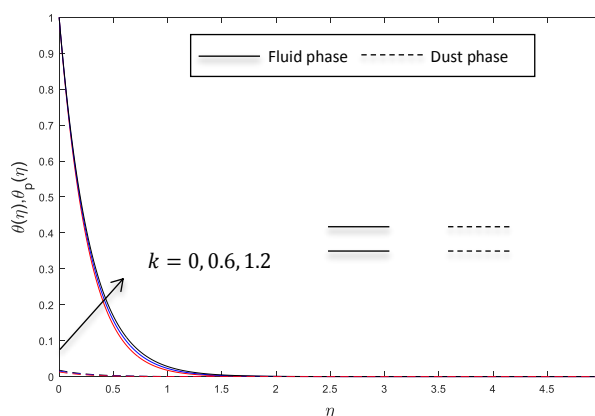

Fig. 7 Temperature profiles for fluid phase $\theta(\eta)$ and dust phase $\theta_p(\eta)$ for different values of k.

Figs. 8 and 9 show the effect of mass concentration ρ_r on velocity and temperature It is obvious from the figure that as ρ_r increases, the velocities for fluid and dust decrease. This is because the

larger mass concentration will slow the movement of fluid and dust. However, the temperature shows only a slight increase as ρ_r increases.
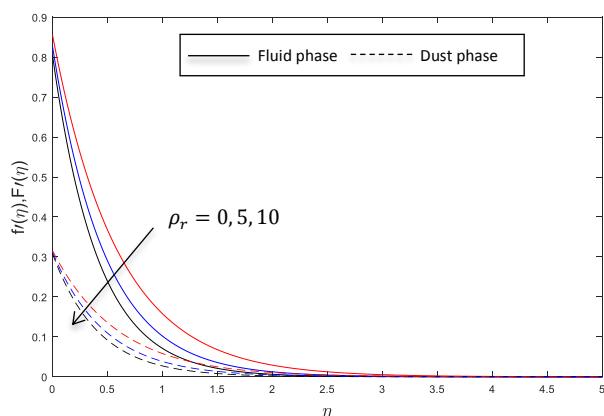

Fig. 8 Velocity profiles for fluid phase $f'(\eta)$ and dust phase $F'(\eta)$ for different values of ρ_r.
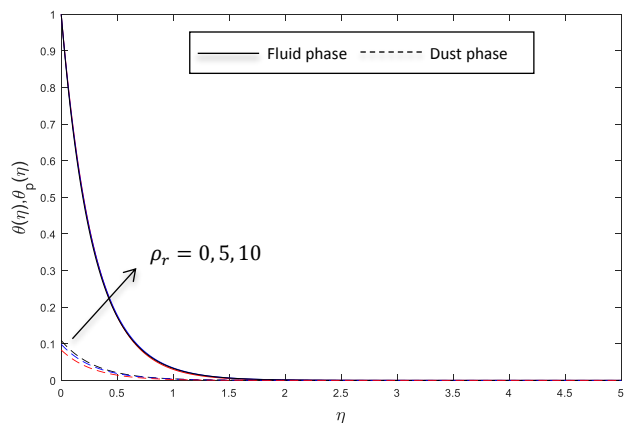

Fig. 9 Temperature profiles for fluid phase $\theta(\eta)$ and dust phase $\theta_p(\eta)$ for different values of ρ_r.

The temperatures for different values of Prandtl number Pr are shown in Fig. 10. We observed that the increasing number of Pr implies the decreasing temperature of fluid and dust phase which implies the thicker momentum boundary layer than the thermal boundary layer.
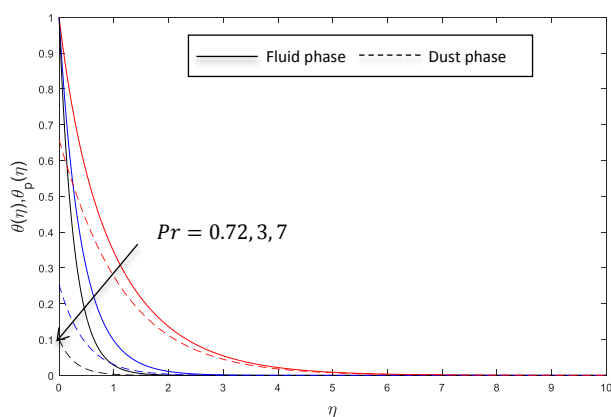

Fig. 10 Temperature profiles for fluid phase $\theta(\eta)$ and dust phase $\theta_p(\eta)$ for different values of Pr.

Table 2 displays the velocity gradient $-f''(0)$ and temperature gradient $-\theta'(0)$. Examining the table reveals that the increasing values of M, ρ_r, k and β will decrease the skin friction coefficient and the

increasing values of ρ_r, β and Pr decrease the rate of heat transfer at the surface.

Table 2 Values of velocity gradient $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for different M, β, ρ_r, k and Pr.

M	ρ_r	β	k	Pr	$-f''(o)$	$-\theta'(0)$
0					1.8158	3.7388
0.5	10	0.5	1	7	1.8863	3.7010
1.0					1.9528	3.6651
	0				1.4455	3.4716
1	5	0.5	1	7	1.7314	3.5503
	10				1.9528	3.6651
		0.1			1.6190	3.4761
1	10	0.5	1	7	1.9528	3.6651
		1.0			2.1153	3.9388
			0		1.8158	3.7388
1	10	0.5	0.5	7	1.8863	3.7010
			1.0		1.9528	3.6651
				0.72	1.9528	1.2152
1	10	0.5	1	3	1.9528	2.5157
				7	1.9528	3.6651

CONCLUSION

The effect of some parameters on hydromagnetic boundary layer flow of a dusty fluid in a porous medium over a stretching sheet with slip effect is investigated. The partial differential equations governing the problem were transformed into a set of nonlinear ordinary differential equations by using similarity transformation. The influence of different parameters including fluid particle interaction parameter β, magnetic parameter M, porosity parameter k, Prandtl number Pr and mass concentration ρ_r on velocity, temperature and gradients of both velocity and temperature are shown.. In conclusion,:

- The effects of M, k and ρ_r is to decrease the momentum boundary layer thickness.
- The effect of β is to increase the momentum boundary layer thickness of fluid. However, it reduces the momentum boundary layer thickness of dust..
- Increasing the Pr decreases the thermal boundary layer thickness.
- Increasing values of M, ρ_r, k and β decrease the skin friction.
- Increment of ρ_r, β and Pr decrease the rate of heat transfer.

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