

RADIATIVE HEAT TRANSFER IN MHD MIXED CONVECTION FLOW OF
NANOFLUIDS ALONG A VERTICAL CHANNEL

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TO MY FAMILY

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ABSTRACT

Over the past few decades, nanofluids have emerged as a promising technology for the enhancement of the intrinsic thermophysical properties of many convective heat transfer fluids such as water and oil. Many researchers have been investigating the merits of dispersing nanometer-sized particles into base fluids to enhance heat transfer, thermal conductivity and viscosity of the fluids. Therefore, this research focused on radiative heat transfer in magneto-hydrodynamics mixed convection flow in a channel filled with nanofluids containing different type of nanoparticles. Five types of nanoparticles (Al_2O_3 , Fe_3O_4 , Cu , TiO_2 , and Ag) with five different shapes (platelet, blade, cylinder, brick and spherical) were used in water (H_2O) and ethylene glycol ($C_2H_6O_2$), as conventional base fluid. An important subtype of nanofluids called ferrofluids (Fe_3O_4 in water based nanofluids) was also studied. Four different problems were modelled as partial differential equations with physical boundary conditions. In the first three problems, the channel walls were taken rigid, while the fourth problem the walls were chosen permeable where suction or injection was taking place. Perturbed type analytical solutions for velocity and temperature were obtained and discussed graphically in various graphs. Results for skin friction and Nusselt number were also computed and presented in tabular forms. This study showed that $C_2H_6O_2$ was the better convective base fluid compared to H_2O because of the higher viscosity and thermal conductivity. Ag nanoparticles had the highest thermal conductivity and viscosity compared to other type of nanoparticles. Increasing nanoparticles size had caused variation in velocity. It was also observed that, variation in velocity for Ag nanoparticles was obtained at low volume concentration, whereas for Al_2O_3 nanoparticles, this variation was observed only at high volume concentration. Velocity increases with increasing Grashof number, radiation, heat generation and permeability parameters, but decreases with increasing magnetic parameter and volume fraction of nanoparticles. However, the effects of these parameters were quite different in the case of suction and injection. Results had also shown that, temperature increases with increasing radiation and heat generation parameters. In this study, the temperature of ferrofluids was found smaller when compared to the temperature of nanofluids.

ABSTRAK

Sejak beberapa dekad yang lalu, bendalir nano telah muncul sebagai suatu teknologi yang berpotensi untuk meningkatkan sifat-sifat termofizikal intrinsik dalam kebanyakan bendalir pemindahan haba yang lazim seperti air dan minyak. Ramai penyelidik telah mengkaji merit penguraian partikel bersaiz nanometer kepada bendalir asas untuk meningkatkan pemindahan haba, kekonduksian terma dan kelikatan bendalir. Oleh itu, penyelidikan ini memberi tumpuan kepada pemindahan haba sinaran di dalam aliran olakan campuran hidrodinamik magnet di dalam saluran yang dipenuhi dengan bendalir nano mengandungi pelbagai jenis partikel nano. Lima jenis partikel nano (Al_2O_3 , Fe_3O_4 , Cu , TiO_2 , dan Ag) dengan lima bentuk yang berbeza (platelet, bilah, silinder, bata dan sfera) telah digunakan di dalam air, (H_2O) dan etilena glikol ($C_2H_6O_2$), sebagai bendalir asas lazim. Subjenis penting dalam bendalir nano dikenali sebagai ferobendalir (Fe_3O_4 di dalam bendalir nano berasaskan air) juga dikaji. Empat masalah yang berbeza telah dimodelkan sebagai persamaan pembezaan separa berserta syarat sempadan fizikal. Dalam tiga masalah yang pertama, dinding saluran adalah tegar, manakala dalam masalah keempat dinding saluran telap dipilih bagi membolehkan berlakunya sedutan atau suntikan. Penyelesaian analitik jenis usikan bagi halaju dan suhu telah diperoleh dan dibincangkan secara grafik dalam pelbagai graf. Keputusan bagi geseran kulit dan nombor Nusselt juga dikira dan dipersembahkan dalam bentuk jadual. Kajian ini menunjukkan bahawa, bendalir asas lazim $C_2H_6O_2$ adalah lebih baik berbanding H_2O kerana kelikatan dan kekonduksian terma adalah lebih tinggi. Ag partikel nano mempunyai kelikatan dan kekonduksian terma yang paling tinggi berbanding jenis partikel nano yang lain. Peningkatan saiz partikel nano menyebabkan berlakunya perbezaan dalam halaju. Dapat diperhatikan bahawa, perubahan dalam halaju untuk partikel nano Ag telah diperoleh ketika isipadu kepekatan rendah, manakala bagi partikel nano Al_2O_3 variasi ini diperhatikan hanya ketika isipadu kepekatan tinggi. Halaju meningkat dengan peningkatan nombor Grashof, parameter sinaran, parameter penjanaan haba dan parameter kebolehtelapan, tetapi berkurangan dengan peningkatan parameter magnet dan pecahan isipadu partikel nano. Namun, kesan bagi semua parameter ini agak berbeza untuk kes sedutan dan suntikan. Keputusan juga menunjukkan bahawa, suhu meningkat dengan peningkatan parameter sinaran dan parameter penjanaan haba. Dalam kajian ini, suhu bagi ferobendalir didapati lebih kecil berbanding dengan suhu bendalir nano.

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LIST OF ABBREVIATIONS

EG	-	Ethylene Glycol
MHD	-	Magnetohydrodynamic
DI	-	Deionized water
RBC	-	Rotary Blade Coupling

LIST OF MATTERS

Al_2O_3	-	Alumina oxide
CuO	-	Copper oxide
Ag	-	Silver
Fe_3O_4	-	Ferric oxide
TiO_2	-	Titanium Dioxide
Cu	-	Copper
$C_2H_6O_2$	-	Ethylene glycol
H_2O	-	Water
$CoFe_3O_4$	-	Cobalt ferrite
$MnBi$	-	Manganese bismuth
Ni	-	Nickel
Fe	-	Iron
Gd	-	Gadolinium
$Mn-ZnFe_3O_4$	-	Manganese-Zinc ferrite

LIST OF SYMBOLS

Roman Letters

a, b	-	Constants depend on shape of nanoparticles
A	-	Surface area of the control volume
B_0	-	Applied magnetic field
B_0	-	Magnitude of applied magnetic field
B	-	Total magnetic field
\underline{b}	-	Induced magnetic field
$(c_p)_s$	-	Heat capacity of solid nanoparticles
$(c_p)_f$	-	Heat capacity of base fluids
$(c_p)_{nf}$	-	Heat capacity of nanofluids
D	-	Rate of strain tensor
d_p	-	Diameter of solid nanoparticles
E	-	Total electric field
e	-	Internal energy per unit volume
\exp	-	Exponential function
F	-	Force
f	-	Function of temperature and volume fraction etc.
Gr	-	Thermal Grashof number
g	-	Gravitational acceleration
$H(.)$	-	Heaviside function
H	-	Total Momentum of the system
I	-	Identity tensor
i	-	Cartesian unit vector in the x -direction

J	-	Current density
$\mathbf{J} \times \mathbf{B}$	-	Lorentz force
j	-	Cartesian unit vector in the y –direction
K	-	Dimensionless Permeability parameter
k_s	-	Thermal conductivity of solid nanoparticles
k_f	-	Thermal conductivity of base fluids
k_{nf}	-	Thermal conductivity of nanofluids
k_b	-	Boltzmann constant
k	-	Cartesian unit vector in the z –direction
M	-	Magnetic parameter
m	-	Mass of the flow of fluids
N	-	Radiation parameter
Nu	-	Nusselt number
n	-	Empirical shape factors
p	-	Pressure
p_h	-	Hydrostatic pressure
p_d	-	Dynamic pressure
Pe	-	Peclet number
Q	-	Heat generation parameter
\mathbf{q}_r	-	Radiant flux vector
q_r	-	Magnitude of radiant heat flux
\mathbf{q}''	-	Heat conduction per unit area
q''	-	Magnitude of heat conduction per unit area
Re	-	Reynold’s number
r_c	-	Radius of gyration
S	-	Surface of the control volume
T	-	Cauchy stress tensor
T	-	Temperature
t	-	Time
u	-	Velocity in x –direction

U_0	-	Reference velocity
$\nabla\nabla$	-	Dyadic tensor
\mathbf{V}	-	Velocity vector field
\forall	-	Control volume
V	-	Magnitude of velocity
v_0	-	Constant velocity in y -direction
W	-	Work done

Greek Letters

ρ_s	-	Density of solid nanoparticles
ρ_f	-	Density of base fluids
ρ_{nf}	-	Density of nanofluids
β_s	-	Volumetric coefficient of thermal expansion of solid nanoparticles
β_f	-	Volumetric coefficient of thermal expansion of base fluids.
β_{nf}	-	Volumetric coefficient of thermal expansion of nanofluids
μ_s	-	Dynamic viscosity of solid nanoparticles
μ_f	-	Dynamic viscosity of base fluids
μ_{nf}	-	Dynamic viscosity of solid nanofluids
$\underline{\beta}$	-	Modeling function
ϕ	-	Volume fraction of solid nanoparticles
Ψ	-	Sphericity
ω	-	Oscillating parameter
ε	-	Perturbed parameter
λ	-	Williamson parameter
α_0	-	Mean absorption coefficient
σ_{nf}	-	Electrical conductivity of nanofluids
σ_{nf}	-	Electrical conductivity of nanofluids

σ_{nf}	-	Electrical conductivity of nanofluids
μ_m	-	Magnetic permeability
∇	-	Delta function
τ_1	-	Skin friction
τ	-	Viscous stress tensor

Subscripts

w	-	condition on the wall
∞	-	free stream condition

CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter is intended to provide the research background, problem statement, research objectives, scope of research, significance of the study, research methodology and thesis outline. The research background describes a brief summary of research and embarks on the study of the flow of radiative heat transfer in MHD mixed convection flow of nanofluids along a vertical channel. The problem statement includes some questions about the mathematical formulation, solutions and influence of various parameters on the flow problem. Research objectives provide the problems tackled in this research together with scope and significance of the study.

1.2 Research Background

Fluids are generally consists of liquids and gases, which are two different phases of matter. Fluid is a substance that continuously deforms under an applied shear stress. There are various types of fluids. However, they are mainly divided into two types knows as Newtonian and non-Newtonian fluids. There are two ways for a fluid to be Newtonian or non-Newtonian. The first way for a fluid to be non-

Newtonian depends on the Cauchy stress tensor used in the constitutive equation of motion. The second way depends on the additional nanoparticles and the volume fraction of nanoparticles, added to a base fluid. This research focuses on the second type of non-Newtonian fluids where the non-Newtonian behavior comes not because of the Cauchy stress tensor but due to the additional nanoparticles to the base fluids. Only on Newtonian fluid in which the shear stress is directly proportional to shear strain. More exactly, in this research Newtonian fluid is used as base fluid and various types of nanoparticles are suspended inside it. This mixture forms is called as nanofluids (Das *et al.*, 2008). Nanofluids on the other hand are liquids or conventional base fluids such as water, ethylene glycol, acetone, decene and oils, containing suspensions of solid nanoparticles with sizes typically of 1-100 *nm*. The thermal conductivity and viscosity of nanofluids are much higher than the conventional base fluids. Even for very small volume fraction of nanoparticles, a large amount of increase in thermal conductivity is observed. Due to this reason, the interests of researchers in investigating nanofluids are increasing day by day.

Different parameters are responsible for the enhancement of thermal conductivity and viscosity of nanofluids such as base fluids, volume fraction, size, shape, effect of particles material, PH value and clustering of nanoparticles. Besides, heat transfer in fluids containing nanoparticles has superior thermo physical properties than the conventional base fluid in terms of thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficient. The reason is that, the conventional heat transfer fluids have inherently poor thermal conductivity compared to solids. Therefore, scientists have tried to make fluids, which enhance the poor thermal conductivity of these conventional heat transfer fluids using uniform dispersion and stable suspension of solid nanoparticles. Further, the researchers are getting interested in nanofluids because of their importance in industry. Some of its applications are found in crystal silicon mirror cooling used in high intense x-ray sources. X-ray sources create a large amount of heat which is controlled by these mirrors. This advanced cooling technology was established by Lee and Choi (1996). Chien *et al.* (2003) were the first to used gold nanoparticles in electronic cooler and enhanced its heat transfer performance. Tsai *et al.* (2004) improved the quality of deionized water (DI) by using gold nanoparticles for meshed circular heat pipe. Heat

pipe was constructed as a heat spreader for desktop or CPU and wire of 200-mesh was used inside the heat pipe. It was observed by Tsai *et al.* (2004) that thermal resistance of the meshed circular heat pipe was reduced by using nanofluids. Silver (Ag) nanoparticles were used inside DI and improved the heat transfer performance of grooved circular heat pipe (Kang *et al.*, 2006). In powerful transmission system, Rotary Blade Coupling (RBC) in four wheel drive vehicle easily attains a high local temperature at high rotating speed. This high thermal stress can damage the rotating components of RBC which is not fixable and should be knocked out. Therefore, Tzeng *et al.* (2005) was the first to use alumina oxide (Al_2O_3) and copper oxide (CuO) nanoparticles in transmission fluids to improve the cooling performance of RBC. Xuan and Li (2003a) and Yu *et al.* (2007) worked to improve the heat transfer performance of transformer oils. They found that if the oils of transformer are replaced by nanofluids then the transformer size can be reduced with the same efficiency. This work is still challenging. Other dynamic applications are found in biomedical processes. Recent investigations proved that cylindrical shaped nanoparticles are seven times more deadly than traditional spherical shaped nanoparticles in the delivery of drug to breast cancer cells. Magnetite nanoparticles are used in cancer therapy to produce high temperature and damage the cancer cells. Nanoparticles can also be used as a safer surgery by cooling around the surgical area (Jordan *et al.*, 1999).

The idea of using small-sized solid particles inside fluids to increase their thermal conductivity was initially given by Maxwell (1873). This idea was based on suspension of micro-sized or milli-sized solid particles inside fluids. Subsequently, it was realized that large sized particles in the milli-scale or even micro-sized particles cause several technical problems. For example, (i) faster settling time, (ii) clogging micro-channels of devices, (iii) abrasion of surfaces, (iv) erosion of pipelines and (v) increasing drop in pressure (Das *et al.*, 2008). Bruggeman (1935) proposed a model to estimate the thermal conductivities of nanoparticles at higher particle concentrations. However, this model was only applicable for spherical shape of nanoparticles. Hamilton and Crosser (1962) extended the Maxwell model to incorporate the effect of the different shapes of the solid particles. Both Maxwell, and Hamilton and Crosser models were derived for the suspension of micro-or milli-

sized solid particles inside the fluids. Currently, these models are frequently used for the study of nanofluids due to their simplicity. Initially, Choi (1995) gives the idea of improving thermal conductivity using nano-sized particles. More specifically, it was experimentally verified in this work that addition of nanoparticles in conventional based fluids enhances the thermal conductivity. Apart from higher thermal conductivity, the addition of nano-sized particles over micro-sized particles to conventional base fluid was preferred due to several valid scientific reasons such as (i) longer suspension time (more stable), (ii) larger surface area/volume ration (1000 times larger), (iii) lower erosion and clogging, (iv) lower demand for pumping power (v) reduction in inventory of heat transfer fluid, and (vi) significant energy saving. Several other theoretical models are available in the literature for calculating the effective thermal conductivity and viscosity of nanofluids (Einstein, 1906; Xuan *et al.*, 2003b; Koo and Kleinstreuer, 2004; Abbaspoursani *et al.*, 2011; Corcione, 2011).

Khanafer *et al.* (2003) studied the buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. The role of Brownian motion in the enhanced thermal conductivity of nanofluids was investigated by Jang and Choi (2004). Chang *et al.* (2005) analysed rheology of *CuO* nanoparticle suspension. Tiwari and Das (2007) studied heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. Temperature and particle size dependent viscosity data for water based nanofluids hysteresis phenomenon was investigated by Nguyen *et al.* (2007). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids was studied by Oztop and Abu-Nada (2008). Timofeeva *et al.* (2009) analyzed particle shape effect on thermophysical properties of alumina nanofluids. Prasad *et al.* (2010) studied the effect of variable fluid properties on the magnetohydrodynamic (MHD) flow and heat transfer over a non-linear stretching sheet.

Khan and Pop (2010) investigated boundary-layer flow of a nanofluids past a stretching sheet. Ahmad and Pop (2010) focused on mixed convection boundary layer flow of nanofluids from a vertical flat plate embedded in a porous medium. Kuznetsov and Nield (2010) investigated natural convective boundary-layer flow of

nanofluids past a vertical plate. In two other investigations Nield and Kuznetsov (2009) and Nield and Kuznetsov (2011) analysed the Cheng-Minkowycz problem for natural convection flow and double diffusive natural flow of nanofluids past a vertical plate embedded in a porous medium. Radiation effect on viscous nanofluids with three different types of spherical shapes of nanoparticles over a nonlinearly stretching sheet was investigated by Hady *et al.* (2012) using shooting technique. They found that ethylene glycol (EG) ($C_2H_6O_2$) has the highest cooling performance than nanoparticles in water (H_2O) base nanofluids. Free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with nanofluids was investigated by Khan and Pop (2011). Bachok *et al.* (2010a) provided numerical solutions for the boundary-layer flow of nanofluids over a moving surface in a flowing fluid. By taking the porosity and MHD effects together, Zhang *et al.* (2015) studied radiation heat transfer in nanofluids containing Cu , Al_2O_3 and Ag past a flat plate having variable surface heat flux and the first-order chemical reaction is also considered.

MHD or magneto-fluid-dynamics (MFD) is the field of fluid mechanics which deals with the dynamics of an electrically conducting fluid under the influence of magnetic field. First time, Hannes Alfvén introduced MHD, and received Noble Prize in 1970 in the field of physics. MHD is described by a set of equations which is the combination of Navier-Stokes and Maxwell equations. Currently, the study of heat transfer by mixed convection in a MHD fluid through a porous channel has garnered the attention and interest of several researchers. This is primarily attributed to the plethora of its applications in the field of science of technology, for instance the heat exchange between atmosphere and soil to form heat beds, beds of fossil fuels; the leaching of salt into soil; the distribution of chemical pollutants into saturated soil; the collection of solar power; insulation of nuclear reactors; moisture migration in fibrous insulation; underground disposal of nuclear waste; the extraction of geothermal energy; chemical catalytic reactors; the storage of grain and many more. Taking into account the significance of MHD in nanofluids, Mansur *et al.* (2015) conducted a study to explore the MHD stagnation point flow of nanofluids over a stretching/shrinking sheet with suction. Colla *et al.* (2012) investigated water-based nanofluids characterization, thermal conductivity and viscosity measurements

and correlation. Abareshi *et al.* (2010) studied fabrication, characterization and measurement of thermal conductivity of nanofluids. Borglin *et al.* (2000) studied experimentally the flow of magnetic nanofluids in porous media.

Effects of a transverse magnetic field and radiative heat transfer on the mixed convection unsteady oscillatory flow of a viscous fluid in a channel filled with porous medium was studied by Makinde and Mhone (2005). Mehmood and Ali (2007) extended their work by taking into account the slip condition. However, such studies for nanofluids in the presence of magnetic field and porous medium are not available. Maghrebi *et al.* (2012) investigated forced convection heat transfer of nanofluids in a porous channel. Mahdi *et al.* (2014) studied the influence of geometrical shapes on mixed convection through open-cell aluminium foam filled with nanofluids.

The above study shows that, using fluids such as water, ethylene glycol, and mineral oils are found to have poor thermal characteristics when compared with metals, non-metals and their oxides. Due to this, it was noticed that the flow analysis of nanofluids with the interaction of magnetic field have increased enormously. There are three categories which describe how a material is equivalently affected by a magnetic field. (i) Diamagnetism: materials such as copper, lead, quartz, water, acetone, and carbon dioxide are diamagnetic and are very weakly affected by magnetic fields, (ii) Paramagnetism: materials such as sodium, oxygen, iron oxide, and platinum are paramagnetic. They are affected somewhat more strongly than diamagnetic materials, and become polarized parallel to a magnetic field (iii) Ferromagnetic: ferromagnetic materials include gadolinium, iron, iron oxide (magnetite), and nickel, cobalt ferrite and manganese bismuth. These materials are strongly affected by magnetic fields. In addition, they become strongly polarized in the direction of the magnetic field and retain their polarization state after the magnetic field is removed (Scherer and Figueiredo Neto, 2005)

Amongst these three types, ferromagnetic materials produce a strong magnetic field. The resulting fluid is called ferrofluids which is also known as

magnetic fluid or magnetite nanofluids. More specifically, ferrofluids are colloidal suspensions of small magnetic particles in a carrier liquid. Some important uses of ferrofluids are found in mechanical damping in loudspeakers and in heat exchangers. In the present research, nanoparticles of magnetite (Fe_3O_4), being the most commonly used magnetic work and water is chosen as a conventional base fluid.

Based on the importance of ferromagnetic materials, Qasim *et al.* (2014) examined MHD flow with slip condition in the presence of heat transfer in ferrofluids with magnetite (Fe_3O_4) nanoparticles over a stretched cylinder with given heat flux. Khan *et al.* (2014) tackled a stagnation-point flow problem of ferrofluids along a stretching sheet with viscous dissipation and heat transfer. They considered ferroparticles of three types: Fe_3O_4 , cobalt ferrite ($CoFe_3O_4$), and $Mn-Zn$ ferrite ($Mn-ZnFe_3O_4$). However, they selected two types of base fluid, water and kerosene and found some interesting results for these two types of base fluids after using implicit finite-difference method with quasi-linearization technique as the solution to a resultant problem. Sheikholeslami and Ganji (2014) analysed ferrohydrodynamic and magnetohydrodynamic effects on ferrofluids flow and convective heat transfer.

Hamad *et al.* (2011a) studied the magnetic field effects on free convection flow of nanofluids past a vertical semi-infinite flat plate. Then, followed by Hamad (2011b) where analytical solution of natural convection flow of nanofluids over a linearly stretching sheet in the presence of magnetic field has been obtained. The conjugate phenomenon of heat and mass transfer of nanofluids over a moving permeable surface with convective boundary conditions has been analyzed by Qasim *et al.* (2013). Mahajan and Sharma (2014) embark on convection in magnetic nanofluids in porous media.

The problems discussed above are mostly carried out either using experimental, numerical or any approximate scheme. Exact solutions for nanofluids are very rare. The first exact solution for nanofluids seem to be that obtained by Loganathan *et al.* (2013) using the Laplace transform method. Turkyilmazoglu

(2014) observed the unsteady convection flow of some nanofluids past a moving vertical flat plate with heat transfer. The governing equations are solved for exact solutions using two types of boundary conditions namely prescribed uniform wall temperature (PST) and prescribed uniform heat flux (PHF). Asma *et al.* (2015) obtained exact solutions for the MHD flow of nanofluids using the Laplace transform method.

It is also noticed from the above discussion, researchers have conducted many experimental or numerical investigations that the heat transfer enhancement through nanofluids either due to free or forced convections in different geometrical configurations. However, limited analytical studies on mixed convection flows of nanofluids in vertical channels have been carried out. Such studies are even scarce in the presence of MHD and porous medium. Therefore, this project mainly focuses on the analytical study of nanofluids and ferrofluids passing through a vertical channel together with heat transfer due to mixed convection. The effects of MHD and porosity are also considered.

1.3 Problem Statement

This study explains the following questions. How the Newtonian based nanofluids and ferrofluids models behave in the problem of heat transfer in MHD mixed convection flow inside a vertical channel? How does the mathematical model behave in this problem involving heat transfer? How does the presence of some parameters including porosity, MHD, heat generation, and some fluids parameters including shape, size, base fluid, particle material, volume fraction and clustering of nanoparticles affect the fluid motion? How does the mixed convection phenomenon occurs in a vertical channel with wall transpiration? How do the analytical solutions for heat transfer in mixed convection flow inside a vertical channel under different effects can be obtained? Specifically, the problems of nanofluids and ferrofluids investigated in this research are:

Problem I. MHD mixed convection flow of a ferrofluids along a vertical channel.

Problem II. Radiation and heat generation effects on MHD mixed convection flow of nanofluids along a vertical channel.

Problem III. MHD mixed convection flow of nanofluids in a vertical channel filled with saturated porous medium.

Problem IV. MHD Mixed convection flow of nanofluids in a porous channel with permeable walls.

1.4 Research Objectives

This theoretical investigation studies the effect of radiation on MHD mixed convection flow of nanofluids and ferrofluids along vertical plate, as mentioned in problem statement. The objectives of this research are:

- i) to derive the mathematical models of the problems which consists of continuity, momentum and energy equations.
- ii) to solve the dimensionless governing equations analytically by using perturbation method.
- iii) to obtain the results of velocity and temperature profiles as well as skin friction and Nusselt number for each of the problem mentioned in problem statement.
- iv) to analyse the results obtained graphically and via tabulated results for different physical conditions namely radiation parameter, magnetic parameter, heat generation parameter, permeability parameter, Prandtl number and Grashof number as well as different types of nanoparticles, shapes, sizes and volume fractions.

1.5 Scope of the Study`

This thesis is focused on the unsteady MHD mixed convection flow of nanofluids inside a vertical channel. Nanoparticles are suspended inside regular fluids where water and ethylene glycol are chosen for this purpose. Nanofluids or ferrofluids are introduced by using several models and equations. Three different driving forces have been considered, which are responsible for inducing the motion into the fluid. These are buoyancy force, external pressure gradient and boundary wall. The first problem emphasized on MHD mixed convection flow of ferrofluids passing through a vertical channel with stationary walls. The second problem focuses on the influence of radiation and heat generation effects on MHD mixed convection flow of nanofluids along a vertical channel. The third problem explores the MHD mixed convection flow of nanofluids in a vertical channel filled with porous medium together with stationary and oscillating boundary conditions. The fourth problem highlights the study of mixed convection flow of nanofluids in a porous channel filled with permeable walls. Perturbation technique has been used to solve the governing linear partial differential equations. Analytical solutions for velocity and temperature are obtained for all the proposed problems and plotted through various graphs. A computational software namely Mathcad has been used for plotting graphs and for computing tabulated results. Further, the limiting cases of the present results give the published results in literature.

1.6 Significance of the study

1. The results obtained in this research enable to enhance the knowledge of the MHD mixed convection flow and heat transfer characteristics through porous medium, with different fluid parameters for nanofluids in a vertical channel.
2. Thermo-physical properties of liquids play a vital role in heating as well cooling applications. Thermal conductivity of a liquid decides its heat transfer performance, due to which it has been regarded as one of the important thermophysical property.

3. The results obtained in this project for Newtonian based nanofluids can be used as bases for complex flow problems frequently occurring in engineering and applied sciences. This idea can be extend for other.
4. Heat transfer is one of the important process in many industrial, consumer products, power generation, microelectronics, air conditioning and transportation.
5. Convection in porous medium and heat generation effects plays an important role in many applications such as geothermal energy storage and flow through filtering devices.

1.7 Research Methodology

This section intends to provide the current development of research which contains two sub-sections, mathematical analysis and numerical computations.

1.7.1 Mathematical Analysis

Mathematical formulation of the problem is done where the equation of momentum and energy are derived for the problems mentioned in Section 1.3. Fluid motion is originated due to buoyancy force together with external pressure gradient of oscillatory form. Water and EG are used as a conventional base fluids. Nanoparticles of magnetite (Fe_3O_4), silver (Ag) in spherical and aluminium oxide (Al_2O_3) in four different shapes namely cylinder, platelet, brick and blade shape are used. The problems are modelled in term of Partial Differential Equations (PDE's) with physical boundary conditions. Perturbation technique has been used to solve the governing problems. Based on the boundary conditions, three different flow situations are discussed.

1.7.2 Numerical Computations

Analytical solutions for velocity and temperature are obtained and plotted through various graphs. The results are discussed for different parameters such as magnetic, radiation, heat generation, permeability, types of nanoparticles, volume fraction, and Grashof number. Water based nanofluids have been compared with EG based nanofluids. Influence of different shapes and sizes of nanoparticles has also been analysed. A computational software namely Mathcad has been used for plotting graphs and for computing tabulated results. Further, it is found that the limiting results give the published results in literature.

1.8 Thesis Organization

This thesis includes total 7 Chapters. Chapter 1 is an introductory chapter which includes the research background, problem statements, objectives and scope of the research, research methodology, significance of the study and finally thesis outlines. Chapter 2 provides the literature review.

Chapter 3 discussed the first problem on MHD mixed convection flow of ferrofluids along a vertical channel. This chapter contains four sections including introduction, mathematical formulation of the problem, solution of the problem and results and discussion. Introduction includes a brief discussion of the problem. Mathematical formulation of the problem is performed where the equations of continuity, momentum and energy are derived. Analytical solutions of velocity and temperature are obtained using the perturbation technique. Expressions for skin friction and rate of heat transfer are also computed. The results are plotted and discussed for different parameters of interest.

Chapter 4, extends the idea of Chapter 3 to the case when the temperature equation takes into account the heat generation parameter. In addition different

models for finding thermal conductivity and viscosity of nanofluids are used to evaluate the effect of sizes of nanoparticles on the flow problem. Same procedure as in Chapter 3 is used for finding velocity and temperature. Results are plotted and discussed for various embedded parameters.

The third problem is discussed in Chapter 5. This problem deals with the MHD mixed convection flow of nanofluids in a channel filled with saturated porous medium. Darcy's law is incorporated in momentum equation. Water and EG are used as conventional base fluids. The energy equation is the same as in Chapter 3. However, here three different flow situations are discussed. Similar to Chapter 3, the solutions of velocity and temperature are obtained by using the perturbation technique. They satisfy all imposed boundary conditions. Further, it is found that the limiting results give the published results in literature. Different from Chapter 3, various models of viscosity and thermal conductivity has been used and based on them, the results of velocity and temperature are computed for three different flow problems depending on the boundary conditions. The shape-based viscosity and Hamilton and Crosser model (1962) of thermal conductivity are used to incorporate the shape effects of nanoparticles. In first case, both of the bounding walls of the channel are at rest. In the second case, the upper wall of the channel is set into oscillatory motion in its own plane whereas the third case extends this idea when both of the channel walls are set into oscillatory motion. Similar to Chapter 3, the associated expressions for skin friction and rate of heat transfer are also evaluated. The graphical results are displayed to see the effects of various embedded parameters on the velocity and temperature profiles.

In Chapter 6, the problem of mixed convection flow of nanofluids in a porous channel with permeable walls is studied. The focal point of this chapter is to study the influence of permeable walls on momentum and heat transfers. The permeable parameter which physically corresponds to suction and injection is incorporated in both the momentum and energy equations. As in previous chapters, solutions of the problem are obtained by using the perturbation technique. Expressions for velocity and temperature are obtained. Effects of various parameters such as thermal Grashof number, volume fraction, different types of nanoparticles, radiation, suction and

injection are studied in different plots. Finally, in Chapter 7, summary of the research and future recommendation are included.

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