MANAGING EMERGING MARKET OPERATIONS

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ABSTRACT

YING ZHANG: MANAGING EMERGING MARKET OPERATIONS.
(Under the direction of Jayashankar M. Swaminathan.)

Emerging markets have been a critical part of global business, with high share of global GDP and rapid economy growth. My dissertation research focuses on studying risks and opportunities in emerging market operations. One critical characteristic of emerging markets is that agriculture remains an essential sector. The world looks to emerging countries to meet the increasing food demand. However, the output remains significantly below the potential due to limited financial, technology and policy support. Scientific agriculture such as effective planting and mechanization could potentially help farmers achieve higher yields. In the first chapter of my dissertation, we study the optimal seeding policy under rainfall uncertainty. Utilizing field weather data from Southern Africa, we investigate the advantage of the optimal planting schedule and the impact of climate conditions on this advantage in a real-size large-scale problem. Another critical characteristic of emerging markets is the low labor cost. This makes emerging markets attractive bases for global manufacturing and service operations. However, the globalization of supply chains complicates the logistics and procurement operations. In the second chapter, we focus on the warehouse outsourcing strategy in global supply chains. We establish the optimal warehousing strategy and demonstrate that excluding the logistics dynamics from contracting and making warehousing decisions unilaterally afterwards can lead to a suboptimal warehousing strategy for the retailer. Furthermore, a variety of threats such as supplier failure and transportation disruption could delay or even disrupt the operations, offsetting the low-cost benefit of emerging economies. In the third chapter, we study the optimal sourcing strategy under disruption in global supply chains. We establish the optimal sourcing strategy and provide insights on the roles of the nearshore supplier in response to supply chain disruption. Overall, my dissertation concentrates on the application of scientific methods to planting and farm machinery procurement to improve agricultural productivity in Africa and leveraging low-cost benefits in emerging markets.

Genuine gratefulness and warmest regards to my academic advisor, Dr. Jayashankar Swaminathan, who guided me through this journey with great passion and patience and phenomenal ideas and feedback all the time.

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CHAPTER 1: INTRODUCTION

Emerging markets are a critical part of global business, with share of 60% of global GDP and economy growth that is three times faster than developed economies (Chakravarty 2016). As a result, multinationals have set their sights on emerging markets for business opportunities. However, the operations strategies that firms apply in developed economies no longer fit for emerging economies that are often characterized by traditional economy, high volatility and limited government support (Amadeo 2016). Tailoring operations strategies to emerging economies is critical for multinationals to increase productivity and profit margin as well as achieve flexibility and responsiveness in emerging market operations. Research studies on managing emerging market operations could provide valuable insights and help multinationals achieve these objectives. Taking one step in this direction, this work explores critical issues on agriculture operations in emerging countries and managing global supply chains that involve emerging markets. Towards the end, my research aims to shed light on the impact of various risks and opportunities on optimal operations strategies in emerging markets.

The first chapter focuses on agriculture operations in emerging markets. Agriculture remains an essential sector in emerging economies, accounting for more than 20% of GDP in many emerging countries (World Bank 2016). With more than 60% of the global uncultivated arable land, emerging countries are regarded as a critical player to feed the world in the near future (Obasanjo 2012). However, the output of agricultural products in many emerging markets is significantly below the potential due to outdated machines and farming techniques and uncertainty in rainfall (Lopes 2014). Scientific agriculture in the form of effective planting, fertilizing, irrigation, pest control and mechanization could potentially help farmers achieve higher yields. In this chapter, we study the optimal seeding policy in rain-fed agriculture in Africa and explore the benefit of the optimal policy over commonly used heuristics in practice using field weather data from Southern Africa.

The second chapter studies warehousing solutions in emerging countries in global sourcing. Low labor cost in emerging countries makes them attractive bases for global manufacturing and service operations. However, due to long distance logistics associated with offshore sourcing, firms face dramatic increase in transportation cost, inventory cost and warehousing cost (Belanger and Leclerc 2013). Warehousing solutions in emerging countries near suppliers attempt to address the cost escalation issue and therefore have become more prevalent in offshore production (Robinson, C.H. 2015). Although the benefit of such warehousing solutions is intuitive, firms need to understand the implications of logistics operations on supply chain contracting to achieve the optimal warehousing strategy. We study the optimal warehousing strategy in offshore procurement and compare the contracting that incorporates the total landed cost in contract negotiation to that commonly studied in previous literature.

The third chapter studies another key challenge in global supply chain management: supply chain disruption in offshore sourcing. In offshore procurement from emerging markets, a variety of threats such as supply and delivery uncertainty and regulatory changes could delay or even disrupt the operations, thus offsetting the low-cost benefit of the emerging economies. Therefore firms start to move production from offshore countries to nearshore countries (Culp 2013). However, this does not always guarantee a higher profit and diversification of suppliers is suggested as an effective strategy (Mann 2014, Jain et al. 2013). In this chapter, we study the optimal dual-sourcing strategy in global sourcing and explore the roles of suppliers (mitigation and contingency) in response to supply chain disruption.

1.1 Optimal Seedling Policy under Rainfall Uncertainty

In the first chapter, we study the optimal planting schedule and explore the benefit of this schedule on crop yields in small-scale farming. This is a critical area of farming operations as increased agricultural productivity is often cited as a solution to the impending global food shortage problem. The demand for agricultural products is increasing due to world's growing population. By 2050, food production must double to meet the demand of the world (United Nations 2009). With roughly 60% of the global total uncultivated arable land, African countries are regarded as a critical player to feed the world (Obasanjo 2012). However, agricultural output in Africa mainly depends on rainfall, as irrigation is too expensive for resource-poor farmers in this semi-arid area

(Foti et al. 2008). Due to outdated machines and farming techniques and uncertainty in rainfall, planting yields in Africa are far below the developing world average. As a result Africa generates only 10% of the global agricultural output (Lopes 2014) and hunger affects about 240 million African people (Munang and Andrews 2014). Furthermore, due to severe climate conditions, crop yields are estimated to decrease by 10% to 20% by the middle of this century (Munang and Andrews 2014). Scientific agriculture in the form of effective planting, fertilizing, irrigation, pest control and mechanization could potentially help farmers achieve higher yields.

Many multi-national firms in the agriculture industry are striving to develop innovative solutions to help small-scale farmers become more effective. Our motivation for this work comes from interactions with AGCO, the third largest manufacturer of farm equipment in the world, that is introducing modern farming practices in Africa. In its efforts to introduce mechanization it needs to quantify the benefits of scientific farming to funding agencies that could then finance those efforts. In order to do that one needs to understand the optimal approach to planting seeds under uncertain rainfall and compare the benefits in relation to commonly used heuristics in practice.

In this paper, we model a farmer's planting problem for a single crop under rainfall uncertainty as a finite-horizon stochastic dynamic program. We use the cumulative biomass production to measure the crop yield and estimate the daily biomass production as the minimum of the values by two methods each day, one limited by water available for transpiration and the other limited by radiant energy. Utilizing earlier work in plant physiology related to biomass production, we further assume that the biomass production is zero during rainy periods and proportional to water transpiration during sunny periods, that in turn is dependent on the soil water content (Patteron 2018). In our model, a farmer needs to decide whether to plant a seed in each period in the planting horizon given the soil water content. We show that the structure of the optimal schedule is a time dependent threshold-type policy where the farmer should plant when the seed amount on hand is above the optimal threshold. This threshold depends on the soil water content and remaining time in the horizon. Furthermore we provide conditions under which the threshold is non-increasing in the soil water content. Mechanization can increase the speed at which seeds could be planted. We extend our model to mechanization by considering a scenario where a farmer could plant up to m ($m > 1$) seeds in each period. For this scenario, we show that the optimal planting schedule is still a time dependent threshold-type policy where the farmer should plant down to an optimal level that depends on soil water content, planting capacity and remaining time in the horizon. This optimal plant-down-to level is non-decreasing in the planting capacity. To the best of our knowledge this is the first model and analysis that incorporates the knowledge from plant physiology literature related to soil water content and seed growth in an optimal decision making framework.

In our computational study we utilize field weather data from Southern Africa to investigate the impact of climate conditions on the relative biomass production advantage of the optimal planting schedule over commonly used heuristics in practice. The relative biomass production advantage of the optimal schedule varies with the initial soil water content and could be as high as 16.88%. Even when the initial soil water content is very low, the relative biomass production advantage of the optimal schedule is 8.88%. Generally crop yields suffer significantly when the main rainfall starts later (Mugalavai et al. 2008), when the expected length of dry spell before the main rainfall becomes longer (Dennett 1987) or when the within-season variability of rainfall becomes higher (Stern and Cooper 2011). We find that the advantages of the optimal planting schedule are higher under these conditions. This indicates that the adoption of the optimal planting schedule could mitigate the risk of crop yield drop due to severe climate conditions.

1.2 Locating Warehouses in An Emerging Country - A Win-Win Proposition?

In the second chapter, we study the retailer's warehousing strategy in global sourcing. Due to long distance logistics associated with offshore sourcing, firms face dramatic increase in transportation cost, inventory cost and warehousing cost (Belanger and Leclerc 2013). Warehouse solutions in emerging countries near suppliers attempt to address the cost escalation issue and therefore have become more prevalent in offshore production (Robinson, C.H. 2015). Setting up such warehouses assists firms to achieve cost efficiency as well as demand responsiveness. For instance, Black Diamond Equipment started a global distribution center in China to locate inventory closer to various OEM providers and its own manufacturing facilities. This allows Black Diamond to consolidate freight, reduce overall inventory holding cost and become more responsive to demand change (Black Diamond 2009). Similarly, Ace Hardware Corporation holds goods from more than fifty suppliers in a global distribution warehouse in China. This enables Ace Hardware to reduce logistics cost and delivery time (China Daily 2006).

Although the benefit of locating warehouses in emerging countries appears intuitive, firms often ignore the implications of logistics operations on supply chain contracting (Kumar et al. 2010). Traditional contracting literature related to offshore sourcing studies the wholesale-price contract and does not include the logistics operations costs in the retailer's and supplier's profits under contract negotiation (Feng and Lu 2013). In fact a retailer's logistics cost structure will change substantially when she sets up an emerging-country warehouse to keep second-tier cycle stock, which in turn will influence the supplier's logistics cost. As a result, excluding logistics operations costs from contracting and making warehousing decisions unilaterally afterwards could lead to a suboptimal warehousing strategy for the retailer.

The motivation of this work comes from our interaction with a large retailer in Australia. The retailer used to have products shipped directly from their Chinese suppliers to the retail locations. Recently they have started to hold second-tier cycle stock at the Chinese warehouse to reduce inventory cost and delivery time. In order to make the optimal warehousing decision, the retailer needs to understand the potential cost advantage or disadvantage of using the Chinese warehouse and the implications of the total landed cost (including logistics operations costs) on contracting and the warehousing decisions.

In this chapter, we study supply chain contracting of a single product between a retailer in a developed country and a supplier in an emerging country. The retailer faces stochastic lead time and stochastic demand. She can hold cycle stock and safety stock at the retail location in the developed country (developed country warehousing). In that case, the supplier delivers products to the exporting harbor and from there the retailer directly ships products to the retail location. Instead, in addition to cycle stock and safety stock at the retail location, the retailer can also hold second-tier cycle stock in a warehouse in the emerging country (emerging country warehousing). In that case, the supplier delivers products to the emerging-country warehouse where the retailer breaks an inbound shipment into small batches. These small batches are then shipped to the retail location sequentially. In both cases, the supplier incurs fixed and variable costs for each batch he ships out. The retailer incurs procurement cost, overseas shipping cost, order processing cost and inventory holding cost. Conditional on the retailer's warehousing decision, the supplier and retailer negotiate over the wholesale price and order batch size.

Using the Nash bargaining framework, we establish the retailer's optimal warehousing strategy by providing a threshold on the holding cost at the emerging-country warehouse below which the retailer should used the emerging-country warehouse. This threshold is increasing in lead time reduction due to the warehouse and could be higher than the holding cost at the retail location if the lead time reduction is high. We show that while the emerging country warehousing is more profitable, the retailer could agree on a higher wholesale price if the holding cost at the warehouse is low and the lead time reduction due to the warehouse is high. This property holds even when the retailer's bargaining power is close to one. If her bargaining power is low, she could still ask for a discount on the wholesale price when the warehouse holding cost is low and the lead time reduction is low.

Under the traditional contract, the negotiated wholesale price is not dependent on the warehousing decision of the retailer as the logistics operations costs are not taken into account in contracting and the warehousing decision is made unilaterally by the retailer after negotiation. In our model, however, the negotiated wholesale price is dependent on the retailer's warehousing decision, which leads to individual profits and warehousing decision different from those under the traditional contract. When the retailer uses the emerging-country warehouse under both contracts, her warehouse inventory level is higher under the contract including the logistics cost. Our results indicate that incorporating the logistics costs into contract negotiation could impact the retailer's warehousing strategy if the warehouse holding cost is low and the lead time reduction by the warehouse is low, or the warehouse holding cost is high and the lead time reduction is high. Finally, we show that for any bargaining power of the retailer, there exists a threshold of the warehouse holding cost below which the retailer's profit is higher under the contract including logistics costs.

1.3 Role of the Nearshore Supplier under Supply Chain Disruption Uncertainty

In the third chapter, we study the optimal sourcing strategy and the role of the nearshore supplier in response to supply chain disruption. Firms start to move production from offshore countries to nearshore countries due to cost increase in offshore countries and increasingly complex disruption in global supply chains (Culp 2013). For instance, Japanese automakers such as Honda, Mazda and Nissan have shifted production from Asian countries to Mexico to serve the market in North America. By doing this, they gain fatter cost margins and improve product availability (Greimel 2014).

However, moving production facilities closer to markets does not always lead to a higher profit. Otis Elevator lost $60m in 2013 due to moving production back to the United States in South Carolina (Mann 2014). Successful examples (e.g. Forever 21 and Mattel) suggest a good strategy of using both offshore and nearshore suppliers to achieve cost efficiency and product availability under the disruption risk of offshore supply chain (Iyer 2010, Render 2012). Jain et al. (2013) also provide empirical evidence that diversification of global suppliers leads to lower inventory investment.

Firms need to consider multiple factors comprehensively to make the optimal decisions in global sourcing. Offshore orders bring cost advantage due to low labor and material cost of the offshore supplier. However, offshore outsourcing is regarded as one of the top causes of supply chain disruption (Zurich Insurance Group 2013), as it brings external threats (e.g. natural disasters), system vulnerabilities (e.g. oil dependence), quality issues and lack of flexibility (Accenture 2013, Anderson 2013). Furthermore, firms need increasing flexibility and responsiveness to prepare for demand fluctuations (Lacity and Rottman 2012). Hence it is difficult for firms to figure out the optimal global sourcing strategy under the risk of supply chain disruption.

A nearshore supplier is often regarded as a contingency supplier when firms adopt a diversified supplier base in response to supply chain disruption. They only order from the nearshore supplier when disruption occurs (Tomlin 2006). Allowing for the dual-sourcing option, we analyze the role of the nearshore supplier: whether it is a purely contingency supplier or also serves as inventory safeguard.

In this chapter, we study a dual-sourcing problem for a single product under the risk of supply chain disruption as a finite-horizon stochastic dynamic program. A firm can order from an offshore supplier and a nearshore supplier each period based on her demand forecast and disruption information to minimize the expected total cost. The nearshore supplier is expensive but reliable and the offshore order is cheap but may meet supply chain disruption. The disruption state determines the probability of disruption and evolves in a Discrete Time Markov Chain (DTMC) every period. The lead time of an offshore order is 2 and that of a nearshore order is 1. The demand forecast evolves following a Martingale Model of Forecast Evolution (MMFE) every period.

We show that the optimal outsourcing strategy is a state-dependent two-threshold base-stock policy. Every period the firm should place a nearshore order up to the optimal nearshore threshold, and place an offshore order additionally up to the optimal offshore threshold, whenever the inventory level allows. If the nearshore threshold is higher than the offshore threshold, she only orders from the nearshore supplier up to the offshore threshold level. We provide conditions on cost parameters and disruption risk under which the firm should use a sole- or dual-sourcing strategy and investigate the impact of cost, disruption and demand forecast on the two thresholds.

In our numerical study, we investigate the impact of various factors on the firm's strategy in response to supply chain disruption. Firms often apply contingency or mitigation tactics to prepare for supply chain disruption and demand fluctuations. Contingency tactics mean that firms take actions after disruption occurs, such as ordering from a backup supplier; mitigation tactics mean that firms take actions in advance of disruption, such as building up enough inventory safeguard (Tomlin 2006). We define two measures to represent the firm's dependence on the nearshore supplier and the role of nearshore orders: a contingency plan or a mitigation plan. An asymptotically optimal heuristics algorithm is developed based on Infinitesimal Purtubation Analysis (IPA) and sample path algorithm to search for the optimal order decisions. Our results indicate that rather then purely serving as a contingency plan, nearshore orders also build up inventory safeguard under specific conditions. We find that compared with long and infrequent disruption, under short and frequent disruption, a larger portion of nearshore orders are contingency orders. Furthermore, although firms shift to nearshore production due to cost increase of offshore orders, they should only do that when the disruption risk is sufficiently high.

CHAPTER 2: OPTIMAL SEEDING POLICY UNDER RAINFALL UNCERTAINTY

2.1 Introduction

The demand for agricultural products is increasing due to world's growing population. By 2050, food production must double to meet the demand of the world (United Nations 2009). With roughly 60% of the global total uncultivated arable land, African countries are regarded as a critical player to feed the world (Obasanjo 2012). However, agricultural output in Africa mainly depends on rainfall, as irrigation is too expensive for resource-poor farmers in this semi-arid area (Foti et al. 2008). Due to outdated machines and farming techniques and uncertainty in rainfall, planting yields in Africa are far below the developing world average. As a result Africa generates only 10% of the global agricultural output (Lopes 2014) and hunger affects about 240 million African people (Munang and Andrews 2014). Furthermore, due to severe climate conditions, crop yields are estimated to decrease by 10% to 20% by the middle of this century (Munang and Andrews 2014). Scientific agriculture in the form of effective planting, fertilizing, irrigation, pest control and mechanization could potentially help farmers achieve higher yields.

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In this paper, we model a farmer's planting problem for a single crop under rainfall uncertainty as a finite-horizon stochastic dynamic program. We use the cumulative biomass production to measure the crop yield and estimate the daily biomass production as the minimum of the values by two methods each day, one limited by water available for transpiration and the other limited by radiant energy. Utilizing earlier work in plant physiology related to biomass production, we further assume that the biomass production is zero during rainy periods and proportional to water transpiration during sunny periods, that in turn is dependent on the soil water content (Patteron 2018). In our model, a farmer needs to decide whether to plant a seed in each period in the planting horizon given the soil water content. We show that the structure of the optimal schedule is a time dependent threshold-type policy where the farmer should plant when the seed amount on hand is above the optimal threshold. This threshold depends on the soil water content and remaining time in the horizon. Furthermore we provide conditions under which the threshold is non-increasing in the soil water content. Mechanization can increase the speed at which seeds could be planted. We extend our model to mechanization by considering a scenario where a farmer could plant up to m ($m > 1$) seeds in each period. For this scenario, we show that the optimal planting schedule is still a time dependent threshold-type policy where the farmer should plant down to an optimal level that depends on soil water content, planting capacity and remaining time in the horizon. This optimal plant-down-to level is non-decreasing in the planting capacity. To the best of our knowledge this is the first model and analysis that incorporates the knowledge from plant physiology literature related to soil water content and seed growth in an optimal decision making framework.

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The rest of the paper is organized as follows. §2.2 discusses the related literature. §2.3 studies the manual planting model and §2.4 analyzes the mechanized planting model. In §2.5 we conduct an extensive computational study and explore the relative biomass production advantage of the optimal planting schedule over commonly used heuristics in practice. In §2.6 we present model variants that consider seed death in the growth as well as availability of water tank irrigation. We conclude in §2.7.

2.2 Related Literature

Our work is in the area of agricultural operations. Lowe and Preckel (2004) review applications of planting models and decision technology to agriculture problems related to operations management. Recent papers in agricultural operations study irrigation resource allocation (Dawande et al. 2013, Huh and Lall 2013), harvest risk (Allen and Schuster 2004, Lejeune and Kettunen 2017), capacity and production planning with random yield and demand (Kazaz 2004, Kazaz and Webster 2011, Tan and Çömden 2012, Hu and Wang 2017, Boyabatlı et al. 2017), crop planning (Maatman et al. 2002, Boyabatli et al. 2018), food gleaning operations (Ata et al. 2017), contracting (Boyabatli et al. 2011, Ferreira et al. 2017), government policy (Gupta et al. 2017, Alizamir et al. 2018), agriculture market in developing economies (An et al. 2015, Tang et al. 2015) and data-driven agriculture operations (Devalkar et al. 2018). Among these, Tan and Çömden (2012) study the optimal farm area and seeding time of multiple farms to maximize the profit under uncertain demand. The unit crop yield is modeled to be purely dependent on the seeding time for a specific farm. Kazaz (2004) and Kazaz and Webster (2011) study pricing and production planning in a two-stage stochastic programming framework and in their models crop yield is dependent on the seeded amount. These papers do not study seed scheduling and ignore the impact of uncertain rainfall on crop yield. In contrast, we focus on seed planting process in rain-fed agriculture and model the seeding problem in a finite-horizon stochastic dynamic program. Maatman et al. (2002) model farmers' strategies of production, consumption, selling, purchasing and storage in a two-stage stochastic programming framework with the objective to minimize deficits of various nutrients over multiple farming seasons. The production decision is dependent on observed rainfall that determines the number of days available for sowing. In our model, the objective is to maximize the expected yield and we establish the optimal seeding policy. The seeding decision is dependent on uncertain rainfall as the rainfall determines the soil water content, which in turn determines the growth of planted seeds and survival rate of seeds after planting. Ata et al. (2017) study the dynamic staffing policy in gleaning operations under uncertain food and labor supply with the objective to maximize the gleaning organization's net payoff and show that the optimal policy is a nested threshold policy. In the context of farming operations, we study the optimal seeding policy under uncertain rainfall (water supply) with the objective to maximize the crop yield and show that the optimal policy is a threshold-type policy.

Production scheduling in manufacturing industry has been extensively studied in operations management (Graves 1981). Most of the production schedule models focus on minimizing total inventory cost during the planning horizon. In each period, inventory cost is incurred due to leftover inventory or unsatisfied demand, that carries over to the next period. The optimal schedule minimizes the expected total cost. For the planting scheduling problem, however, in each period a seed planted generates an expected yield that depends on the soil water content, rainfall and sunny days in the remaining horizon. The optimal planting schedule maximizes the cumulative biomass production at the end of the horizon. Our work is also related to scheduling problems in agriculture research, that includes production scheduling (Burt and Allison 1963), harvesting scheduling (Chen et al. 1980) and fertilizer scheduling (Thornton and MacRobert 1994). Most of these scheduling problems ignore the stochasticity in growth rate due to external factors such as rainfall that determines the final yield. One exception is Burt and Allison (1963) who formulate the crop-rotation planting schedule and model the dynamics of soil water content that is influenced by annual planting or fallowing decision. We also model the dynamics of soil water content and seed growth but additionally take uncertain rainfall into consideration. In our model, the soil water content evolution and seed growth are determined by the weather rather than planting decisions. Different from their work, we demonstrate the structure of the optimal planting schedule and show its advantage over commonly used heuristics in practice under varying climate conditions.

2.3 Basic Planting Model

In this section we present a planting model for a single crop in a finite horizon. A planting horizon consists of N periods with reverse time indexing, i.e., the first period is period N, followed by $N-1$, $N-2$ and so on. The weather in the planting horizon is characterized by vector $\mathbf{p}^r$. p_t^r is the probability that it is rainy in period t. $1-p_t^r$ is the probability that it is sunny in period t. As African countries receive many hours of sunshine on average and high intensity of solar radiation, we assume that the weather is sunny when it does not rain (SOLA 2013). We assume that at the beginning of any period t, the farmer knows whether period t would be rainy or not. This assumption is reasonable since the local weather forecast information is available to most farmers nowadays. The farmer cannot plant in a period if it rains and thus the decision for the farmer is to decide whether to plant in each sunny period. To simplify the analysis, we assume that only one seed can be planted in one period (in §2.4, we generalize this model). Note that, for simplicity we assume that each period represents one day. However, one could consider the period that represents half a day or smaller intervals as well.

We assume that fertilizing, pest control and harvesting processes are automatically optimized by the farmer given the planting schedule and do not explicitly model these decisions. Fertilizers are often used to strengthen the root and leaf growth, blossom formation and fruit production. Therefore the effectiveness of fertilizing is highly dependent on the planting time and growth stage of crops (Grant 2018). Similarly the schedule of insecticide use is dependent on the planting date because treatments are required to target specific growth stages and a time window shortly before or after the planting date (Allen et al. 2017). Harvesting is often scheduled to start some time after the planting date and the time gap between planting and harvesting is determined by the crop species and geographical characteristics (NASS and USDA 1997). Therefore scheduling any of these operations is dependent on and coordinated with the planting schedule.

After a seed is planted, it begins to germinate and establish the seedling under favorable conditions. Whether a seed survives after planting and successfully establishes the seedling is significantly dependent on the soil water content in a short period after planting, about six days for maize in Africa (George and Rice 2016, du Plessis 2003). As this is a small proportion of the growing cycle, say 100 or 120 days for maize in Africa, we ignore the time of seedling emergence and thus assume that the survival probability of a seed after planting is a function of soil water content at the beginning of the planting period. We use sw_t to denote the soil water content at the beginning of period t. If a seed is planted in period t, the probability that the seed survives is denoted by $sv(sw_t)$

where $sv(.)$ is the survival probability function of a seed. We assume that once a seed survives the planting period, it will survive the rest of the planting horizon. We discuss the variant that seeds could die after the planting period in §2.6 and the extension where multiple seeds could be planted in §2.4.

Crop yield is considered as the product of biomass production (also referred as total dry matter or above-ground biomass) and harvest index, where the latter often varies with crop species and genotypes (Atwell 1999). As we consider the planting schedule for a single crop, we use the cumulative biomass production to measure the crop yield.

To estimate daily biomass production, we use the same method as in Agricultural Production Systems Simulator (APSIM). The biomass production is the minimum of the values by two methods each day, one limited by water available for transpiration and the other limited by radiant energy, biomass production = min{transpiration × transpiration efficiency, radiation interception × radiation use efficiency} (APSIM 1996, Kumar 2011). Let BM_t denote the daily biomass production in period t by a seed living in the ground. Then $BM_t = \min\{BM_t^{tp}(tp_t), BM_t^{ri}(ri_t)\}$ where $BM_t^{tp}(tp_t)$ is the biomass production in period t calculated through plant transpiration in that period tp_t and $BM_t^{ri}(ri_t)$ is the biomass production in period t calculated through radiation interception in that period ri_t.

The biomass production estimation method (transpiration or radiation interception) that limits the biomass production is dependent on the weather, rainy or sunny. During rainy days, the relative humidity of the air is high and this results in minimal transpiration level (Taiz and Zeiger 2010). We assume that the water transpiration is zero during a rainy period. Therefore, if period t is rainy, $BM_t = \min\{BM_t^{tp}(0), BM_t^{ri}(ri_t)\} = BM_t^{tp}(0) = 0$.

During sunny days, the plant could intercept abundant radiation in Africa (SOLA 2013). Meanwhile, the plant also incurs water loss through transpiration in the high temperature (Taiz and Zeiger 2010). Hence during a sunny day, $BM_t = \min\{BM_t^{tp}(tp_t), BM_t^{ri}(ri_t)\} = BM_t^{tp}(tp_t)$. The biology and agronomy literature shows that biomass production is linear in cumulative transpiration (de Wit 1958). Since previous work measures the cumulative transpiration, we consider stationary transpiration efficiency and assume that $BM_t^{tp}(.)$ is stationary and independent on t. Further, the literature shows that daily transpiration is a piece-wise linear function of soil water content (Gard-

Table 2.1: NOTATIONS

	Notations for the Basic Planting Model
s_t	system state at the beginning of period t
gsd_t	amount of seeds living in the ground at the beginning of period t
asd_t	amount of seeds available on hand for future planting at the beginning of period t
cbm_t	cumulative biomass production by all seeds up to the beginning of period t
sw_t	soil water content at the beginning of period t
$prec_t$	precipitation amount in period t given it rains
p_t^r	probability of rainfall in period t
$\omega_t(.)$	transition of soil water content from period t to $t-1$ as a function of soil water content at the beginning of period t and precipitation amount (zero if sunny in period t)
$sv(.)$	probability of seed survival after planting as a function of soil water content
$bm(.)$	daily biomass production of a single seed as a function of soil water content
$V_t(.)$	maximum expected biomass production by seeds in the ground and seeds available on hand with t periods to go
$Q_t(.)$	expected biomass production by seeds in the ground with t periods to go
$U_t(.)$	maximum expected biomass production by seeds available on hand with t periods to go
	Additional Notations for the Mechanized Planting Model
m	planting capacity
s_t^m	system state at the beginning of period t where the planting capacity is m
i_t	decision variable, the amount of seeds to plant in period t given period t is not rainy
$V_t^m(.)$	maximum expected biomass production by seeds in the ground and seeds available on hand under planting capacity m with t periods to go

ner and Ehlig 1963). Therefore, $BM_t^{tp}(.)$ can be expressed as a stationary function of soil water content. Let $bm(.)$ denote the biomass production by a seed living in the ground during a sunny period. Given period t is sunny, $BM_t = BM_t^{tp}(tp_t) = bm(sw_t)$. If a seed survives the planting period t, we assume that it contributes the biomass production of $bm(sw_t)$ in period t as any other seed living in the ground does. We can show that our results still hold when a seed that survives the planting period only starts to contribute the biomass production from the next period.

Let $s_t = (gsd_t, asd_t, cbm_t, sw_t)$ denote the system state at the beginning of period t and $S = \{(gsd, asd, cbm, sw) | cbm, sw \in \mathcal{R}^+ \bigcup \{0\}, gsd, asd \in \mathcal{N}\}$. gsd_t is the number of seeds living in the ground at the beginning of period t; asd_t is the number of seeds available on hand for future planting at the beginning of period t; cbm_t is the cumulative biomass production from all seeds living in the ground up to the beginning of period t; sw_t is the soil water content at the beginning of period t. We use $prec_t$ to denote the conditional precipitation amount given it rains in period t and $\omega_t(.)$ to denote the transition function of soil water content. Given that the soil water content at the beginning of period t is sw_t, the soil water content at the beginning of period $t-1$ is $sw_{t-1} = \omega_t(sw_t, 0)$ if it does not rain in period t and $sw_{t-1} = \omega_t(sw_t, prec_t)$ if it rains in period t.

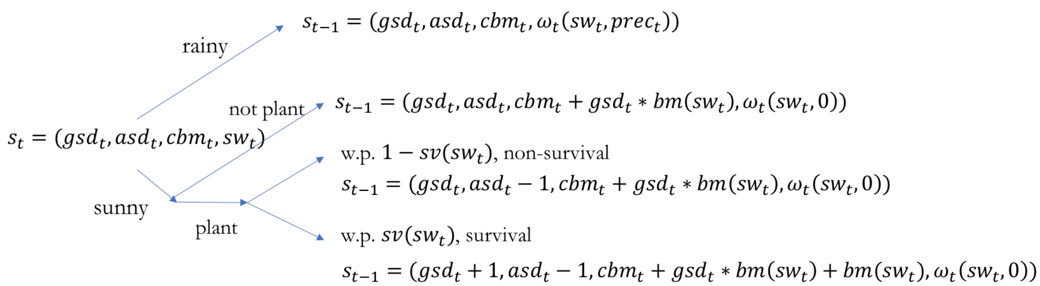

Figure 2.1: STATE TRANSITION FOR SINGLE CROP PLANTING MODEL

Table 2.1 summarizes our notations and Figure 2.1 illustrates the state transition. At the beginning of period t, if it is rainy, the farmer cannot plant and no decision needs to be made. The amount of seeds living in the ground, available on hand and the cumulative biomass production remain the same till the beginning of period $t-1$. If it is sunny in period t, the farmer needs to decide whether to plant a seed or not. If the decision is not to plant, the amount of seeds living in the ground and the number of seeds available on hand remain the same till the beginning of period $t-1$. The seeds living in the ground contribute $gsd_t * bm(sw_t)$ to the cumulative biomass production. If the decision is to plant, the amount of seeds available on hand decreases by one. With probability $sv(sw_t)$, the seed survives after planting. In this case the amount of seeds living in the ground increases by one and the cumulative biomass production increases by $gsd_t * bm(sw_t) + bm(sw_t)$, where the latter part is from the newly planted seed. With probability $1 - sv(sw_t)$, the seed does not survive after planting. In this case the amount of seeds living in the ground does not change and the cumulative biomass production increases by $gsd_t * bm(sw_t)$.

Our objective is to find a planting schedule that maximizes the expected biomass production at the end of the planting horizon. Let $V_t(.)$ denote the maximum expected total biomass production with t periods to go, as a function of system state $s_t = (gsd_t, asd_t, cbm_t, sw_t)$. Then,

$$
\begin{aligned}
&V_t(gsd_t, asd_t, cbm_t, sw_t) = p_t^r V_{t-1}(gsd_t, asd_t, cbm_t, \omega_t(sw_t, prec_t)) + (1 - p_t^r) * \max \big\{ V_{t-1}(gsd_t, asd_t, \\
&cbm_t + gsd_t * bm(sw_t), \omega_t(sw_t, 0)), sv(sw_t) * V_{t-1}(gsd_t + 1, asd_t - 1, cbm_t + (gsd_t + 1) * bm(sw_t), \\
&\omega_t(sw_t, 0)) + (1 - sv(sw_t)) * V_{t-1}(gsd_t, asd_t - 1, cbm_t + gsd_t * bm(sw_t), \omega_t(sw_t, 0)) \big\}, \\
&V_0(gsd_0, asd_0, cbm_0, sw_0) = cbm_0.
\end{aligned}
$$

Lemma 2.1 (Separable Property of Biomass Production). *The biomass production value function $V_t(gsd_t, asd_t, cbm_t, sw_t)$ can be expressed as the sum of cbm_t, a function of (gsd_t, sw_t) and a function of (asd_t, sw_t), i.e.,*

$$V_t(gsd_t, asd_t, cbm_t, sw_t) = cbm_t + gsd_t * Q_t(sw_t) + U_t(asd_t, sw_t), \text{ where} \tag{2.1}$$

$$Q_t(sw_t) = p_t^r Q_{t-1}(\omega_t(sw_t, prec_t)) + (1 - p_t^r)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))), \quad Q_0(sw_0) = 0, \tag{2.2}$$

$$\begin{aligned} U_t(asd_t, sw_t) &= p_t^r U_{t-1}(asd_t, \omega_t(sw_t, prec_t)) + (1 - p_t^r) \max\big\{U_{t-1}(asd_t, \omega_t(sw_t, 0)), \\ sv(sw_t)&(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) + U_{t-1}(asd_t - 1, \omega_t(sw_t, 0))\big\}, \quad U_0(asd_0, sw_0) = 0. \end{aligned} \tag{2.3}$$

Lemma 2.1 shows that for any period t, seeds that are living in the ground (gsd_t) and seeds available on hand (asd_t) independently contribute to cumulative biomass production. $Q_t(sw_t)$ is the expected future biomass production of one seed living in the ground with t periods to go given that the soil water content at the beginning of period t is sw_t. It is dependent on the weather and precipitation amount in the future periods as shown in Equation (2.2). $U_t(asd_t, sw_t)$ is the maximum expected biomass production of all seeds available on hand (asd_t) with t periods to go given that the soil water content at the beginning of period t is sw_t. Since the future biomass production of all seeds living in the ground $(gsd_t * Q_t(sw_t))$ is not dependent on the future planting decisions of the farmer, it is sufficient to focus on Equation (2.3) to explore the optimal planting decision.

Theorem 2.1. *Given state $s_t = (gsd_t, asd_t, cbm_t, sw_t)$ in period t,*

1. *the optimal planting decision is dependent on the amount of seeds available on hand (asd_t) and soil water content (sw_t);*

2. *there exists a threshold $SD_t(sw_t)$ that the optimal decision is to plant if $asd_t > SD_t(sw_t)$ and not to plant otherwise.*

For a sunny period t, the number of seeds available on hand (asd_t) and the soil water content (sw_t) are the determinants of the optimal planting decision. Planting early allows seeds to contribute biomass production for a long time. However the soil water content at planting may be low and this results in low seed survival after planting. On the other hand, seeds can only contribute biomass production for a short period if they are planted late. But they would survive with a high chance as the soil water content tends to become higher at the late stage of the planting horizon. For sunny period t, the optimal decision is to plant if the contribution of biomass production by planting one seed in period t $(sv(sw_t)*(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))))$ is higher than the marginal contribution of biomass production by reserving this seed in the next period $(U_{t-1}(asd_t,\omega_t(sw_t,0)) - U_{t-1}(asd_t - 1,\omega_t(sw_t,0)))$. The optimal policy can be characterized by the optimal planting threshold $SD_t(sw_t)$. Given soil water content sw_t, the optimal decision is to plant if the seed amount available on hand is higher than the optimal threshold $(asd_t > SD_t(sw_t))$. This is because the expected biomass production of planting one seed $(sv(sw_t)*(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))))$ remains constant with varying seed amount on hand (asd_t) while the marginal biomass production of reserving this seed in the next period $(U_{t-1}(asd_t,\omega_t(sw_t,0)) - U_{t-1}(asd_t-1,\omega_t(sw_t,0)))$ is non-increasing in asd_t.

Proposition 2.1. *Assume* $\omega_t(sw_t, prec_t) = \delta * sw_t + prec_t$, $\delta \in (0,1)$, $bm(sw), sv(sm)$ *are continuous and three-times differentiable and* $p_t^r \omega_{t+1}(sw_{t+1}, \frac{prec_t}{\delta}) + (1-p_t^r)\omega_{t+1}(sw_{t+1},0) \geq sw_{t+1}, \forall t$. $\boldsymbol{SD_t(sw_t)}$ ***is non-increasing in*** $\boldsymbol{sw_t}$ *if (1)* $bm(sw)$ *is non-decreasing and convex in* sw *and has third order derivative non-negative for any* $sw > 0$ *and (2)* $sv(sw)*bm(sw)$ *and* $sv(sw)*bm(\delta*sw)$ *are non-decreasing and concave in* sw *and have third order derivative non-positive for any* $sw > 0$.

Proposition 2.1 provides conditions on the survival probability function $sv(.)$ and the biomass production function $bm(.)$ under which the optimal planting threshold $SD_t(sw_t)$ is non-increasing in the soil water content given that the dynamics soil water content $\omega_t(.)$ takes a widely used form (see details in §2.5) and the expected soil water content is non-decreasing. Generally higher soil water content leads to both higher biomass production of planting one seed in the current period $(sv(sw_t)(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))))$ and higher marginal biomass production by reserving this seed in the next period $(U_{t-1}(asd_t,\omega_t(sw_t,0)) - U_{t-1}(asd_t-1,\omega_t(sw_t,0)))$. The conditions in Proposition 2.1 guarantee that with higher soil water content, the biomass production of planting a seed increases more than that of reserving this seed in the next period. Therefore the farmer is more willing to plant under higher soil water content.

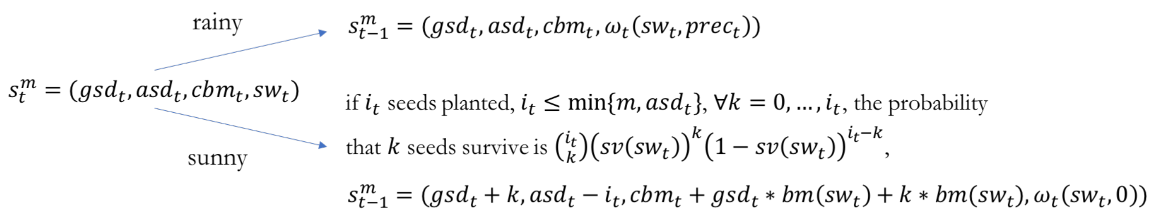

Figure 2.2: STATE TRANSITION FOR MECHANIZED PLANTING MODEL

2.4 Mechanized Planting Model

Mechanization can increase the speed of planting. In this section, we study a mechanized planting problem where the farmer can plant up to m ($m > 1, m \in \mathcal{N}$) seeds in each period and m is the capacity of mechanized planting. Let s_t^m denote the system state and $V_t^m(.)$ denote the maximum expected biomass production with t periods to go and planting capacity m. Note that the mechanized planting problem can be regarded as a general case of the manual planting problem in §2.3. For any sunny period, the farmer needs to decide how many seeds to plant. Suppose i_t seeds are planted in period t, the number of seeds available on hand would decrease by i_t, $asd_{t-1} = asd_t - i_t$. We assume that the probability function of seed survival amount is a binomial function and the probability that k seeds would survive after planting is $(sv(sw_t))^k(1-sv(sw_t))^{1-k}$, $k = 0, 1, \ldots, i_t$. In this case, the amount of seeds living in the ground would increase by k, $gsd_{t-1} = gsd_t + k$, and the cumulative biomass production would increase by $gsd_t * bm(sw_t) + k * bm(sw_t)$, where the former part is the biomass production contributed by seeds living in the ground and the latter part by the newly planted seeds. Figure 2.2 illustrates the state transition in the mechanized planting problem.

$$
\begin{aligned}
&V_t^m(gsd_t, asd_t, cbm_t, sw_t) \\
=&p_t^r V_{t-1}^m(gsd_t, asd_t, cbm_t, \omega_t(sw_t, prec_t)) + (1 - p_t^r) * \max_{i_t=0,\ldots,\min\{asd_t, m\}} \Big\{ \sum_{k=0}^{i_t} \binom{k}{i_t} (sv(sw_t))^k \\
&* (1 - sv(sw_t))^{i_t-k} * V_{t-1}^m(gsd_t + k, asd_t - i_t, cbm_t + (gsd_t + k)bm(sw_t), \omega_t(sw_t, 0)) \Big\} \\
&V_0^m(gsd_0, asd_0, cbm_0, sw_0) = cbm_0
\end{aligned}
$$

We can show that $V_t^m(.)$ satisfies the separable properties as in Lemma 2.1 and hence the optimal planting decision is dependent on the amount of seeds available on hand and the soil water content for given planting capacity. Let $a_t^m(asd_t, sw_t)$ denote the optimal planting decision for a sunny period t under capacity m.

Theorem 2.2. *Given state $s_t^m = (gsd_t, asd_t, cbm_t, sw_t)$ in period t,*

1. *the optimal planting decision is dependent on the amount of seeds available on hand (asd_t), the soil water content (sw_t) and the planting capacity (m);*

2. *there exists a threshold $SD_t^m(sw_t)$ that the optimal decision $a_t^m(asd_t, sw_t)$ satisfies*

$$a_t^m(asd_t, sw_t) = \begin{cases} \min\{m, asd_t - SD_t^m(sw_t)\} & \text{if } asd_t > SD_t^m(sw_t) \\ 0 & \text{otherwise} \end{cases}; \tag{2.4}$$

3. *assume $\omega_t(sw_t, prec_t) = \delta * sw_t + prec_t$, $\delta \in (0, 1)$, $bm(sw), sv(sm)$ are continuous and three-times differentiable and $p_t^r \omega_{t+1}(sw_{t+1}, \frac{prec_t}{\delta}) + (1 - p_t^r)\omega_{t+1}(sw_{t+1}, 0) \geq sw_{t+1}, \forall t$. $SD_t^m(sw_t)$ is non-increasing in sw_t if (1) $bm(sw)$ is non-decreasing and convex in sw and has third order derivative non-negative and (2) $sv(sm) * bm(sw)$ and $sv(sm) * bm(\delta * sw)$ are non-decreasing, concave in sw and have third order derivative non-positive for any $sw > 0$.*

The optimal planting schedule can be described as a capacitated plant-down-to policy, as shown in Figure 2.3. Given the soil water content $sw_t = 2$ and planting capacity $m = 4$, the optimal decision is not to plant when the amount of available seeds (asd_t) is less than or equal to 5. If the amount of available seeds is 6 to 9, the optimal decision is to plant 1 to 4 seeds respectively. When the amount of available seeds is more than 9, the optimal planting amount is 4 as the planting capacity is 4 and no more than 4 seeds can be planted in each period. The optimal decision in each period can be described by a threshold $SD_t^4(sw_t)$, that is dependent on the soil water content and planting capacity. In a sunny period, if the available seed amount is higher than the threshold $SD_t^4(sw_t)$, the optimal decision is to plant down to $SD_t^4(sw_t)$ unless limited by the capacity. Otherwise the optimal decision is to hold seeds to the next period. As a result, given the current optimal decision is to plant, if the available seed amount on hand increases by one unit, the optimal planting amount also increases by one unit, as long as the planting capacity allows. This

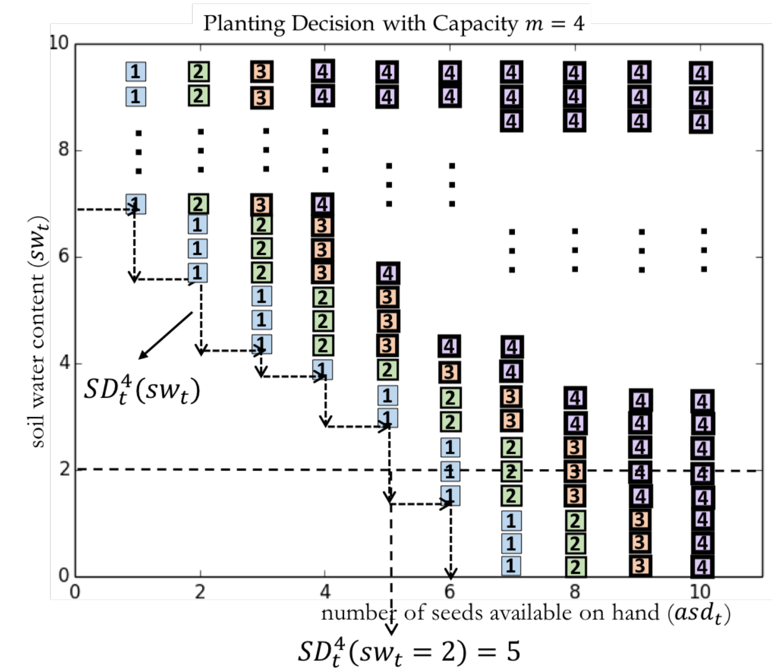

Figure 2.3: AN EXAMPLE OF OPTIMAL PLANTING POLICY UNDER CAPACITY 4 WITH t PERIODS TO GO

result is similar to the modified base stock policy for inventory problems under capacity constraint (Federgruen and Zipkin 1986).

Proposition 2.2. *$SD_t^m(sw_t)$ is non-decreasing in m. Furthermore, when $SD_t^m(sw_t) > 0$, $SD_t^m(sw_t) + 1 \leq SD_t^{m+1}(sw_t)$.*

The optimal plant-down-to level is non-decreasing in the planting capacity m as stated in Proposition 2.2. Obviously under unlimited capacity, the optimal plant-down-to threshold characterizes the optimal planting decision. Under limited planting capacity, the planting amount may not reach the optimal plant-down-to threshold as under unlimited capacity, harming the expected biomass production. In order to mitigate this loss due to capacity limitation in future periods, the optimal planting amount tends to be larger under a lower planting capacity than under a higher capacity. This result is consistent with the literature of capacitated inventory system where the order-up-to level tends to be non-increasing in the production capacity (Federgruen and Zipkin 1986).

2.5 Computational Study

Our computational study is aimed to (i) investigate the benefit of using an optimal planting policy in comparison to commonly used heuristics in practice and (ii) identify climate conditions where using the optimal policies might be more beneficial.

Seed Survival Function

We assume a logit model of seed survival after planting, where a logit-transformation of the survival probability of a planted seed is a linear function of the soil water content, i.e. $sv(sw_t) = \frac{\beta + e^{\gamma * sw_t}}{1+\beta+e^{\gamma * sw_t}}$. The logit model is consistent with the generalized linear mixed model (GLMM) that is commonly used to analyze the impact of soil water content on seed survival and seedling emergence (Bolker et al. 2009). We set $\beta = 0$ for simplicity and set $\gamma = 0.05$ to allow for a large range of survival probabilities throughout the computational study.

Biomass Production Function

As the crop physiology literature indicates, we assume that the biomass production is a linear function of soil water content $bm(sw_t) = \alpha * sw_t, \alpha > 0$ (Gardner and Ehlig 1963, de Wit 1958). Because $bm(sw_t)$ is proportional to sw_t, when comparing two planting policies (Θ_1 and Θ_2), the relative difference of the biomass production between the two policies is irrelevant to α, as stated in Proposition 2.3. Without loss of generality we set $\alpha = 1$.

Proposition 2.3. *For a planting horizon with N periods, let $BM^{\Theta_1}(asd_N, sw_N)$ denote the cumulative biomass production over the planting horizon under policy Θ_1 and $BM^{\Theta_2}(asd_N, sw_N)$ under policy Θ_2, where at the beginning of the planting horizon the number of seeds available on hand is asd_N and the soil water content is sw_N. If $bm(sw_t) = \alpha * sw_t$, $\alpha > 0$, then the relative difference of the biomass production under Θ_1 and Θ_2, $\frac{BM^{\Theta_2}(asd_N,sw_N)-BM^{\Theta_1}(asd_N,sw_N)}{BM^{\Theta_1}(asd_N,sw_N)}$, is independent on the value of α.*

Dynamics of Soil Water Content

To characterize the dynamics of soil water content, we apply a widely used Antecedent Precipitation Index (API) model to describe the impact of water run-off and precipitation on the soil water content (Kohler and Linsley 1951). In this model, the transition function of soil water content is $\omega_t(sw_t, prec_t) = \delta * sw_t + prec_t$ where δ is a recession factor that describes the water run-off. Although the factor δ depends on the geographical characteristics of the studied area, studies show that the value of this factor minimally differs among different areas (Kohler and Linsley 1951). Pellarin et al. (2009) estimate a daily recession factor $\delta = 0.7788$ in West Africa. Therefore in the computational study we set the recession factor $\delta = 0.8$.

2.5.1 Weather Data

We use daily weather data from 124 weather stations that span nine countries in Southern Africa (Zambia, Malawi, Zimbabwe, Botswana, Mozambique, Namibia, South Africa, Lesotho and Swaziland) from *www.wunderground.com*. The data coverage is from September 2010 to May 2017 and we consider weather stations that have at least one full year of records. The data records indicate whether a given day was rainy or not at a given station as well as the amount of rain. We use this data set to calculate the probability of rain for a specific date (month, day) (p_t^r) and the conditional precipitation amount for a specific date given that day is rainy ($prec_t$).

We use an example to illustrate how to calculate p_t^r and $prec_t$ for each date in the planting horizon. For a specific station, suppose we have six observations for January 1^{st} - two rainy observations and four sunny observations - and the precipitation amount for the two rainy observations are 12 mm and 10 mm. Then the probability of rain for January 1^{st} is, $p_t^r = 2/6 = 0.33$. The conditional precipitation amount for January 1^{st} given it is rainy is $(12mm + 10mm)/2 = 11mm$.

2.5.2 The Real Size Problem

In the real problem that motivated this work, the season typically goes from November to May in a six-month period. As crops need time to grow to maturity after planting, we assume no planting after March 1^{st} in our study. This is consistent with practice in Southern Africa that cereal

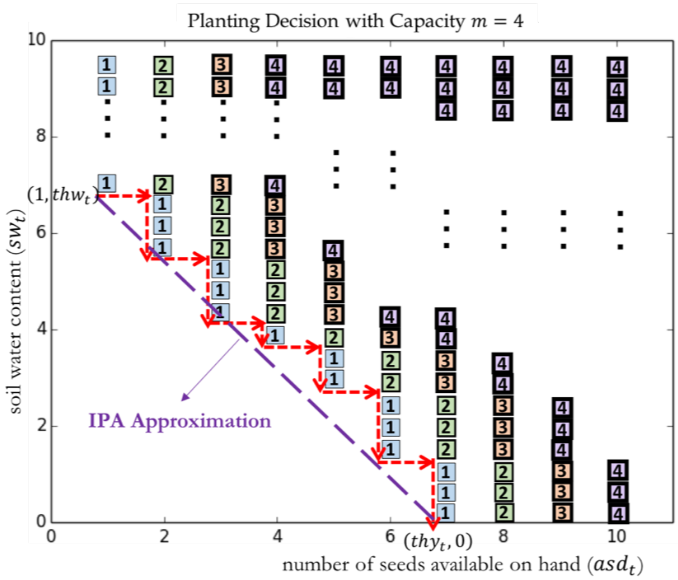

Figure 2.4: ILLUSTRATION OF IPA APPROXIMATION

planted in early November starts complete photosynthesis through leaves in February and farmers stop planting around the beginning of March (FEWS NET 2017). The planting horizon ends in February thereby having 120 periods (days) in which decisions need to be made. Although we can use backward induction to search for the optimal planting schedule and obtain final cumulative biomass production values, the running time increases exponentially in the length of the planting horizon. In order to handle such a large-scale problem and find the optimal solution, we develop an approximation based on IPA (Infinitesimal Perturbation Analysis). Based on Theorem 2.2, in each period the optimal plant-down-to policy can be illustrated in a stair structure as represented by the arrows in Figure 2.4. For each period we use a line segment to approximate the stair structure, as indicated by the dashed line in Figure 2.4. We then apply IPA to search for the optimal line segments associated with all periods under the assumption of continuous action space. For each period, searching for the optimal line segment is equivalent to searching for the optimal decision variables (thw_t, thy_t in Figure 2.4) that characterize the line segment and we apply gradient search in this process. IPA guarantees that the expectation of sample-path gradient we obtain converges to the gradient of the expected biomass production value. Each time we evaluate the expected gradient of the biomass production value with respect to the threshold decision variables, we take the average of 300 sample-path gradients. Once the thresholds are searched out, the biomass

Table 2.2: RELATIVE BIOMASS PRODUCTION DEVIATION FROM THE BENCHMARK SETTING

tol	7%	3%	1%	0.5%	*grtol*	15%	10%	7%	3%	1%
	-0.28%	0.81%	0.87%	0.91%		0.07%	0.10%	0.06%	0.11%	0.13%

production values are calculated by taking the average of $5,000$ sample-path biomass production values, each obtained following the plant-down-to policy with thresholds computed.

To make sure that the convergence happens properly, we impose two stopping criteria in searching for the optimal thresholds. The first criteria is that, in two consecutive search iterations, if the absolute value of the relative average gradient change with respect to any decision variable is smaller than $grtol$, then we stop searching. The second criteria is that, in two consecutive search iterations, if the absolute value of the relative biomass production change for any combination of initial seed amount on hand and initial soil water content (asd_N, sw_N) is smaller than tol, then we stop searching. In our computational study, we use $grtol = 5\%$ and $tol = 5\%$. Setting this as the benchmark, we calculate the relative deviation from the biomass production values when $tol = \{7\%, 3\%, 1\%, 0.5\%\}$ and $grtol = \{15\%, 10\%, 7\%, 3\%, 1\%\}$. Our results in Table 2.2 demonstrate that the average biomass production values does not deviate significantly when the values of $grtol$ and tol deviate from the benchmark setting.

2.5.3 Relative Advantage of Optimal Planting Schedule using Approximation

We consider the cases where the planting capacity $m \in \{1, \ldots, 8\}$ and investigate the relative biomass production advantage of the optimal planting schedule over commonly used myopic heuristics in practice. In Africa, farmers commonly start to plant after observing several consecutive days of rain or enough cumulative precipitation. For the staple product, maize, one of the rules of thumb is to start planting after observing 25 mm of cumulative precipitation in a 10-day period starting from November 1^{st} and then keep planting at full capacity until all seeds are depleted (Tadross et al. 2009). Based on this convention, we adopt the commonly used heuristics in practice using the rainfall data and compare the final biomass production with that generated by the optimal schedule. We use the relative biomass production advantage of the optimal planting schedule over commonly used heuristics, $\frac{\text{Biomass Production of Optimal Planting Schedule} - \text{Biomass Production of Commonly Used Heuristics}}{\text{Biomass Production of Commonly Used Heuristics}}$,

Table 2.3: RELATIVE BIOMASS PRODUCTION IMPROVEMENT OF THE OPTIMAL POLICY

planting capacity	1	2	3	4
avg. soil water content	12.75%	15.05%	15.05%	15.10%
min. soil water content	8.88%	14.02%	13.95%	13.96%
max. soil water content	15.04%	16.81%	16.78%	16.81%
planting capacity	5	6	7	8
avg. soil water content	15.10%	15.08%	15.07%	15.14%
min. soil water content	14.05%	14.05%	14.03%	14.03%
max. soil water content	16.88%	16.87%	16.77%	16.86%

to measure the benefit from adopting the optimal planting schedule. The biomass production values under commonly used heuristics are obtained by taking the average of 5,000 sample-path yields, each obtained under the $25mm$ heuristics. Note that under commonly used heuristics with planting capacity $m > 1$, when the decision is to plant, the amount of seeds planted is set to be the maximum planting capacity m.

We consider three cases of initial soil water content from the field weather data: the average soil water content (10.39 mm), the minimum soil water content (2.26 mm) and the maximum soil water content (24.53 mm). For each of the seven planting horizons between September 2010 and May 2017, we calculate the soil water content on November 1^{st} using the function of soil water dynamics $\omega_t(sw_t, prec_t) = 0.8 * sw_t + prec_t$ and the rainfall data in September and October before the start of that planting horizon, assuming that the soil water content at the beginning of September 1^{st} is zero. The average (minimum, maximum) case is the average (minimum, maximum) soil water content on November 1^{st} over the seven planting horizons.

Table 2.3 demonstrates the relative biomass production advantage of the optimal schedule over commonly used myopic heuristics when the planting capacity under both planting policies are the same. When the initial soil water content level is very low, the final biomass production under the optimal schedule gets hurt since it takes a long time for the soil water content to reach the optimal threshold for planting. In spite of that adopting the optimal schedule leads to improvement in the final biomass production even under minimum initial soil water content. The relative biomass production improvement of the optimal policy under manual planting ($m = 1$) is 12.75% under average initial soil water content, 8.88% under minimum initial soil water content and 15.04%

Table 2.4: RELATIVE BIOMASS PRODUCTION IMPROVEMENT OF THE OPTIMAL POLICY AND MECHANIZED PLANTING

planting capacity	1	2	3	4
avg. soil water content	12.75%	125.64%	245.49%	360.89%
min. soil water content	8.88%	116.68%	242.04%	356.05%
max. soil water content	15.04%	129.96%	250.26%	367.26%

under maximum initial soil water content. Under mechanized planting ($m > 1$), the relative biomass production improvement of the optimal policy minimally changes as the planting capacity changes. The relative improvement is about 15% under average initial soil water content, 14% under minimal soil water content and 17% under maximum soil water content.

Table 2.4 illustrates the relative biomass production advantage of mechanized ($m > 1$) or manual ($m = 1$) planting under the optimal policy over manual planting ($m = 1$) under commonly used myopic heuristics. When moving from manual planting ($m = 1$) to mechanized planting ($m > 1$), we are increasing the number of seeds available to the farmer proportional to the increased speed. By adopting mechanization ($m = 3$) under the optimal policy, the final biomass production can be improved by nearly 250% regardless of the initial soil water content. This improvement by adopting both the optimal policy and mechanization is consistent with the observations made in the pilot study in Zambia by AGCO (Swaminathan 2018).

2.5.4 Out-of-Sample Testing

We conduct out-of-sample testing to validate our results. Our dataset contains seven years of daily rainfall data, where each data point (month, date) contains the average probability of rainfall and the average conditional precipitation amount given it rains over all weather stations. Let $p_t^{[i],r}$ and $prec_t^{[i]}, i = 1, ..., 7; t = 1, ..., 181$ denote the average probability of rainfall in day t of the i^{th} planting horizon and average conditional precipitation amount in day t of the i^{th} planting horizon given it rains. To conduct the out-of-sample testing, we search for the optimal planting thresholds using the data of the first three years (November 2010 - April 2013), where the daily rainfall probability and conditional precipitation amount are calculated by $p_t^r = \frac{\sum_{j=1}^{3} p_t^{[j],r}}{3}, prec_t = \sum_{j=1}^{3} \frac{p_t^{[j],r} prec_t^{[j]}}{3p_t^r}$. Then for year k in the rest four years (November 2013 - April 2017), we use the daily rainfall probability of year k, $p_t^{[k],r}$ to generate $5,000$ sample paths where each sample

Table 2.5: RELATIVE BIOMASS PRODUCTION IMPROVEMENT OF THE OPTIMAL POLICY IN OUT-OF-SAMPLE TESTING ($m = 1$)

year	2014	2015	2016
avg.soil water content	8.35%	8.23%	10.31%
min. soil water content	7.09%	2.29%	6.75%
max. soil water content	9.94%	11.19%	11.84%

path characterizes whether it rains in each day of the planting horizon. If it rains in period t, the precipitation amount is $prec_t^{[k]}$. The biomass production under the optimal policy is calculated by taking the average of the $5,000$ sample-path biomass production values, each obtained following the plant-down-to policy with planting thresholds searched out based on the first three years' rainfall data. The biomass production under the commonly used heuristics is calculated by taking the average of the $5,000$ sample-path values, each obtained following the $25mm$ heuristics.

Table 2.5 shows that the biomass production from the optimal policy is higher than that from commonly used heuristics in practice in the out-of-sample testing. The relative improvement of the optimal policy is close to but lower than the full information case ($m = 1$) in Table 2.3.

2.5.5 Advantage of Optimal Planting Schedule under Climate Change

Next we explore the impact of severe climate conditions on the advantage of optimal planting schedule over commonly used heuristics in practice under manual planting ($m = 1$). In practice, farmers may adopt different rules to determine the onset planting date under different climate conditions. In calculating the relative advantage of the optimal policy under different climate conditions, we compare the biomass production under the optimal policy to the best commonly used myopic heuristics policy and take the average relative advantage of the optimal policy. We use $[x, y]$ to denote the heuristics under which the farmer would start planting at full capacity after observing x mm of precipitation in a y-day period starting from November 1^{st} and keep planting whenever possible till the depletion of seeds or the end of the planting horizon, whichever occurs earlier. We enumerate the myopic policies in set $H = \{[x, y] | x = 10, 15, 20, 25, 30; y = 5, 10, 15, 20, 25, 30, 35\}$.

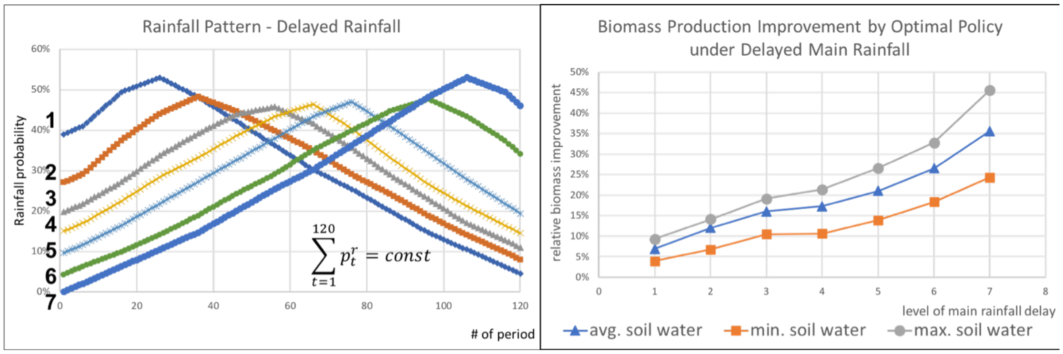

Figure 2.5: RELATIVE BIOMASS PRODUCTION IMPROVEMENT OF THE OPTIMAL POLICY UNDER DELAYED MAIN RAINFALL

Delayed Main Rainfall:

We first explore the impact of delayed main rainfall on the relative biomass production advantage of the optimal planting schedule. Our weather data shows that both the rainfall probability and average precipitation tend to increase from November and starts to drop around mid-January. In recent years, however, extensive areas in Southern Africa have seen delays in the onset of the planting season of up to five and six weeks due to late start of the main rainfall (Stern and Cooper 2011). We shift the evolution pattern of rainfall probability to indicate early or late start of the main rainfall, as shown in the left panel of Figure 2.5. A larger index indicates a later start of the main rainfall. The average rainfall probability over the planting horizon remains the same among all rainy patterns (constant $\frac{\sum_{t=1}^{120} p_t^r}{120}$).

Our numerical study demonstrates that the loss in harvest due to delayed rainfall could be mitigated by applying the optimal planting schedule, as in the right panel of Figure 2.5. Generally crop yields suffer significantly with a late onset date (Mugalavai et al. 2008). When the main rainfall comes later, under commonly used myopic heuristics, the farmer would wait for a longer time until sufficient cumulative precipitation is observed. In our computational results, the delayed rainfall leads to increasing time window in the best myopic policy: from $[10, 5]$ in pattern 1 to $[10, 20]$ in pattern 7. Planting starts later if the main rainfall starts later and all seeds may not be depleted at the end of the horizon. However, the optimal planting schedule is created uniquely for each of the weather patterns. Therefore, if the rainfall gets delayed, the optimal planting thresholds tend to

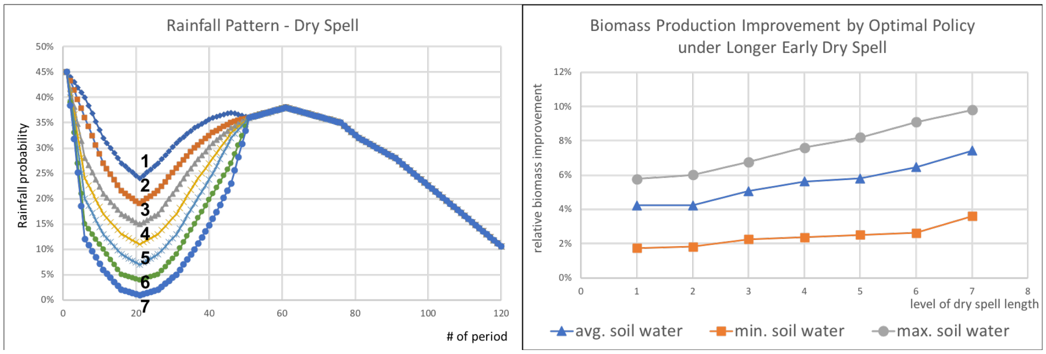

Figure 2.6: RELATIVE BIOMASS PRODUCTION IMPROVEMENT OF THE OPTIMAL POLICY UNDER LONGER EARLY DRY SPELL

decrease and the farmer will start to plant at a lower soil water content. More seeds can be planted to grow during the main rainfall, leading to a higher final yield than commonly used heuristics.

Dry Spell Before Main Rainfall:

Another critical agricultural issue that African countries have faced in recent years, especially with El Niño events, is that a long dry spell often occurs before the main rainfall. This type of drought often aggravates circumstances. Early rainfall is often followed by a long dry spell before the main rainfall comes. This early rainfall allows the planting process to start, but it is insufficient for crop establishment and thus leads to drop in yields (Dennett 1987). We represent the increasing expected length of dry spells by decreasing the rainfall probabilities before the main rainfall, as shown in the left panel of Figure 2.6. A larger index indicates a longer expected length of dry spell before the main rainfall.

The right panel of Figure 2.6 shows that the relative biomass production advantage of the optimal schedule increases as the expected length of early dry spell becomes longer. Under commonly used myopic heuristics, the farmer starts to plant after observing early rainfall and keeps planting during the dry spell. The longer expected dry spell leads to a longer time window in the best myopic policy: from $[10, 5]$ in pattern 1 to $[10, 10]$ in pattern 7. Longer early dry spell leads to later onset planting date and more biomass production loss under the myopic policy. Under the optimal schedule, however, planting may not continue after early rainfall if a dry spell encountered in the following periods results in low soil water content. When the early dry spell becomes longer, the

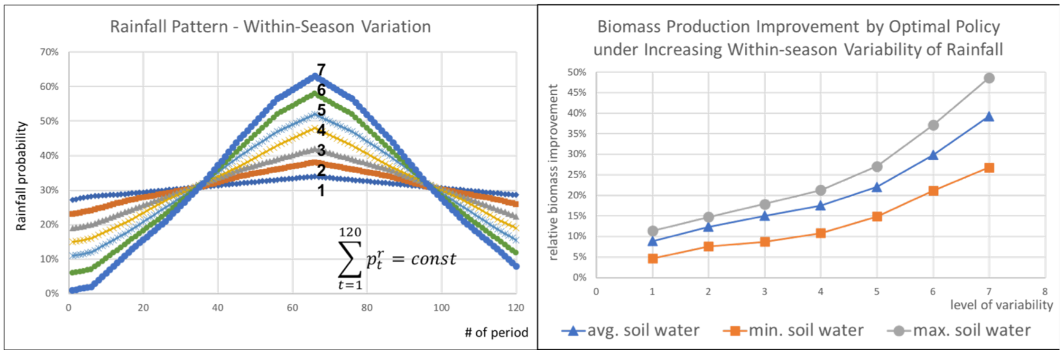

Figure 2.7: RELATIVE BIOMASS PRODUCTION ADVANTAGE OF THE OPTIMAL POLICY UNDER INCREASING RAINFALL VARIABILITY

optimal planting thresholds of the early periods become higher and planting occurs at a higher soil water content. When the initial soil water content is low, the farmer starts planting later when the dry spell is about to end. If the initial soil water content is high and the farmer starts planting when the early rainfall is observed, she may stop planting during the dry spell if it results in low soil water content and continue to plant later when more rainfall occurs. Therefore the biomass production loss due to longer early dry spell could be mitigated by adopting the optimal schedule.

Within-season Variability of Rainfall:

The within-season variability of rainfall distribution usually leads to lower crop yields (Stern and Cooper 2011). In Africa, the distribution of daily precipitation is often highly skewed, with 23% of rainy days contributing 80% of total rainfall (Dennett 1987). In our model, we use the flatness of the evolution pattern of the rainfall probability to indicate within-season variability of the rainfall, as in the left panel of Figure 2.7. A smaller index indicates a more flat distribution and a lower within-season variability of rainfall. The average rainfall probability over the horizon remains the same among all rainy patterns (constant $\frac{\sum_{t=1}^{120} p_t^r}{120}$).

The right panel of Figure 2.7 illustrates that the relative biomass production advantage of the optimal planting schedule increases as the within-season variability of rainfall increases. Under such conditions, the optimal planting thresholds of early periods decrease and those of later periods increase. Therefore the farmer starts to plant earlier and seeds contribute more biomass production. However, under commonly used heuristics, planting starts later when the within-season rainfall variability increases. The time window in the best myopic policy increases from pattern 1 ($[10, 5]$) to pattern 7 ($[10, 15]$). When the rainfall peak comes, fewer seeds have been planted and the planting process may be impeded as farmers cannot plant when it rains. Therefore with increasing within-season rainfall variability, the final biomass production under commonly used heuristics tends to become lower and thus the relative advantage of the optimal policy becomes higher.

2.5.6 Advantage of Optimal Policy under Varying Seed Quality

Poor crop establishment has been identified as a major cause of low yields in Southern Africa. With inadequate soil water content and poor land preparation methods, sowing good-quality seeds could significantly improve crop yields. In plant physiology literature, seed quality refers to three aspects: seed germination, vigor and size and it influences final crop yields during different growing stages (Ellis 1992). In this section, we investigate the impact of seed quality on the advantage of the optimal policy over commonly used heuristics in practice.

Seed size is considered of particular importance at early seedling stages and studies have provided evidence that larger seed size would lead to higher survival rate and emergence percentage (Lloret et al. 1999). Seeds with higher vigor have been demonstrated to provide a higher survival rate in the field of many crops such as corn and soybean (Ellis 1992). Seed priming - soaking seeds into water or other solution before sowing - often leads to higher germination percentage (Foti et al. 2008). Thus high seed quality indicates high seed survival probability after planting. To investigate the impact of seed quality on the advantage of the optimal schedule, we use the probability function of seed survival $sv(sw_t, \eta) = \frac{e^{\gamma * sw_t}}{\eta + e^{\gamma * sw_t}}$ and evaluate the relative biomass production advantage of the optimal planting schedule under varying $\eta \in \{1, \ldots, 10\}$ (varying γ minimally changes the value of survival probability). A larger η represents lower seed quality. We calculate the relative biomass production advantage of the optimal policy over the commonly used heuristics ($25mm$ heuristics) with both policies under manual planting ($m = 1$).

Figure 2.8 shows the impact of seed quality (η) on the advantage of the optimal policy. Both the absolute (left panel) and relative (right panel) biomass production advantage of the optimal policy are increasing in seed quality (decreasing in η). We observe that the relative biomass production advantage of the optimal policy is minimally increasing in seed quality (decreasing in η) when the

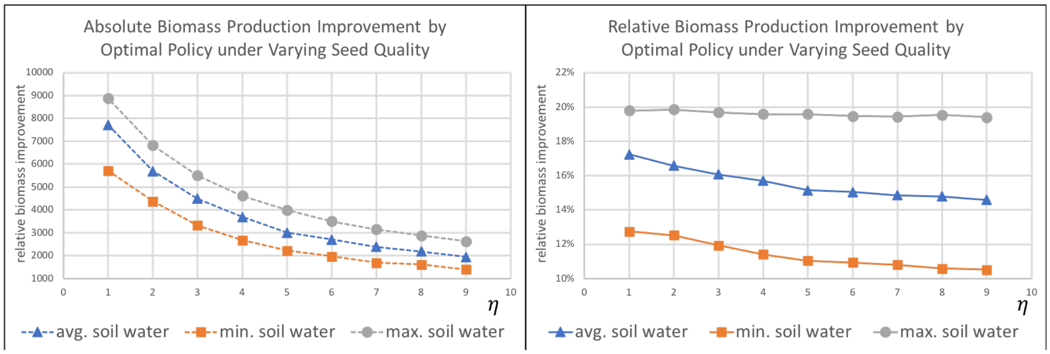

Figure 2.8: IMPACT OF SEED QUALITY ON THE RELATIVE BIOMASS PRODUCTION ADVANTAGE OF THE OPTIMAL POLICY

soil water content is high. Because the high soil water content guarantees high survival rate of seeds even when the seed quality is low, varying the seed quality does not significantly influence the relative biomass production advantage of the optimal policy.

2.6 Model Variants

In this section, we study two variants of the planting model to discuss the cases when some assumptions are relaxed.

2.6.1 Plant Death during Growth

In practice even if a seed survives the planting day, it could die later in the rest of the planting horizon due to drought (Stern and Cooper 2011). We use $sd(sw_t)$ to denote the probability that a seed living in the ground dies in period t where sw_t is the soil water content at the beginning of period t. We assume that the probability function of seed death amount is a binomial function and seed death only occurs in sunny periods as it is often associated with drought. When a seed dies, the biomass production generated by this seed is also lost and we use the average biomass production of a single seed up to that point (cbm_t/gsd_t) to approximate the biomass production loss due to the death of that seed. We use superscript d to represent the variant with seed death and let $V_t^{d,m}(.)$ denote the maximum expected biomass production with t periods to go. Then,

$$
\begin{aligned}
&V_t^{d,m}(gsd_t, asd_t, cbm_t, sw_t)\\
=&p_t^r V_t^{d,m}(gsd_t, asd_t, cbm_t, \omega_t(sw_t, prec_t)) + (1-p_t^r) \max_{i_t=0,\ldots,\min\{asd_t,m\}} \Big\{ \sum_{k=0}^{i_t} \binom{k}{i_t} (sv(sw_t))^k\\
&* (1-sv(sw_t))^{i_t-k} * \Big(\sum_{j=0}^{gsd_t} \binom{j}{gsd_t} (sd(sw_t))^j (1-sd(sw_t))^{gsd_t-j} * V_{t-1}^{d,m}(gsd_t - j + k,\\
&asd_t - i_t, cbm_t + (gsd_t - j + k) * bm(sw_t) - j\frac{cbm_t}{gsd_t}, \omega_t(sw_t, 0))\Big)\Big\}.
\end{aligned}
$$

We can show that the optimality of the plant-down-to policy still holds in the variant with seed death. For given soil water content in period t, the optimal planting threshold tends to become lower if the seed death probability becomes lower. This is because a lower seed death probability provides a higher integrated survival chance of a seed, leading to a higher expected biomass production of a planted seed and thus higher planting amount. Proposition 2.4 states this result.

Proposition 2.4. *Assume $\omega_t(.)$, $sv(.)$ and $bm(.)$ satisfies the conditions in Proposition 2.1. Let $sd_1(.)$, $sd_2(.)$ denote two seed death probability functions and $SD_{t,1}^{d,m}(.)$, $SD_{t,2}^{d,m}(.)$ the associated optimal planting thresholds for period t, $t = 1, \ldots, N$. If $sd_1(sw_t) \leq sd_2(sw_t)$, $\forall sw_t > 0$, then $SD_{t,1}^{d,m}(sw_t) \leq SD_{t,2}^{d,m}(sw_t)$.*

2.6.2 Water Tank Irrigation

In some areas in Africa, although irrigation is rarely available, communities build water tank to reserve rainfall water for irrigation purposes. Farmers could use the water for irrigation when rainfall does not occur for a long time. As the administrator of the community makes irrigation schedule decisions rather than the farmer, we assume fixed irrigation schedule and explore the biomass production advantage of the water tank irrigation under the optimal policy.

Let irr_t denote the amount of irrigation water in period t when it does not rain. The farmer could use irr_t amount of water to irrigate the field if period t is sunny and we assume that watering the field does not interfere her planting process. Given the soil water content at the beginning of period t is sw_t, the soil water content in the next period is $sw_{t-1} = \omega_t(sw_t, irr_t)$ and we use $\omega_t(sw_t, irr_t) = 0.8 * sw_t + irr_t$ in the computational study. We explore the advantage of

Table 2.6: RELATIVE BIOMASS PRODUCTION ADVANTAGE OF WATER TANK IRRIGATION UNDER THE OPTIMAL POLICY ($m = 1$)

θ%	5%	10%	15%	20%	25%
avg. soil water content	4.28%	8.45%	12.59%	16.88%	21.02%
min. soil water content	4.26%	8.57%	12.27%	17.38%	21.55%
max. soil water content	4.06%	8.13%	12.26%	16.44%	20.89%

water tank irrigation under the optimal policy and manual planting ($m = 1$) in the real size problem. We set irr_t equal to θ percentage of the average daily precipitation (2.89 mm) and use $\theta \in \{5, 10, 15, 20, 25\}$. We compare the biomass production with irr_t water irrigation to the no-irrigation case under the optimal policy and manual planting. Table 2.6 illustrates the relative biomass production improvement of water tank irrigation under the optimal policy. It indicates that the relative biomass production improvement of water tank irrigation is approximately linearly increasing in the amount of irrigation water.

2.7 Concluding Remarks

Increase in human population has brought a lot of attention to agriculture in African countries. With the average planting yields far below the developing world average, farmers in Africa need to adopt advanced planting techniques to increase crop yields. Further, agriculture in Africa faces severe issues due to delay of the main rainfall, long dry spells before the main rainfall, and high within-season variability of rainfall. As climate conditions become more severe, the crop yield is seriously harmed.

In this paper we study the planting schedule problem of a single crop under rainfall uncertainty as a finite-horizon stochastic dynamic program. Planting early may allow the seeds to start contributing biomass production early, but higher soil water content later on could lead to a higher chance of seed survival. We show that the optimal planting schedule is a time dependent threshold-type policy, where the farmer should plant down to the optimal threshold.

In practice, farmers start to plant each year after observing enough cumulative rainfall. Utilizing field weather data collected from nine countries in Southern Africa, we show that adopting the optimal schedule could significantly improve final biomass production. Furthermore, our results demonstrate that the risk from the severe climate conditions can be significantly mitigated by adopting the optimal planting schedule. The more severe the climate conditions the higher the relative biomass production advantage of the optimal planting schedule.

In this work we only focus on the planting schedule of seeds and assume that other decisions such as fertilizer addition and pest control are done optimally. In many real situations those aspects can also be difficult to adopt. In Africa, farmers start to obtain access to advanced technology in farming such as soil and solar sensors, satellite data and plant growth monitors. Application of these technologies would influence the optimal seeding policy. Besides, we study the planting schedule problem in a dynamic programming framework. Other models such as robust dynamic programming or Bayesian approach could also be applied to characterize the optimal planting policy. Further, we only consider one crop in this work. Sometimes, crop rotation has an important impact on the yields of seeds and in such cases that needs to be incorporated. Finally, we do not consider any budget constraints that a farmer might face for seed procurement, automation, fertilizers or pest control. All the above issues are ripe for future studies in this area.

Acknowledgements

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CHAPTER 3: LOCATING WAREHOUSES IN AN EMERGING COUNTRY - A WIN-WIN PROPOSITION?

3.1 Introduction

Due to long distance logistics associated with offshore sourcing, firms face dramatic increase in transportation cost, inventory cost and warehousing cost (Belanger and Leclerc 2013). Warehousing solutions in emerging countries near suppliers try to address the cost escalation and therefore have become more prevalent in offshore production (Robinson, C.H. 2015). Setting up such warehouses assists firms to achieve cost efficiency as well as demand responsiveness. For instance, Black Diamond Equipment started a global distribution center in China to locate inventory closer to various OEM providers and its own manufacturing facilities. This allows Black Diamond to consolidate freight, reduce overall inventory holding cost and become more responsive to demand change (Black Diamond 2009). Similarly, Ace Hardware Corporation holds goods from more than fifty suppliers in a global distribution warehouse in China. This enables Ace Hardware to reduce logistics cost and delivery time (China Daily 2006).

Although the benefit of locating warehouses in emerging countries appears intuitive, firms often ignore the implications of logistics operations on supply chain contracting (Kumar et al. 2010). Traditional contracting literature related to offshore sourcing studies the wholesale-price contract and does not include the logistics operations costs in the retailer's and supplier's profits under contract negotiation (Feng and Lu 2013). In fact a retailer's logistics cost structure will change substantially when she sets up an emerging-country warehouse to keep second-tier cycle stock, which in turn will influence the supplier's logistics cost. As a result, excluding logistics operations costs from contracting and making warehousing decisions unilaterally afterwards could lead to a suboptimal warehousing strategy for the retailer.

The motivation of this work comes from our interaction with a large retailer in Australia. The retailer used to have products shipped directly from their Chinese suppliers to the retail locations.

Recently they have started to hold second-tier cycle stock at the Chinese warehouse to reduce inventory cost and delivery time. In order to make the optimal warehousing decision, the retailer needs to understand the potential cost advantage or disadvantage of using the Chinese warehouse and the implications of the total landed cost (including logistics operations costs) on contracting and the warehousing decisions.

In this chapter, we study supply chain contracting of a single product between a retailer in a developed country and a supplier in an emerging country. The retailer faces stochastic lead time and stochastic demand. She can hold cycle stock and safety stock at the retail location in the developed country (developed country warehousing). In that case, the supplier delivers products to the exporting harbor and from there the retailer directly ships products to the retail location. Instead, in addition to cycle stock and safety stock at the retail location, the retailer can also hold second-tier cycle stock in a warehouse in the emerging country (emerging country warehousing). In that case, the supplier delivers products to the emerging-country warehouse where the retailer breaks an inbound shipment into small batches. These small batches are then shipped to the retail location sequentially. In both cases, the supplier incurs fixed and variable costs for each batch he ships out. The retailer incurs procurement cost, overseas shipping cost, order processing cost and inventory holding cost. Conditional on the retailer's warehousing decision, the supplier and retailer negotiate over the wholesale price and order batch size.

Using the Nash bargaining framework, we establish the retailer's optimal warehousing strategy by providing a threshold on the holding cost at the emerging-country warehouse below which the retailer should used the emerging-country warehouse. This threshold is increasing in lead time reduction due to the warehouse and could be higher than the holding cost at the retail location if the lead time reduction is high. We show that while the emerging country warehousing is more profitable, the retailer could agree on a higher wholesale price if the holding cost at the warehouse is low and the lead time reduction due to the warehouse is high. This property holds even when the retailer's bargaining power is close to one. If her bargaining power is low, she could still ask for a discount on the wholesale price when the warehouse holding cost is low and the lead time reduction is low.

Under the traditional contract, the negotiated wholesale price is not dependent on the retailer's warehousing decision as the logistics operations costs are not taken into account in contracting and the warehousing decision is made unilaterally by the retailer after negotiation. In our model, however, the negotiated wholesale price is dependent on the retailer's warehousing decision, which leads to individual profits and warehousing decision different from those under the traditional contract. When the retailer uses the emerging-country warehouse under both contracts, her warehouse inventory level is higher under the contract including the logistics cost. Our results indicate that incorporating the logistics costs into contract negotiation could impact the retailer's warehousing strategy if the warehouse holding cost is low and the lead time reduction by the warehouse is low, or the warehouse holding cost is high and the lead time reduction is high. Finally, we show that for any bargaining power of the retailer, there exists a threshold of the warehouse holding cost below which the retailer's profit is higher under the contract including logistics costs.

The remainder of this paper is organized as follows. The next section surveys the related literature. Our model is presented in §3.3. §3.4 discusses the retailer's warehousing decision and §3.5 analyzes the implications of the retailer's warehousing decision on the negotiation outcomes. In §3.6, we compare the retailer's optimal warehousing decision under the traditional contract and the contract including logistics costs. Our concluding remarks are presented in §3.7.

3.2 Related Literature

Our work is in the area of production outsourcing and adds to the aspect of supply chain contracting. Previous research has discussed various issues related to outsourcing decisions such as contract type (Van Mieghem 1999), scale economies (Cachon and Harker 2002), demand risk allocation (Ülkü et al. 2007), industry structure (Feng and Lu 2012, Feng and Lu 2013) and learning-by-doing (Gray et al. 2009). Among previous papers studying offshore outsourcing, many of them ignore the embedded risk in long-distance supply chain due to long and uncertain lead time and stochastic demand. In this chapter, we model the safety stock at the retail location to represent the retailer's risk from stochastic market demand coupled with overseas shipping. Furthermore, previous work on outsourcing contract often models the retailer's cost as variable cost (Feng and Lu 2012) or the sum of variable cost and fixed cost for each order batch (Cachon and Harker 2002). Logistics operations costs such as inventory holding cost and transportation cost are ignored

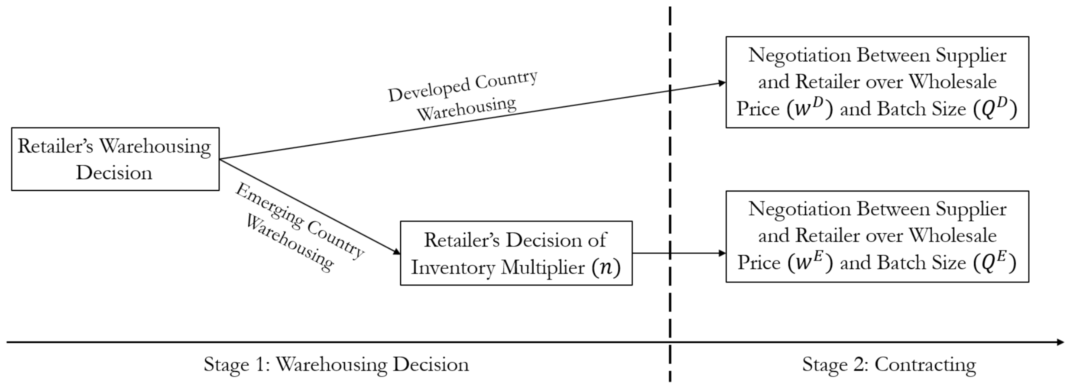


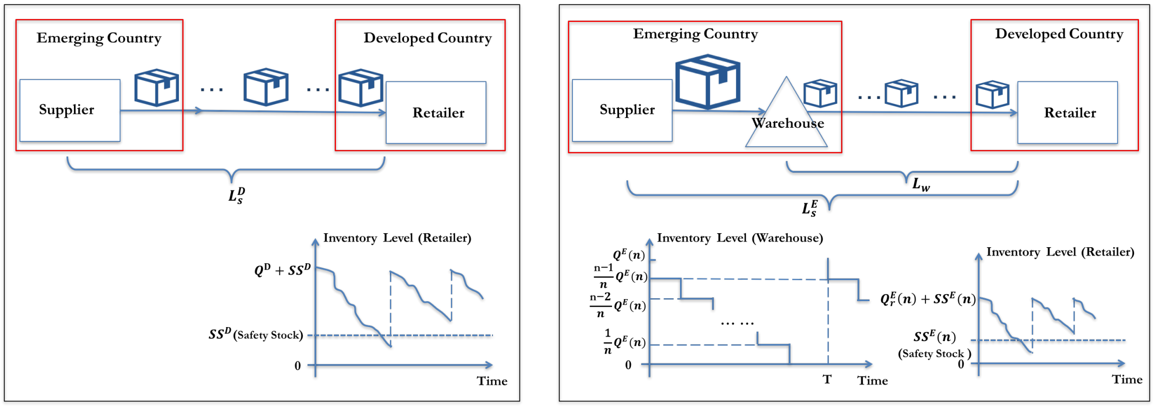
Figure 3.1: EVENT SEQUENCE

in contract negotiation and therefore the negotiation outcomes are independent of the retailer's warehousing strategy. In contrast, we study a detailed logistics cost model that includes lot-sizing shipment cost and inventory holding cost. Hence the retailer's warehousing decision would impact the negotiated outcomes such as wholesale price and order batch size.

Our paper builds on the inventory theory of multi-echelon systems. The retailer manages a multi-echelon inventory system when she uses an emerging-country warehouse as she holds inventory at the warehouse as well as the retail location. Previous literature has studied the optimal inventory policy in a multi-echelon system under various settings such as periodic or continuous review inventory system (Clark and Scarf 1960, De Bodt and Graves 1985), stochastic or constant market demand (Schwarz and Schrage 1975, Chen and Zheng 1994) and echelon stock or installation stock inventory policy (Badinelli 1992, Axsäter and Rosling 1993). This stream of research has demonstrated the optimality of the echelon-stock policy and most papers assume the nested inventory policy. For tractability of our model, we apply a nested echelon-stock policy and adopt the approximation of the retailer's long-run average cost from De Bodt and Graves (1985). Using this approximation, we further study the contracting problem.

3.3 Model

We study supply chain contracting of a single product between a retailer in a developed country and a supplier in an emerging country. The market price p is fixed and the market demand is stochastic and stationary with normal distribution $N(\mu, \sigma^2)$. The retailer faces a two-stage



(*a*) (*b*)

Figure 3.2: ORDER TRANSSHIPMENT AND INVENTORY LEVEL IN DEVELOPED COUNTRY WAREHOUSING (a) AND EMERGING COUNTRY WAREHOUSING (b)

process: the warehousing decision stage and the contracting stage and Figure 3.1 illustrates the event sequence. At the first stage, she determines the warehousing strategy: developed country warehousing or emerging country warehousing. In developed country warehousing setting, the supplier ships products to the exporting harbor and from there the retailer ships products directly to the retail location, as illustrated in (a) of Figure 3.2. In this case, the retailer holds cycle stock and safety stock at the retail location. In emerging country warehousing setting, however, the supplier ships products to the warehouse in the emerging country and the retailer takes over afterwards. As in (b) of Figure 3.2, an inbound shipment is broken into n smaller batches at the warehouse and these small batches are shipped to the retail location sequentially. $n \in \mathcal{N}^+$ is called *inventory multiplier* in inventory literature (De Bodt and Graves 1985) and determined by the retailer at this stage. After the batches are depleted, the warehouse gets the next replenishment from the supplier. In this case, in addition to safety stock and cycle stock at the retail location, the retailer holds second-tier cycle stock at the warehouse. Conditional on the retailer's warehousing decision, at the second stage, she negotiates with the supplier over the wholesale price and order batch size.

3.3.1 Supplier's Profit

The supplier's cost structure remains the same in both developed country warehousing setting and emerging country warehousing setting. He incurs variable and fixed costs for every batch he ships out and the retailer pays him for procurement. Let Q^D denote the batch size in developed country warehousing setting and $Q^E(n)$ in emerging country warehousing setting with n as the *inventory multiplier*. The supplier incurs cost of $c_s + cQ^D$ in developed country warehousing setting and $c_s + cQ^E(n)$ in emerging country warehousing setting for each batch that he ships out where c represents the variable production cost and c_s can be interpreted as fixed cost such as local shipping cost or customs declaration cost. Without loss of generality, we normalize c to zero. Let w^D denote the wholesale price in developed country warehousing setting and $w^E(n)$ in emerging country warehousing setting. The supplier's long-run average profit in developed country warehousing setting Π_s^D and that in emerging country warehousing setting $\Pi_s^E(n)$ are given by

$$\Pi_s^D = (w^D - \frac{c_s}{Q^D})\mu, \quad \Pi_s^E(n) = (w^E(n) - \frac{c_s}{Q^E(n)})\mu. \tag{3.1}$$

3.3.2 Retailer's Profit in Developed Country Warehousing

In developed country warehousing setting, the retailer takes care of the products once she receives shipments at the exporting harbor. She obtains sales revenue at p per unit and pays the supplier at w^D per unit. For every batch shipped to the retail location, the retailer incurs cost $c_w + c_h Q^D$, with c_w representing the fixed cost for each batch and c_h the variable cost for each unit. To measure the long-run average inventory cost of the retailer, we consider a continuous review inventory system at the retail location, consisting of cycle stock and safety stock. The average inventory level due to cycle stock is $Q^D/2$. Let L_s^D denote the retailer's lead time: the time from order receiving at the supplier to order arrival at the retail location and l_s and $(\sigma_s)^2$ denote the mean and variance of L_s^D. Thus the safety stock at the retail location is $SS^D = k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})$, where k is the service factor at the retail location (Eppen and Martin 1988). Let h_r denote the inventory holding cost at the retail location. The retailer's long-run average profit Π_r^D is given by

$$\Pi_r^D = (p - w^D - c_h - \frac{c_w}{Q^D})\mu - h_r(\frac{Q^D}{2} + k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})). \tag{3.2}$$

3.3.3 Retailer's Profit in Emerging Country Warehousing

In emerging country warehousing setting, the retailer takes care of the products once she receives shipments at the emerging-country warehouse. Let $Q_r^E(n)$ denote the batch size of the outbound shipment of the warehouse to the retail location and $Q_r^E(n) = \frac{Q^E(n)}{n}$ as an inbound shipment to the warehouse is broken into n outbound shipments. For every outbound batch of the warehouse, the retailer incurs logistics cost of $c_w + c_h^E Q_r^E(n)$. As the variable cost (c_h in developed country warehousing setting, c_h^E in emerging country warehousing setting) is often associated with transportation and labor intensive tasks such as sorting, packaging, labeling and loading and these costs are usually lower in emerging country warehousing setting, we assume $c_h^E = 0$ without loss of generality.

The retailer manages a two-echelon inventory system as she holds inventory at the warehouse as well as the retail location. We assume that the retailer adopts a nested inventory policy since it leads to a stationary policy and it is easy to control and evaluate (De Bodt and Graves 1985). A nested policy means that whenever a stage orders, all its downstream stages also order. In our model, a nested inventory policy indicates that every time an inbound batch ($Q^E(n)$) arrives at the emerging-country warehouse, immediately an outbound batch ($Q_r^E(n)$) is shipped out overseas. The remaining batches at the warehouse are shipped out sequentially to the retail location until depletion and then the next inbound shipment arrives, as shown in (b) of Figure 3.2. The average inventory level at the emerging-country warehouse is $\frac{n-1}{2n}Q^E(n)$ and the average cycle stock level at the retail location is $\frac{Q_r^E(n)}{2} = \frac{Q^E(n)}{2n}$.

We assume that the retailer adopts an echelon-stock inventory policy that concerns echelon stock levels rather than installation stock levels. Installation stock is the on-hand inventory while echelon stock is the installation stock plus all downstream installation stock and in-transit inventory (De Bodt and Graves 1985). To calculate the expected safety stock under a nested echelon-stock policy, we adopt the approximation of expected safety stock in De Bodt and Graves (1985) who use the expected net inventory level just before replenishment to approximate the average safety stock. Let L_w denote the lead time of the retail location: the time from order receiving at the warehouse to order arrival at the retail location, with mean l_w and variance $(\sigma_w)^2$. Let L_s^E denote the overall lead time of the retailer: the sum of the time from order receiving at the supplier to order arrival at the warehouse and the time from order receiving at the warehouse to order arrival at the retail location (L_w). To keep the analysis simple, we assume that the mean and variance of L_s^D and L_s^E are the same. Thus the safety stock level at the retail location is given by

$$SS^E(n) = k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))$$

(see De Bodt and Graves (1985) and Mitra and Chatterjee (2004) for detailed derivation).

We assume that $l_s > l_w$ and $\sigma_s > \sigma_w$ as the lead time of the retail location becomes shorter and more stable when the product batches are shipped to the retail location from the warehouse rather than the supplier. L_w includes the time on order processing at the warehouse, overseas shipping and local shipping in the developed country. L_w is shorter on average ($l_w < l_s$) and more stable ($\sigma_w < \sigma_s$) because it does not include stochastic time on production, shipping within the emerging country and customs declaration at the exporting harbor. In other words, the retailer's safety stock level becomes lower in emerging country warehousing setting.

Let h_w denote the inventory holding cost at the warehouse. The retailer's long-run average profit $\Pi_r^E(n)$ is given by

$$\begin{aligned}\Pi_r^E(n) =&(p - w^E(n) - \frac{c_w n}{Q^E(n)})\mu - h_w\frac{n-1}{2n}Q^E(n) - \\ &- h_r(\frac{Q^E(n)}{2n} + k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))).\end{aligned} \tag{3.3}$$

3.3.4 Negotiation Outcomes

We adopt a *Nash bargaining* framework to model the negotiation between the supplier and the retailer. Supply chain management literature has pointed out that the bargaining framework is more appropriate to model procurement contract than the Stakelberg game (Lovejoy 2010, Feng and Lu 2013). We adopt the *asymmetric Nash bargaining solution* to determine the negotiation outcomes. Let θ denote the bargaining power of the retailer and then the bargaining power of the supplier is $1-\theta$, $\theta \in (0,1)$. In each of the two warehousing settings, if the supplier and retailer cannot reach an agreement, we assume that both parties achieve zero profit. We further assume that the optimal supply chain profit in developed country warehousing setting is positive.

Let (π_r^D, π_s^D) denote the optimal *Nash bargaining solution* and $\Pi_{sc}^D = \Pi_r^D + \Pi_s^D$ the supply chain profit in developed country warehousing setting. (π_r^D, π_s^D) maximizes $(\Pi_s^D)^{1-\theta} * (\Pi_r^D)^\theta$ over (Π_s^D, Π_r^D). Taking the first order condition of $(\Pi_s^D)^{1-\theta} * (\Pi_r^D)^\theta$ with respect to Π_r^D and Π_s^D, we have $\pi_r^D = \theta\Pi_{sc}^D$, $\pi_s^D = (1-\theta)\Pi_{sc}^D$. Let $(\pi_r^E(n), \pi_s^E(n))$ denote the optimal *Nash bargaining solution* and $\Pi_{sc}^E(n) = \Pi_r^E(n) + \Pi_s^E(n)$ the supply chain profit in emerging country warehousing setting. Similarly we have $\pi_r^E(n) = \theta\Pi_{sc}^E(n)$, $\pi_s^E(n) = (1-\theta)\Pi_{sc}^E(n)$. That is, the individual profit is proportional to the supply chain profit and the coefficient is the bargaining power. Therefore maximizing the individual profit is equivalent to maximizing the supply chain profit.

In our model, as the logistics costs are taken into account in contracting, the negotiation is over both wholesale price and order batch size and thus the optimal negotiated wholesale price is dependent on the optimal batch size. To maximize individual profits, the optimal order batch size maximizes the supply chain profit. In developed country warehousing setting, the supply chain profit Π_{sc}^D is,

$$\Pi_{sc}^D = \Pi_s^D + \Pi_r^D = (p - c_h - \frac{c_s + c_w}{Q^D})\mu - h_r(\frac{Q^D}{2} + k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})). \tag{3.4}$$

Let q^D denote the optimal batch size in this setting and $q^D = \sqrt{\frac{2\mu(c_w+c_s)}{h_r}}$ as Π_{sc}^D is concave in Q^D. The associated negotiated wholesale price and long-run average profit of the retailer are,

$$w^D = (1-\theta)(p-c_h) - (2(1-\theta)c_w + (1-2\theta)c_s)\sqrt{\frac{h_r}{2\mu(c_s+c_w)}} - (1-\theta)\frac{h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})}{\mu}, \tag{3.5}$$

$$\pi_r^D = \theta((p-c_h)\mu - \sqrt{2h_r\mu(c_s+c_w)} - h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})). \tag{3.6}$$

In emerging country warehousing setting, the supply chain profit $\Pi_{sc}^E(n)$ is given by,

$$\begin{aligned}\Pi_{sc}^E(n) = \Pi_s^E(n) + \Pi_r^E(n) =& (p - \frac{c_w n + c_s}{Q^E(n)})\mu - (h_w \frac{n-1}{2n} + h_r \frac{1}{2n})Q^E(n) - \\ & - h_r k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})).\end{aligned} \tag{3.7}$$

Let $q^E(n)$ denote the optimal batch size in this setting and $q^E(n) = \sqrt{\frac{2\mu n(c_w n + c_s)}{(n-1)h_w + h_r}}$ as $\Pi_{sc}^E(n)$ is concave in $Q^E(n)$. The associated negotiated wholesale price and long-run average profit of the retailer are,

$$\begin{aligned}w^E(n) =& (1-\theta)p - (2(1-\theta)c_w n + (1-2\theta)c_s)\sqrt{\frac{(n-1)h_w + h_r}{2n\mu(c_w n + c_s)}} - \\ & - (1-\theta)\frac{h_r k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))}{\mu},\end{aligned} \tag{3.8}$$

$$\begin{aligned}\pi_r^E(n) =& \theta(p\mu - \sqrt{\frac{2(c_w n + c_s)(h_w(n-1) + h_r)\mu}{n}} - \\ & - h_r k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))).\end{aligned} \tag{3.9}$$

3.4 Warehouse Outsourcing Decision

The retailer would choose emerging country warehousing if her profit is higher than that in developed country warehousing setting, i.e. $\pi_r^E(n) \geq \pi_r^D$. We define the retailer's benefit of emerging country warehousing over developed country warehousing by $\Delta_{\pi_r}(n) = \pi_r^E(n) - \pi_r^D$.

$$\begin{aligned}\Delta_{\pi_r}(n) =& \theta c_h \mu + \theta h_r k \frac{n-1}{n}(\sigma\Delta_L + \sqrt{\mu}\Delta_\sigma) + \theta(\sqrt{2h_r(c_s + c_w)\mu} - \\ & - \sqrt{2(c_w h_w n + \frac{c_s(h_r - h_w)}{n} + c_s h_w + c_w(h_r - h_w))\mu}),\end{aligned} \tag{3.10}$$

where $\Delta_L = \sqrt{l_s} - \sqrt{l_w}$, $\Delta_\sigma = \sigma_s - \sigma_w$. $\Delta_{\pi_r}(n)$ represents the cost advantage of emerging country warehousing setting with *inventory multiplier* set to be n over developed country warehousing setting. Using the emerging-country warehouse brings potential cost advantage from three aspects. First, the retailer achieves labor cost advantage in the emerging country ($\theta c_h \mu$). Second, a shorter and more stable lead time reduces the safety stock holding cost at the retail location ($\theta h_r k \frac{n-1}{n}(\sigma\Delta_L + \sqrt{\mu}\Delta_\sigma)$). Third, the retailer obtains potential benefit from shifting some cycle stock from the retail location to the emerging-country warehouse as second-tier cycle stock $(\theta(\sqrt{2h_r(c_s+c_w)\mu} - \sqrt{2(c_w h_w n + \frac{c_s(h_r-h_w)}{n} + c_s h_w + c_w(h_r-h_w))\mu}))$.

$\Delta_{\pi_r}(n)$ helps understand the impact of demand patterns on the retailer's warehousing strategy. $\Delta_{\pi_r}(n)$ is increasing in demand uncertainty (σ) as it is positively related to the safety stock reduction by using the emerging-country warehouse. When the demand fluctuates more dramatically, the retailer has an incentive to use the emerging-country warehouse to mitigate the risk from demand uncertainty. However, $\Delta_{\pi_r}(n)$ may be increasing or decreasing in demand mean (μ). A higher μ enlarges the absolute value of each of the three parts that make up $\Delta_{\pi_r}(n)$ in Equation (3.10): labor cost advantage, safety stock cost reduction and cycle stock cost advantage of using the emerging-country warehouse. If the third component of $\Delta_{\pi_r}(n)$ is negative, it is decreasing in μ.

Lemma 3.1. *The retailer's warehousing strategy (developed country warehousing or emerging country warehousing) is independent of her bargaining power.*

As the retailer obtains θ proportion of the supply chain profit, $\Delta_{\pi_r}(n)$ equals to θ proportion of the supply chain profit advantage of emerging country warehousing over developed country warehousing. In other words, the retailer's choice between developed country warehousing and emerging country warehousing is independent of her bargaining power.

Proposition 3.1. *(i) There exists a threshold of inventory multiplier n^f such that $\Delta_{\pi_r}(n) \geq 0$ when $n \leq n^f$. Moreover, n^f is non-increasing in h_w and non-decreasing in Δ_L.*

(ii) Let n^* denote the optimal inventory multiplier that maximizes $\Delta_{\pi_r}(n)$. Then $n^* \leq n^f$. Moreover, n^* is non-increasing in h_w and non-decreasing in Δ_L.

The retailer would choose emerging country warehousing if the threshold $n^f > 1$. In that case, setting $n \leq n^f$ results in a higher profit in emerging country warehousing setting. A larger n indicates more cycle stock at the warehouse and less cycle stock and safety stock at the retail location. If n is high, although the retailer keeps little inventory at the retail location, high warehouse inventory level and frequent delivery to the retail location lead to high cost. This could even offset the cost benefit of low inventory level at the retail location and scale economy at the supplier, resulting in a lower supply chain profit. The value of n that best balances the inventory at the emerging-country warehouse and the retail location results in the optimal profit for the retailer.

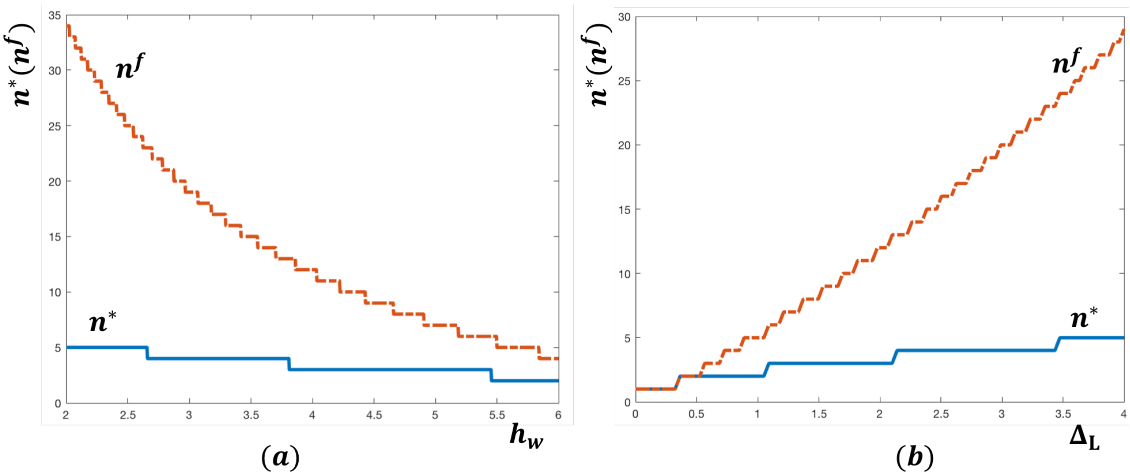


Figure 3.3: EFFECT OF WAREHOUSE HOLDING COST h_w (a) AND LEAD TIME REDUCTION BY EMERGING-COUNTRY WAREHOUSE Δ_L (b) ON n^f AND n^*, $c_w = 5, c_s = 50, h_r = 4, c_h = 0$

Both the threshold n^f and the optimal *inventory multiplier* n^* are non-increasing in the holding cost at the emerging-country warehouse (h_w) and non-decreasing in the lead time reduction by using the warehouse (Δ_L), as demonstrated in Figure 3.3. When the warehouse holding cost is low, the retailer achieves high cost advantage by holding inventory at the warehouse. In this case, emerging country warehousing outweighs developed country warehousing even if the retailer holds high inventory level at the warehouse (large n). Therefore n^f is high. Moreover, to achieve the optimal profit with low warehouse holding cost, the retailer would keep most of her cycle stock at the warehouse and low inventory level at the retail location. Therefore n^* is non-increasing in h_w. When the lead time reduction by using the warehouse is high, the retailer observes high safety stock cost reduction by using the warehouse. In this case, even if the retailer sets high warehouse inventory level, the benefit from safety stock reduction could offset the possible cost disadvantage due to high cycle stock. Therefore n^f is high. Moreover, to maximize the advantage of high lead time reduction due to the warehouse, the retailer needs to benefit from high safety stock reduction that requires frequent delivery to the retail location. Therefore n^* is non-decreasing in Δ_L.

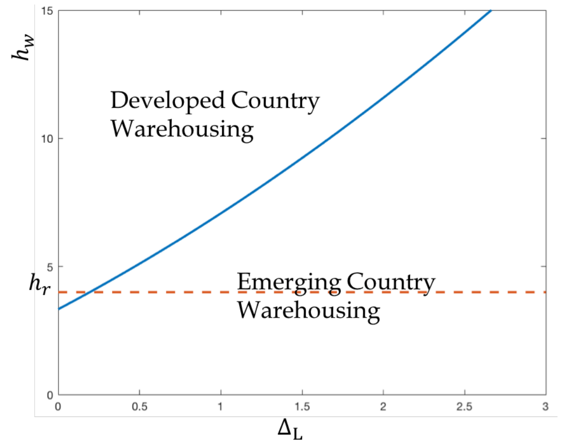


Figure 3.4: IMPACT OF WAREHOUSE HOLDING COST (h_w) AND LEAD TIME REDUCTION (Δ_L) ON THE RETAILER'S OPTIMAL WAREHOUSE DECISION, $c_w = 10, c_s = 50, h_r = 4, c_h = 0$.

Proposition 3.2. *There exists a threshold r^h such that if $h_w/h_r < r^h$, where*

$$r^h = \left(\sqrt{1 + \frac{c_s}{2c_w + c_s}} + \frac{kh_r(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu}}{2\sqrt{h_r(2c_w + c_s)}} \right)^2 - 1, \tag{3.11}$$

then $\Delta_\pi(n^*) > 0$.

The retailer should use the emerging-country warehouse when the warehouse holding cost ratio h_w/h_r is lower than the threshold r^h. Figure 3.4 shows that the retailer's optimal warehouse strategy under varying warehouse holding cost (h_w) and lead time reduction by the warehouse (Δ_L). When Δ_L is large, $r^h > 1$ and emerging country warehousing setting is preferred even when $h_w > h_r$. In this case, even though shifting cycle stock from the retail location to the warehouse incurs additional inventory holding cost, it is more than offset by cost savings in safety stock reduction by using the warehouse.

Figure 3.5 illustrates that the threshold of the warehouse holding cost (r^h) is increasing in the supplier's fixed cost (c_s) if the lead time reduction due to the warehouse is low (Figure 3.5 (a)) and decreasing if the lead time reduction is high (Figure 3.5 (b)). When the lead time reduction (Δ_L) is low, the cost advantage of the retailer's logistics operations cost mostly comes from holding much inventory at the warehouse. Meanwhile the supplier obtains benefit from scale economy due to large order quantities from the retailer. When the fixed cost of the supplier (c_s) becomes higher, using

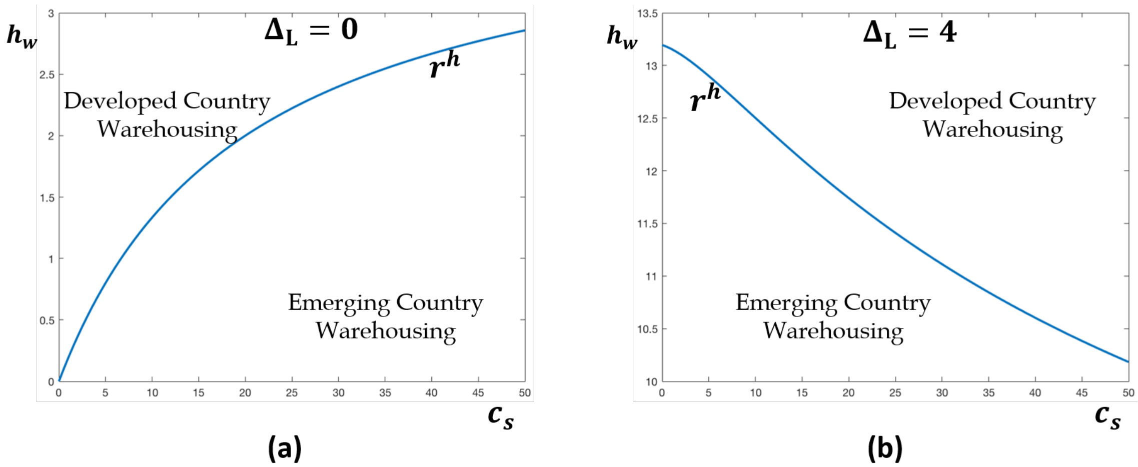


Figure 3.5: IMPACT OF WAREHOUSE HOLDING COST (h_w) AND SUPPLIER'S FIXED COST (c_s) ON THE RETAILER'S OPTIMAL WAREHOUSE DECISION, $c_w = 10, h_r = 4, c_h = 0$.

the warehouse brings more benefit from scale economy to the supplier. Therefore emerging country warehousing could still be more profitable with higher warehouse holding cost (higher r^h). On the other hand, when the lead time reduction (Δ_L) is high, the retailer obtains high cost advantage from safety stock reduction. In this case, the retailer holds low inventory at the warehouse and orders in small quantities from the supplier. When the fixed cost of the supplier (c_s) becomes higher, the supplier's cost disadvantage is more severe. Therefore the warehouse holding cost has to be lower to ensure that emerging country warehousing is more profitable (lower r^h).

3.5 Implications on Negotiation Outcomes

To analyze the impact of warehousing strategy on the negotiated wholesale price, we define the increase in the wholesale price from developed country warehousing setting to emerging country warehousing setting as $\Delta_w(n) = w^E(n) - w^D$,

$$\begin{aligned}\Delta_w(n) =& (1-\theta)c_h + (1-\theta)c_w\left(\sqrt{\frac{2h_r}{(c_w+c_s)\mu}} - \sqrt{\frac{2((n-1)h_w+h_r)n}{(c_wn+c_s)\mu}}\right) + \\ &+ (1-2\theta)c_s\left(\sqrt{\frac{h_r}{2(c_w+c_s)\mu}} - \sqrt{\frac{(n-1)h_w+h_r}{2(c_wn+c_s)n\mu}}\right) + \\ &+ (1-\theta)h_rk\frac{(n-1)(\sigma\Delta_L+\sqrt{\mu}\Delta_\sigma)}{n\mu}.\end{aligned} \tag{3.12}$$

$\Delta_w(n) \leq 0$ indicates that the negotiated wholesale price becomes lower when the emerging-country warehouse is used.

Proposition 3.3. *There exists a threshold $n^w(\theta)$, such that*

(i) $\Delta_w(n) \leq 0$ *when* $n \geq n^w(\theta)$*;*

(ii) if $h_w \leq h_r$*, then* $\Delta_w(n) \geq 0$ *for all* $n \leq n^w(\theta)$*;*

(iii) if $h_w \leq h_r$*, then* $n^w(\theta) \leq n^f$*, therefore* $\Delta_{\pi_r}(n) \geq 0$ *for all* $n \leq n^w(\theta)$*;*

(iv) if $r^w(\theta) \leq h_w/h_r \leq 1$*, where*

$$r^w(\theta) =$$

$$((1 - \frac{2(1-\theta)c_w}{4(1-\theta)c_w + (1-2\theta)c_s})\sqrt{\frac{2(2c_w + c_s)}{c_w + c_s}} + \frac{kh_r(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu}}{\sqrt{h_r}(4c_w + \frac{(1-2\theta)}{1-\theta}c_s)/\sqrt{2c_w + c_s}})^2 - 1,$$

then $n^w(\theta) = 1$*, therefore* $\Delta_w(n) \leq 0$ *for all* $n \in \mathcal{N}^+$*.*

While using the emerging-country warehouse with large n, the retailer keeps high inventory level at the warehouse and orders in a large batch size from the supplier. The supplier achieves benefit in scale economy since he does not have to ship as many lots while the high inventory level at the warehouse results in high total inventory holding cost for the retailer. In this case, the supplier has to offer a discount on the wholesale price. On the other hand, with small n the retailer keeps low inventory level at warehouse and orders in small quantities from the supplier. The supplier obtains little benefit in scale economy as he has to ship frequently to the warehouse while the retailer obtains cost reduction in safety stock and possible advantage of warehouse inventory holding cost. In this case, to compensate the supplier, the retailer has to agree on a higher wholesale price. When the warehouse holding cost ratio is above the threshold $r^w(\theta)$ ($h_w/h_r \geq r^w(\theta)$), even if the order quantities from the supplier are small, the retailer achieves limited benefit in total inventory holding cost from using the emerging-country warehouse. Therefore the she always asks for a discount on the wholesale price while using the emerging-country warehouse, regardless how she sets the *inventory multiplier*.

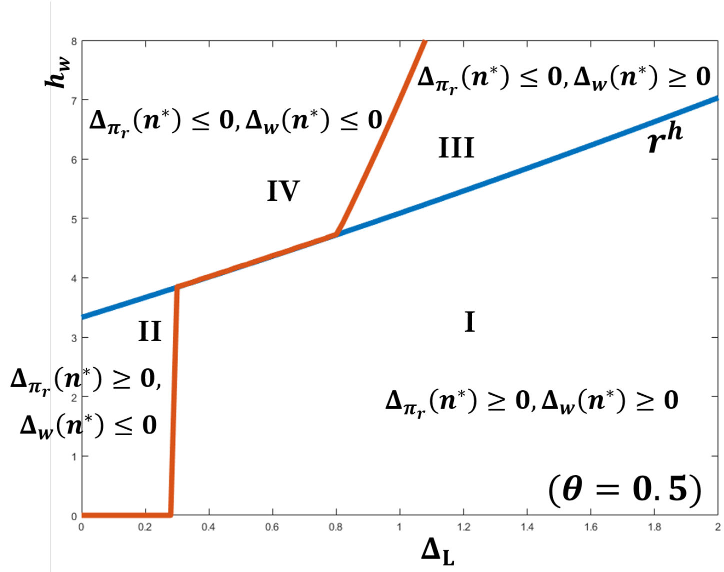


Figure 3.6: IMPACT OF WAREHOUSE HOLDING COST (h_w) AND LEAD TIME REDUCTION BY THE EMERGING-COUNTRY WAREHOUSE (Δ_L) ON THE SIGN OF $\Delta_{\pi_r}(n^*)$ AND $\Delta_w(n^*)$, $c_w = 5, c_s = 50, h_r = 4, c_h = 0$.

Figure 3.6 demonstrates different cases of $(\Delta_{\pi_r}(n^*), \Delta_w(n^*))$, i.e. whether the change in the retailer's optimal profit $(\Delta_{\pi_r}(n^*))$ and the associated wholesale price $(\Delta_w(n^*))$ are positive or negative when the retailer shifts from developed country warehousing to emerging country warehousing. There are two factors in play. The benefit from the emerging-country warehouse increases in Δ_L (lead time reduction) and reduces with h_w (warehouse holding cost). In Zone I, both the wholesale price and individual profits become higher when the retailer uses the emerging-country warehouse $(\Delta_{\pi_r}(n^*) \geq 0, \Delta_w(n^*) \geq 0)$. In this zone, the retailer obtains benefit from safety stock reduction. When the warehouse holding cost is low, she also takes cost advantage by keeping stock at the warehouse, therefore $\Delta_{\pi_r}(n^*) \geq 0$. In this case, the retailer needs to agree on a higher wholesale price to share the benefit from using the warehouse. When the warehouse holding cost is high and the safety stock reduction is also high, the benefit from safety stock reduction outweighs the cost disadvantage at the warehouse and using the emerging-country warehouse is still profitable $(\Delta_{\pi_r}(n^*) \geq 0)$. In this case, the retailer keeps low stock level at the warehouse due to high warehouse holding cost and the order quantities from the supplier are small, which leads to high logistics cost for the supplier. To compensate the supplier, the retailer needs to agree on a higher

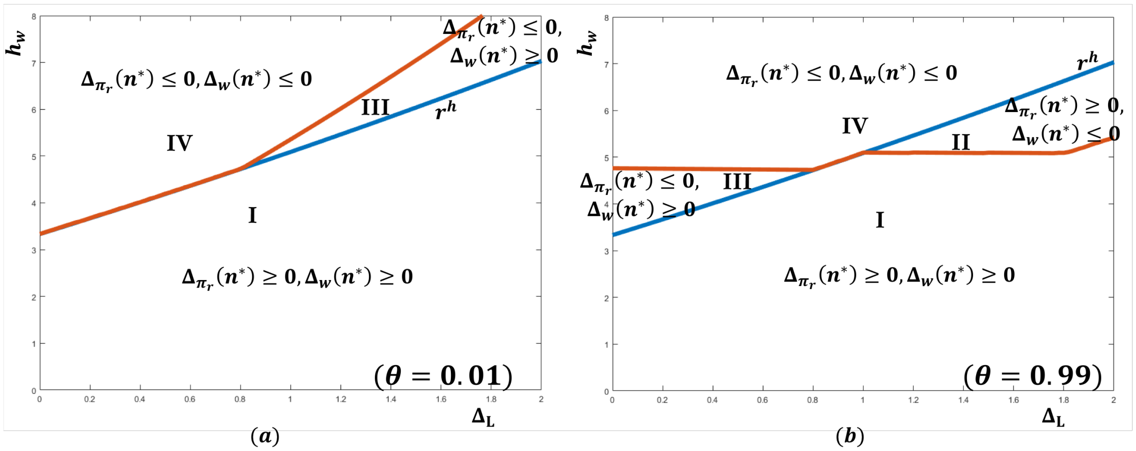


Figure 3.7: IMPACT OF WAREHOUSE HOLDING COST (h_w), LEAD TIME REDUCTION BY THE EMERGING-COUNTRY WAREHOUSE (Δ_L) AND THE RETAILER'S BARGAINING POWER (θ) ON THE SIGN OF $\Delta_\pi(n^*)$ AND $\Delta_w(n^*)$, $c_w = 5, c_s = 50, h_r = 4, c_h = 0$.

wholesale price ($\Delta_w(n^*) \ge 0$).

In contrast, in Zone IV, both the wholesale price and individual profits become lower when the retailer uses the emerging-country warehouse ($\Delta_{\pi_r}(n^*) \le 0, \Delta_w(n^*) \le 0$). In this region, the reduction in lead time is low and the warehouse holding cost is high. Using the emerging-country warehouse is not profitable. To share the cost disadvantage, the supplier has to offer a discount on the wholesale price.

In Zone II ($\Delta_{\pi_r}(n^*) \ge 0, \Delta_w(n^*) \le 0$), the lead time reduction of the warehouse (Δ_L) is low but the warehouse holding cost (h_w) is also low. Since the benefit from safety stock reduction is low, the retailer gets most of the advantage from keeping high stock level at the warehouse. In this case, the retailer orders from the supplier in large quantities, which in turn decreases the supplier's logistics cost. Therefore the retailer requires the supplier to offer a discount on the wholesale price.

In Zone III ($\Delta_{\pi_r}(n^*) \le 0, \Delta_w(n^*) \ge 0$), the warehouse holding cost is high and the increased holding cost by using the emerging-country warehouse outweighs the benefit from lead time reduction ($\Delta_{\pi_r}(n^*) \le 0$). In this case, the retailer benefits from lead time reduction, but also decreases the quantity purchased, leading to low inventory level at the warehouse and increased logistics cost for the supplier. Therefore, she compensates the supplier through a higher wholesale price.

The bargaining power influences the distribution of the four zones, as shown in Figure 3.7. When the retailer's bargaining power is low ($\theta = 0.01$ in Figure 3.7 (a)), Zone II no longer exists. In Zone II in Figure 3.6, the retailer requires a discount on wholesale price while using the warehouse because she achieves most of the cost advantage from keeping high stock level at the warehouse while the supplier's gets reduced logistics cost due to large order quantities. When the retailer's bargaining power is low ($\theta = 0.01$), however, the retailer can only obtain a small proportion of supply chain benefit from using the warehouse and thus has to agree on a higher wholesale price. Therefore Zone II as in Figure 3.6 no longer exists in Figure 3.7 (a). In Zone III, using the emerging-country warehouse is not profitable and the retailer has to agree on a higher wholesale price. Compared with larger bargaining power of the retailer as in Figure 3.6, the area of region III is smaller and the supplier tends to offer a discount on the wholesale price when the emerging-country warehouse is used (larger area of Zone IV). This is because, although using the warehouse results in benefit loss for the supply chain, the retailer only takes a small proportion of this loss when her bargaining power is small and thus tends to require a discount on the wholesale price.

When the retailer's bargaining power is high ($\theta = 0.99$ in Figure 3.7 (b)), the distributions of Zone II and Zone III as in Figure 3.6 get exchanged. In Zone II, using the emerging-country warehouse brings benefit to both parties and the retailer asks for a discounted wholesale price. In this zone, the retailer achieves most of the benefit from high lead time reduction by the warehouse and holds low inventory stock at the warehouse. Although the supplier does not achieve high scale economy due to small order quantities, he still has to offer a discount on the wholesale price because the retailer obtains a large proportion of supply chain profit ($\theta = 0.99$). In Zone III, on the other hand, using the emerging-country warehouse is not profitable and the retailer has to agree on a higher wholesale price. In this zone, the retailer achieves some benefit from lead time reduction by the warehouse and keeps low stock level at the warehouse, while the supplier's logistics cost is high due to small order quantities. Since the retailer has to take a large proportion of the profit loss while using the warehouse ($\theta = 0.99$), she has to offer a higher wholesale price to compensate the supplier.

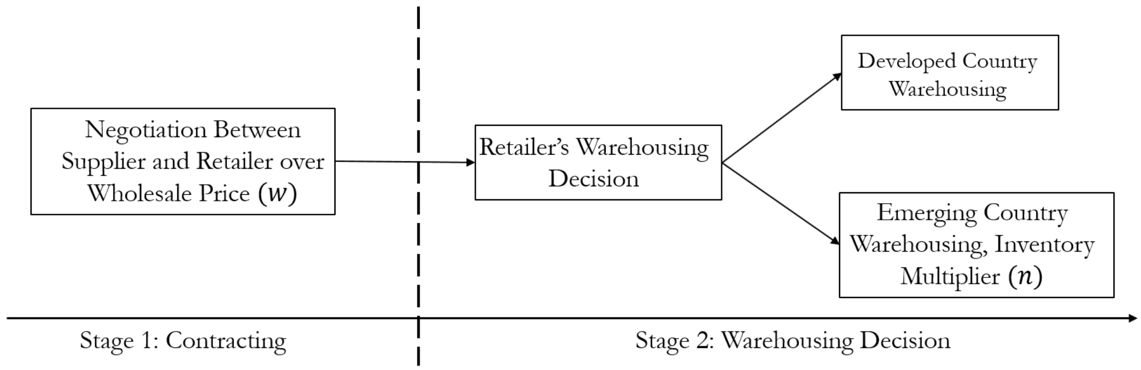


Figure 3.8: EVENT SEQUENCE UNDER TRADITIONAL CONTRACTING

3.6 Comparison with Traditional Contract Design

In offshore sourcing, firms do not consider the total landed cost (including logistics costs) during contract negotiation and make decisions of the logistics operations in a responsive way afterwards (Kumar et al. 2010). Figure 3.8 illustrates the event sequence in this case. The retailer still faces a two-stage decision process. In the first stage, the retailer and the supplier negotiate over the wholesale price. As introduced earlier, in procurement contracting literature the cost structure is often modeled in the form of unit variable cost. The logistics operations costs are not included in contracting. In the second stage, the retailer makes the warehousing decision. The order batch size, whether to use the emerging-country warehouse and the *inventory multiplier* if the emerging-country warehouse is used are determined by the retailer unilaterally. In this section, we analyze the differences in outcome from such analysis.

Let w denote the wholesale price in the traditional contract. The retailer's profit under negotiation is $(p-w)\mu$ and the supplier's profit under negotiation is $(w-c)\mu$ with c normalized to zero. Compared with the contract discussed in §3.3, the supplier's profit under negotiation does not include her logistics cost ($\frac{c^s}{Q^D}\mu$ in developed country warehousing setting and $\frac{c^s}{Q^E}\mu$ in emerging country warehousing setting) and the retailer's profit under negotiation does not include her transportation, inventory and warehousing costs ($(c_h+\frac{c_w}{Q^D})\mu+h_r(\frac{Q^D}{2}+k(\sigma\sqrt{l_s}+\sigma_s\sqrt{\mu}))$ in developed country warehousing setting and $\frac{c_w n}{Q^E(n)}\mu+h_w\frac{n-1}{2n}Q^E(n)+h_r(\frac{Q^E(n)}{2n}+k(\frac{n-1}{n}(\sigma\sqrt{l_w}+\sigma_w\sqrt{\mu})+\frac{1}{n}(\sigma\sqrt{l_s}+\sigma_s\sqrt{\mu})))$ in emerging country warehousing setting). Let w^t denote the negotiated wholesale price and we use superscript t to represent the traditional contract in this section. As the disagreement profits for both parties are zero, the negotiated wholesale price is $w^t = (1-\theta)p$. The contract assigns $(1-\theta)p\mu$ to the supplier and $\theta p\mu$ to the retailer.

As the retailer takes care of her logistics costs (transportation, inventory and warehousing costs) by herself, she determines the order batch size by minimizing her total logistics cost and this decision is independent on the negotiation outcomes. Let $Q^{D,t}$ denote the order batch size in developed country warehousing setting and $Q^{E,t}(n)$ in emerging country warehousing setting with *inventory multiplier* n. In developed country warehousing setting, the retailer's profit is given by,

$$\Pi_r^{D,t} = (\theta p - c_h)\mu - (\frac{c_w \mu}{Q^{D,t}} + \frac{h_r}{2} Q^{D,t}) - h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})$$

and the optimal order quantity is $q^{D,t} = \sqrt{2c_w\mu/h_r}$. Hence the retailer's optimal profit is

$$\pi_r^{D,t} = (\theta p - c_h)\mu - \sqrt{2c_w\mu h_r} - h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}). \tag{3.13}$$

In emerging country warehousing setting, the retailer's profit is given by,

$$\begin{aligned}\Pi_r^{E,t}(n) = &\theta p\mu - (\frac{c_w n\mu}{Q^{E,t}(n)} + \frac{(n-1)h_w + h_r}{2n} Q^{E,t}(n)) - \\ &- h_r k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))\end{aligned}$$

and the optimal order batch size is $q^{E,t}(n) = n\sqrt{\frac{2c_w\mu}{(n-1)h_w+h_r}}$. Hence the retailer's optimal profit is

$$\pi_r^{E,t}(n) = \theta p\mu - \sqrt{2c_w\mu((n-1)h_w + h_r)} - h_r k(\frac{n-1}{n}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})). \tag{3.14}$$

The retailer chooses emerging country warehousing setting if $\Delta_{\pi_r}^t(n) = \Pi_r^{E,t}(n) - \Pi_r^{D,t} \geq 0$, where

$$\Delta_{\pi_r}^t(n) = \sqrt{2c_w\mu h_r} - \sqrt{2c_w\mu((n-1)h_w + h_r)} + h_r k\frac{n-1}{n}(\sigma\Delta_L + \Delta_\sigma\sqrt{\mu}) + c_h\sqrt{\mu}.$$

Proposition 3.4. *Let n^t denote the optimal inventory multiplier that maximizes $\pi_r^{E,t}(n)$.*

(i) There exists a threshold $r^{h,t}$ such that if $h_w/h_r \le r^{h,t}$, where

$$r^{h,t} = \left(1 + \frac{kh_r(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu}}{2\sqrt{2h_r c_w}}\right)^2 - 1,$$

then $\Delta^t_{\pi_r}(n^t) \ge 0$.

(ii) There exists a threshold Δ^t_L such that if $\Delta_L \le \Delta^t_L$, then $r^{h,t} \le r^h$. Therefore, $\Delta_{\pi_r}(n) \ge 0$ for all n such that $\Delta^t_{\pi_r}(n) \ge 0$.

Under the traditional contract, the retailer uses the emerging-country warehouse if the warehouse holding cost is sufficiently low. Note that this threshold of holding cost ratio (h_w/h_r) has different expression from that in Proposition 3.2, which could lead to the warehousing decision under this contract different from that under the contract including logistics costs as in §3.3, as illustrated in Figure 3.9. We use (X, X^t) to denote the warehousing strategy as the lead time reduction (Δ_L) and the warehouse holding cost (h_w) vary, with D indicating developed country warehousing and E emerging country warehousing. The first coordinate in the parenthesis represents the warehousing strategy under the contract that includes logistics costs in negotiation and the second coordinate with superscript t represents the warehousing strategy under the traditional contract that excludes logistics operations costs in negotiation. Under the traditional contract, the retailer incurs logistics cost $\sqrt{2c_w h_r \mu} + h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}) + c_h\mu$ if developed country warehousing setting is chosen and $\sqrt{2c_w\mu((n^t-1)h_w + h_r)} + h_r k(\frac{n^t-1}{n^t}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n^t}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))$ if emerging country warehousing setting is chosen, and obtains $\theta p\mu$ from selling the products, regardless of her warehousing decision. Under the contract including logistics costs, the retailer's effective logistics cost is θ proportion of the supply chain logistics cost, i.e. $\theta(c_h\mu + \sqrt{2h_r(c_w + c_s)\mu} + h_r k(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu}))$ if developed country warehousing setting and $\theta(\sqrt{\frac{2(c_w n^* + c_s)(h_w(n^*-1) + h_r)\mu}{n^*}} + h_r k(\frac{n^*-1}{n^*}(\sigma\sqrt{l_w} + \sigma_w\sqrt{\mu}) + \frac{1}{n^*}(\sigma\sqrt{l_s} + \sigma_s\sqrt{\mu})))$ if emerging country warehousing setting is chosen, and obtains $\theta p\mu$ from selling the products, regardless of her warehousing decision. Therefore the optimal warehousing decision under the traditional contract minimizes the retailer's logistics cost while the optimal warehousing decision under the contract including logistics costs minimizes the supply chain logistics cost.

In Zone I, when the holding cost at the emerging-country warehouse is low, the retailer takes advantage of low inventory cost and safety stock reduction ($\Delta^t_{\pi_r}(n^t) \ge 0$). Moreover, as the retailer

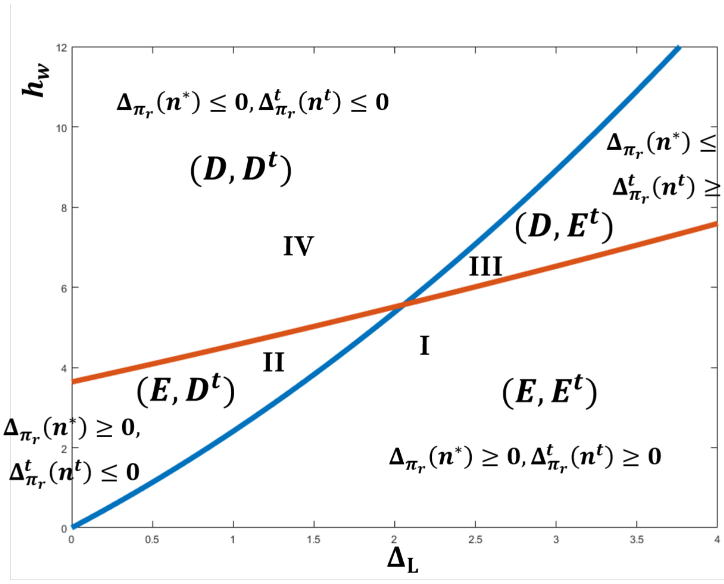


Figure 3.9: WAREHOUSING STRATEGIES UNDER CONTRACTS INCLUDING AND EXCLUDING LOGISTICS COSTS, $c_w = 10, c_s = 200, h_r = 4, k = 1.2, c_h = 0$

keeps high inventory level at the warehouse and orders in large quantities, the supplier benefits from scale economy and thus the supply chain logistics cost becomes lower ($\Delta_{\pi_r}(n^*) \geq 0$). When the warehouse holding cost is high and the lead time reduction due to the warehouse is also high, with respect to the supply chain, the benefit from safety stock reduction more than offsets the disadvantage of high inventory holding cost at the warehouse and high logistics cost of the supplier due to small order quantities ($\Delta_{\pi_r}(n^*) \geq 0$). In this case, as the supplier's logistics cost is not considered in the optimal warehousing decision under the traditional contract, the retailer also achieves a lower logistics cost while using the warehouse ($\Delta^t_{\pi_r}(n^t) \geq 0$).

In contrast, in Zone IV, the benefit from lead time reduction cannot offset the cost disadvantage of holding inventory at the warehouse. Therefore developed country warehousing is preferred under the traditional contract ($\Delta^t_{\pi_r}(n^t) \leq 0$). Further, smaller order quantities due to low stock level at the warehouse lead to higher logistics cost for the supplier when the emerging-country warehouse is used and thus even higher supply chain logistics cost. Therefore developed country warehousing is preferred under the contract including logistics costs as well ($\Delta_{\pi_r}(n^*) \leq 0$).

In Zone II, the retailer uses the emerging-country warehouse under the contract including logistics costs but not under the traditional contract ($\Delta_{\pi_r}(n^*) \geq 0$, $\Delta^t_{\pi_r}(n^t) \leq 0$). In this zone, the retailer achieves some benefit from safety stock reduction but it cannot offset the cost disadvantage by holding inventory at the warehouse. Therefore under the traditional contract using the warehouse is not profitable ($\Delta^t_{\pi_r}(n^t) \leq 0$). Under the contract including logistics costs, the retailer stocks much at the warehouse and orders in large quantities from the supplier, which in turn leads to low logistics cost for the supplier. Although the retailer's logistics cost becomes higher when she uses the warehouse, the overall supply chain logistics cost gets reduced and therefore the retailer would use the emerging-country warehouse.

In Zone III ($\Delta_{\pi_r}(n^*) \leq 0$, $\Delta^t_{\pi_r}(n^t) \geq 0$), the lead time reduction by the emerging-country warehouse (Δ_L) is high but the warehouse holding cost (h_w) is also high. In this case, the retailer's cost advantage from lead time reduction outweighs the cost disadvantage from warehouse holding cost. Therefore under the traditional contract, using the emerging-country warehouse brings benefit to the retailer ($\Delta^t_{\pi_r}(n^t) \geq 0$). However, under the contract including logistics costs, the retailer keeps low stock level at the warehouse and orders in small quantities, which in turn leads to high logistics cost for the supplier. Although using the warehouse brings benefit to the retailer, it leads to cost disadvantage for the supply chain. Therefore developed country warehousing is preferred under the contract including logistics costs. In both Zone II and Zone III, the strategic decision is impacted by whether logistics costs are take into account in contract negotiation.

Proposition 3.5. *If $h_w/h_r \leq r^h$, then $q^E(n^*) \geq q^{E,t}(n^t)$ and $n^* \geq n^t$.*

Under the traditional contract, the optimal order batch size minimizes the logistics cost of the retailer. However, under the contract including logistics costs, the optimal order batch size minimizes the logistics cost of the supply chain, i.e. the sum of the retailer's logistics cost and the supplier's logistics cost. Since the supplier's logistics cost is decreasing in the batch size, the optimal batch size under the contract including logistics costs is larger ($q^E(n^*) \geq q^{E,t}(n^t)$). That is, while using the emerging country warehouse, the retailer holds more cycle stock at the warehouse and less cycle stock at the retail location under the contract including logistics costs. Therefore, the retailer ships more frequently from the warehouse to the retail location and holds less safety stock at the retail location ($n^* \geq n^t$).

Proposition 3.6. *Let $\pi^*_r = \max\{\pi^D_r, \pi^E_r(n^*)\}$ denote the retailer's optimal profit under the contract*

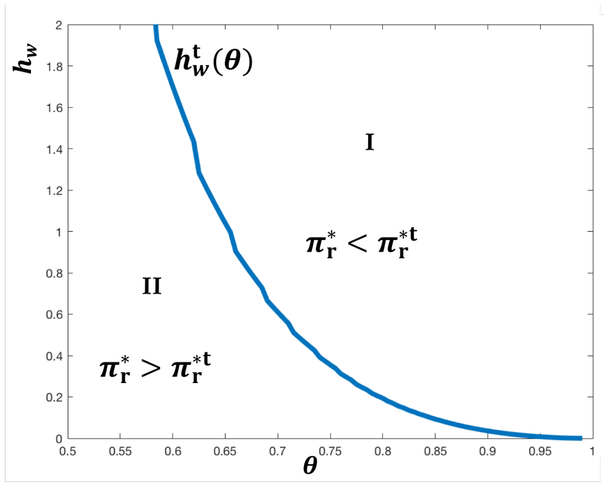


Figure 3.10: IMPACT OF BARGAINING POWER (θ) AND WAREHOUSE HOLDING COST (h_w) ON RETAILER'S OPTIMAL PROFITS UNDER CONTRACTS INCLUDING AND EXCLUDING LOGISTICS OPERATIONS COST, $c_w = 10, c_s = 50, h_r = 4, k = 1.2, c_h = 0, l_w = 1, 1_s = 2$

including logistics operations costs and $\pi_r^{*t} = \max\{\pi_r^{D,t}, \pi_r^{E,t}(n^t)\}$ *denote the retailer's optimal profit under the traditional contract.*

1. *There exists a threshold* $\theta^t(h_w)$ *such that if* $\theta \leq \theta^t(h_w)$*, then* $\pi_r^{*t} \leq \pi_r^*$*;*

2. *There exists a threshold* $h_w^t(\theta)$ *such that if* $h_w \leq h_w^t(\theta)$*, then* $\pi_r^{*t} \leq \pi_r^*$*.*

In Figure 3.10 we compare the retailer's optimal profits under the traditional contract and the contract including logistics costs. The retailer's optimal profit under the contract including logistics costs is higher (lower) if her logistics cost is lower (higher). Note that the retailer's logistics cost under the contract including logistics costs is proportional to her bargaining power (θ) while that under the traditional contract is independent on her bargaining power. Hence there exists a threshold $\theta^t(h_w)$ such that the retailer's profit is higher under the contract including logistics costs when θ is smaller than $\theta^t(h_w)$. In this case, since the retailer only needs to take care a small proportion of the supply chain logistics cost under the contract including logistics costs, her logistics cost is lower than that under the traditional contract ($\pi_r^* > \pi_r^{*t}$).

Figure 3.10 shows that for any θ, there exists a threshold $h_w^t(\theta)$ such that the retailer's profit is higher under the contract including logistics costs ($\pi_r^* > \pi_r^{*t}$) when the warehouse holding cost is lower than that threshold ($h_w \leq h_w^t(\theta)$). In region I, when the warehouse holding cost is high, the supply chain logistics cost is high. As the bargaining power of the retailer is high, her (effective) logistics cost is higher under the contract including logistics costs. Therefore $\pi_r^* < \pi_r^{*t}$. When the warehouse holding cost is low and the retailer uses the emerging-country warehouse, the inventory level at the warehouse is higher under the contract including logistics costs (as explained after Proposition 3.5). Therefore as the warehouse holding cost decreases, the inventory holding cost at the warehouse decreases more under the contract including logistics costs than that under the traditional contract. This indicates that the supply chain logistics cost under the contract including logistics costs decreases more than the retailer's logistics cost under the traditional contract. When the warehouse holding cost is lower than the threshold $h_w^t(\theta)$, her logistics cost under the contract including logistics costs (θ proportion of supply chain logistics cost) would be lower than that under the traditional contract ($\pi_r^* > \pi_r^{*t}$). This threshold $h_w^t(\theta)$ goes to zero when θ goes to one. If the retailer's bargaining power is low as in region II, her logistics cost is lower under the contract including logistics costs ($\pi_r^* > \pi_r^{*t}$) when the warehouse holding cost is high and developed country warehousing is chosen under both contracts. When the warehouse holding cost is low, the emerging-country warehouse is used. In this case, as discussed above, the supply chain logistics cost under the contract including logistics costs decreases faster (with respect to warehouse holding cost) than the retailer's logistics cost under the traditional contract. In other words, for low θ, $\pi_r^* > \pi_r^{*t}$ when h_w is high and as h_w decreases, π_r^* increases more than π_r^{*t}. Therefore $\pi_r^* > \pi_r^{*t}$ still holds when h_w is low.

3.7 Concluding Remarks

Due to long and uncertain lead time and high inventory level in long-distance supply chains, transportation, inventory and warehousing costs increase dramatically when firms globalize their supply chains. In order to achieve cost efficiency and demand responsiveness in global sourcing, firms have started to locate warehouses in emerging countries near their offshore suppliers. Previous academic literature ignores logistics operations costs in supply chain contracting. In this chapter, we incorporate logistics operations costs in contracting between a retailer in a developed country and a supplier in an emerging country. We explore the implications of logistics costs on the retailer's optimal warehouse decision and demonstrates that ignoring logistics operations costs in contracting could lead to suboptimal warehousing decisions.

We show that if a retailer could achieve short and stable lead time from the emerging-country warehouse, she may use the emerging-country warehouse even when the warehouse holding cost does not bring cost advantage. When the emerging-country warehouse brings low warehouse holding cost and/or high lead time reduction, the retailer would agree on a higher wholesale price to the supplier while using the emerging-country warehouse. Further we demonstrate that including logistics costs in contract negotiation impacts the retailer's warehousing strategy. Finally, we show that when the emerging-country warehouse provides low holding cost, the retailer could achieve a higher profit by including logistics costs in contracting.

CHAPTER 4: ROLE OF THE NEARSHORE SUPPLIER UNDER SUPPLY CHAIN DISRUPTION UNCERTAINTY

4.1 Introduction

Firms start to move production from offshore countries to nearshore countries due to cost increase in offshore countries and increasingly complex disruption in global supply chains (Culp 2013). For instance, Japanese automakers such as Honda, Mazda and Nissan have shifted production from Asian countries to Mexico to serve the market in North America. By doing this, they gain fatter cost margins and improve product availability (Greimel 2014).

However, moving production facilities closer to markets does not always lead to a higher profit. Otis Elevator lost $60m in 2013 due to moving production back to the United States in South Carolina (Mann 2014). Successful examples (e.g. Forever 21 and Mattel) suggest a good strategy of using both offshore and nearshore suppliers to achieve cost efficiency and product availability under the disruption risk of offshore supply chain (Iyer 2010, Render 2012). Jain et al. (2013) also provide empirical evidence that diversification of global suppliers leads to lower inventory investment.

Firms need to consider multiple factors comprehensively to make the optimal decisions in global sourcing. Offshore orders bring cost advantage due to low labor and material cost of the offshore supplier. However, offshore outsourcing is regarded as one of the top causes of supply chain disruption (Zurich Insurance Group 2013), as it brings external threats (e.g. natural disasters), system vulnerabilities (e.g. oil dependence), quality issues and lack of flexibility (Accenture 2013, Anderson 2013). Furthermore, firms need increasing flexibility and responsiveness to prepare for demand fluctuations (Lacity and Rottman 2012). Hence it is difficult for firms to figure out the optimal global sourcing strategy under the risk of supply chain disruption.

A nearshore supplier is often regarded as a contingency supplier when firms adopt a diversified supplier base in response to supply chain disruption. A firm only orders from the nearshore supplier after disruption occurs (Tomlin 2006). Allowing for the dual-sourcing option, we analyze the role of the nearshore supplier: whether it is a purely contingency supplier or also serves as inventory safeguard.

In this chapter, we study a dual-sourcing problem for a single product under the risk of supply chain disruption as a finite-horizon stochastic dynamic program. A firm can order from an offshore supplier and a nearshore supplier each period based on her demand forecast and disruption information to minimize the expected total cost. The nearshore supplier is expensive but reliable and the offshore order is cheap but may meet supply chain disruption. The disruption state determines the probability of disruption and evolves in a Discrete Time Markov Chain (DTMC) every period. The lead time of an offshore order is two and that of a nearshore order is one. The demand forecast evolves following a Martingale Model of Forecast Evolution (MMFE) every period.

We show that the optimal outsourcing strategy is a state-dependent two-threshold base-stock policy. Every period the firm should place a nearshore order up to the optimal nearshore threshold, and place an offshore order additionally up to the optimal offshore threshold, whenever the inventory level allows. If the nearshore threshold is higher than the offshore threshold, she only orders from the nearshore supplier up to the offshore threshold level. We provide conditions on cost parameters and disruption risk under which the firm should use a sole- or dual-sourcing strategy and investigate the impact of cost, disruption and demand forecast on the two thresholds.

In our numerical study, we investigate the impact of various factors on the firm's strategy in response to supply chain disruption. Firms often apply contingency or mitigation tactics to prepare for supply chain disruption and demand fluctuations. Contingency tactics mean that firms take actions after disruption occurs, such as ordering from a backup supplier; mitigation tactics mean that firms take actions in advance of disruption, such as building up enough inventory safeguard (Tomlin 2006). We define two measures to represent the firm's dependence on the nearshore supplier and the role of nearshore orders: a contingency plan or a mitigation plan. An asymptotically optimal heuristics algorithm is developed based on Infinitesimal Purtubation Analysis (IPA) and sample path algorithm to search for the optimal order decisions. Our results indicate that rather than purely serving as a contingency plan, nearshore orders also build up inventory safeguard under specific conditions. We find that compared with long and infrequent disruption, under short and frequent disruption, a larger portion of nearshore orders are contingency orders. Furthermore, although firms shift to nearshore production due to cost increase of offshore orders, they should only do that when the disruption risk is sufficiently high.

The remainder of this paper is organized as follows. §4.2 surveys the related literature. We analyze the basic model in §4.3. A model with general lead time is presented in §4.4. In §4.5, we develop an efficient heuristics and investigate the effect of various parameters on the optimal strategy and the role of the suppliers in a numerical study.

4.2 Related Literature

Our work is in the area of supply chain disruption and adds to the aspect of dual sourcing. Previous literature has covered various issues under supply chain disruption, such as sourcing decisions in competitive setting (Wu and Zhang 2014, Yang et al. 2012) and non-competitive setting (Song and Zipkin 2009, Silbermayr and Minner 2014) and pricing decisions (Gong et al. 2014, Feng 2010). Supply chain disruption has been modeled in the form of supplier availability (Parlar et al. 1995), supply uncertainty (Anupindi and Akella 1993), stochastic lead times (Song and Zipkin 2009), supplier with possible system breakdown (Tomlin 2006, Chen et al. 2012), etc. We consider two characteristics of supply chain disruption. First, we allow for time non-homogeneity of the disruption risk. The majority of previous papers only focus on deterministic disruption risks except a few modeling the evolution of disruption length (Tomlin 2006, Saghafian and Van Oyen 2016) or the evolution of up and down state (Gong et al. 2014). We model the evolution of disruption state as a Discrete Time Markov Process (DTMC). Second, we consider disruption uncertainty when making decisions and different disruption states represent different probabilities of disruption occurrence. Previous studies often assume observed disruption (up or down) before ordering, that is, the probability of disruption is either zero or one (Tomlin 2006, Gong et al. 2014). Therefore our model can be regarded as a generalization of those in previous papers.

Dual-souring in both finite and infinite horizon settings have been extensively studied in operations management (Minner 2003, Veeraraghavan and Scheller-Wolf 2008). Previous multi-sourcing problems with forecast updates focus on various issues such as optimal policies with or without fixed cost (Sethi et al. 2003, Sethi et al. 2001) and under stochastic lead times (Song and Zipkin

2009), heuristics policies (Allon and Van Mieghem 2010), capacity planning (Li and Debo 2009, Peng et al. 2012), etc. Similar to many papers (Peng et al. 2012, Sethi et al. 2001, Sethi et al. 2003, Feng et al. 2005), we adopt the Martingale Model of Forecast Evolution (MMFE) to model demand forecast update. The power of MMFE is first illustrated in Heath and Jackson (1994). It reflects the forecast from a lot of forecasting methods and captures the demand evolution from aggregated information.

4.3 Model

We study a dual-sourcing problem for a single product under the risk of supply chain disruption in a finite horizon. Every period the firm orders from a nearshore supplier and an offshore supplier to minimize the expected total cost over the planning horizon. The market demand is continuous, stochastic and stationary with mean μ and unsatisfied demand is backlogged. Let N denote the number of periods in the planning horizon.

Nearshore orders are reliable and take one period to arrive. Offshore orders face possible complete disruption and take two periods to arrive. Let F_t and S_t denote the fast and slow order quantities in period t, $t = 1, \ldots, N$. When placing orders, the firm incurs cost of $c_t^f(F_t)$ for the nearshore order and prepayment of $c_t^s(S_t)$ for the offshore order. If the offshore order would arrive, the firm incurs additional cost of $c_t(S_t)$. The firm incurs a prepayment on placing offshore orders because overseas subcontractors usually ask for some proportion of total payment at order submission in order to mitigate their financial risk (Wang 2013). We assume that $c_f^f(.)$, $c_t^s(.)$ and $c_t(.)$ are convex. Let h_t denote the unit inventory holding cost and π_t the unit backlog penalty cost. With x the on-hand inventory level at the beginning of period t and D the demand in period t, the expected inventory cost at the end of period t is $H_t(x - D) = \mathbb{E}_D[h_t(x - D)^+ + \pi_t(x - D)^-]$.

We use MMFE to model the demand forecast evolution. Let D_t denote the stochastic demand in period t. We use $\boldsymbol{D}_t = \{D_{t,t}, \ldots, D_{t,N}\}$ to represent the demand forecast obtained at the beginning of period t and $\boldsymbol{\epsilon}_t = \{\epsilon_{t,t}, \ldots, \epsilon_{t,N}\}$ to represent the demand forecast update at the end of period t with mean zero and covariance matrix Σ_t. Hence the realized demand at the end of period t is $d_t = D_{t+1,t} = D_{t,t} + \epsilon_{t,t}$ and the demand forecast at the beginning of period $t + 1$ is $\boldsymbol{D}_{t+1} = \{D_{t,t+1} + \epsilon_{t,t+1}, \ldots, D_{t,N} + \epsilon_{t,N}\}$. We assume that $\boldsymbol{\epsilon}_t$ is independent of $\boldsymbol{D}_t$ and $\boldsymbol{\epsilon}_t, \boldsymbol{\epsilon}_s$, $s \neq t$

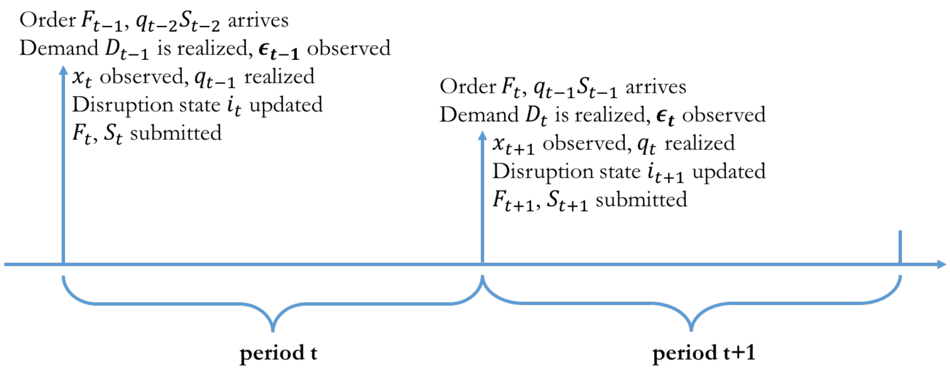


Figure 4.1: EVENT SEQUENCE

are independent.

Let i_t denote the disruption state of period t. With probability P_i, the offshore order placed at period t, S_t would meet complete disruption and cannot arrive. The assumption of complete disruption is not as restrictive as it seems to be. For example, common external factors (e.g. natural disaster, strike events, customs inspection) and internal factors (e.g. lacking communication with suppliers, inappropriate product design, outdated technology) all lead to complete disruption. Without loss of generality, we label the disruption states such that $\forall i > j, P_i < P_j$, i.e. the larger the state number is, the lower the disruption risk is. The disruption state evolves in a Discrete Time Markov Chain (DTMC) with transition matrix denoted by $\{\mathcal{P}_{ij}\}$, i.e. $Prob\{i_{t+1} = j | i_t = i\} = P_{ij}$. We assume that i_t is observed at the beginning of period t. Let $q_t \in \{0, 1\}$ denote the disruption indicator of S_t, where $q_t = 0$ indicates complete disruption and $q_t = 1$ indicates no disruption. We assume that the firm observes q_t at the beginning of period $t+1$ as often the time information about the order state would be available some time after the order is placed. At the beginning of period $t+1$, the firm knows that $q_t S_t \in \{0, S_t\}$ would arrive at the end of period $t+1$.

Let x_t denote the on-hand inventory at the beginning of period t. Figure 4.1 illustrates the event sequence. Note that we do not differentiate between the end of period t and the beginning of period $t+1$.

1. At the beginning of period t, the firm observes the on-hand inventory x_t and the disruption indicator of the last offshore order q_{t-1}. The disruption state i_t is observed;

2. The firm orders F_t from the nearshore supplier at cost $c_t^f(F_t)$ and S_t from the offshore supplier with

prepayment $c_t^s(S_t)$;

3. At the end of period t, F_t arrives. If $q_{t-1}=1$, the firm receives S_{t-1} and incurs cost $c_t(S_t)$; otherwise she does not receive S_{t-1} nor incurs cost;

4. The firm observes demand updates $\boldsymbol{\epsilon}_t = \{\epsilon_{t,t}, \ldots, \epsilon_{t,N}\}$ at the end of period t. The demand in period t is realized through $D_{t+1,t} = D_{t,t} + \epsilon_{t,t}$ and demand forecast is updated through $\boldsymbol{D}_{t+1} = \{D_{t,t+1} + \epsilon_{t,t+1}, \ldots, D_{t,N} + \epsilon_{t,N}\}$. The firm incurs holding or backlogging cost.

Let $s_t = (x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ denote the system state in period t and $V_t(s_t)$ denote the optimal cost-to-go function in period t at state s_t, $t = 1, \ldots, N$. We model the problem in a finite-horizon dynamic programming framework and the Bellman's equations are,

$$\begin{aligned} V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) &= q_{t-1}c_{t-1}(S_{t-1}) + \inf_{F_t, S_t \geq 0} \Big\{ c_t^f(F_t) + c_t^s(S_t) + H_t(x_t + F_t + q_{t-1}S_{t-1} \\ &- D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \mathcal{P}_{i_t,i_{t+1}} \big(P_{i_t} V_{t+1}(X_{t+1}, S_t, 0, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) V_{t+1}(X_{t+1}, S_t, 1, i_{t+1}, \\ &\boldsymbol{D}_{t+1}) \big) \Big\}, \quad t = 1, \ldots, N, \\ V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) &= q_{t-1}c_{t-1}(S_{t-1}), \quad t = N+1. \end{aligned} \tag{4.1}$$

Proposition 4.1. *Let $w_t = x_t + q_{t-1}S_{t-1}, t = 2, \ldots, N$, $w_1 = x_1$. $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ can be transformed to a convex function of w_t, denoted by $U_t(w_t, i_t, \boldsymbol{D}_t)$. Specifically, for $t = 1, \ldots, N$,*

$$U_t(w_t, i_t, \boldsymbol{D}_t) = V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) - q_{t-1}c_{t-1}(S_{t-1}), \tag{4.2}$$

$$\begin{aligned} U_t(w_t, i_t, \boldsymbol{D}_t) &= \inf_{z \geq y \geq w_t} \Bigg\{ H_t(y - D_{t+1,t}) + (1 - P_{i_t})c_t(z - y) + c_t^f(y - w_t) + c_t^s(z - y) + \\ &+ \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \Big[P_{i_t} U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) U_{t+1}(z - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big] \Bigg\}, \\ U_t(w_t, i_t, \boldsymbol{D}_t) &= 0, \quad t = N+1. \end{aligned} \tag{4.3}$$

w_t is the inventory position the firms observes at the beginning of period t before ordering. Rather than keeping track of x_t, q_{t-1} and S_{t-1} separately, it is sufficient to keep track of the inventory position w_t. We redefine the system state as $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$. $y = x_t + q_{t-1}S_{t-1} + F_t$ in Equation (4.3) is the inventory position after placing the order from the nearshore supplier and

$z = x_t + q_{t-1}S_{t-1} + F_t + S_t$ in Equation (4.3) is the inventory position after placing orders from both suppliers. Searching for the optimal F_t and S_t in Equation (4.1) is equivalent to searching for the optimal y and z in Equation (4.3). We call y the nearshore base-stock level and z the offshore base-stock level.

Theorem 4.1. *Assume the slow order cost functions are linear, i.e., $c_t^s(x) = c_t^s * x, c_t(x) = c_t * x$. Given $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$, two base-stock levels y_t^*, z_t^* characterize the optimal ordering policy as follows:*

i. $y_t^* < z_t^*$

$$(F_t^*, S_t^*) = \begin{cases} (y_t^* - w_t, z_t^* - y_t^*) & w_t < y_t^* \\ (0, z_t^* - w_t) & y_t^* \leq w_t < z_t^* \\ (0, 0) & o.w. \end{cases}$$

ii. $y_t^* \geq z_t^*$

$$(F_t^*, S_t^*) = \begin{cases} (z_t^* - w_t, 0) & w_t < z_t^* \\ (0, 0) & o.w. \end{cases}$$

Consistent with the dual-sourcing literature (Sethi et al. 2001), the optimal policy in our model is a state-dependent two-threshold base-stock policy. The firm should order up to $\min(y_t^*, z_t^*)$ from the nearshore supplier. If $y_t^* < z_t^*$, she should order additionally up to z_t^* from the offshore supplier. The assumption that q_{t-1} is known at the beginning of period t is critical to the optimality of the two-threshold base-stock policy. It guarantees that the inventory position w_t is observed at the beginning of every period. We can show the optimality of the state-dependent two-threshold base-stock policy still holds when we adopt Markovian modulated demand rather than MMFE or assume partial disruption rather than complete disruption.

Our model considers uncertain disruption where the supply chain disruption cannot be observed at ordering from the offshore supplier. If the firm knows whether S_t would meet disruption or not before ordering, as assumed in previous studies (Tomlin 2006), she would place nearshore orders only after disruption occurs and offshore orders otherwise. Thus the firm only orders from one supplier each period. Nearshore orders always serve as a contingency plan in response to disruption. In contrast, if the firm does not know whether S_t would meet disruption or not as in our model, she may order from both suppliers in one period. Nearshore orders may serve as a mitigation plan as well as a contingency plan under some circumstances. We illustrate this result further in §4.5.

Although the structure of the optimal policy appears similar to the dual-sourcing problems in literature (Sethi et al. 2001), the difference occurs in evolution of base-stock levels. With stationary demand and costs but no disruption, $y_t^* = y^* + D_{t,t} - \mu$ and $z_t^* = z_t^*(D_{t,t}, D_{t,t+1})$. The firm only makes additive modifications on the two thresholds based on the demand forecast update $\boldsymbol{\epsilon}_t$ every period. In contrast, under the risk of supply chain disruption, the firm needs to update the optimal thresholds based on the disruption state evolution and may switch between sole- and dual-sourcing from period to period.

Proposition 4.2. *Assume* $c_t^k(x) = c_{t+1}^k(x) = c^k x, k \in \{f, s\}, c_t(x) = c_{t+1}(x) = cx$. *Given* $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$,

i. $y_t^* \geq z_t^*$ *if* $c^f - \frac{c^s}{1-P_{i_t}} - c \leq 0$ *and* $y_t^* < z_t^*$ *if* $c^f - \frac{c^s}{1-P_{i_t}} - c > 0$;

ii. $z_t^* - y_t^*$ *increases with* $c^f - \frac{c^s}{1-P_{i_t}} - c$.

The effective purchase cost difference ($c^f - \frac{c^s}{1-P_{i_t}} - c$) captures the expected cost difference between nearshore and offshore orders. It determines which supplier(s) the firm should order from in period t. In period t, if $y_t^* < z_t^*$, the firm may order from both suppliers; if $y_t^* \geq z_t^*$, she only orders from the nearshore supplier. The higher the expected cost difference is, the more the firm tends to order from the offshore supplier. The firm orders only from the nearshore supplier when the offshore order shows no effective cost advantage and only from the offshore supplier when the offshore order shows sufficiently high effective cost advantage. If the effective cost advantage is moderate, the firm would order from both suppliers.

4.3.1 Impact of Disruption and Demand Forecast

In this section we investigate the impact of supply chain disruption and demand forecast on optimal thresholds. Throughout this section, we consider linear and stationary purchase cost as in Proposition 4.2. We assume that the DTMC describing the disruption states is stochastically monotone.

Definition 4.1 (Definition of Stochastically Monotone (Daley 1968))**.** A real-valued Markov chain with stationary one-step transition function $\mathcal{P}(.,.)$ is stochastically monotone when for every set $B_y = (-\infty, y] \cap \mathcal{X}$ and every pair $x_1, x_2 \in \mathcal{X}$ with $x_1 < x_2$, $\mathcal{P}(x_1, B_y) \geq \mathcal{P}(x_2, B_y)$.

The assumption of stochastically monotone DTMC indicates that, for states i, j such that $i < j$, it is more likely for the less reliable state i, compared to the more reliable state j, to transit to less reliable states.

Definition 4.2. For two offshore suppliers with same set of disruption state I, the transition matrix $\mathcal{P}$ is larger than $\mathcal{P}'$ in stochastic order ($\mathcal{P} \geq_{st} \mathcal{P}'$) if $\sum_{j<k} \mathcal{P}_{i,j} > \sum_{j<k} \mathcal{P}'_{i,j}, \forall i, k \in I$.

When comparing two offshore suppliers, the supplier with a larger transition matrix in stochastic order indicates higher reliability of the supply system: the transition probability from any state to a more reliable state is higher than the other system. This definition describes another dimension of supply chain disruption other than the risk of a disruption state: supply chain reliability. Supply chain reliability refers to the possibility that the supply chain would stay in a relatively reliable state while the disruption risk indicates the possibility that the disruption occurs.

Proposition 4.3. *For two systems with the same cost parameters and* $\boldsymbol{\epsilon}_t$ *independent and identically distributed indexed by* $[1], [2]$*, given* $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$*, consider the following conditions:*

(a) $P_{i_t}^{[1]} \geq P_{i_t}^{[2]}$*; (b)* $i_t^{[1]} \leq i_t^{[2]}$*; (c)* $\mathcal{P}^{[1]} \leq_{st} \mathcal{P}^{[2]}$*.*

i. If any of $(a) \sim (c)$ *holds,* $y_t^{*,[1]} \geq y_t^{*,[2]}$*;*

ii. If $z_t^* > y_t^*$ *holds in both systems and any of* $(a) \sim (c)$ *holds,* $z_t^{*,[1]} \leq z_t^{*,[2]}$*;*

iii. If $z_t^* < y_t^*$ *holds in both systems and any of* $(b), (c)$ *holds,* $z_t^{*,[1]} \geq z_t^{*,[2]}$*.* (a) *is irrelevant to comparison of* $z_t^{*,[1]}$ *and* $z_t^{*,[2]}$*.*

iv. If $z_t^* \leq y_t^*$ *in system* $[1]$*,* $z_t^* \geq y_t^*$ *in system* $[2]$ *and any of* $(a) \sim (c)$ *holds,* $z_t^{*,[1]} \geq z_t^{*,[2]}$*.*

(i) and (ii) discuss the case where the firm would order from both suppliers in the two systems in period t. Condition (a) and (b) compare the disruption risk of the current state in the two systems. Under more severe disruption risk in a less reliable state, the firm orders more from the nearshore supplier and less from the offshore supplier. The total order quantity becomes lower, because the firm tends to over-order from the offshore supplier due to potential disruption but not from the nearshore supplier. Condition (c) compares the supply chain reliability of the two systems. Although the disruption risk of each disruption state remains the same in the two systems, system [2] is more reliable than system [1]. In a more reliable system, the firm should order more from the offshore supplier and less from the nearshore supplier, as it is more possible to transit to a more reliable state in a more reliable system. More severe disruption risk, less state reliability and less supply chain reliability strengthen the attractiveness of the nearshore supplier.

(i) and (iii) discuss the case where the firm only orders from the nearshore supplier in period t in both systems. In a more reliable system or state, although the nearshore supplier is always reliable, the nearshore order quantity tends to decrease. In system [1], where the firm tends to transit to a less reliable state, she needs more inventory safeguard to mitigate potential disruption risk. If the current disruption state or supply system becomes sufficiently reliable, the firm even starts to order from the offshore supplier.

Definition 4.3. Consider two random variables X and Y such that $\mathbb{E}[\varphi(X)] \leq \mathbb{E}[\varphi(Y)]$ for all convex functions φ, provided expectation exists. Then X is said to be smaller than Y in the convex order denoted as $X \leq_{cx} Y$.

Proposition 4.4. *Assume that $U_t(w_t, i_t, \boldsymbol{D}_t)$ is continuously twice differentiable and Σ_t is a diagonal matrix, $t = 1, \ldots, N$. Consider two systems with the same cost parameters and demand mean, indexed by $[1], [2]$. If $\epsilon_{t,s}^{[1]} \leq_{cx} \epsilon_{t,s}^{[2]}$, then $y_s^{*[1]} \leq y_s^{*[2]}$, $z_s^{*[1]} \leq z_s^{*[2]}$, and $U_s(w, i, \boldsymbol{D}_s)^{[1]} \leq U_s(w, i, \boldsymbol{D}_s)^{[2]}$, $\forall i$, $s = t, \ldots, N$.*

Larger forecast in convex order means lower forecast accuracy. The firm would order more from the nearshore supplier and in total under lower forecast accuracy. Although it appears that the firm relies more on the reliable supplier under such conditions, this is not always true as whether the slow order quantity would increase or decrease is not obvious. We illustrate this effect in §4.5.

4.3.2 Bounds of Optimal Thresholds

As no closed-form solutions exist for the optimal base-stock levels, we provide upper and lower bounds of the two thresholds with stationary cost parameters: the myopic thresholds and infinite-horizon thresholds. The myopic thresholds y^o, z^o ignore the effect of future periods and minimizes the single-period expected cost. The infinite-horizon thresholds y^*, z^* optimize an associated infinite-horizon problem, minimizing the long-run average expected cost.

Proposition 4.5. *Let* $(y^*(i_t, \boldsymbol{D}_t), z^*(i_t, \boldsymbol{D}_t))$ *and* $(y^o(i_t, \boldsymbol{D}_t), z^o(i_t, \boldsymbol{D}_t))$ *denote the optimal infinite-horizon threshold levels and the myopic threshold levels with stationary linear cost functions such that* $c_t^k(x) = c^k x, k \in \{f, x\}, c_t(x) = cx$, $c^f - \frac{c^s}{1-P_{i_t}} - c > 0$, $t = 1, \ldots, N$. *Then for* $(y_t^*(i_t, \boldsymbol{D}_t), z_t^*(i_t, \boldsymbol{D}_t))$,

1. $y^*(i_t, \boldsymbol{D}_t) > y_t^*(i_t, \boldsymbol{D}_t) > y^o(i_t, \boldsymbol{D}_t)$;
2. $z^*(i_t, \boldsymbol{D}_t) > z_t^*(i_t, \boldsymbol{D}_t) > z^o(i_t, \boldsymbol{D}_t)$.

The optimal thresholds are higher than the myopic thresholds because the latter ignores future demand. Similarly the optimal thresholds are lower than the infinite-horizon thresholds. The optimal thresholds tend to increase with the length of planning horizon and converges to the infinite-horizon thresholds.

4.4 Extended Model

In this section, we discuss the optimal policy with general fixed lead times. Let l_f denote the lead time of the nearshore supplier and l_s the lead time of the offshore supplier. The firm observes the nearshore and offshore orders in-transit before ordering every period.

Theorem 4.2. *If the lead times of the two suppliers differ by one period, i.e.,* $l_f = l, l_s = l+1$, *and* q_t *is realized at the beginning of period* $t+1$, *the optimal policy is a state-dependent two-threshold base-stock policy with the structure stated in Theorem 4.1.*

Our result is consistent with the literature (Minner 2003) that, for a dual-sourcing problem without disruption and demand forecast, a two-threshold base-stock policy is no longer optimal if the lead time difference is larger than one. The critical assumption of q_{t-1} ensures that the

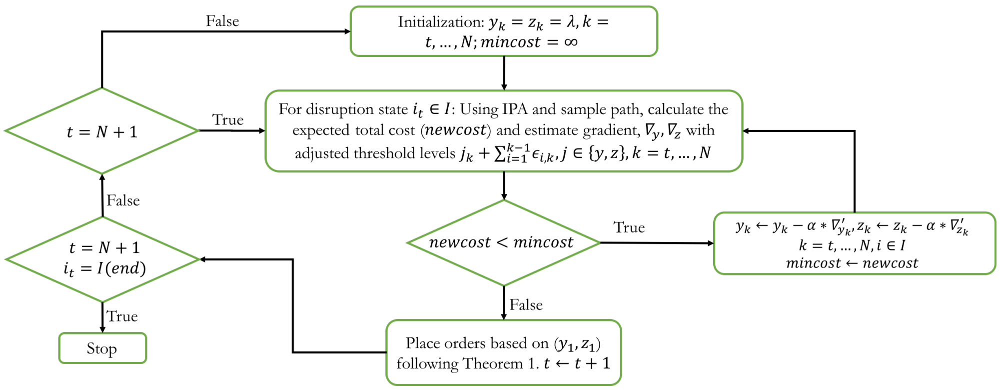


Figure 4.2: ILLUSTRATION OF THE HEURISTICS ALGORITHM

firm is able to keep track of her inventory position before placing orders. The optimality of the two-threshold base-stock policy breaks down if this assumption does not hold. We use a simple counter example to illustrate this.

A Counter Example Suppose we have a 3-period problem with $l_f = 2, l_s = 3$, and q_1 is realized at the beginning of period 3. Following the notation defined in §4.3, the inventory positions in periods 1, 2, 3 are $x_1, x_2 + F_1 + q_1 S_1, x_3 + F_2 + q_1 S_1$. However, the firm doesn't observe q_1 at the beginning of period 2. In period 2, the firm cannot observe the order-up-to levels after placing orders, as $y_2 = x_2 + F_1 + q_1 S_1 + F_2, z_2 = y_2 + S_2$. Hence in period 2, we cannot characterize the optimal ordering policy by base-stock levels.

4.5 Computational Study

4.5.1 A Heuristic Algorithm

We develop an asymptotically optimal heuristics algorithm to search for the base-stock levels and calculate expected order quantities and cost. We use Infinitesimal Perturbation Analysis (IPA) (Glasserman 1991) in gradient search to search for the optimal base-stock levels and a sample-path algorithm to calculate expected order quantities and cost. Although IPA has been widely applied for base-stock levels (Glasserman and Tayur 1995), few studies apply IPA in two-threshold base-stock policies. We illustrate the procedure of the heuristics algorithm in Figure 4.2.

- *Step 0*: Set $t = 1$ and i to be the disruption state with the smallest value. Let *min-cost* denote the current minimum cost and *new-cost* denote the newly calculated expected cost. Choose the step size, α and the tolearance δ in gradient search.

- *Step 1*: For disruption state i, assuming $\sum_{j=1}^{t-1} \epsilon_{j,t} = 0$, use the following steps to search for the base-stock levels for period t, y_t^*, z_t^*.
  - *Step 1.1* Initialization: set $y_k = \mu, z_k = \mu, k = t, t+1, \ldots, N$, *min-cost*=$\infty$;
  - *Step 1.2* For disruption state i, use IPA to estimate the gradient of total expected cost with respect to $y_k, z_k, k = t, t+1, \ldots, N$, denoted by $\nabla y_k, \nabla z_k, k = t, t+1, \ldots, N$;
  - *Step 1.3* Use sample path algorithm to calculate expected cost (*new-cost*) with newly updated $y_k, z_k, k = t, t+1, \ldots, N$. When placing orders, adjust optimal base-stock levels dependent on $\sum_{i=j}^{k-1} \epsilon_{j,k}, k = t, t+1, \ldots, N$ generated by sample paths: $y_k' \leftarrow y_k + \sum_{j=1}^{k-1} \epsilon_{j,k}, z_k' \leftarrow z_k + \sum_{j=1}^{k-1} \epsilon_{j,k}, k = t, t+1, \ldots, N$;
  - *Step 1.4* If $|$*new-cost*$-$*min-cost*$| < \delta$, set $y_k \leftarrow y_k - \alpha * \nabla y_k, z_k \leftarrow z_k - \alpha * \nabla z_k, k = t, t+1, \ldots, N$, *min-cost*$\leftarrow$*new-cost* and go back to the second *Step 1.2*; otherwise, $y_t^* \leftarrow y_t, z_t^* \leftarrow z_t$, and go to *Step 2*.
- *Step 2* Adjust threshold levels based on $\sum_{j=1}^{k-1} \epsilon_{j,k}, k = t, t+1, \ldots, N$ as stated in *Step 1.3*. The threshold levels for disruption state i in period t, $y_t^*(i), z_t^*(i)$ are y_t^* and z_t^*. If all disruption states in period t has been traversed, go to *Step 3*; otherwise set i to the next disruption state (next smallest value) and go to *Step 1*.
- *Step 3* $t \leftarrow t+1$. If $t = N+1$, stop; otherwise, go to *Step 1*.

We search for optimal threshold levels $(y_t^*(i), z_t^*(i))$ regardless of the threshold difference for different disruption states in future periods. Let I denote the set of disruption states. It would be more accurate to search for optimal $y_t^*(i), z_t^*(i), i \in I$ based on $y_k(i), z_k(i), i \in I, k = t+1, \ldots, N$. However the number of threshold levels in future periods, $\sum_{k=t+1}^{N} \sum_{i \in I} (y_k(i) + z_k(i))$, would increase exponentially as N increases, leading to significant loss of computation efficiency.

Ever period the firm incurs purchase cost $c_t^f F_t + c_t^s S_t + c_{t-1} q_{t-1} S_{t-1}$ and inventory cost $h(x_{t+1})^+ + \pi(x_{t+1})^-$. To calculate ∇y_k and ∇z_k, $k = t, \ldots, N$ in *Step 1.2*, we need $\frac{\partial F_t}{\partial s}$, $\frac{\partial S_t}{\partial s}$ and $\frac{\partial x_{t+1}}{\partial s}$, $\forall s \in \{y_t, \ldots, y_N, z_t, \ldots, z_N\}$ in each period. Applying IPA, the partial derivatives can be derived based on the following iterative equations,

$$\frac{\partial F_t}{\partial s} = \begin{cases} (\frac{\partial y_t}{\partial s} - \frac{\partial x_t}{\partial s} - q_{t-1}\frac{\partial S_{t-1}}{\partial s})\mathcal{I}_{\{y_t > x_t + q_{t-1}S_{t-1}\}}, & y_t < z_t \\ (\frac{\partial z_t}{\partial s} - \frac{\partial x_t}{\partial s} - q_{t-1}\frac{\partial S_{t-1}}{\partial s})\mathcal{I}_{\{z_t > x_t + q_{t-1}S_{t-1}\}}, & z_t < y_t \end{cases} \tag{4.4}$$

$$\frac{\partial S_t}{\partial s} = \begin{cases} (\frac{\partial z_t}{\partial s} - \frac{\partial x_t}{\partial s} - q_{t-1}\frac{\partial S_{t-1}}{\partial s})\mathcal{I}_{\{y_t < x_t + q_{t-1}S_{t-1} < z_t\}} + (\frac{\partial z_t}{\partial s} - \frac{\partial y_t}{\partial s})\mathcal{I}_{\{y_t > x_t + q_{t-1}S_{t-1}\}}, & y_t < z_t \\ 0, & z_t < y_t \end{cases} \tag{4.5}$$

$$\frac{\partial x_{t+1}}{\partial s} = \frac{\partial x_t}{\partial s} + \frac{\partial F_t}{\partial s} + q_{t-1}\frac{\partial S_{t-1}}{\partial s}, \tag{4.6}$$

where $s \in \{y_t, \ldots, y_N, z_t, \ldots, z_N\}$. Equation (4.4) and Equation (4.5) follow the two-threshold base-stock policy with thresholds y_t, z_t. Equation (4.6) follows on-hand inventory level evolution. With Equation (4.4)∼Equation (4.6), we can use $\mathbb{E}\frac{\partial F_t}{\partial s}$ ($\mathbb{E}\frac{\partial S_t}{\partial s}$, $\mathbb{E}\frac{\partial x_t}{\partial s}$) to estimate $\frac{\partial \mathbb{E}F_t}{\partial s}$ ($\frac{\partial \mathbb{E}S_t}{\partial s}$, $\frac{\partial \mathbb{E}x_t}{\partial s}$) in calculating the expected total cost. Since $\mathbb{E}\frac{\partial F_t}{\partial s}$ ($\mathbb{E}\frac{\partial S_t}{\partial s}$, $\mathbb{E}\frac{\partial x_t}{\partial s}$) converges to $\frac{\partial \mathbb{E}F_t}{\partial s}$ ($\frac{\partial \mathbb{E}S_t}{\partial s}$, $\frac{\partial \mathbb{E}x_t}{\partial s}$) with probability 1, the heuristics algorithm is asymptotically optimal.

Proposition 4.6. *If $\{\epsilon_{t,s}\}, s = t, \ldots, N$ are independent and each D_t has a density on $(0, \infty)$, $\forall t = 1, \ldots, N$, then the followings hold:*

i. For $t = 1, \ldots, N$, each of F_t, S_t, x_{t+1} is differentiable at $(y_1, \ldots, y_N, z_1, \ldots, z_{N-1})$ with respect to each $y_t, z_t, t = 1, \ldots, N-1$, with probability one. Moreover, the derivatives satisfy Equation (4.4)∼Equation (4.6) (Assuming $S_N = 0$);

ii. If in addition $\mathbb{E}[D_t] < \infty$ for all t, then $\frac{\partial \mathbb{E}F_t}{\partial s}, \frac{\partial \mathbb{E}S_t}{\partial s}, \frac{\partial \mathbb{E}x_{t+1}}{\partial s}$ exist and equal $\mathbb{E}\frac{\partial F_t}{\partial s}, \mathbb{E}\frac{\partial S_t}{\partial s}, \mathbb{E}\frac{\partial x_{t+1}}{\partial s}$ correspondingly, $s \in \{y_1, \ldots, y_N, z_1, \ldots, z_{N-1}\}$.

4.5.2 Numerical Study

In the numerical study, we explore the firm's reliance on different suppliers and investigate which type(s) of strategies that the firm should apply in response to potential disruption: contingency and mitigation strategies. We study a 20-period problem and generate $M = 2,000$ sample paths in each instance to calculate total nearshore and offshore order quantities. Let $\boldsymbol{P} = [P_r, P_u]$ denote the disruption probability vector, where r represents the reliable state and u represents the unreliable state. We consider stationary cost parameters. The basic parameter setting is $c^f \in \{4.5, 6, 9, 9.9\}, c^s = 1, c = 2, b = 10, h \in \{0.5, 1, 2, 3\}$ where b is the stationary unit backlog cost and h is the stationary unit holding cost. The transition matrix of the disruption DTMC is $\mathcal{P} = [0.7, 0.3; 0.3, 0.7]$.

We use $\bar{P} = \frac{P_r+P_u}{2}$ to represent the disruption level and $\hat{P} = P_u - P_r$ to represent the disruption stability. Disruption level emphasizes the severity of the average disruption risk. We enumerate $\boldsymbol{P}$ in $S1 = \{[0.1, 0.2], [0.2, 0.3], \ldots, [0.7, 0.8], [0.8, 0.9]\}$ to discover the impact of disruption level on the firm's strategy. In $S1$, the disruption becomes more and more severe as $\bar{P}$ increases from $\boldsymbol{P} = [0.1, 0.2]$ to $\boldsymbol{P} = [0.8, 0.9]$ but $\hat{P}$ remains the same. Disruption stability explains how close the two disruption states are. We enumerate $\boldsymbol{P}$ in $S2 = \{[0.5, 0.5], [0.4, 0.6], \ldots, [0.1, 0.9]\}$ to discover the impact of disruption stability on the firm's strategy. In $S2$, the disruption becomes more and more decentralized as $\hat{P}$ changes from $\boldsymbol{P} = [0.5, 0.5]$ to $\boldsymbol{P} = [0.1, 0.9]$ but the average disruption risk ($\bar{P}$) remains the same.

We define **DOM** to measure the percentage of nearshore orders among all orders, where

$$\mathbf{DOM} = \frac{\textit{Total Fast Order Quantity}}{\textit{Total Fast Order Quantity} + \textit{Total Slow Order Quantity}}. \tag{4.7}$$

DOM explains the firm's dependence on the nearshore supplier. We define **MIT** to measure the perentage of mitigation orders among nearshore orders, where

$$\mathbf{MIT} = \frac{\textit{Total Mitigation Order Quantity}}{\textit{Total Mitigation Order Quantity} + \textit{Total Contingency Order Quantity}}. \tag{4.8}$$

MIT explains the firm's reliance on mitigation orders among nearshore orders. It indicates whether the firm relies on the nearshore supplier to build inventory safeguard besides the offshore supplier. In each period, the contingency order occurs when the last offshore order cannot arrive due to disruption. Thus we use the minimum of nearshore order quantity and the last offshore order quantity under such circumstances as a proxy of contingency nearshore order quantity and the

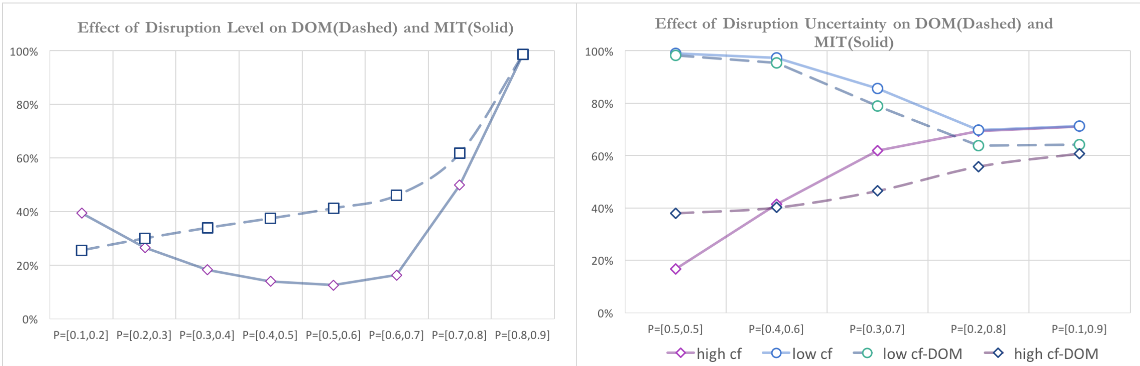


Figure 4.3: EFFECT OF DISRUPTION RISK ON **DOM** AND **MIT**, $cv = 0.05, h = 2$

difference of the two quantities as a proxy of mitigation nearshore order quatity, i.e.,

$$Total\ Mitigation\ Order\ Quantity = \sum_{m=1}^{M} \frac{1}{M} \sum_{t=2}^{N} \left(F_t^m * \mathcal{I}_{\{q_{t-1}^m=1\}} + (F_t^m - S_{t-1}^m)^+ * \mathcal{I}_{\{q_{t-1}^m=0\}} \right),$$

$$Total\ Contingency\ Order\ Quantity = \sum_{m=1}^{M} \frac{1}{M} \sum_{t=2}^{N} \left(\min\{F_t^m, S_{t-1}^m\} * \mathcal{I}_{\{q_{t-1}^m=0\}} \right),$$

where N is the length of planning horizon, M is the number of sample paths and superscript m indicates the m-th sample path.

4.5.2.1 Effect of Supply Chain Disruption

The left panel of Figure 4.3 shows that **DOM** increases with the disruption level, as indicated by Proposition 4.3. **MIT** decreases with the disruption level ($\bar{P}$) when it is low and increases with it when it is high. When the disruption of the offshore supply chain is not severe, the firm mainly orders from the offshore supplier. Most of the nearshore orders are placed when the last offshore order meet disruption. As the disruption becomes more severe, the firm places contingency orders more frequently. Hence the nearshore supplier serves more of a backup supplier. However, when the disruption is severe, the firm mainly depends on the nearshore supplier to satisfy the demand. Therefore, the nearshore supplier mainly serves as a normal supplier rather than a backup supplier. Furthermore, the threshold of the disruption level above which **MIT** starts to increase with the disruption level tends to increase with the nearshore order cost. This is because the firm benefits

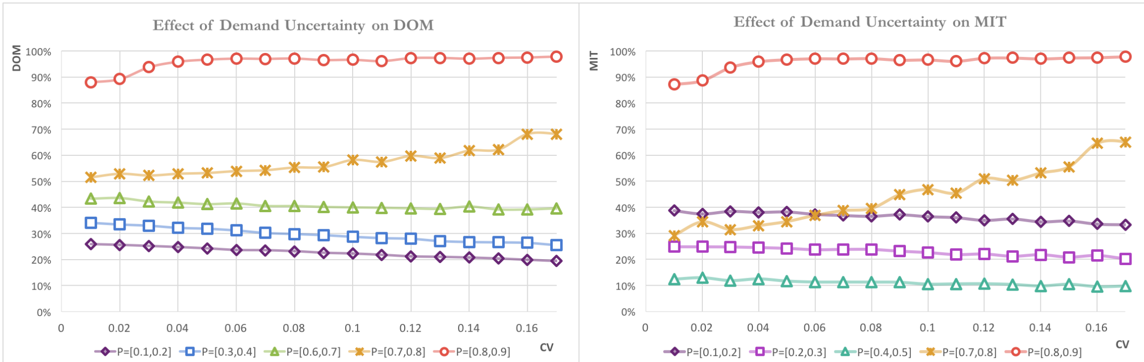


Figure 4.4: EFFECT OF FORECAST ACCURACY ON **DOM** AND **MIT** UNDER VARIOUS DISRUPTION RISK, $c^f = 9.9, h = 0.5$

more from the offshore supplier when its cost advantage becomes higher.

The right panel of Figure 4.3 shows that both **MIT** and **DOM** increase with the disruption risk uncertainty ($\hat{P}$) when the nearshore order is expensive and decrease with $\hat{P}$ otherwise. When the disruption is highly decentralized (e.g. $\boldsymbol{P} = [0.1, 0.9]$), in the reliable state the firm almost only places contingency orders from the nearshore supplier. Hence both **MIT** and **DOM** are not influenced much by the cost difference between nearshore and offshore orders. On the other hand, when the disruption becomes centralized (e.g. $\boldsymbol{P} = [0.5, 0.5]$), with great cost advantage of the offshore supplier the firm rarely orders from the offshore supplier and all nearshore orders are mitigation orders. As the cost advantage of offshore orders increases, the firm orders more from the offshore supplier and places more contingency orders from the nearshore supplier. Therefore, compared with centralized disruption, under decentralized disruption the optimal decision is more sensitive to cost advantage of offshore orders.

4.5.2.2 Effect of Forecast Accuracy

We use $cv = \frac{\mathbb{V}ar(\epsilon_{t,t})}{\mathbb{E}D_t}$ to measure the forecast accuracy and investigate the effect of cv by enumerating $cv \in \{0.01, \ldots, 0.20\}$, as firms require the forecast error below 10% to 15%. When cv increases, the forecast accuracy decreases. For normally distributed forecast error, we set the forecast accuracy k periods ahead as $cv_k = \frac{\mathbb{V}ar(\epsilon_{t,t+k})}{\mathbb{E}D_{t+k}} = cv * (k+1)$.

As Figure 4.4 exhibits, **DOM** decreases with cv under low $\bar{P}$ values while increases with cv under high $\bar{P}$ values. When forecast becomes less accurate, the firm needs to prepare more inventory

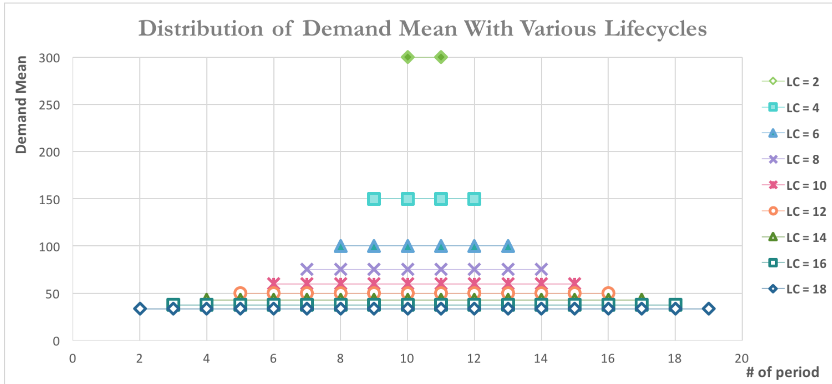


Figure 4.5: DISTRIBUTION OF DEMAND MEAN FOR DIFFERENT PRODUCT LIFE CYCLES, LC(LIFE CYCLE)$\in \{2, 4, \ldots, 18\}$

safeguard in advance. This additional inventory is contributed by the offshore supplier under unsevere disruption and by the nearshore supplier under severe disruption. Hence when cv increases, the firm relies less on the nearshore supplier under unsevere disruption and more on the nearshore supplier under severe disruption. For the same reason, **MIT** also decreases in cv under unsevere disruption and increases in cv under severe disruption. In addition, our numerical results show that the effect of the forecast accuracy on the optimal ordering decision is significantly influenced by the disruption level rather than the disruption uncertainty.

4.5.2.3 Effect of Product Life Cycle

We explore the impact of product life cycle in a 20-period planning horizon. For life cycle LC, demand occurs in periods $(20 - LC)/2 + 1$ to $(20 + LC)/2$ with mean $600/LC$, as illustrated in Figure 4.5. For instance, with $LC = 2$ the mean demand in period 10, 11 is 300 and that in other periods is 0. Products with long life cycles could indicate daily consumed products, while those with short life cycles could indicate seasonal products.

DOM increases with product lifecycle only under high offshore order advantage (both high offshore order cost advantage and unsevere disruption), as illustrated in Figure 4.6 (see left panel, $\mathbf{P} = [0.2, 0.3]$). It decreases with product lifecycle when the advantage of the offshore order is not sufficiently high (severe disruption or low offshore order cost advantage). Under such conditions, with a longer lifecycle, the firm can accumulate more cheap order deliveries. Thus she needs

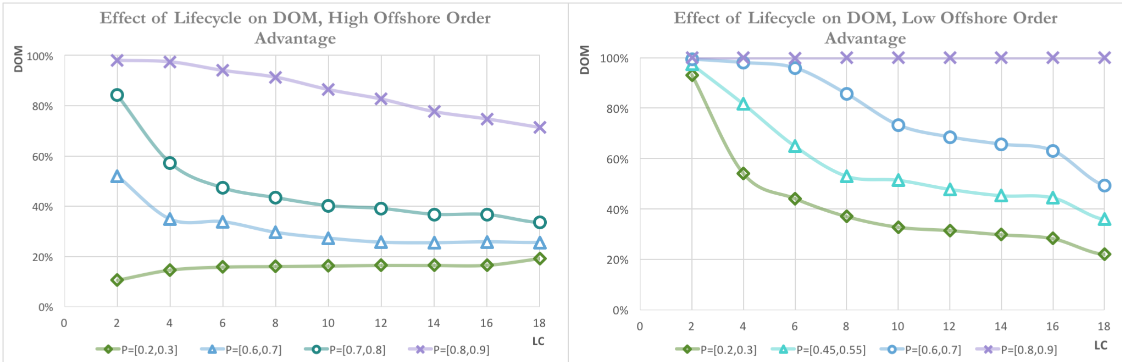


Figure 4.6: EFFECT OF PRODUCT LIFE CYCLE ON **DOM** UNDER VARIOUS DISRUPTION RISK, $c^f = 9, h = 0.5$ (LEFT), $c^f = 6, h = 2$ (RIGHT), $LC \in \{2, 4, \ldots, 18\}$

less orders from the nearshore supplier. When the advantage of offshore orders is high, however, nearshore orders are almost only needed during disruption. With a longer lifecycle, the contingency order quantity increases as more periods are exposed to disruption. In addition, the firm orders less from the offshore supplier in advance due to low disruption risk. Therefore **DOM** increases with product lifecycle.

In addition, our results show that, under more cetralized disruption risk, **DOM** and **MIT** are more sensitive to the change of product lifecycle. This is because, the firm mainly relies on the nearshore supplier under severe disruption risk and on the offshore supplier under unsevere disruption risk, but not on both suppliers in either case. Hence under more centralized disruption risk, the allocation of nearshore and offshore orders (the allocation of mitigation and contingency orders among nearshore orders) is more sensitive to the product lifecycle.

4.5.2.4 Effect of Disruption Type

We investigate how firms should prepare for and respond to supply chain disruption facing different types of disruption: long and infrequent disruption and short and frequent disruption. For instance, disruption such as natural disaster belongs to long and infrequent disruption while disruption such as shipping delay and customs detention belong to short and frequent disruption. We consider the disruption states where $\boldsymbol{P} \in S_3 = \{[0,1], [0.001, 0.999], [0.01, 0.99], [0.05, 0.95], [0.1, 0.9]\}$. In the unreliable (reliable) state, it is almost sure that the disruption would (would not) occur. We use the transition matrix of disruption states in the form of $\mathcal{P} = [p, 1-p; 1-p, p]$ with extreme p

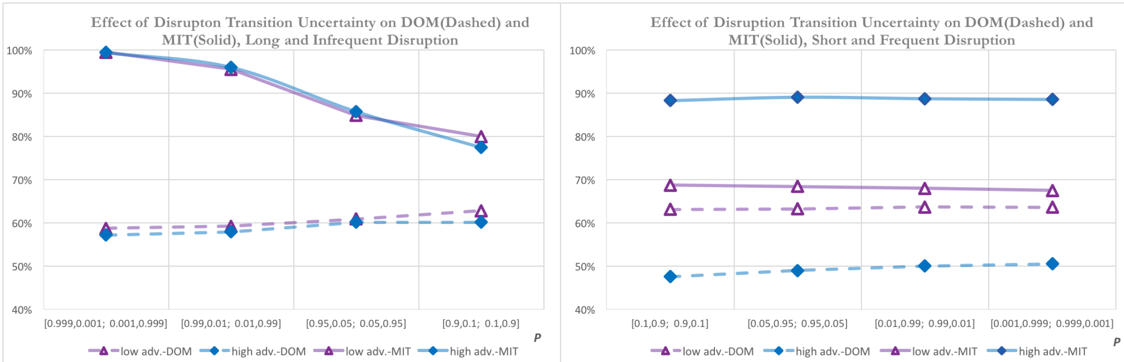


Figure 4.7: EFFECT OF DISRUPTION TRANSITION UNCERTAINTY ON **DOM** AND **MIT** UNDER LONG AND INFREQUENT DISRUPTION (LEFT) AND SHORT AND FREQUENT DISRUPTION (RIGHT), LOW ADVANTAGE: $c^f = 4.5, h = 2$, HIGH ADVANTAGE: $c^f = 9.9, h = 1.5$, $\boldsymbol{P} = [0.001, 0.999]$

values. The disruption uncertainty (whether the disruption would occur or not) is low with such p values. A transition matrix with p close to 0, $p \in S_4 = \{0.1, 0.05, 0.01, 0.001\}$, indicates short and frequent disruption. From $p = 0.1$ to $p = 0.001$, the disruption becomes shorter and more infrequent on average and the transition uncertainty decreases. A transition matrix with p close to 1, $p \in S_5 = \{0.9, 0.95, 0.99, 0.999\}$, indicates long and infrequent disruption. From $p = 0.9$ to $p = 0.999$, the disruption becomes longer and less frequent and the transition uncertainty decreases.

As illustrated in Figure 4.7, the disruption uncertainty and cost advantage of offshore orders have different effect on **DOM** and **MIT** under different types of disruption. With low disruption uncertainty, the firm mainly places nearshore orders in the unreliable state and offshore orders in the reliable state. Under long and infrequent disruption, the firm mainly relies on one supplier to build inventory safeguard. The more uncertain the disruption is, the more contingency orders are needed. Thus neither **DOM** nor **MIT** are significantly influenced by the cost advantage of offshore orders but sensitive to transition uncertainty. Under short and frequent disruption, however, the firm needs contingency orders in the unreliable state and tends to "over-order" in the reliable state. Hence the cost advantage of offshore orders significantly influences **DOM** and **MIT** but the transition uncertainty does not.

We then explore the value of transition uncertainty information, **VTI**, defined as the relative cost reduction when ordering with accurate disruption information, i.e. $\frac{Total\ Cost(\mathcal{P}_b) - Total\ Cost(\mathcal{P})}{Total\ Cost(\mathcal{P})}$, $\mathcal{P} \in$

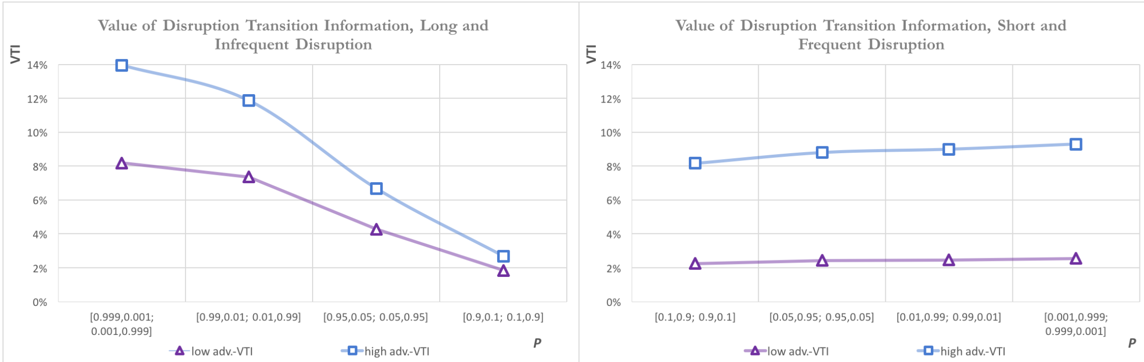


Figure 4.8: EFFECT OF DISRUPTION TRANSITION UNCERTAINTY ON **VTI** UNDER LONG AND INFREQUENT DISRUPTION (LEFT) AND SHORT AND FREQUENT DISRUPTION (RIGHT), LOW ADVANTAGE: $c^f = 9.9, h = 1.5$, HIGH ADVANTAGE: $c^f = 4.5, h = 1.5$, $\boldsymbol{P} = [0.01, 0.09]$

$S_4 \bigcup S_5$, where $\mathcal{P}_b = [0.5, 0.5; 0.5, 0.5]$. Given that the true transition matrix is $\mathcal{P}$, *Total Cost*$(\mathcal{P}_b)$ is the total cost when using $\mathcal{P}_b$ to place orders and *Total Cost*$(\mathcal{P})$ is the total cost when using the correct transition matrix $\mathcal{P}$ to place orders. Thus **VTI** is the relative cost increase when ordering without any information about disruption state transition.

Because of the effect of disruption transition uncertainty on **DOM** and **MIT** is significant under long and infrequent disruption and not significant under short and frequent disruption, the effect on **VTI** shows the similar pattern. The more certain the true transition is, the higher **VTI** is. Furthermore, **VTI** is higher under higher cost advantage of offshore orders.

4.5.2.5 Effect of Offshore Order Cost Advantage

As firms claim that cost increase is an important cause for nearshore production, we explore how sensitive firms should be to this cost increase. We regard $(c^s + c)/c^f$ as a proxy of the cost advantage of offshore orders and enumerate $(c^s + c)/c^f \in \{\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}\}$ to investigate how this ratio influences **DOM** and **MIT**. Our numerical results show that unless the disruption level is sufficiently high, increasing $(c^s + c)/c^f$ does not significantly influence **DOM** or **MIT**. When the disruption level is sufficiently high ($\mathbf{P} \in \{[0.7, 0.8], [0.8, 0.9]\}$), however, **DOM** and **MIT** become quite sensitive to $(c^s + c)/c^f$. We also vary the prepayment ratio $(c^s/(c^s + c))$ from 10% to 100% and observe that this change does not significantly influence **DOM** or **MIT** either, except under sufficiently high disruption level.

4.6 Concluding Remarks

With increasingly frequent disruptions and increasing cost in long-distance supply chains, firms have shifted manufacturing facilities from offshore countries to nearshore countries in recent years. However, this shift may not always bring them expected benefit. Firms need to balance the trade-off between reliable but expensive operations in local markets and unreliable but cost efficient operations in offshore countries. In this chapter, we study a dual-sourcing problem with an offshore and a nearshore supplier available. We identify the optimal sourcing strategy and explore the firm's behavior in preparation for and response to supply chain disruption.

We show that the optimal policy is a two-threshold base-stock policy and explore the effect of effective cost difference, disruption parameters and forecast error on base-stock levels. We develope an IPA-based heuristics algorithm to calculate average order quantities. By analyzing two measures (**DOM** and **MIT**), we explore the firm's reliance on the nearshore supplier and the mitigation order percentage among nearshore orders. Our results indicate that rather than a pure backup supplier, firms should also use nearshore orders to build inventory safeguard in advance under some conditions. As the disruption in long distance supply chain becomes more severe, the nearshore orders serve more of a contingency plan if the disruption is not sufficiently severe. If the disruption is sufficiently severe, firms would regard the nearshore supplier as the main supplier.

Finally we address some related issues in global sourcing that we do not cover in this chapter. The product quality from offshore outsourcing is another important factor that drives firms to nearshore manufacturing facilities. We do not specifically model the quality issue of offshore suppliers, although we can regard unqualified products as a special type of disruption. In addition, firms value quick-response supply chain as it determines how frequently they can update and modify the product designs (Anderson 2013). In that sense, lead time determines the innovation frequency of product design. Extended research could explore the impact of innovation frequency on the optimal strategy in global sourcing.

CHAPTER 5: CONCLUSION AND FUTURE RESEARCH

Research studies on operations strategies tailored to emerging markets are critical for multinationals to improve competitiveness in emerging markets. In this dissertation, we address key issues in agriculture operations in emerging countries and global supply chain management that involves emerging markets. This work provides insights on how to leverage the emerging markets to increase productivity and profit margin as well as to achieve flexibility and responsiveness in emerging market operations. The study in the three chapters demonstrates the advantage of scientific methods in improving agricultural productivity in emerging countries and contributes to the understanding of the impact of various risks and opportunities on the low-cost benefits in emerging markets.

In the first chapter we study a planting schedule problem of a single crop under rainfall uncertainty as a finite-horizon stochastic dynamic program. Planting early may allow the seeds to start contributing biomass production early, but planting later with higher soil water content could lead to a higher chance of seed survival. We show that the optimal planting schedule is a time dependent threshold-type policy, where the farmer should plant down to the optimal threshold.

In practice, farmers start to plant after observing enough cumulative rainfall in the planting season. Utilizing field weather data from Southern Africa, we show that the risk of crop yield drop due to severe climate conditions can be significantly mitigated by adopting the optimal planting schedule. The more severe the climate conditions the higher the relative yield advantage of the optimal planting schedule. Furthermore, for the real size large-scale problem, we show that adopting the optimal schedule could significantly improve the crop biomass production.

In this work we only focus on the planting schedule of seeds and assume that other decisions such as fertilizer addition and pest control are done optimally. In many real situations those aspects can also be difficult to adopt. Further, we only consider one crop in this work. Sometimes, crop rotation has an important impact on the yields of seeds and in such cases that needs to be incorporated. Finally, we do not consider any budget constraints that a farmer might face for seed procurement, automation, fertilizers or pest control. All the above issues are ripe for future studies in this area.

In the second chapter, we study supply chain contracting that incorporates total landed cost between a retailer in a developed country and a supplier in an emerging country. We explore the implications of logistics costs on the retailer's optimal warehouse decision and demonstrates that ignoring logistics operations costs in contracting could lead to suboptimal warehousing decisions.

We show that if a retailer could achieve short and stable lead time from the emerging-country warehouse, she may use the emerging-country warehouse even when the warehouse holding cost does not bring cost advantage. When the emerging-country warehouse leads to low warehouse holding cost and/or high lead time reduction, the retailer would agree on a higher wholesale price while using the emerging-country warehouse. Further we demonstrate that including logistics costs in contract negotiation impacts the retailer's warehousing strategy. Finally, we show that when the emerging-country warehouse provides low holding cost, the retailer could achieve a higher profit by including logistics costs in contracting.

In the third chapter, we study a dual-sourcing problem with an offshore supplier and a nearshore supplier available and explore the role of the nearshore supplier in response to supply chain disruption. We show that the optimal policy is a two-threshold base-stock policy and explore the effect of effective cost difference between the nearshore and offshore suppliers, disruption parameters and forecast error on base-stock levels.

In the computational study, we develop two measures to explore the firm's reliance on the nearshore supplier: the percentage of nearshore order quantity over total order quantity (**DOM**) and the percentage of mitigation order quantity over total nearshore order quantity (**MIT**). Our results indicate that rather than regarding the nearshore supplier as a pure backup supplier, firms should also use nearshore orders to build inventory safeguard in advance under some conditions.

Finally we address some related issues in global sourcing that we do not cover in this chapter. The product quality from offshore outsourcing is another important factor that drives firms to nearshore manufacturing facilities. We do not specifically model the quality issue of offshore suppliers, although we can regard unqualified products as a special type of disruption. In addition, firms value quick-response supply chain as it determines how frequently they can update and modify the product designs (Anderson 2013). In that sense, offshore sourcing would result in infrequent product innovation due to the long lead time. Extended research could explore the impact of innovation frequency on the optimal strategy in global sourcing.

This dissertation takes one step to explore critical issues in tailoring operations strategies to emerging economies and leveraging the emerging markets. The studies have opened up avenues of promising and exciting research agendas. Future work concerns alternative applications of mechanization and scientific methods in agriculture operations and diverse analysis of global supply chain management that involves emerging markets.

In emerging countries, lacking access to farming machinery results in agricultural output far below the developing-world average. Adopting mechanization is a fundamental and sustainable approach to increase agricultural productivity through improved timeliness of farming operations and expansion of cultivated area. Research studies on the application of mechanization in agriculture could help governments, charity foundations and multinationals promote affordable farming machinery suitable for emerging countries. Future research would focus on the investment of mechanization in agriculture. As the procurement of mechanization and crop seeds is constrained by limited budget, a study that explores the optimal investment under limited budget and the strategy of sharing farming machinery among farmers would provide administrative advice on the adoption of mechanization. Moreover, as farmers could be strategic in pricing the agricultural products based on the crop yield and previous price information, extended research studies the impact of strategic farmers on the optimal investment of mechanization.

In the context of global supply chain management, we discuss the prevalent practices of offshore warehousing and nearshore sourcing in global sourcing. Research extensions on the optimal warehousing strategy with both nearshore and offshore suppliers and the impact of competition between retailers on the optimal warehousing strategy would contribute to the understanding of the implications of logistics operations on the global procurement strategies.

Furthermore, after the adoption of the Sustainability Development Goals (SDGs), global food security issues have brought great attention to governments and companies. The emerging markets play an important role in combating global food insecurity as they are major producers of many agricultural products while being relatively underfed and malnourished (Fan and Brzeska 2010). Future research concerning critical issues on food safety, food waste reduction and nutrient improvement would provide insights on managing sustainable global food supply chains and facing the challenge of feeding the world.

APPENDICES

Appendix I

In this section, we present appendix for Chapter 2.

Proof of Results in §2.3

Proof of Lemma 2.1. We prove this result by induction. For $t = 0$, since $Q_0(sw_0) = 0$, $U_0(asd_0, sw_0) = 0$ and $V_0(gsd_0, asd_0, cbm_0, sw_0) = cbm_0$, obviously Equation (2.1) holds. Assume that Equation (2.1), Equation (2.2) and 2.3 hold for $Q_t(sw_t)$, $U_t(asd_t, sw_t)$ and $V_t(gsd_t, asd_t, cbm_t, sw_t)$. For period $t+1$,

$$V_{t+1}(gsd_{t+1}, asd_{t+1}, cbm_{t+1}, sw_{t+1}) = p^r_{t+1} V_t(gsd_{t+1}, asd_{t+1}, cbm_{t+1}, \omega_{t+1}(sw_{t+1}, prec_{t+1}))+$$
$$(1-p^r_{t+1}) * \max\big\{V_t(gsd_{t+1}, asd_{t+1}, cbm_{t+1} + gsd_{t+1} * bm(sw_{t+1}), \omega_{t+1}(sw_{t+1}, 0)), sv(sw_{t+1})$$
$$* V_t(gsd_{t+1}+1, asd_{t+1}-1, cbm_{t+1} + (gsd_{t+1}+1) * bm(sw_{t+1}), \omega_{t+1}(sw_{t+1}, 0))+$$
$$(1 - sv(sw_{t+1})) * V_t(gsd_{t+1}, asd_{t+1}-1, cbm_{t+1} + gsd_{t+1} * bm(sw_{t+1}), \omega_{t+1}(sw_{t+1}, 0))\big\}$$
$$= p^r_{t+1}(cbm_{t+1} + gsd_{t+1} * Q_t(\omega_{t+1}(sw_{t+1}, prec_{t+1})) + U_t(asd_{t+1}, \omega_{t+1}(sw_{t+1}, prec_{t+1})))+$$
$$(1-p^r_{t+1}) \max\big\{cbm_{t+1} + gsd_{t+1} * bm(sw_{t+1}) + gsd_{t+1} * Q_t(\omega_{t+1}(sw_{t+1}, 0)) + U_t(asd_{t+1},$$
$$\omega_{t+1}(sw_{t+1}, 0)), sv(sw_{t+1})(cbm_{t+1} + (gsd_{t+1}+1) * bm(sw_{t+1}) + (gsd_{t+1}+1)*$$
$$Q_t(\omega_{t+1}(sw_{t+1}, 0)) + U_t(asd_{t+1}-1, \omega_{t+1}(sw_{t+1}, 0))) + (1 - sv(sw_{t+1}))(cbm_{t+1}+$$
$$gsd_{t+1} * bm(sw_{t+1}) + gsd_{t+1} * Q_t(\omega_{t+1}(sw_{t+1}, 0)) + U_t(asd_{t+1}-1, \omega_{t+1}(sw_{t+1}, 0)))\big\}$$
$$= cbm_{t+1} + gsd_{t+1} * \big(p^r_{t+1} Q_t(\omega_{t+1}(sw_{t+1}, prec_{t+1})) + (1-p^r_{t+1})(bm(sw_{t+1}) + Q_t(\omega_{t+1}($$
$$sw_{t+1}, 0)))\big) + \big(p^r_{t+1} U_t(asd_{t+1}, \omega_{t+1}(sw_{t+1}, prec_{t+1})) + (1-p^r_{t+1}) \max\{U_t(asd_{t+1}, \omega_{t+1}($$
$$sw_{t+1}, 0)), sv(sw_{t+1})(bm(sw_{t+1}) + Q_t(\omega_{t+1}(sw_{t+1}, 0))) + U_t(asd_{t+1}-1, \omega_{t+1}(sw_{t+1}, 0))\}\big)$$
$$= cbm_{t+1} + gsd_{t+1} * Q_{t+1}(sw_{t+1}) + U_{t+1}(asd_{t+1}, sw_{t+1})$$

where $Q_{t+1}(sw_{t+1})$ and $U_{t+1}(asd_{t+1}, sw_{t+1})$ follow Equation (2.2) and Equation (2.3) for period $t+1$. □

Proof of Theorem 2.1. The first result can be easily shown using Lemma 2.1 as the maximizing operation in $V_t(gsd_t, asd_t, cbm_t, sw_t)$ lies only in $U_t(asd_t, sw_t)$ but neither cbm_t nor $gsd_t * Q_t(sw_t)$. To show the second result, it is sufficient to show that $U_t(asd_t, sw_t)$ is concave in asd_t, i.e. the incremental difference of $U_t(asd_t, sw_t)$ with respect to asd_t is non-increasing. We show this result by induction. Let $\Delta U_t(asd_t, sw_t) = U_t(asd_t+1, sw_t) - U_t(asd_t, sw_t)$ denote the incremental indifference of $U_t(asd_t, sw_t)$ with respect to asd_t and we need to show that $\Delta U_t(asd_t, sw_t) \geq \Delta U_t(asd_t+1, sw_t)$. For $t = 0$, obviously this result holds based on Equation (2.3). Assume that $\Delta U_{t-1}(asd_{t-1}, sw_{t-1}) \geq \Delta U_{t-1}(asd_{t-1}+1, sw_{t-1})$. Note that $SD_t(sw_t)$ is the largest integer value of asd_t that satisfies

$$\Delta U_{t-1}(asd_t - 1, \omega_t(sw_t, 0)) \geq sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))).$$

Then

$$\begin{aligned}\Delta U_t(asd_t, sw_t) =& p_t^r \Delta U_{t-1}(asd_t, \omega_t(sw_t, prec_t)) + (1-p_t^r)* \\ & \begin{cases} \Delta U_{t-1}(asd_t - 1, \omega_t(sw_t, 0)) & asd_t > SD_t(sw_t) \\ \Delta U_{t-1}(asd_t, \omega_t(sw_t, 0)) & asd_t < SD_t(sw_t) \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) & asd_t = SD_t(sw_t) \end{cases},\end{aligned}$$

$$\begin{aligned}\Delta U_t(asd_t+1, sw_t) =& p_t^r \Delta U_{t-1}(asd_t+1, \omega_t(sw_t, prec_t)) + (1-p_t^r)* \\ & \begin{cases} \Delta U_{t-1}(asd_t, \omega_t(sw_t, 0)) & asd_t > SD_t(sw_t) - 1 \\ \Delta U_{t-1}(asd_t+1, \omega_t(sw_t, 0)) & asd_t < SD_t(sw_t) - 1 \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) & asd_t = SD_t(sw_t) - 1 \end{cases}.\end{aligned}$$

If $asd_t \geq SD_t(sw_t) + 1$, $\Delta U_t(asd_t+1, sw_t) \leq \Delta U_t(asd_t, sw_t)$ is non-positive by assumption; If $asd_t \leq SD_t(sw_t) - 2$, $\Delta U_t(asd_t+1, sw_t) \leq \Delta U_t(asd_t, sw_t)$ is non-positive by assumption; If $asd_t = SD_t(sw_t) - 1$, $\Delta U_t(asd_t+1, sw_t) - \Delta U_t(asd_t, sw_t)$ is

$$\begin{aligned}& p_t^r(\Delta U_{t-1}(asd_t+1, \omega_t(sw_t, prec_t) - \Delta U_{t-1}(asd_t, \omega_t(sw_t, prec_t)))) + \\ & (1-p_t^r)(sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) - \Delta U_{t-1}(asd_t, \omega_t(sw_t, 0))),\end{aligned}$$

which is non-positive because the optimal decision is not to plant when the seed amount $asd_t + 1$

equals to $SD_t(sw_t)$; If $asd_t = SD_t(sw_t)$, $\Delta U_t(asd_t+1, sw_t) - \Delta U_t(asd_t, sw_t)$ is

$$p_t^r(\Delta U_{t-1}(asd_t+1, \omega_t(sw_t, prec_t) - \Delta U_{t-1}(asd_t, \omega_t(sw_t, prec_t))))+$$
$$(1-p_t^r)(\Delta U_{t-1}(asd_t, \omega_t(sw_t, 0)) - sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))),$$

which is non-positive because the optimal planting decision is to plant when the seed amount $asd_t + 1$ equals to $SD_t(sw_t) + 1$. □

Proof of Proposition 2.1. We first show that if $sv(sw_t)$ and $bm(sw_t)$ are non-decreasing and convex in sw_t, $SD_t(sw_t)$ is non-increasing in sw_t. Note that $SD_t(sw_t)$ is the largest integer value of asd_t that satisfies

$$\Delta U_{t-1}(asd_t - 1, \omega_t(sw_t, 0)) \geq sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))),$$

and $\Delta U_{t-1}(asd_t - 1, \omega_t(sw_t, 0))$ is non-increasing in asd_t. Hence it is sufficient to show that

$$\Delta U_{t-1}(asd_t, \omega_t(sw_t, 0)) - sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) \tag{A.1}$$

is non-increasing in sw_t, $\forall \beta \geq 0$. For $t = 1$, it is obvious that (A.1) is non-increasing in sw_t. Assume that (A.1) is non-increasing in sw_t. Then for a $(t+1)$-period problem, we want to show that

$$\Delta U_t(asd_{t+1}, \omega_{t+1}(sw_{t+1}, 0)) - sv(sw_{t+1}) * (bm(sw_{t+1}) + Q_t(\omega_{t+1}(sw_{t+1}, \beta))) \tag{A.2}$$

is non-increasing in sw_{t+1}, where

$$\begin{aligned}
&sv(sw_{t+1})(bm(sw_{t+1}) + Q_t(\omega_{t+1}(sw_{t+1}, 0))) \\
=&sv(sw_{t+1})(bm(sw_{t+1}) + p_t^r Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, 0), prec_t))+ \\
&(1-p_t^r)(bm(\omega_{t+1}(sw_{t+1}, 0)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, 0), 0))))
\end{aligned}$$

Since (A.1) is non-increasing in sw_t, we have

$$\begin{aligned}
&\Delta U_{t-1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},0), prec_t)) - sv(\omega_{t+1}(sw_{t+1},0) + \frac{prec_t}{\delta})(bm(\omega_{t+1}(sw_{t+1},0)+ \\
&\frac{prec_t}{\delta}) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0) + \frac{prec_t}{\delta}, 0))), \\
&\Delta U_{t-1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},0),0)) - sv(\omega_{t+1}(sw_{t+1},0))(bm(\omega_{t+1}(sw_{t+1},0))+ \\
&Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))), \\
&\Delta U_{t-1}(asd_{t+1}-1, \omega_t(\omega_{t+1}(sw_{t+1},0),0)) - sv(\omega_{t+1}(sw_{t+1},0))(bm(\omega_{t+1}(sw_{t+1},0))+ \\
&Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0)))
\end{aligned} \tag{A.3}$$

are non-increasing in $\omega_{t+1}(sw_{t+1},0)$ by assumption. As $\omega_{t+1}(sw_{t+1},0)$ is non-decreasing in sw_{t+1}, the expressions in (A.2) are non-increasing in sw_{t+1}. Note that

$$\begin{aligned}
&\Delta U_t(asd_{t+1}, \omega_{t+1}(sw_{t+1},0)) \\
=&p_t^r \Delta U_{t-1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},0), prec_t)) + (1-p_t^r)* \\
&\begin{cases}
\Delta U_{t-1}(asd_{t+1}-1, \omega_t(\omega_{t+1}(sw_{t+1},0),0)) & asd_{t+1} > SD_t(\omega_{t+1}(sw_{t+1},0)) \\
\Delta U_{t-1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},0),0)) & asd_{t+1} < SD_t(\omega_{t+1}(sw_{t+1},0)) \\
sv(\omega_{t+1}(sw_{t+1},0)) * (bm(\omega_{t+1}(sw_{t+1},0)) & \\
+Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))) & asd_{t+1} = SD_t(\omega_{t+1}(sw_{t+1},0))
\end{cases}.
\end{aligned}$$

Since the expressions in (A.3) are non-increasing in sw_{t+1}. Then it remains to show that

$$\begin{aligned}
&p_t^r sv(\omega_{t+1}(sw_{t+1},0) + \frac{prec_t}{\delta})(bm(\omega_{t+1}(sw_{t+1},0) + \frac{prec_t}{\delta}) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+ \\
&\frac{prec_t}{\delta}, 0))) + (1-p_t^r) sv(\omega_{t+1}(sw_{t+1},0))(bm(\omega_{t+1}(sw_{t+1},0)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))) \\
&- sv(sw_{t+1})(bm(sw_{t+1}) + p_t^r Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0), prec_t)) + (1-p_t^r)(bm(\omega_{t+1}(sw_{t+1},0))+ \\
&Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))))
\end{aligned} \tag{A.4}$$

is non-increasing in sw_{t+1}. As $sw_{t+1} \in \mathcal{R}$ and $SD_t(\omega_{t+1}(sw_{t+1},0)) \in \mathcal{N}$, in proving the result, we only consider the incremental of sw_{t+1} that will not change the value of $SD_t(\omega_{t+1}(sw_{t+1},0))$. We first show that

$$p_t^r sv(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})bm(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})+(1-p_t^r)sv(\omega_{t+1}(sw_{t+1},0))* \\ bm(\omega_{t+1}(sw_{t+1},0))-sv(sw_{t+1})bm(sw_{t+1}) \tag{A.5}$$

is non-increasing in sw_{t+1}. Since $sv(sw)*bm(\delta*sw)$ has third order derivative negative, $sv(sw)*bm(sw)$ is concave and $p_t^r\omega_{t+1}(sw_{t+1},\frac{prec_t}{\delta})+(1-p_t^r)\omega_{t+1}(sw_{t+1},0)\geq sw_{t+1}$,

$$p_t^r sv(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})bm(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})+(1-p_t^r)sv(\omega_{t+1}(sw_{t+1},0))* \\ bm(\omega_{t+1}(sw_{t+1},0))-sv(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})bm(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})+ \\ sv(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})bm(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})-sv(sw_{t+1})bm(sw_{t+1})$$

is non-increasing in sw_{t+1} (first order derivative of $sv(sw)*bm(\delta*sw)$ is concave and first order derivative of $sv(sw)*bm(sw)$ is decreasing). We then show that

$$p_t^r sv(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta},0))+(1-p_t^r)sv(\omega_{t+1}(sw_{t+1}, \\ 0))Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))-sv(sw_{t+1})(p_t^rQ_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),prec_t))+(1-p_t^r)* \\ Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))) \tag{A.6}$$

is non-increasing in sw_{t+1}. Since $sv(sw)bm(\delta*sw)$ is convex and has third order derivative negative, so does $sv(sw)Q_{t-1}(\omega_t(sw,0))$. Since $bm(sw)$ is concave and has third order derivative positive, so does $Q_{t-1}(\omega_t(sw,0)$. Hence

$$p_t^r sv(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta})Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+\frac{prec_t}{\delta},0))+(1-p_t^r)sv(\omega_{t+1}(sw_{t+1}, \\ 0))Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0))-sv(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+ \\ p_t^r\frac{prec_t}{\delta},0))+sv(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta})Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta},0))- \\ sv(sw_{t+1})Q_{t-1}(\omega_t(sw_{t+1},0))+sv(sw_{t+1})(Q_{t-1}(\omega_t(sw_{t+1},0))-Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+ \\ p_t^r\frac{prec_t}{\delta},0))+Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0)+p_t^r\frac{prec_t}{\delta},0))-p_t^rQ_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),prec_t))- \\ (1-p_t^r)Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},0),0)))$$

is non-increasing in sw_{t+1} (the first order derivative of $sv(sw)Q_{t-1}(\omega_t(sw,0))$ is concave, $sv(sw)Q_{t-1}(\omega_t(sw,0))$ is concave, $Q_{t-1}(sw)$ is convex, $sv(sw)$ is positive and increasing in sw and the first order derivative of $Q_{t-1}(\omega_t(sw,0))$ is convex). Therefore we show that (A.4) is non-increasing in sw_{t+1} as $sv(sw) * bm(\omega_{t+1}(sw,0))$ is non-decreasing in sw. □

Proof of Results in §2.4

Proof of Theorem 2.2. Following the procedure in Lemma 2.1, we can show that the separable property holds for the mechanized planting problem, that is

$$V_t^m(gsd_t, asd_t, cbm_t, sw_t) = cbm_t + gsd_t * Q_t(sw_t) + U_t^m(asd_t, sw_t), \tag{A.7}$$

where

$$\begin{aligned} Q_t(sw_t) =& p_t^r Q_{t-1}(\omega_t(sw_t, prec_t)) + (1-p_t^r)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t,0))), \\ Q_0(sw_0) =& 0, \end{aligned} \tag{A.8}$$

$$\begin{aligned} U_t^m(asd_t, sw_t) =& p_t^r U_{t-1}^m(asd_t, \omega_t(sw_t, prec_t)) + (1-p_t^r) * \max_{i_t=\{0,\ldots,\min\{m,asd_t\}\}} \big\{ i_t * sv(sw_t) * \\ & (bm(sw_t) + Q_{t-1}(\omega_t(sw_t,0))) + U_{t-1}^m(asd_t - i_t, \omega_t(sw_t,0)) \big\}, \\ U_0^m(asd_0, sw_0) =& 0. \end{aligned} \tag{A.9}$$

Note that the $Q_t(.)$ is independent on the capacity m as it is the expected biomass production of a single plant living in the ground with t periods to go. To show the optimality of the threshold policy, it is sufficient to show that $\Delta U_t^m(asd_t, sw_t) = U_t^m(asd_t+1, sw_t) - U_t^m(asd_t, sw_t)$ is non-increasing in asd_t. For $t=0$, obviously $\Delta U_t^m(asd_t, sw_t)$ is non-increasing in asd_t. Assume that $\Delta U_{t-1}^m(asd_{t-1}, sw_{t-1})$ is non-increasing in asd_{t-1}. Then

$$\begin{aligned} \Delta U_t^m(asd_t, sw_t) =& p_t^r \Delta U_{t-1}^m(asd_t, \omega_t(sw_t, prec_t)) + (1-p_t^r) * \\ & \begin{cases} \Delta U_{t-1}^m(asd_t - m, \omega_t(sw_t,0)) & asd_t \geq SD_t^m(sw_t) + m \\ \Delta U_{t-1}^m(asd_t, \omega_t(sw_t,0)) & asd_t < SD_t^m(sw_t) \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t,0))) & o.w. \end{cases}, \end{aligned}$$

$$\Delta U_t^m(asd_t+1, sw_t) = p_t^r \Delta U_{t-1}^m(asd_t+1, \omega_t(sw_t, prec_t)) + (1-p_t^r)*$$

$$\begin{cases} \Delta U_{t-1}^m(asd_t+1-m, \omega_t(sw_t,0)) & asd_t \geq SD_t^m(sw_t)+m-1 \\ \Delta U_{t-1}^m(asd_t+1, \omega_t(sw_t,0)) & asd_t < SD_t^m(sw_t)-1 \\ sv(sw_t)*(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))) & o.w. \end{cases}$$

If $asd_t \geq SD_t^m(sw_t)+m$, $\Delta U_t^m(asd_t+1, sw_t) \leq \Delta U_t^m(asd_t, sw_t)$ by assumption; If $asd_t \leq SD_t^m(sw_t)-2$, $\Delta U_t^m(asd_t+1, sw_t) \leq \Delta U_t^m(asd_t, sw_t)$ by assumption; If $asd_t = SD_t^m(sw_t)-1$, $\Delta U_t^m(asd_t+1, sw_t) - \Delta U_t^m(asd_t, sw_t)$ is

$$p_t^r(\Delta U_{t-1}^m(asd_t+1, \omega_t(sw_t, prec_t) - \Delta U_{t-1}^m(asd_t, \omega_t(sw_t, prec_t))))+$$
$$(1-p_t^r)(sv(sw_t)(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))) - \Delta U_{t-1}^m(asd_t, \omega_t(sw_t,0))),$$

which is non-positive because the optimal decision is not to plant when the seed amount asd_t+1 is $SD_t^m(sw_t)$; If $asd_t = SD_t^m(sw_t)+c-1$, $\Delta U_t^m(asd_t+1, sw_t) - \Delta U_t^m(asd_t, sw_t)$ is

$$p_t^r(\Delta U_{t-1}^m(asd_t+1, \omega_t(sw_t, prec_t) - \Delta U_{t-1}^m(asd_t, \omega_t(sw_t, prec_t))))+$$
$$(1-p_t^r)(\Delta U_{t-1}^m(asd_t+1-m, \omega_t(sw_t,0)) - sv(sw_t)(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0)))),$$

which is non-positive because the optimal planting decision is to plant when the seed amount asd_t+1 is $SD_t^m(sw_t)+m$. If $SD_t^m(sw_t) \leq asd_t \leq SD_t^m(sw_t)+m-2$, $\Delta U_t^m(asd_t+1, sw_t) - \Delta U_t^m(asd_t, sw_t)$ is zero and it is obviously non-positive.

Similar to Proposition 2.1, to show that $SD_t^m(sw_t)$ is non-increasing in sw_t, it is sufficient to show that

$$\Delta U_{t-1}^m(asd_t, \omega_t(sw_t,0)) - sv(sw_t)(bm(sw_t)+Q_{t-1}(\omega_t(sw_t,0))) \tag{A.10}$$

is non-increasing in sw_t. Following the same procedure as in the proof of Proposition 2.1, we can show that (A.10) is non-increasing in sw_{t+1}. □

Proof of Proposition 2.2. To show that $SD_t^m(sw_t) \leq SD_t^{m+1}(sw_t)$, it is sufficient to show that

$$\begin{aligned} &U_{t-1}^m(asd_t+1, \omega_t(sw_t,\beta)) - U_{t-1}^m(asd_t, \omega_t(sw_t,\beta)) \\ \leq &U_{t-1}^{m+1}(asd_t+1, \omega_t(sw_t,\beta)) - U_{t-1}^{m+1}(asd_t, \omega_t(sw_t,\beta)), \end{aligned} \tag{A.11}$$

$\beta > 0$. For $t = 1$, it is easy to show that (A.11) holds. We assume that (A.11) holds for $t-1$. Note that $SD_t^m(sw_t)$ is the largest integer that satisfies

$$\Delta U_{t-1}^m(asd_t - 1, \omega_t(sw_t, 0)) \geq sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) \tag{A.12}$$

and $\Delta U_{t-1}^m(asd_t - 1, \omega_t(sw_t, 0))$ is non-decreasing in m by assumption. Therefore at $asd_t = SD_t^m(sw_t)$, we have $\Delta U_{t-1}^{m+1}(asd_t - 1, \omega_t(sw_t, 0)) \geq sv(sw_t)((bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))))$. $SD_t^{m+1}(sw_t)$ is the largest integer that satisfies

$$\Delta U_{t-1}^{m+1}(asd_t - 1, \omega_t(sw_t, 0)) \geq sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) \tag{A.13}$$

and $\Delta U_{t-1}^{m+1}(asd_t - 1, \omega_t(sw_t, 0))$ satisfies (A.12) at $asd_t = SD_t^{m+1}(sw_t)$, $SD_t^{m+1}(sw_t) \geq SD_t^m(sw_t)$ by definition. When $SD_t^c(sw_t) = 0$, obviously $SD_t^{m+1}(sw_t) \geq SD_t^m(sw_t)$ holds.

We then show that $\Delta U_t^m(asd_{t+1}, \omega_{t+1}(sw_{t+1}, \beta))$ is non-decreasing in m. For a t-period problem with capacity m, we have

$$\Delta U_t^m(asd_{t+1}, \omega_{t+1}(sw_{t+1}, \beta)) = p_t^r \Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), prec_t)) + (1 - p_t^r) *$$
$$\begin{cases} \Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} \geq SD_t^m(\omega_{t+1}(sw_{t+1}, \beta)) + m \\ \Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} < SD_t^m(\omega_{t+1}(sw_{t+1}, \beta)) \\ sv(\omega_{t+1}(sw_{t+1}, \beta)) * (bm(\omega_{t+1}(sw_{t+1}, \beta)) & \\ +Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) & o.w. \end{cases}.$$

For a t-period problem with capacity $m + 1$, we have

$$\Delta U_t^{m+1}(asd_{t+1}, \omega_{t+1}(sw_{t+1}, \beta)) = p_t^r \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), prec_t)) + (1 - p_t^r) *$$
$$\begin{cases} \Delta U_{t-1}^{m+1}(asd_{t+1} - m - 1, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} > SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) + m \\ \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} < SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \\ sv(\omega_{t+1}(sw_{t+1}, \beta)) * (bm(\omega_{t+1}(sw_{t+1}, \beta)) & \\ +Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) & o.w. \end{cases}.$$

We define

$$\Lambda_{t-1}^{m}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) = \frac{\Delta U_t^m - p_t^r \Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), prec_t))}{1 - p_t^r}.$$

By assumption, $\Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), prec_t))$ is non-decreasing in m. It remains to show that $\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))$ is non-decreasing in m. We consider the following three cases of $SD_t^m(sw_t)$ as in the proof of Theorem 2.2.

1. $SD_t^m(\omega_{t+1}(sw_{t+1}, \beta)) \leq \max\{0, asd_{t+1} - m\}$,

$$\Lambda_{t-1}^{m}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) =$$
$$\begin{cases} \Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} \geq m \\ sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta))+ & \\ Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) & asd_{t+1} < m \end{cases},$$

if $SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \leq \max\{0, asd_{t+1} - m - 1\}$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) =$$
$$\begin{cases} \Delta U_{t-1}^{m+1}(asd_{t+1} - m - 1, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) & asd_{t+1} \geq m+1 \\ sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta))+ & \\ Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) & asd_{t+1} < m+1 \end{cases},$$

if $\max\{0, asd_{t+1} - m\} \leq SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \leq asd_{t+1}$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) =$$
$$sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))),$$

if $SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \geq asd_{t+1} + 1$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) = \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0));$$

2. $\max\{0, asd_{t+1}+1-m\} \leq SD_t^m(\omega_{t+1}(sw_{t+1},\beta)) \leq asd_{t+1}$,

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) =$$
$$sv(\omega_{t+1}(sw_{t+1},\beta))(bm(\omega_{t+1}(sw_{t+1},\beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},\beta),0)))$$
$$\textit{if } SD_t^{m+1}(\omega_{t+1}(sw_{t+1},\beta)) \geq asd_{t+1}+1,$$
$$\Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) = \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)),$$
$$\textit{if } SD_t^{m+1}(\omega_{t+1}(sw_{t+1},\beta)) \leq asd_{t+1}, \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) =$$
$$sv(\omega_{t+1}(sw_{t+1},\beta))(bm(\omega_{t+1}(sw_{t+1},\beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},\beta),0)));$$

3. $SD_t^m(\omega_{t+1}(sw_{t+1},\beta)) \geq asd_{t+1}+1$,

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) = \Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)),$$
$$\Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) = \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)).$$

For case 1, if $SD_t^m(\omega_{t+1}(sw_{t+1},\beta)) \leq \max\{0, asd_{t+1}-m\}$,

$$\Delta U_{t-1}^m(asd_{t+1}-m, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) \leq \Delta U_{t-1}^{m+1}(asd_{t+1}-m, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0))$$

by assumption and

$$\Delta U_{t-1}^{m+1}(asd_{t+1}-m, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) \leq \Delta U_{t-1}^{m+1}(asd_{t+1}-m-1, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0))$$

as $\Delta U_{t-1}^{m+1}(asd_{t+1}-m, \omega_t(\omega_{t+1},\beta),0)$ is non-increasing in asd_{t+1}. For $asd_{t+1}=m$, as

$$\Delta U_{t-1}^m(asd_{t+1}-m, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0))$$
$$\leq sv(\omega_{t+1}(sw_{t+1},\beta))(bm(\omega_{t+1}(sw_{t+1},\beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1},\beta),0)))$$

when $SD_t^m(\omega_{t+1}(sw_{t+1},\beta)) \leq \max\{0, asd_{t+1}-m\}$. Therefore

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1},\beta),0)).$$

If $\max\{0, asd_{t+1} - m\} \leq SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \leq asd_{t+1}$,

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))$$

as

$$\Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) -$$
$$sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) < 0$$

when $asd_{t+1} \geq m$. If $SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \geq asd_{t+1} + 1$,

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))$$

as

$$\Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) -$$
$$sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) > 0.$$

Hence for case 1, $\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))$. For case 2, as

$$\Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) -$$
$$sv(\omega_{t+1}(sw_{t+1}, \beta))(bm(\omega_{t+1}(sw_{t+1}, \beta)) + Q_{t-1}(\omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))) > 0$$

if $SD_t^{m+1}(\omega_{t+1}(sw_{t+1}, \beta)) \geq asd_{t+1} + 1$,

$$\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)).$$

For case 3, by assumption

$$\Delta U_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Delta U_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)).$$

Hence $\Lambda_{t-1}^m(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0)) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}, \omega_t(\omega_{t+1}(sw_{t+1}, \beta), 0))$. Therefore we show that $\Delta U_t^m(asd_{t+1}, \omega_{t+1}(sw_{t+1}, \beta))$ is non-decreasing in m, i.e. $U_t^m(asd_t, sw_t)$ is supermodular in (asd_t, m).

Finally to show that $SD_t^m(sw_t) + 1 \leq SD_t^{m+1}(sw_t)$ when $SD_t^m(sw_t) > 0$, we only need to consider the case where $SD_t^m(sw_t)$ is the largest $j \in \mathcal{N}^+$ that satisfies $\Delta U_{t-1}^m(j-1, \omega_t(sw_t, 0)) \geq sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))$ and $SD_t^{m+1}(sw_t)$ is the largest $j \in \mathcal{N}^+$ that satisfies

$$\Delta U_{t-1}^{m+1}(j-1, \omega_t(sw_t, 0)) \geq sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))).$$

If we can show that $\Delta U_{t-1}^{m+1}(SD_t^m(sw_t), \omega_t(sw_t, 0)) \geq sv(sw_t)(bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))$, then we can show that $SD_t^m(sw_t) + 1 \leq SD_t^{m+1}(sw_t)$. It is sufficient to show that

$$\Delta U_{t-1}^m(asd_t, sw_t) \leq \Delta U_{t-1}^{m+1}(asd_t + 1, sw_t)$$

for all $asd_t \in \mathcal{N}$. We show this by induction. For $t = 1$, it is obvious that $\Delta U_{t-1}^m(asd_t, sw_t) \leq \Delta U_{t-1}^{m+1}(asd_t + 1, sw_t)$. Assume that $\Delta U_{t-1}^m(asd_t, sw_t) \leq \Delta U_{t-1}^{m+1}(asd_t + 1, sw_t)$. For period t,

$$\Delta U_t^m(asd_{t+1}, sw_t) = p_t^r \Delta U_{t-1}^m(asd_{t+1}, \omega_t(sw_t, prec_t)) + (1 - p_t^r) * \begin{cases} \Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(sw_t, 0)) & asd_{t+1} \geq SD_t^m(sw_t) + m \\ \Delta U_{t-1}^m(asd_{t+1}, \omega_t(sw_t, 0)) & asd_{t+1} < SD_t^m(sw_t) \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) & o.w. \end{cases},$$

By assumption $\Delta U_{t-1}^m(asd_{t+1}, \omega_t(sw_t, prec_t)) \leq \Delta U_{t-1}^{m+1}(asd_{t+1} + 1, \omega_t(sw_t, prec_t))$. Then we need to show that $\Lambda_{t-1}^m(asd_{t+1}, sw_t) \leq \Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t)$. We consider the following cases. Note that $SD_t^m(sw_t) \leq SD_t^{m+1}(sw_t)$.

1. $SD_t^m(sw_t) \le \max\{0, asd_{t+1} - m\}$,

$$\Lambda_{t-1}^m(asd_{t+1}, sw_t) = \begin{cases} \Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(sw_t, 0)) & asd_{t+1} \ge m \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) & asd_{t+1} < m \end{cases},$$

if $SD_t^{m+1}(sw_t) \le \max\{0, asd_{t+1} - m\}$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = \begin{cases} \Delta U_{t-1}^{m+1}(asd_{t+1} + 1 - m, \omega_t(sw_t, 0)) & asd_{t+1} \ge m \\ sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) & asd_{t+1} < m \end{cases},$$

if $\max\{0, asd_{t+1} - m + 1\} \le SD_t^{m+1}(sw_t) \le asd_{t+1} + 1$,

$\Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))$,

if $SD_t^{m+1}(sw_t) \ge asd_{t+1} + 2, \Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = \Delta U_{t-1}^{m+1}(asd_{t+1} + 1, \omega_t(sw_t, 0))$;

2. $\max\{0, asd_{t+1} + 1 - m\} \le SD_t^m(sw_t) \le asd_{t+1}$,

$$\Lambda_{t-1}^m(asd_{t+1}, sw_t) = sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))),$$

if $SD_t^{m+1}(sw_t) \ge asd_{t+1} + 2$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = \Delta U_{t-1}^{m+1}(asd_{t+1} + 1, \omega_t(sw_t, 0)),$$

if $SD_t^{m+1}(sw_t) \le asd_{t+1} + 1$,

$$\Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)));$$

3. $SD_t^m(sw_t) \ge asd_{t+1} + 1$,

$$\Lambda_{t-1}^m(asd_{t+1}, sw_t) = \Delta U_{t-1}^m(asd_{t+1}, \omega_t(sw_t, 0)),$$

$$\Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) = \Delta U_{t-1}^{m+1}(asd_{t+1} + 1, \omega_t(sw_t, 0)).$$

For case 1, when $asd_{t+1} \le m - 1$, it is obvious that $\Lambda_{t-1}^m(asd_{t+1}, sw_t) \le \Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t)$ as $\Delta U_{t-1}^{m+1}(asd_{t+1} + 1, sw_t) \ge sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))$ when $SD_t^{m+1}(sw_t) \ge asd_{t+1} + 2$. When $asd_{t+1} \ge m$, as $\Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(sw_t, 0)) \le sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0)))$ when $SD_t^m(sw_t) \le \max\{0, asd_{t+1} - m\}$ and by assumption $\Delta U_{t-1}^m(asd_{t+1} - m, \omega_t(sw_t, 0)) \le \Delta U_{t-1}^{m+1}(asd_{t+1} - m, \omega_t(sw_t, 0))$, $\Lambda_{t-1}^m(asd_{t+1}, sw_t) \le \Lambda_{t-1}^{m+1}(asd_{t+1} + 1, sw_t)$. For case 2, as $\Delta U_{t-1}^{m+1}(asd_{t+1} +$

$1, \omega_t(sw_t, 0)) - sv(sw_t) * (bm(sw_t) + Q_{t-1}(\omega_t(sw_t, 0))) \geq 0$ when $SD_t^{m+1}(sw_t) \geq asd_{t+1} + 2$, $\Lambda_{t-1}^{m}(asd_{t+1}, sw_t) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}+1, sw_t)$. For case 3, $\Delta U_{t-1}^{m}(asd_{t+1}, \omega_t(sw_t, 0)) \leq \Delta U_{t-1}^{m+1}(asd_{t+1}+1, \omega_t(sw_t, 0))$ by assumption. Thus we show that $\Lambda_{t-1}^{m}(asd_{t+1}, sw_t) \leq \Lambda_{t-1}^{m+1}(asd_{t+1}+1, sw_t)$. □

Proof of Results in §2.5

We develop a heuristics algorithm for the mechanized planting problem by using a line segment to approximate the stairs as illustrated in Figure A.1. In order to apply Infinitesimal Perturbation Analysis (IPA, see Glasserman 1991), we loose the constraint of integer planting amount and set the action space with state (asd_t, sw_t) to be $[1, \min\{asd_t, c\}]$ ($[0,1]$ for $c = 1$). Let $(SPR_t, SPS_t)_{t=1,\ldots,N}$ denote a sample path of an N-period problem that describes the weather of each period and whether sunny season starts at that period if not rainy. In this section, as we generate sample paths in the analysis, the index is no longer in reverse order, i.e. index with t indicates the t^{th} period from the start of the planting horizon. Let $H_N^{SP}(\boldsymbol{thw}, \boldsymbol{ths})$ denote the sample path yield under weather sample path SP. The threshold of period t is the line segment between $(1, thw_t)$ ($(0, thw_t)$ for $c = 1$) and $(ths_t, 0)$, where the first argument in the parentheses is the seed amount and the second argument is the soil moisture. The partial derivative of the sample path yield function $H_N^{SP}(\boldsymbol{thw}, \boldsymbol{ths})$ with respect to any threshold $s_k, s = thw, ths; k = 1, \ldots, N$ can be derived based on the following iterative equations. We use sw_t^{SP} and asd_t^{SP} to denote the soil water content and amount of seeds available in period t on sample path SP, $t = 1, \ldots, N$ and $\frac{\partial asd_t^{SP}}{\partial s_k}$, $\frac{\partial H_t^{SP}(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k}$ to denote the path-wise derivatives of asd_t^{SP}, $H_t^{SP}(\boldsymbol{thw}, \boldsymbol{ths})$ with respect to threshold s_k, $s = thw, ths; t = 1, \ldots, N$.

$$\frac{\partial asd_{t+1}^{SP}}{\partial s_k} = \begin{cases} \frac{\partial asd_t^{SP}}{\partial s_k}\mathcal{I}_{\{asd_t^{SP} > c\}} & sw_t^{SP} \geq thw_t \ or \ thw_t * ths_t = 0; sw_t^{SP} < thw_t, \\ & \bar{y}_t^{SP} < ths_t - sw_t^{SP}(ths_t - minplant)/thw_t \\ \frac{\partial asd_{t+1}^{SP}}{\partial thw_k} = \frac{sw_t^{SP} ths_t}{(thw_t)^2}\mathcal{I}_{\{k \leq t\}}, & sw_t^{SP} < thw_t, \bar{y}_t^{SP} - (ths_t - minplant)* \\ \frac{\partial asd_{t+1}^{SP}}{\partial ths_k} = (1 - \frac{sw_t^{SP}}{thw_t})\mathcal{I}_{\{k \leq t\}} & (1 - sw_t^{SP}/thw_t) \in (minplant, c) \\ \frac{\partial asd_t^{SP}}{\partial s_k} & o.w. \end{cases}, \tag{A.14}$$

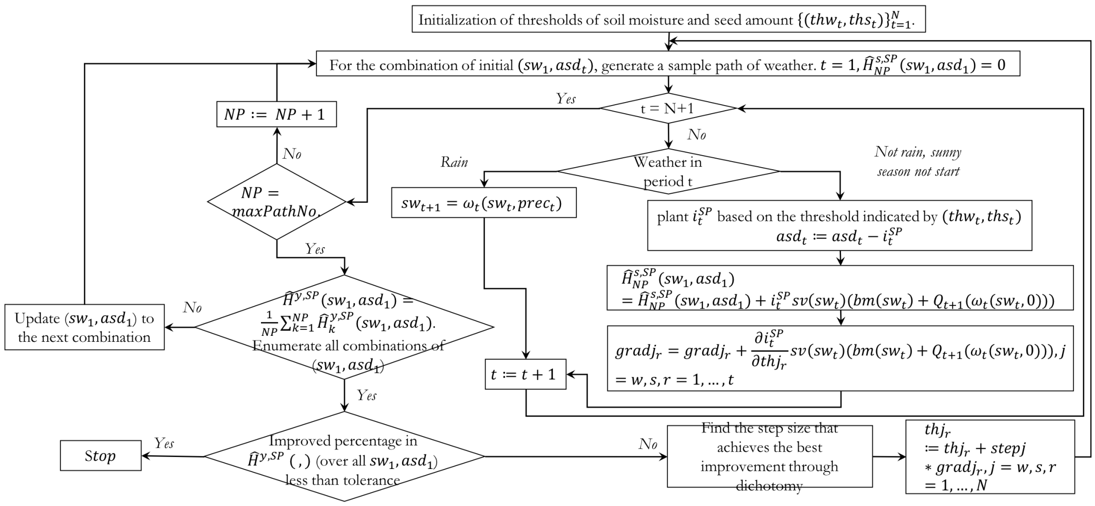


Figure A.1: HEURISTICS ALGORITHM BASED ON IPA AND SAMPLE PATH

where $minplant = 0$ if $c = 1$ and $minplant = 1$ otherwise. $\forall s = thw, ths$,

$$\begin{aligned}
\frac{\partial H_{t+1}^{SP}(\boldsymbol{thw}, \boldsymbol{ths})}{\partial s_k} =& \frac{\partial H_t^{SP}(\boldsymbol{thw}, \boldsymbol{ths})}{\partial s_k} + \frac{\partial (asd_t^{SP} - asd_{t+1}^{SP})}{\partial s_k} g^s(sw_t^{SP}) Q_t^{SP}(1,1), \\
\frac{\partial H_0^{SP}(\boldsymbol{thw}, \boldsymbol{ths})}{\partial s_k} =& 0,
\end{aligned} \tag{A.15}$$

Proposition A.1. *If $(SPR_t, SPS_t), t = 1, \ldots, N$ are independent and each sw_t has a density on $(0, \infty)$, $\forall t = 1, \ldots, N$, then the followings hold:*

1. *For $t = 1, \ldots, N$, each of asd_t^{SP} and $H_t^{SP}(\boldsymbol{thw}, \boldsymbol{ths})$ is, with probability one, differentiable at $s_k, s = thw, ths; k = 1, \ldots, N$. Moreover, the derivatives satisfy Equation* (A.14) *and Equation* (A.15)*;*

2. $\frac{\partial \mathbb{E}_{SP} asd_t^{SP}}{\partial s_k}$ *and* $\frac{\partial \mathbb{E}_{SP} H_t^{SP}(\boldsymbol{thw}, \boldsymbol{ths})}{\partial s_k}$ *exist and equal* $\mathbb{E}_{SP} \frac{\partial asd_t^{SP}}{\partial s_k}$ *and* $\mathbb{E}_{SP} \frac{\partial H_t^{SP}(\boldsymbol{thw}, \boldsymbol{ths})}{\partial s_k}$, $s = thw, ths; k = 1, \ldots, N$.

We use $\mathbb{E}_{SP}\{\partial H_N^{SP}(\boldsymbol{thw}, \boldsymbol{ths})/\partial s_k\}$ to estimate $\partial \mathbb{E}_{SP}\{H_N^{SP}(\boldsymbol{thw}, \boldsymbol{ths})\}/\partial s_k$ in our heuristics, $s = thw, ths; k = 1, \ldots, N$, as the former converges to the latter. The gradient search is used to improve the thresholds until the deviation falls in the pre-determined tolerance. Figure A.1 demonstrates the design of the algorithm.

Lemma A.1 ((Glasserman and Tayur (1995)))**.** *Let $\{X(s), s \in S\}$ be a random function with*

S and open subset of $\boldsymbol{S}$. Suppose that $\mathbb{E}[X(s)] < \infty$ for all $s \in S$. Suppose, further, that X is differentiable at $s_0 \in S$ with probability one, and that X is almost surely Lipschitz with modulus K_X satisfying $\mathbb{E}K_X < \infty$. Then $\mathbb{E}[X(s_0)]'$ exists and equals $\mathbb{E}[X'(s_0)]$.

See the proof of the lemma in Glasserman and Tayur (1995), Lemma 3.2. For $t = 1, \ldots, N$, given thresholds thw_t, ths_t and weather sample path SP, the amount of seeds available to plant, asd_t^{SP} evolves with the following iterative equation.

$$asd_t - asd_{t+1} = \begin{cases} asd - \min\{asd_t, c\} & sw_t \geq thw_t \text{ or } thw_t * ths_t = 0 \\ \min\{c, asd_t - & asd_t \geq (ths_t - minplant)* \\ (ths_t - minplant)(1 - sw_t/thw_t)\} & (1 - sw_t/thw_t), sw_t < thw_t \\ 0 & o.w. \end{cases} \tag{A.16}$$

Proof of Proposition A.1. Let i_t, $t = 1, \ldots, N$ denote the planting amount of period t and $U_t^x(1,1)$, $t = 1, \ldots, N$ is the biomass production of one unit of seed planted in period t. Then $i_t = asd_t - asd_{t+1}$. Let $H_N(\boldsymbol{thw}, \boldsymbol{ths})$ denote the expected yield with thresholds $\boldsymbol{thw}$ and $\boldsymbol{ths}$.

$$\begin{aligned}
&H_N(\boldsymbol{thw}, \boldsymbol{ths}) \\
=&\sum_{j=1}^{N}\Bigg(\sum_{t=1}^{j-1}\mathbb{E}(sv(sw_t)i_t(Q_{N-t+1}(sw_t) + bm(sw_t))\Bigg) \\
=&\sum_{j=1}^{N}\Bigg(\sum_{t=1}^{j-1}\mathbb{E}((asd_t - asd_{t+1})sv(sw_t)Q_{N-t+1}(sw_t) + bm(sw_t))\Bigg) \\
=&\sum_{j=1}^{N}\Bigg(\sum_{t=1}^{j-1}\mathbb{E}\Big(\min\left\{asd_t + (ths_t - minplant)\min\{\frac{sw_t}{thw_t} - 1, 0\}, c\right\}* \\
&*\mathcal{I}_{\{asd_t - (ths_t - minplant)(1 - \frac{sw_t}{thw_t}) > minplant\}} * sv(sw_t)(Q_{N-t+1}(sw_t) + bm(sw_t))\Big)\Bigg) \\
=&\sum_{j=1}^{N}\Bigg(\sum_{t=1}^{j-1}\mathbb{E}\Big(c\mathcal{I}_{\{asd_t + (ths_t - minplant)\min\{\frac{w_t}{thw_t} - 1, 0\} > m\}} + (asd_t + (ths_t - minplant)* \\
&(\frac{sw_t}{thw_t} - 1)\mathcal{I}_{\{\frac{sw_t}{thw_t} - 1 < 0\}}) * \mathcal{I}_{\{minplant < asd_t + (ths_t - minplant)\min\{\frac{sw_t}{thw_t} - 1, 0\} < c\}} * \\
&* sv(sw_t)(Q_{N-t+1}(sw_t) + bm(sw_t))\Big)\Bigg)
\end{aligned} \tag{A.17}$$

As the probability that the sunny season starts at period j is $(1-P^r_j)P^s_j$, the derivative of the final yield with respect to some threshold s_k, $s = thw, ths$, $t = 1,\dots,N$ is,

$$\frac{\partial H_N(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k}$$
$$=\sum_{t=1}^{j-1}\frac{\partial \mathbb{E} asd_t \mathcal{I}_{\{minplant<asd_t+(ths_t-minplant)\min\{\frac{sw_t}{thw_t}-1,0\}<c\}} sv(sw_t)(Q_{N-t+1}(sw_t)+bm(sw_t))}{\partial s_k}$$
$$+\sum_{t=1}^{j-1}\frac{\partial \mathbb{E}(ths_t-minplant)(\frac{sw_t}{thw_t}-1)\mathcal{I}_{\{\frac{sw_t}{thw_t}<1, minplant<asd_t+(ths_t-minplant)(\frac{sw_t}{thw_t}-1)<c\}} sv(w_t)}{\partial s_k}$$
$$*\left(Q_{N-t+1}(sw_t)+bm(sw_t))\right). \tag{A.18}$$

The state variables asd_t are differentiable up to the starting of sunny season if the quantities in Equation (A.16) are uniquely attained in all periods. As we assume continuous available seed amount asd_t in all periods in the algorithm and the action space is also continuous, the probability of having $\mod(asd_N, c) = 0$ converges zero. Hence with probability one, asd_t is differentiable with respect to s_k. As the amount of seeds available at the beginning of the planting horizon is finite and $sv(sw_t) \le 1$, the expectation of asd_t is finite. The operations min, max and addition are Lipschitz, so each $\bar{y}_t$ is a composition of Lipschitz functions and therefore Lipschitz. Therefore the results $\mathbb{E}\frac{\partial asd^{SP}_t}{\partial s_k} = \frac{\partial \mathbb{E} asd_t}{\partial s_k}$ and $\mathbb{E}\frac{\partial H^{SP}_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k} = \frac{\partial \mathbb{E} H_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k}$, $s = thw, ths; k, t = 1,\dots,N$ follow from Lemma A.1.

Let $i^{SP}_t(\boldsymbol{thw},\boldsymbol{ths})$ denote the planting decision on sample path SP under sample path SP with respect to threshold s_k, $s = thw, ths; k, t = 1,\dots,N$ and $i_t(\boldsymbol{thw},\boldsymbol{ths})$ the planting decision under these thresholds. To prove that the expectation of the sample path derivatives converge to the derivatives of the expectation, it is sufficient to show

$$\lim_{SP\to\infty}\frac{1}{SP}\sum_{l=1}^{SP}\frac{\partial H^l_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k} = \mathbb{E}\frac{\partial H^l_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k}$$
$$\Leftrightarrow \lim_{SP\to\infty}\frac{1}{SP}\sum_{l=1}^{SP}\frac{\partial i^l_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k} = \mathbb{E}\frac{\partial i_t(\boldsymbol{thw},\boldsymbol{ths})}{\partial s_k}.$$

Hence it is sufficient to show that

$$\lim_{SP\to\infty}\frac{1}{SP}\sum_{l=1}^{SP}\frac{\partial asd_t^l}{\partial s_k}=\mathbb{E}\frac{\partial asd_t}{\partial s_k}.$$

$$\begin{aligned}
&\lim_{SP\to\infty}\frac{1}{SP}\sum_{l=1}^{SP}\frac{\partial asd_{t+1}^l}{\partial s_k}\\
=&\lim_{SP\to\infty}\frac{1}{SP}\Big\{\sum_{l=1}^{SP}\frac{\partial asd_t^l}{\partial s_k}\Big\{\mathcal{I}_{\{sw_t^l<thw_t, asd_t^l<minplant+(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})\}}+\mathcal{I}_{\{sw_t^l\geq thw_t,\bar{y}_t^l>c\}}+\\
&+\mathcal{I}_{\{sw_t^l<thw_t, asd_t^l-(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})>c\}}\Big\}+\frac{\partial(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})}{\partial s_k}*\\
&*\mathcal{I}_{\{sw_t^l<thw_t, minplant<asd_t^l-(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})<c\}}\Big\}\\
=&\lim_{SP\to\infty}\frac{1}{SP}\Big\{\sum_{l=1}^{SP}\frac{\partial asd_t^l}{\partial s_k}\{\mathcal{I}_{\{sw_t^l\geq thw_t, asd_t^l>c\}}+\\
&+\mathcal{I}_{\{sw_t^l<thw_t, asd_t^l-(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})\notin(minplant,c)\}}\}+\\
&+\frac{\partial(ths_t-minplant)}{\partial s_k}\mathcal{I}_{\{sw_t^l<thw_t, minplant<asd_t^l-(ths_t-minplant)(1-\frac{sw_t^l}{thw_t})<c\}}-\\
&-\frac{\partial\frac{(ths_t-minplant)}{thw_t}}{\partial s_k}\mathbb{E}[sw_t\mathcal{I}_{\{sw_t<thw_t, minplant<asd_t^l-(ths_t-minplant)(1-\frac{w_t}{thw_t})<m\}}]\Big\}
\end{aligned}$$

Note that $\frac{\partial ths_t}{\partial s_k}=1$ if $s_k=ths_t$ and 0 otherwise; $\frac{\partial\frac{ths_t}{thw_t}}{\partial s_k}=\frac{1}{thw_t}$ if $s_k=ths_t$, $-\frac{ths_t}{(thw_t)^2}$ if $s_k=thw_t$ and 0 otherwise. Hence it remains to show the derivative of asd_t with respect to s_k. That is, the average of the sample path derivative of asd_{t+1} with respect to s_k converges to the expectation of the derivative of asd_{t+1} with respect to s_k when the number of sample path goes to infinity as long as this result holds for period t. We can use induction to show that $\lim_{SP\to\infty}\frac{1}{SP}\sum_{l=1}^{SP}\frac{\partial asd_t^l}{\partial s_k}=\mathbb{E}\frac{\partial asd_t}{\partial s_k}, \forall t$ as it is obvious that this result holds for $t=1$. $\square$

To show that the approximation of line segment does not lead to large deviation in yield values, we compare the yield values under the heuristics based on IPA approximation to the optimal values in a 13-period problem with varying parameters. In IPA approximation, we take the average of 500 sample path when evaluating the gradient as well as calculating the final yields. The optimal values are calculated through value iteration. The absolute relative deviation of the heuristics to the optimal value function ((optimal value − value from heuristics)/optimal value) is minimal.

Appendix II

In this section, we present appendix for Chapter 3.

Lemma B.2. *Let $\Delta_{\pi_r}(0) = 0$. There exists an $n^* \in \mathcal{N}^+$ that $\Delta_{\pi_r}(n) > \Delta_{\pi_r}(n-1)$ when $n \leq n^*$ and $\Delta_{\pi_r}(n) \geq \Delta_{\pi_r}(n+1)$ when $n \geq n^*$.*

Proof of Lemma B.2. Let $\bar{\Delta}_{\pi_r}(n) = \sqrt{n + \frac{a}{n} + c} + \frac{b}{n}$ where $a = \frac{c_s(h_r - h_w)}{c_w h_w}$, $c = \frac{c_s h_w + c_w(h_r - h_w)}{c_w h_w}$, $b = \frac{h_r k(\sigma\Delta_L + \sqrt{\mu}\Delta_\sigma)}{\sqrt{2c_w h_w \mu}}$. Let $\Delta_{\pi_r}(n)$ denote the value of Δ_{π_r} when the *inventory multiplier* is n, $n \in \mathcal{N}^+$. So $\theta\sqrt{2c_w h_w \mu}(\bar{\Delta}_{\pi_r}(1) - \bar{\Delta}_{\pi_r}(n)) + \theta c_h \mu = \Delta_{\pi_r}(n)$. For $n \in \{x : x > 1, x \in \mathcal{N}^+\}$,

$$\begin{aligned}
\Delta_{\pi_r}(n) > \Delta_{\pi_r}(n-1) &\Leftrightarrow \bar{\Delta}_{\pi_r}(n) < \bar{\Delta}_{\pi_r}(n-1) \\
&\Leftrightarrow \sqrt{n + \frac{a}{n} + c} + \frac{b}{n} < \sqrt{n - 1 + \frac{a}{n-1} + c} + \frac{b}{n-1} \\
&\Leftrightarrow \frac{1 - \frac{a}{n(n-1)}}{\sqrt{n + \frac{a}{n} + c} + \sqrt{n - 1 + \frac{a}{n-1} + c}} < \frac{b}{n(n-1)} \\
&\Leftrightarrow n - 1 - \frac{a}{n} < b\left(\sqrt{\frac{1}{n} + \frac{a}{n^3} + \frac{c}{n^2}} + \sqrt{\frac{n-1}{n^2} + \frac{a}{n^2(n-1)} + \frac{c}{n^2}}\right).
\end{aligned}$$

It is easy to show that the left hand side is increasing in n and the right hand side is decreasing in n with $n \in \mathcal{N}^+$. So there exists $n^* \in \mathcal{N}^+$ that $\bar{\Delta}_{\pi_r}(n) < \bar{\Delta}_{\pi_r}(n-1)$ if $n \leq n^*$ and $\bar{\Delta}_{\pi_r}(n) \leq \bar{\Delta}_{\pi_r}(n+1)$ otherwise ($n^* = 1$ if $\bar{\Delta}_{\pi_r}(n)$ is always decreasing in n). Note that $\bar{\Delta}_{\pi_r}(x), x \geq 1$ is continuous and differentiable at x when $x \in \mathcal{R}^+$. Hence we use $\bar{\Delta}_{\pi_r}(x), x \geq 1$ to analyze the behavior of $\bar{\Delta}_{\pi_r}(n), n \in \mathcal{N}^+$ in following analysis. □

Proof of Proposition 3.1. Lemma B.2 shows that $\Delta_\pi(n)$ is increasing in n when $n \leq n^o$ and is decreasing in n when $n \geq n^o$. n^o is the integer that maximizes $\Delta_\pi(n)$. In addition, $\Delta_\pi(1) = c_h\mu$ and $\Delta_\pi(n) \to -\infty$ as $n \to \infty$. Hence there exists some n^f that $\Delta_\pi(n) \geq 0$ if $1 \leq n \leq n^f$ and $\Delta_\pi(n) < 0$ if $n > n^f$, where n^f is the largest integer n satisfying $\Delta_\pi(n) \geq 0$. Obviously $n^f > n^o$.

For $n = n^f$, it is the largest integer that satisfies

$$\begin{aligned}&\sqrt{c_w h_w n^f + \frac{c_s(h_r - h_w)}{n^f} + c_s h_w + c_w(h_r - h_w)} - \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}}(1 - \frac{1}{n^f})\\ \leq&\sqrt{h_r(c_s + c_w)} + c_h\sqrt{\mu}.\end{aligned}$$

Note that the left hand side $\sqrt{c_w h_w n^f + \frac{c_s(h_r-h_w)}{n^f} + c_s h_w + c_w(h_r - h_w)} - \frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2}}(1 - \frac{1}{n^f})$ is decreasing in n at $n = n^f$ based on Lemma B.2. As this expression is decreasing in Δ_L or Δ_σ, n^f changes to $n^f + 1$ when the expression value is decreased enough. Rearranging the terms, it is easy to see that $\sqrt{c_w h_w n^f + \frac{c_s(h_r-h_w)}{n^f} + c_s h_w + c_w(h_r - h_w)}$ is increasing in h_w. If the increase is high enough, n^f changes to $n^f - 1$.

For $n = n^o$, it satisfies

$$\begin{aligned}&\sqrt{c_w h_w n^o + \frac{c_s(h_r - h_w)}{n^o} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}n^o}\\ \leq&\sqrt{c_w h_w(n^o + 1) + \frac{c_s(h_r - h_w)}{(n^o + 1)} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}(n^o + 1)},\\ &\sqrt{c_w h_w n^o + \frac{c_s(h_r - h_w)}{n^o} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}n^o}\\ \leq&\sqrt{c_w h_w(n^o - 1) + \frac{c_s(h_r - h_w)}{(n^o - 1)} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}(n^o - 1)}.\end{aligned}$$

When Δ_L or Δ_σ is increasing, $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2}(n^o-1)}$ is increasing the most, then $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2}n^o}$ and $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2}(n^o+1)}$. If Δ_L or Δ_σ is increasing enough,

$$\begin{aligned}&\sqrt{c_w h_w n^o + \frac{c_s(h_r - h_w)}{n^o} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}n^o}\\ \geq&\sqrt{c_w h_w(n^o + 1) + \frac{c_s(h_r - h_w)}{(n^o + 1)} + c_s h_w + c_w(h_r - h_w)} + \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2}(n^o + 1)}\end{aligned}$$

and n^o increases to $n^o + 1$. At $n = n^o$, $\sqrt{c_w h_w n + \frac{c_s(h_r-h_w)}{n} + c_s h_w + c_w(h_r - h_w)}$ is increasing in n. Note that the in the squared function, the coefficient of h_w is $c_w(n^o - 1) + c_s(1 - \frac{1}{n^o})$. As $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2}n}$ is decreasing in n, $c_w(n^o - 1) + c_s(1 - \frac{1}{n^o})$ is increasing in n when $n \geq n^o - 1$ or n^o minimizes $c_w(n^o - 1) + c_s(1 - \frac{1}{n^o})$. For the former case, when the increase of h_w is sufficiently large, n^o changes to $n^o - 1$. For the latter case, n^o does not change when h_w is increasing. □

Proof of Proposition 3.2. From Proposition 3.1, to have some $n > 1$ that $\Delta_\pi(n) > 0$, the necessary and sufficient condition is $\Delta_\pi(2) \geq 0$.

$$\Delta_\pi(2) \geq 0 \Leftrightarrow \frac{h_w}{h_r} \leq \left(\sqrt{1 + \frac{c_s}{2c_w + c_s}} + \frac{k\sqrt{h_r}(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu/h_r}}{\sqrt{2c_w + c_s}}\right)^2 - 1. \qquad \text{(B.19)}$$

The right hand side of Equation (B.19) increases with c_s when

$$c_s < \left(\frac{2\sqrt{2}c_w}{k\sqrt{h_r}(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu/h_r}}\right)^2 - c_w$$

and decreases with c_s otherwise. □

Proof of Proposition 3.3. When $\theta \in (0, 1)$, we use $\Delta_w(x), x > 1, x \in \mathcal{R}$ to analyze the behavior of $\Delta_w(n), n > 0, n \in \mathcal{N}^+$.

$$\begin{aligned}\Delta_w(x) =&(1-\theta)\Bigg(c_h + c_w\left(\sqrt{\frac{2h_r}{(c_w + c_s)\mu}} - \sqrt{\frac{2((x-1)h_w + h_r)x}{(c_w x + c_s)\mu}}\right) + \\ &+ \frac{1-2\theta}{1-\theta}c_s\left(\sqrt{\frac{h_r}{2(c_w + c_s)\mu}} - \sqrt{\frac{(x-1)h_w + h_r}{2(c_w x + c_s)x\mu}}\right) + h_r k\frac{(x-1)(\sigma\Delta_L + \sqrt{\mu}\Delta_\sigma)}{x\mu}\Bigg).\end{aligned}$$

$\Delta_w(x)$ is continuous and differentiable in x. If we can show that there exists $x^w(\theta)$ such that $\Delta_w(x)$ is increasing in x when $x < x^w(\theta)$ and decreasing in x otherwise, then we can show that there exists $n^w(\theta)$ that $\Delta_w(n)$ is increasing in n when $n \leq n^w(\theta)$ and decreasing in n otherwise. Letting $r_c = \frac{c_s}{c_w}$ and $r_h = \frac{h_r}{h_w}$, we have

$$\begin{aligned}\frac{d\Delta_w(x)}{dx} =&\frac{1-\theta}{x^2}\sqrt{\frac{2c_w h_w}{\mu}}\Bigg(\frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2c_w h_w}} \\ &- \frac{x^3 + 2r_c x^2 + (r_h - 1)r_c x - \frac{1-2\theta}{2(1-\theta)}r_c(x^2 + 2(r_h - 1)x + (r_h - 1)r_c)}{2\sqrt{(x + r_c)(x - 1 + r_h)/x}(x + r_c)}\Bigg).\end{aligned}$$

We want to show that $\frac{d\Delta_w(x)}{dx} > 0$ for n smaller than some $n^w(\theta)$ and $\frac{d\Delta_w(x)}{dx} < 0$ otherwise.

$$\frac{d\Delta_w(x)}{dx} > 0 \Leftrightarrow$$
$$\frac{x^3 + 2r_c x^2 + (r_h - 1)r_c x - \frac{1-2\theta}{2(1-\theta)} r_c \left(x^2 + 2(r_h - 1)x + (r_h - 1)r_c\right)}{2\sqrt{(x - 1 + r_h)(x + r_c)/x}(x + r_c)} < \frac{h_r k(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma)}{\sqrt{2c_w h_w}} \tag{B.20}$$

Note that $\alpha = -\frac{1-2\theta}{2(1-\theta)}$ is increasing in θ and $\alpha \geq -1/2$. To show our results, we discuss how $h(x,\alpha) := \frac{x^3+(2+\alpha)r_c x^2+(1+2\alpha)(r_h-1)r_c x+\alpha r_c^2(r_h-1)}{(x+r_c)\sqrt{(x+r_c)(x+r_h-1)/x}}$ changes with x, $x > 1$, $\alpha \geq -1/2$.

$$\begin{aligned}\frac{\partial h(x,\alpha)}{\partial x} =& \frac{x - 1 + r_h}{2x^3((r_c + x)(x - 1 + r_h)/x)^{\frac{5}{2}}}\Big(\alpha r_c\big[x^4 + r_c^2(r_h - 1)^2 + 2r_c x\big(2x^2 + 3(r_h - 1)x + \\ &+ 2(r_h - 1)^2\big)\big] + x\big[x^3\left(3x + 4(r_h - 1)\right) + 2r_c x^2\left(5(r_h - 1) + 4x\right) + r_c^2\big(8x^2 + \\ &+ 12(r_h - 1)x + 3(r_h - 1)^2\big)\big]\Big).\end{aligned}$$

Since $x > 1$, $\frac{x-1+r_h}{2x^3((r_c+x)(x-1+r_h)/x)^{\frac{5}{2}}} > 0$. To show how $h(x,\alpha)$ changes with x, it remains to show whether

$$\begin{aligned}h_1(x,\alpha) =& \alpha r_c\left[x^4 + r_c^2(r_h - 1)^2 + 2r_c x\left(2x^2 + 3(r_h - 1)x + 2(r_h - 1)^2\right)\right] + x\big[x^3\big(3x + \\ &+ 4(r_h - 1)\big) + 2r_c x^2\left(5(r_h - 1) + 4x\right) + r_c^2\left(8x^2 + 12(r_h - 1)x + 3(r_h - 1)^2\right)\big]\end{aligned}$$

is positive or negative with $x > 1, \alpha \geq -1/2$.

$$\begin{aligned}h_{11}(x,\alpha) =& \frac{\partial h_1(x,\alpha)}{\partial x} = x^3(15x + 16r_h - 16) + 2r_c x^2\big[15r_h + 2(8 + \alpha)x - 15\big] + r_c^2\Big(4\alpha\big[1 + \\ &+ r_h^2 - 3x + 3x^2 + r_h(-2 + 3x)\big] + 3\big[1 + r_h^2 - 8x + 8x^2 + r_h(-2 + 8x)\big]\Big) \\ h_{111}(x,\alpha) =& \frac{\partial h_{11}(x,\alpha)}{\partial x} = 15x^3 + 3x^2(15x + 16r_h - 16) + 4(8 + \alpha)r_c x^2 + 4r_c x\big[15r_h - 15 + \\ &+ 2(8 + \alpha)x\big] + r_c^2\big[4\alpha(6x + 3r_h - 3) + 3(16x + 8r_h - 8)\big].\end{aligned}$$

With $r_h, r_c > 0$, $\alpha > -1/2$ and $x > 1$, it is easy to show that $h_{111}(x,\alpha) > 0$. So $h_{11}(x,\alpha)$ is increasing in x. We have

$$h_{11}(1,\alpha) = 16r_h + 2r_c(16 + 2\alpha + 15r_h - 15) + r_c^2\big[4\alpha(1 + r_h^2 + r_h) + 3(1 + r_h^2 + 6r_h)\big] > 0.$$

So with $r_h, r_c > 0$, $\alpha > -1/2$ and $x > 1$, $h_{11}(x,\alpha) > 0$. This means that $h_1(x,\alpha)$ is increasing in x.

$$h_1(1,\alpha) = 4(r_h-1)+3+2r_c(5(r_h-1)+4)+r_c^2(3(r_h-1)^2+12(r_h-1)+8)+ \\ +\alpha r_c\big[r_c^2(r_h-1)^2+1+2r_c(2(r_h-1)^2+3(r_h-1)+2)\big].$$

We take a look at $h_1(x,\alpha)$. Since

$$x^4+r_c^2(r_h-1)^2+2r_cx\left(2x^2+3(r_h-1)x+2(r_h-1)^2\right)>0$$

always holds, $\min_\alpha h_1(x,\alpha)$ is attained at $\alpha=-1/2$. Obviously as α increases, $h_1(x,\alpha)$ increases for all $x>1$ and given r_c, r_h, when α is sufficiently large, $\frac{\partial h(x,\alpha)}{\partial x} > 0, \forall x > 1$. In this case, since $h(x,\alpha)$ is increasing in x, there exists some value x^w such that $\frac{d\Delta_w(x)}{dx} > 0$ when $x < x^w$ and $\frac{d\Delta_w(x)}{dx} < 0$ otherwise. Since

$$x\left[x^3\left(3x+4(r_h-1)\right)+2r_cx^2\left(5(r_h-1)+4x\right)+r_c^2\left(8x^2+12(r_h-1)x+3(r_h-1)^2\right)\right]\to\infty$$

as $x\to\infty$, the value of α above which $\frac{\partial h(x,\alpha)}{\partial x} > 0, \forall x > 1$ always holds is bounded. Hence it is sufficient to analyze $h_1(x,\alpha)$ with $x=1$ and $\alpha=-1/2$.

$$h_1(1,-1/2)=(r_c^2-0.5r_c^3)(r_h-1)^2+(4+10r_c+9r_c^2)(r_h-1)+(3+7.5r_c+6r_c^2).$$

If $h_1(1,-1/2)>0$, then $h(x,\alpha)$ is increasing in x. Hence we can show that there exists an x^w that $\frac{d\Delta_w(x)}{dx} > 0$ when $1 < x < x^w$ and $\frac{d\Delta_w(x)}{dx} < 0$ otherwise. If $h_1(1,-1/2) < 0$, as x increases, there exists an x^{ww} that $\frac{\partial h(x,\alpha)}{\partial x} < 0$ when $1 < x < x^{ww}$ and $\frac{\partial h(x,\alpha)}{\partial x} > 0$ otherwise. $h(x,\alpha)$ first is decreasing in x and then is increasing in x. Based on the value of $h(x,-1/2)$, three cases of $\Delta_w(x)$ are possible, illustrated in Figure B.2. For $x\in(1,\infty)$,

i. if $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2c_w h_w}} < \min_{x>1} h(x,-1/2)$, $\Delta_w(x)$ is decreasing in x, as illustrated in the upper left panel of Figure B.2;

ii. if $\frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2c_w h_w}} > h(1,-1/2)$, $\Delta_w(x)$ first increases then is decreasing in x, as illustrated in the upper right panel of Figure B.2;

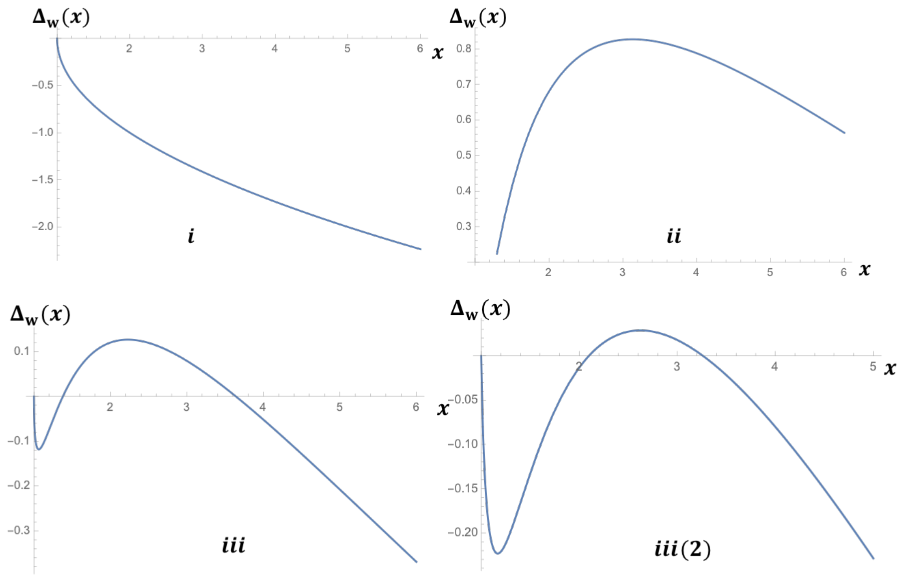


Figure B.2: THREE CASES OF $\Delta_w(x)$ WITH $h_1(1,-1/2)<0$, $c_h=0$

iii. if $\min_{x>1} h(x,-1/2) < \frac{h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}{\sqrt{2c_w h_w}} < h(1,-1/2)$, $\Delta_w(x)$ is first decreasing then increasing then decreasing in x, as illustrated in the bottom panels of Figure B.2.

It easy to see that for all cases, either $\Delta_w(x) < \Delta_w(1)$ for all $x>1$ or there exists $x^w(\theta)$ such that $\Delta_w(x) < \Delta_w(1)$ for all $x > \Delta_w(x)$ and there exists $x < x^w(\theta)$ such that $\Delta_w(x) > \Delta_w(1)$. Note that case *iii* will never happen with $r_h \geq 1$, as $h_1(1,-1/2) > 0$ in this case. Since $\Delta_w(n,\theta) \to -\infty$ as $n \to \infty$, based on the above analysis there exists a threshold $n^w(\theta)$ that $\Delta_w(n) < 0$ when $n > n^w(\theta)$ and $\exists n < n^w(\theta)$ s.t. $\Delta_w(n) \geq 0$. Furthermore, if $r_h \leq 1$, $\Delta_w(n) \geq 0$ for all $n < n^w(\theta)$.

To show that there exists some $n > 1$ such that $\Delta_w(n) > 0$, it is sufficient to show that $\Delta_w(2) > 0$. When $\Delta_w(2) > 0$, we have

$$h_w < h_r\Big(\Big(\big(1 - \frac{2(1-\theta)c_w}{4(1-\theta)c_w + (1-2\theta)c_s}\big)\sqrt{\frac{2(2c_w+c_s)}{c_w+c_s}} + \\ + \frac{k\sqrt{h_r}(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma) + 2c_h\sqrt{\mu/h_r}}{(4c_w + \frac{(1-2\theta)}{1-\theta}c_s)/\sqrt{2c_w+c_s}}\Big)^2 - 1\Big)$$

We then show that $n^w(\theta)$ is decreasing in θ. By rearranging the terms of $\Delta_w(n)$, we know that $\Delta_w(n)$ is decreasing in θ when $\Delta_\pi(n) \geq 0$ and increasing in θ otherwise. As $n^w(\theta) \leq n^f$ when $h_w \leq h_r$, for all $n \leq n^w$, $\Delta_w(n)$ is decreasing in θ. As a result, $n^w(\theta)$ is decreasing in θ in this case.

From result 1 in Proposition 3.3, if $\Delta_w(2) < 0$ for some θ, $\Delta_w(n) < 0$, $\forall n$.

$$\begin{aligned}
&\Delta_w(2) < 0 \\
\Leftrightarrow &\frac{h_r k \sigma \Delta_L}{2\sqrt{2dc_w h_w}} < \left(\sqrt{\frac{r_c(r_c - r_h + 1)}{2 + r_c} + r_h - r_c + 1} - \sqrt{\frac{r_h}{1 + r_c}}\right) + \\
&+ \frac{(1-2\theta)r_c}{2(1-\theta)}\left(\sqrt{\frac{r_h + 1}{2(2 + r_c)}} - \sqrt{\frac{r_h}{1 + r_c}}\right).
\end{aligned}$$

As $\sqrt{\frac{r_h+1}{2(2+r_c)}} - \sqrt{\frac{r_h}{1+r_c}} < 0$ and $\frac{(1-2\theta)r_c}{2(1-\theta)}$ is decreasing in θ, the right hand side is increasing in θ. So there exists some θ_w such that when $\theta \geq \theta_w$, $\Delta_w(n) < 0$, $\forall n$.

To show the last result, we need to show that, for all $\theta \in (0,1)$,

$$\begin{aligned}
&\left(\sqrt{n + \frac{r_c(r_c - r_h + 1)}{n + r_c} + r_h - r_c - 1} - \sqrt{\frac{r_h}{1 + r_c}}\right) + \frac{(1-2\theta)r_c}{2(1-\theta)}\left(\sqrt{\frac{n + r_h - 1}{n(n + r_c)}} - \sqrt{\frac{r_h}{1 + r_c}}\right) \\
\leq &\sqrt{n + \frac{r_c(r_h - 1)}{n} + r_c + r_h - 1} - \sqrt{r_h(r_c + 1)},
\end{aligned} \tag{B.21}$$

where $r_c = \frac{c_s}{c_w}$ and $r_h = \frac{h_r}{h_w} > 1$. Since $\sqrt{\frac{n+r_h-1}{n(n+r_c)}} - \sqrt{\frac{r_h}{1+r_c}} \leq 0$ and $\frac{(1-2\theta)}{2(1-\theta)} < \frac{1}{2}$, it is sufficient to show that inequality B.21 holds when $\frac{(1-2\theta)}{2(1-\theta)} = \frac{1}{2}$,

$$\begin{aligned}
&\sqrt{n + \frac{r_c(r_h - 1)}{n} + r_c + r_h - 1} - \sqrt{n + \frac{r_c(r_c - r_h + 1)}{n + r_c} + r_h - r_c - 1} - \frac{r_c}{2}\sqrt{\frac{n + r_h - 1}{n(n + r_c)}} \\
\leq &\sqrt{r_h(r_c + 1)} - \sqrt{\frac{r_h}{1 + r_c}} - \frac{r_c}{2}\sqrt{\frac{r_h}{1 + r_c}}
\end{aligned} \tag{B.22}$$

When $n = 1$, the left hand side of inequality B.22 equals to the right hand side. Therefore it is sufficient to show that

$$\begin{aligned}
&\delta(x) \\
=&\sqrt{x + \frac{r_c(r_h - 1)}{x} + r_c + r_h - 1} - \sqrt{x + \frac{r_c(r_c - r_h + 1)}{x + r_c} + r_h - r_c - 1} - \frac{r_c}{2}\sqrt{\frac{x + r_h - 1}{x(x + r_c)}}
\end{aligned}$$

is decreasing in x.

$$
\begin{aligned}
\delta'(x) < 0 \Leftrightarrow & \frac{1 - \frac{r_c(r_h-1)}{x^2}}{\sqrt{x + \frac{r_c(r_h-1)}{x} + r_c + r_h - 1}} < \frac{1 - \frac{r_c(r_c-r_h+1)}{(x+r_c)^2} - \frac{x^2+(2x+r_c)(r_h-1)}{x(x+r_c)^2} * \frac{r_c}{2}}{\sqrt{x + \frac{r_c(r_c-r_h+1)}{x+r_c} + r_h - r_c - 1}} \\
\Leftrightarrow & \frac{\sqrt{x + \frac{r_c(r_c-r_h+1)}{x+r_c} + r_h - r_c - 1}}{\sqrt{x + \frac{r_c(r_h-1)}{x} + r_c + r_h - 1}} < \frac{1 - \frac{r_c(r_c-r_h+1)}{(x+r_c)^2} - \frac{x^2+(2x+r_c)(r_h-1)}{x(x+r_c)^2} * \frac{r_c}{2}}{1 - \frac{r_c(r_h-1)}{x^2}} \\
\Leftrightarrow & \frac{x + r_c}{x} < \frac{x^2 + 3xr_c/2 - r_c^2(r_h-1)/(2x)}{x^2 - r_c(r_h-1)} \\
\Leftrightarrow & x^2 + 2x(r_h-1) + r_c(r_h-1) > 0
\end{aligned}
$$

Therefore, Equation (B.21) always holds. As a result, when $h_w \leq h_r$, $\Delta_w(n) > 0$ as long as $\Delta_\pi(n) > 0$. □

Lemma B.3. *Let* $f(x) = \sqrt{1 - \frac{1}{x^2}} + \frac{ss}{x}, x > 1, ss > 0$. *Then* $\exists\ x^o$ *s.t.* $f(x)$ *is increasing in* x *when* $x \leq x^o$ *and is decreasing in* x *otherwise.*

Proof of Lemma B.3. We take the derivative of f, $f(x)' = \frac{1}{\sqrt{x^2-1}} - \frac{\sqrt{x^2-1}+ss}{x^2}$. Then $f(x)' \geq 0$ when $x \leq \sqrt{1 + \frac{1}{ss^2}}$ and $f(x)' > 0$ otherwise. □

Proof of Proposition 3.4. Based on the proof of Proposition 3.2, $\exists\ n^{tr}$ s.t. $\Delta_\pi^{tr} \geq 0$ when $n \leq n^{tr}$ and $\Delta_\pi^{tr} < 0$ when $n \geq n^{tr}$. Therefore, the sufficient and necessary condition that $max_{n\in\mathcal{N}^+}\Delta_\pi^{tr} > 0$ is that $\Delta_\pi^{tr} \geq 0$ at $n = 2$. Plugging in $n = 2$ and we can get $r^{h,tr}$.

Note that $r^{h,tr} = r^h$ if $c_s = 0$. Based on Lemma B.3, $r^{h,tr}$ is increasing in c_s when c_s is sufficiently small or $\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma$ is sufficiently large. To investigate the impact of $\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma$ on the comparison of r^h and $r^{h,tr}$, we compare the first order derivatives of the following two functions:

$$
r_1^{tr} = \frac{kh_r(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu}}{2\sqrt{2h_r c_w}}, \quad r_1 = \frac{kh_r(\sigma\Delta_L/\sqrt{\mu} + \Delta_\sigma) + 2c_h\sqrt{\mu}}{2\sqrt{h_r(2c_w + c_s)}}.
$$

Obviously $\frac{\partial r_1^{tr}}{\partial(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)} \geq \frac{\partial r_1}{\partial(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)}$. r_1^{tr} is increasing faster in Δ_L than r_1 and both are increase to infinity as Δ_L is going to infinity. Therefore there exists Δ_L^{tr} s.t. $r_1^{tr} + 1 \leq r_1 + \sqrt{1 + \frac{c_w}{2c_w+c_s}}$ when $\Delta_L \geq \Delta_L^{tr}$ and $r_1^{tr} + 1 \leq r_1 + \sqrt{1 + \frac{c_w}{2c_w+c_s}}$ otherwise. As comparing $r^{h,tr}$ and r^h

is equivalent to comparing $r_1^{tr}+1$ and $r_1+\sqrt{1+\frac{c_w}{2c_w+c_s}}$, this leads to the first result. Similarly the second result in Proposition 3.4 holds as r_1^{tr} is not decreasing in c_s while r_1 is decreasing in c_s. □

Proof of Proposition 3.5. Let x^{tr} denote the maximizer of $\Pi_r^{E,tr} - \Pi_r^{D,tr}$ on $(0,\infty)$ and x^* the maximizer of Δ_π. Equivalently, x^{tr} minimizes $f^{tr} = \sqrt{2c_w((x-1)h_w+h_r)}+h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)/x$ and x^* minimizes $f^E = \sqrt{2(c_w+c_s/x)((x-1)h_w+h_r)}+h_r k(\sigma\Delta_L/\sqrt{\mu}+\Delta_\sigma)/x$. Obviously f^{tr} is decreasing in x when x is lower than some threshold and is increasing in x otherwise. If $x^{tr}\leq 1$ or x^{tr} is very close to 1 such that $n^{tr}=1$, obviously $n^o \geq n^{tr}$. If $x^{tr}>1$ and $n^{tr}>1$, the derivative of $df^{tr}/dx = 0$ at $x = x^{tr}$. Note that

$$
\begin{aligned}
f^E - f^{tr} =& \sqrt{2(c_w+c_s/x)((x-1)h_w+h_r)} - \sqrt{2c_w((x-1)h_w+h_r)}\\
=&\frac{2c_s(h_w+\frac{h_r-h_w}{x})}{\sqrt{2(c_w+c_s/x)((x-1)h_w+h_r)}+\sqrt{2c_w((x-1)h_w+h_r)}}\\
=&\frac{c_s\sqrt{2*((x-1)h_w+h_r)/xc_w}}{\sqrt{(x+c_s/c_w)}+\sqrt{x}}
\end{aligned}
$$

is non-negative and decreasing in x. As f^E is decreasing faster in x than f^{tr}, f^E is still decreasing when f^{tr} reaches its minimum point, i.e. $x^{tr} < x^*$ as long as $h_w/h_r \leq r^h$. □

Lemma B.4. *Let $f(x)$ be an increasing function of x, $x \in \mathcal{N}^+$ and h, b be positive constant. Then $\sqrt{hf(x)+b}$ is supermodular in (x,h), i.e.*

$$\frac{\partial(\sqrt{hf(x+1)+b}-\sqrt{hf(x)+b})}{\partial h} > 0.$$

Proof of Lemma B.4. We directly take the derivative of $\sqrt{hf(x+1)+b}-\sqrt{hf(x)+b}$ with respect to h, we get

$$\frac{\partial(\sqrt{hf(x+1)+b}-\sqrt{hf(x)+b})}{\partial h} = \frac{1}{2}\left(\frac{f(x+1)}{h+\frac{b}{f(x+1)}} - \frac{f(x)}{h+\frac{b}{f(x)}}\right) > 0.$$

The above expression is positive because $f(x)/(h+b/f(x))$ is positive and increases with x. □

Proof of Proposition 3.6. We first prove the result about the bargaining power. Without generality, we assume that $c_h = 0$. Under traditional contract, the retailer's optimal profit is

$$\pi_r^{*t} = \theta(p-c) - \sqrt{2c_w\mu((n^t-1)h_w + h_r)} - h_r k(\frac{1}{n^t}(\sigma l_s + \sigma_s\sqrt{\mu}) + \frac{n^t-1}{n^t}(\sigma l_w + \sigma_w\sqrt{\mu})).$$

When case D (direct shipping to the developed country) is preferred over case E (using the emerging-country warehouse), $n^t = 1$. Thus we use $n^t = 1$ to represent the case where the emerging-country warehouse is not preferred under traditional contract. Under the contract including logistics operations cost during negotiation, the retailer's optimal profit is

$$\begin{aligned}\pi_r^* =& \theta(p - c - \sqrt{\frac{2(c_w n^* + c_s)((n^*-1)h_w + h_r)}{n^*}} + h_r k(\frac{1}{n^*}(\sigma l_s + \sigma_s\sqrt{\mu}) + \\ &+ \frac{n^*-1}{n^*}(\sigma l_w + \sigma_w\sqrt{\mu}))).\end{aligned}$$

Similarly we use $n^* = 1$ to represent the case where using the emerging-country warehouse is not preferred under the contract including logistics operations cost. As θ does not influence n^* or n^t, we see that

$$\begin{aligned}\pi_r^* \geq \pi_r^{*t} \Leftrightarrow \theta \leq & \frac{\sqrt{2c_w\mu((n^t-1)h_w + h_r)} + h_r k(\frac{1}{n^t}(\sigma l_s + \sigma_s\sqrt{\mu}) + \frac{n^t-1}{n^t}(\sigma l_w + \sigma_w\sqrt{\mu}))}{\sqrt{\frac{2(c_w n^* + c_s)((n^*-1)h_w + h_r)}{n^*}} + h_r k(\frac{1}{n^*}(\sigma l_s + \sigma_s\sqrt{\mu}) + \frac{n^*-1}{n^*}(\sigma l_w + \sigma_w\sqrt{\mu}))} \\ & := \theta^t(h_w).\end{aligned}$$

Thus when $\theta \leq \theta^t(h_w)$, $\pi_r^* \geq \pi_r^{*t}$.

Next we show the result about the warehouse holding cost. Let C_r^{Log} denote the minimized cost of the retailer's logistics operations under traditional contract and C_{sc}^{Log} denote the minimized cost of supply chain logistics operations under contract including logistics operations cost, where

$$\begin{aligned}C_r^{Log} =& \sqrt{2c_w\mu((n^t-1)h_w + h_r)} + h_r k(\frac{1}{n^t}(\sigma\Delta_L + \Delta_\sigma\sqrt{\mu}) + (\sigma l_w + \sigma_w\sqrt{\mu})) \\ C_{sc}^{Log} =& \sqrt{\frac{2\mu(c_w n^* + c_s)((n^*-1)h_w + h_r)}{n^*}} + h_r k(\frac{1}{n^*}(\sigma\Delta_L + \Delta_\sigma\sqrt{\mu}) + (\sigma l_w + \sigma_w\sqrt{\mu})).\end{aligned}$$

To show that there exists $h_w^t(\theta)$ such that when $h_w \leq h_w^t(\theta)$, $\pi_r^* \geq \pi_r^{*t}$, we first consider the case where the emerging-country warehouse is preferred under both contracts. Then it is sufficient to show that as h_w increases, C_{sc}^{Log} increases faster than C_r^{Log}. If $n^* > 1$, when h_w increases to

$h_w + \Delta h$, $\Delta h > 0$, we discuss three cases:

1. both n^* and n^t does not change;

2. n^* decreases to $n^* - 1$, n^t does not change;

3. n^t increases to $n^t - 1$, n^* does not change.

We only consider sufficiently small Δh such that the case where both n^* and n^t decrease by at least one cannot occur. Note that $n^* \geq n^t$ always holds as h_w changes. For the first case where both n^* and n^t does not change,

$$
\begin{aligned}
&C_{sc}^{Log}(h_w + \Delta h) - C_{sc}^{Log}(h_w) \geq C_r^{Log}(h_w + \Delta h) - C_r^{Log}(h_w) \\
\Leftrightarrow &\sqrt{(c_w + c_s/n^*)}\left(\sqrt{(n^* - 1)(h_w + \Delta h) + h_r} - \sqrt{(n^* - 1)h_w + h_r}\right) \\
\geq &\sqrt{c_w}\left(\sqrt{(n^t - 1)(h_w + \Delta h) + h_r} - \sqrt{(n^t - 1)h_w + h_r}\right)
\end{aligned}
$$

Since $(c_w + c_s/n)(n - 1)$ is an increasing function of n, based on Lemma B.4, we have

$$
\begin{aligned}
&\sqrt{(c_w + \frac{c_s}{n^*})(n^* - 1)(h_w + \Delta h) + (c_w + \frac{c_s}{n^*})h_r} - \sqrt{(c_w + \frac{c_s}{n^*})(n^* - 1)h_w + (c_w + \frac{c_s}{n^*})h_r} \\
\geq &\sqrt{(c_w + \frac{c_s}{n^t})(n^t - 1)(h_w + \Delta h) + (c_w + \frac{c_s}{n^*})h_r} - \sqrt{(c_w + \frac{c_s}{n^t})(n^t - 1)h_w + (c_w + \frac{c_s}{n^*})h_r} \\
\geq &\sqrt{(c_w + \frac{c_s}{n^t})(n^t - 1)(h_w + \Delta h) + (c_w + \frac{c_s}{n^t})h_r} - \sqrt{(c_w + \frac{c_s}{n^t})(n^t - 1)h_w + (c_w + \frac{c_s}{n^t})h_r}
\end{aligned}
$$

Then it is sufficient to show that

$$
\begin{aligned}
&\sqrt{(c_w + \frac{c_s}{n^t})((n^t - 1)(h_w + \Delta h) + h_r)} - \sqrt{(c_w + \frac{c_s}{n^t})((n^t - 1)h_w + h_r)} \\
\geq &\sqrt{c_w((n^t - 1)(h_w + \Delta h) + h_r)} - \sqrt{c_w((n^t - 1)h_w + h_r)},
\end{aligned}
$$

which is sufficient to show that

$$
\sqrt{(c_w + \frac{c_s}{n^t})((n^t - 1)(h_w + \Delta h) + h_r)} - \sqrt{(c_w + \frac{c_s}{n^t})((n^t - 1)h_w + h_r)}
$$

is increasing in c_s. As

$$\begin{aligned}&\sqrt{(c_w+\frac{c_s}{n^t})((n^t-1)(h_w+\Delta h)+h_r)}-\sqrt{(c_w+\frac{c_s}{n^t})((n^t-1)h_w+h_r)}\\=&\frac{(c_w+\frac{c_s}{n^t})(n^t-1)\Delta h}{\sqrt{((n^t-1)(h_w+\Delta h)+h_r)}+\sqrt{((n^t-1)h_w+h_r)}},\end{aligned}$$

$\sqrt{(c_w+c_s/n^t)((n^t-1)(h_w+\Delta h)+h_r)}-\sqrt{(c_w+c_s/n^t)((n^t-1)h_w+h_r)}$ is increasing in c_s. Hence we show that when both n^* and n^t does not change when h_w increases to $h_w+\Delta h$, $C_{sc}^{logistis}$ increases faster than C_r^{Log}.

If n^* decreases to n^*-1 as h_w increase to $h_w+\Delta h$ and n^t does not change,

$$\begin{aligned}&C_{sc}^{Log}(h_w+\Delta h)-C_{sc}^{Log}(h_w)\geq C_r^{Log}(h_w+\Delta h)-C_r^{Log}(h_w)\\\Leftrightarrow&\sqrt{(c_w+\frac{c_s}{(n^*-1)})((n^*-2)(h_w+\Delta h)+h_r)}+h_rk\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{\sqrt{2\mu}(n^*-1)}-\\&-\sqrt{(c_w+\frac{c_s}{n^*})((n^*-1)h_w+h_r)}-h_rk\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{\sqrt{2\mu}n^*}\\\geq&\sqrt{c_w}\left(\sqrt{(n^t-1)(h_w+\Delta h)+h_r}-\sqrt{(n^t-1)h_w+h_r}\right)\end{aligned}$$

As n^* minimizes C_{sc}^{Log} at h_w, we have

$$\begin{aligned}&\sqrt{2\mu(c_w+c_s/n^*)((n^*-1)h_w+h_r)}+h_rk\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{n^*}\\\leq&\sqrt{2\mu(c_w+c_s/(n^*-1))((n^*-2)h_w+h_r)}+h_rk\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{n^*-1}.\end{aligned}$$

Thus it is sufficient to show that

$$\begin{aligned}&\sqrt{(c_w+\frac{c_s}{n^*-1})((n^*-2)(h_w+\Delta h)+h_r)}-\sqrt{(c_w+\frac{c_s}{n^*-1})((n^*-2)h_w+h_r)}\\\geq&\sqrt{c_w}\left(\sqrt{(n^t-1)(h_w+\Delta h)+h_r}-\sqrt{(n^t-1)h_w+h_r}\right)\end{aligned}$$

For the first part, we have already show that, with $n^*-1\geq n^t$, the above inequality holds. Hence for this case, when h_w increases to $h_w+\Delta h$, C_{sc}^{Log} increases faster than C_r^{Log}.

If n^* does not change but n^t decreases to n^t-1 as h_w increases to $h_w+\Delta h$,

$$
\begin{aligned}
& C_{sc}^{Log}(h_w+\Delta h)-C_{sc}^{Log}(h_w) \geq C_r^{Log}(h_w+\Delta h)-C_r^{Log}(h_w) \\
\Leftrightarrow & \sqrt{(c_w+\frac{c_s}{n^*})((n^*-1)(h_w+\Delta h)+h_r)}-\sqrt{(c_w+\frac{c_s}{n^*})((n^*-1)h_w+h_r)} \\
\geq & \sqrt{c_w((n^t-2)(h_w+\Delta h)+h_r)}+h_r k\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{\sqrt{2\mu}(n^t-1)}-\sqrt{c_w((n^t-1)h_w+h_r)}- \\
& -h_r k\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{\sqrt{2\mu}n^t}
\end{aligned}
$$

As n^t-1 minimizes C_r^{Log} at $h_w+\Delta h$, we have

$$
\begin{aligned}
& \sqrt{2\mu c_w((n^t-2)(h_w+\Delta h)+h_r)}+h_r k\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{n^t-1} \\
\leq & \sqrt{2\mu c_w((n^t-1)(h_w+\Delta h)+h_r)}+h_r k\frac{\sigma\Delta_L+\Delta_\sigma\sqrt{\mu}}{n^t}.
\end{aligned}
$$

Thus it is sufficient to show that

$$
\begin{aligned}
& \sqrt{(c_w+\frac{c_s}{n^*})((n^*-1)(h_w+\Delta h)+h_r)}-\sqrt{(c_w+\frac{c_s}{n^*})((n^*-1)h_w+h_r)} \\
\geq & \sqrt{c_w((n^t-1)(h_w+\Delta h)+h_r)}-\sqrt{c_w((n^t-1)h_w+h_r)}
\end{aligned}
$$

As $n^*\geq n^t$, the above inequality holds. Thus as n^t decrease to n^t-1 but n^* does not change when h_w increases to $h_w+\Delta h$, C_{sc}^{Log} increases faster than C_r^{Log}. Note that as h_w increases, C_{sc}^{Log} and C_r^{Log} increase faster as we show above. Hence there exists $h_w^t(\theta)\geq 0$ such that $\theta C_{sc}^{logsitics}\leq C_r^{Log}$ when $h_w\leq h_w^t(\theta)$. When h_w goes to zero, both n^* and n^t go to infinity. However, as n^* and n^t is in a squared form of h_w, both C_r^{Log} and C_{sc}^{Log} would go to $\sqrt{2c_w\mu h_r}$. Hence when $h_w\leq h_r*\min\{r^h,r^t\}$, there always exists an $h_w^t(\theta)>0$ such that $\pi_r^*\geq\pi_r^t$ when $h_w\leq h_w^T r$.

Furthermore, for $r^t<h_w/h_r<r^h$, as h_w decreases, π_r^* decreases but π_r^{*t} does not change. If at some $h_w\in(h_r*r^t,h_r*r^h)$ that $\pi_r^*=\pi_r^{*t}$, then $\pi_r^*\geq\pi_r^{*t}$ always holds when $h_w\leq h_r*r^t$, as explained above. Therefore when $h_w\leq h_r*r^h$, as π_r^* always decreases faster than π_r^{*t} as h_w decreases, there must exist an $h_w^t(\theta)\leq r^h*h_r$ such that $\pi_r^*\geq\pi_r^{*t}$ when $h_w\leq h_w^t(\theta)$. $\square$

Appendix III

In this section, we present appendix for Chapter 4.

Proof of Results in §4.3

Lemma C.5. *$V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ is (jointly) convex in (x_t, S_{t-1}), $t = 1, \ldots, N$.*

Proof of Lemma C.5. The Bellman's equation of $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ is

$$V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) = \inf_{\boldsymbol{F}_t, \boldsymbol{S}_t \geq 0} \Big\{ H_t(x_t) + \mathbb{E}\Big[\sum_{l=t}^{N-1} \big[c_l^f(F_l) + c_l^s(S_l) + q_{l-1}c_{l-1}(S_{l-1}) + \\ + H_l(X_{l+1})\big] + c_N^f(F_N) + q_{N-1}c_{N-1}(S_{N-1}) + H_N(X_{N+1})\Big]\Big\}.$$

Let $\boldsymbol{F}_t = (F_t, F_{t+1}, \ldots, F_N)$ denote the fast order quantities from period t to period N, and $\boldsymbol{S}_t = (S_t, S_{t+1}, \ldots, S_{N-1})$ denote the slow order quantities from period t to period $N-1$. $J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ is the expected cost in period t with future order quantities $\boldsymbol{F}_t, \boldsymbol{S}_t$. Let $\boldsymbol{X_t} = (x_t, \ldots\, x_N)$ be a sample path of on hand inventory at the beginning of period t given $(x_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ and $\tilde{\boldsymbol{X}}_t = (\tilde{x}_t, \ldots, \tilde{x}_N,\ \hat{\boldsymbol{X}}_t = (\hat{x}_t, \ldots, \hat{x}_N)$ be two sample paths of on hand inventory at the beginning of period t driven by the same demand sample path given $(x_t, \tilde{\boldsymbol{F}}_t, \tilde{\boldsymbol{S}}_t)$ and $(x_t, \hat{\boldsymbol{F}}_t, \hat{\boldsymbol{S}}_t)$. We denote $J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ as the expected cost from period t to N with orders $\boldsymbol{F}_t, \boldsymbol{S}_t$,

$$J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t) = E\big[\sum_{l=t}^{N-1} \Big[c_l^f(F_l) + c_l^s(S_l) + q_{l-1}c_{l-1}(S_{l-1}) + H_l(X_{l+1})\Big] + \\ + c_N^f(F_N) + q_{N-1}c_{N-1}(S_{N-1}) + H_N(X_{N+1})\big].$$

Then it is sufficient to show that $J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ is (jointly) convex in (x_t, S_{t-1}), as

$$V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) = \inf_{\boldsymbol{F}_t, \boldsymbol{S}_t \geq 0} J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t).$$

We prove the convexity of $J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ in (x_t, S_{t-1}) by induction. Note that $H_t(x)$ is convex in x. For two sets of $(\boldsymbol{X}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$, $(\tilde{\boldsymbol{X}}_t, \tilde{\boldsymbol{F}}_t, \tilde{\boldsymbol{S}}_t)$ and $(\hat{\boldsymbol{X}}_t, \hat{\boldsymbol{F}}_t, \hat{\boldsymbol{S}}_t)$, $\forall \theta \in [0, 1]$, the following inequality is satisfied,

$$\begin{aligned}&\theta H_t(\tilde{X}_t+\tilde{F}_t+q_{t-1}\tilde{S}_{t-1})+(1-\theta)H_t(\hat{X}_t+\hat{F}_t+q_{t-1}\hat{S}_{t-1})\\ \geq &H_t(\theta(\tilde{X}_t+\tilde{F}_t+q_{t-1}\tilde{S}_{t-1})+(1-\theta)(\hat{X}_t+\hat{F}_t+q_{t-1}\hat{S}_{t-1}))\end{aligned}$$

Obviously when $t = N$, $J_N(x_N, S_{N-1}, q_{N-1}, i_N, \boldsymbol{D}_N, F_N)$ is (jointly) convex in (x_N, S_{N-1}) as $c_t^f(.), c_t^s(.), c_t(.)$ are convex. Assume $J_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, \boldsymbol{F}_t, \boldsymbol{S}_t)$ is (jointly) convex for $t = n+1$. Then for $t = n$, we have

$$\begin{aligned}&\theta\Big\{q_{n-1}c_{n-1}(\tilde{S}_{n-1})+c_n^f(\tilde{F}_n)+c_n^s(\tilde{S}_n)+\mathbb{E}\Big\{H_n(\tilde{F}_{n+1})+\sum_{l=n+1}^{N-1}\big[c_l^f(\tilde{F}_l)+c_l^s(\tilde{S}_l)+\\ &+q_{l-1}c_{l-1}(\tilde{S}_{l-1})+H_{l+1}(\tilde{X}_l)\big]+c_N^f(\tilde{F}_N)+q_{N-1}c_{N-1}(\tilde{S}_{N-1})+H_N(\tilde{X}_{l+1})\Big\}\Big\}+(1-\theta)*\\ &*\Big\{q_{n-1}c_{n-1}(\hat{S}_{n-1})+c_n^f(\hat{F}_n)+c_n^s(\hat{S}_n)+\mathbb{E}\Big\{H_n(\hat{F}_{n+1})+\sum_{l=n+1}^{N-1}\big[c_l^f(\hat{F}_l)+c_l^s(\hat{S}_l)+\\ &+q_{l-1}c_{l-1}(\hat{S}_{l-1})+H_l(\hat{X}_{l+1})\big]+c_N^f(\hat{F}_N)+q_{N-1}c_{N-1}(\hat{S}_{N-1})+H_N(\hat{X}_{l+1})\Big\}\Big\}\\ \geq &q_{n-1}c_{n-1}(\theta\tilde{S}_{n-1}+(1-\theta)\hat{S}_{n-1})+c_n^f(\theta\tilde{F}_n+(1-\theta)\hat{F}_n)+c_n^s(\theta\tilde{F}_n+(1-\theta)\hat{F}_n)+\\ &+\mathbb{E}\Big\{H_n(\theta\tilde{X}_{n+1}+(1-\theta)\hat{X}_{n+1})+\sum_{l=n+1}^{N-1}\big[c_l^f(\theta\tilde{F}_l+(1-\theta)\hat{F}_l)+c_l^s(\theta\tilde{S}_l+(1-\theta)\hat{S}_l)+\\ &+q_{l-1}c_l(\theta\tilde{S}_{l-1}+(1-\theta)\hat{S}_{l-1})+H_l(\theta\tilde{X}_{l+1}+(1-\theta)\hat{X}_{l+1})\big]+c_N^f(\theta\tilde{F}_N+(1-\theta)\hat{F}_N)+\\ &+q_{N-1}c_{N-1}(\theta\tilde{S}_{N-1}+(1-\theta)\hat{S}_{N-1})+H_N(\theta\tilde{X}_{N+1}+(1-\theta)\hat{X}_{N+1})\Big\}\\ \Leftrightarrow &\theta J_n(\tilde{x}_n,\tilde{S}_{n-1},q_{n-1},i_n,\boldsymbol{D}_n,\tilde{\boldsymbol{F}}_n,\tilde{\boldsymbol{S}}_n)+(1-\theta)J_n(\hat{x}_n,\hat{S}_{n-1},q_{n-1},i_n,\boldsymbol{D}_n,\hat{\boldsymbol{F}}_n,\hat{\boldsymbol{S}}_n)\\ \geq &J_n(\theta\tilde{x}_n+(1-\theta)\hat{x}_n,\theta\tilde{S}_{n-1}+(1-\theta)\hat{S}_{n-1},q_{n-1},i_{n-1},\boldsymbol{D}_t,\theta\tilde{\boldsymbol{F}}_n+(1-\theta)\hat{\boldsymbol{F}}_n,\theta\tilde{\boldsymbol{S}}_n+\\ &+(1-\theta)\hat{\boldsymbol{S}}_n)\end{aligned}$$

Hence $V_n(x_n, S_{n-1}, q_{n-1}, i_n, \boldsymbol{D}_n)$ is (jointly) convex in (x_n, S_{n-1}). □

Lemma C.6. *$V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ satisfies the optimality equations,*

$$\begin{aligned}V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) =& q_{t-1}c_{t-1}(S_{t-1}) + \inf_{F_t, S_t \geq 0} \Big\{ c_t^f(F_t) + c_t^s(S_t) + H_t(X_{t+1}) + \\ &+ \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \big(P_{i_t} V_{t+1}(X_{t+1}, S_t, 0, i_{t+1}, \boldsymbol{D_{t+1}}) + \\ &+ (1 - P_{i_t}) V_{t+1}(X_{t+1}, S_t, 1, i_{t+1}, \boldsymbol{D_{t+1}}) \big) \Big\}, t = 1, \ldots, N-1; \\ V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) =& q_{t-1}c_{t-1}(S_{t-1}) + \inf_{F_t, S_t \geq 0} \big\{ c_t^f(F_t) + H_t(X_{t+1}) \big\}, t = N.\end{aligned}$$

There are Borel measurable functions that provide optimal fast and slow order quantities,

$$F_n^* = f_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t), t = 1, \ldots, N; S_n^* = s_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t), t = 1, \ldots, N-1.$$

Proof of Lemma C.6. Let $\hat{F}_1 = f_1(x_1, S_0, q_0, i_1, \boldsymbol{D_1}), \hat{S}_1 = s_1(x_1, S_0, q_0, i_1, \boldsymbol{D_1}), \hat{X}_1 = x_1$; $\hat{X}_t = \hat{X}_{t-1} + \hat{F}_{t-1} + q_{t-2}\hat{S}_{t-2} - D_{t+1,t}; \hat{F}_t = f_t(\hat{X}_t, \hat{S}_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t), \hat{S}_t = s_t(\hat{X}_t, \hat{S}_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$. From the definition of $\hat{F}_1, \hat{S}_1$, for the first period,

$$\begin{aligned}& c_1^f(\hat{F}_1) + c_1^s(\hat{S}_1) + \mathbb{E} \sum_{i_2} \mathcal{P}_{i_1, i_2} \big(P_{i_1} V_2(\hat{X}_2, \hat{S}_1, 0, i_2, \boldsymbol{D_2}) + (1 - P_{i_1}) V_2(\hat{X}_2, \hat{S}_1, 1, i_2, \boldsymbol{D_2}) \big) \\ \leq & c_1^f(F_1) + c_1^s(S_1) + \mathbb{E} \sum_{i_2} \mathcal{P}_{i_1, i_2} \big(P_{i_1} V_2(X_2, S_1, 0, i_2, \boldsymbol{D_2}) + (1 - P_{i_1}) V_2(X_2, S_1, 1, i_2, \boldsymbol{D_2}) \big)\end{aligned}$$

For the second period,

$$\begin{aligned}& V_2(X_2, S_1, q_1, i_2, \boldsymbol{D_2}) \\ =& q_1 c_1(S_1) + \inf_{F_2, S_2 \geq 0} \Big\{ c_2^f(F_2) + c_2^s(S_2) + H_2(\hat{X}_3) + \mathbb{E} \sum_{i_3} \mathcal{P}_{i_2, i_3} \big(P_{i_2} V_3(\hat{X}_3, S_2, 0, i_3, \boldsymbol{D}_3) + \\ & + (1 - P_{i_2}) V_3(\hat{X}_3, S_2, 1, i_3, \boldsymbol{D}_3) \big) \Big\} \\ \leq & q_1 c_1(S_1) + c_2^f(F_2) + c_2^s(S_2) + H_2(X_3) + \mathbb{E} \sum_{i_3} P_{i_2, i_3} \big(P_{i_2} V_3(X_3, S_2, 0, i_3, \boldsymbol{D}_3) + \\ & + (1 - P_{i_2}) V_3(X_3, S_2, 1, i_3, \boldsymbol{D}_3) \big)\end{aligned}$$

where X_3 is the on hand inventory at the beginning of period 3 when order quantities in period 2 are $f_2(X_2, S_1, q_1, i_2, \boldsymbol{D_2})$,$s_2(X_1, S_1, q_1, i_2, \boldsymbol{D}_2)$ correspondingly. Combining the above inequalities, we have

$$
\begin{aligned}
& c_1^f(\hat{F}_1) + c_1^s(\hat{S}_1) + H_2(\hat{X}_3) + \mathbb{E}_{X_2} c_2^f(\hat{F}_2) + \mathbb{E}_{X_2} c_2^s(\hat{S}_2) + (1-P_{i_1})c_1(\hat{S}_1) + \mathbb{E}\sum_{i_2} \mathcal{P}_{i_1,i_2}\big(P_{i_1} \\
& \big[\mathbb{E}\sum_{i_3} \mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(\hat{X}_3, \hat{S}_2, 0, i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(\hat{X}_3, \hat{S}_2, 1, i_3, \boldsymbol{D}_3)\big)\big] + (1-P_{i_1})* \\
& * \big[\mathbb{E}\sum_{i_3} \mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(\hat{X}_3, \hat{S}_2, 0, i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(\hat{X}_3, \hat{S}_2, 1, i_3, \boldsymbol{D}_3)\big)\big]\big) \\
\leq & c_1^f(F_1) + c_1^s(S_1) + H_2(X_3) + \mathbb{E}_{X_2} c_2^f(F_2) + \mathbb{E}_{X_2} c_2^s(S_2) + (1-P_{i_1})c_1(S_1) + \mathbb{E}\sum_{i_2} \mathcal{P}_{i_1,i_2}\big(P_{i_1} \\
& \big[\mathbb{E}\sum_{i_3} \mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(X_3, S_2, 0, i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(X_3, S_2, 1, i_3, \boldsymbol{D}_3)\big)\big] + (1-P_{i_1})* \\
& * \big[\mathbb{E}\sum_{i_3} \mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(X_3, S_2, 0, i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(X_3, S_2, 1, i_3, \boldsymbol{D}_3)\big)\big]\big) \\
\Leftrightarrow & \mathbb{E}_{\hat{X}_2} \sum_{l=1}^{2} \big(c_l^f(\hat{F}_l) + c_l^s(\hat{S}_l)\big) + (1-P_{i_1})c_1(\hat{S}_1) + H_2(\hat{X}_3) + \sum_{i_2}\sum_{i_3} \mathcal{P}_{i_1,i_2}\mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(\hat{X}_3, \hat{S}_2, 0, \\
& i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(\hat{X}_3, \hat{S}_2, 1, i_3, \boldsymbol{D}_3)\big) \\
\leq & \mathbb{E}_{X_2} \sum_{l=1}^{2} \big(c_l^f(F_l) + c_l^s(S_l)\big) + (1-P_{i_1})c_1(S_1) + H_2(X_3) + \sum_{i_2}\sum_{i_3} \mathcal{P}_{i_1,i_2}\mathcal{P}_{i_2,i_3}\big(P_{i_2} V_3(X_3, S_2, 0, \\
& i_3, \boldsymbol{D}_3) + (1-P_{i_2}) V_3(X_3, S_2, 1, i_3, \boldsymbol{D}_3)\big)
\end{aligned}
$$

Keeping deriving similar inequalities about total cost from period 1 to period N, eventually we have

$$
\begin{aligned}
& \sum_{l=1}^{N} \mathbb{E}_{X_l}\big(c_l^f(\hat{F}_l) + c_l^s(\hat{S}_l) + H_l(\hat{X}_{l+1})\big) + \sum_{l=1}^{N} \mathcal{P}_{i_1,i_2} \sum_{i_2} \dots \mathcal{P}_{i_{N-1},i_N} \sum_{i_N} (1-P_{i_l}) \mathbb{E}_{\hat{X}_l} c_l(\hat{S}_l) \\
\leq & \sum_{l=1}^{N} \mathbb{E}_{X_l}\big(c_l^f(F_l) + c_l^s(S_l) + H_l(X_{l+1})\big) + \sum_{l=1}^{N} \mathcal{P}_{i_1,i_2} \sum_{i_2} \dots \mathcal{P}_{i_{N-1},i_N} \sum_{i_N} (1-P_{i_l}) \mathbb{E}_{X_l} c_l(S_l).
\end{aligned}
$$

Next we prove that $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ satisfies the optimality equations states in the proposition by induction. Obviously for $t = N$, $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ satisfies the optimality equation by definition. Suppose that for $t = n+1$, $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ satisfies the optimality equations states in the proposition. For $t = n$,

$$
\begin{aligned}
&V_n(x_n, S_{n-1}, q_{n-1}, i_n, \boldsymbol{D}_n) \\
=&q_{n-1}c_{n-1}(S_{n-1}) + \inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + H_n(X_{n+1}) + \sum_{l=n+1}^{N-1} \Big\{ c_l^f(F_l) + c_l^s(S_l) + \\
&+ q_{l-1}c_{l-1}(S_{l-1}) + H_l(X_{l+1}) \Big\} + c_N^f(F_N) + q_{N-1}c_{N-1}(S_{N-1}) + H_N(X_{N+1}) \Big\} \\
=&q_{n-1}c_{n-1}(S_{n-1}) + \inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + \mathbb{E}\Big\{ H_n(X_{n+1}) + q_n c_n(S_n) + \\
&+ \inf_{F_{n+1}, S_{n+1} \geq 0} \Big\{ c_{n+1}^f(F_{n+1}) + c_{n+1}^s(S_{n+1}) + H_{n+1}(X_{n+2}) + \sum_{l=n+2}^{N-1} \Big(c_l^f(F_l) + c_l^s(S_l) + \\
&+ q_{l-1}c_{l-1}(S_{l-1}) + H_l(X_{l+1}) \Big) + c_N^f(F_N) + q_{N-1}c_{N-1}(S_{N-1}) + H_N(X_{N+1}) \Big\} \Big\} \Big\} \\
=&q_{n-1}c_{n-1}(S_{n-1}) + \inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + H_n(X_{n+1}) + \mathbb{E} \sum_{i_{n+1}} \mathcal{P}_{i_n, i_{n+1}} \Big[P_{i_n} V_{n+1}(X_{n+1}, \\
&S_n, 0, i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + (1 - P_{i_n}) V_{n+1}(X_{n+1}, S_n, 1, i_{n+1}, \boldsymbol{D}_{n+1}) \Big] \Big\}
\end{aligned}
$$

By assuption $V_{n+1}(x_{n+1}, S_n, q_n, i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n)$ is (jointly) convex in (x_{n+1}, S_n) and $X_{n+1} = x_n + F_n + q_{n-1}S_{n-1} - D_{n+1,n}$. Hence

$$
(P_{i_n} V_{n+1}(X_{n+1}, S_n, 0, i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + (1 - P_{i_n}) V_{n+1}(X_{n+1}, S_n, 1, i_{n+1}, \boldsymbol{D}_{n+1}))
$$

is (jointly) convex in (F_n, S_n). The optimal choices of F_n, S_n must be bounded by some $Q < \infty$. Following the same reasoning in Sethi et al. (2001), we have two Borel measurable functions that provide the optimal order quantities. □

Lemma C.7. *The optimality equation $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ can be transformed as*

$$
U_t(w_t, i_t, \boldsymbol{D}_t) = V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t) - q_{t-1}c_{t-1}(S_{t-1}),
$$

where $w_t = x_t + q_{t-1}S_{t-1}$. *The transformed optimality equations satisfy*

$$U_t(w_t, i_t, \boldsymbol{D}_t) = \inf_{z \geq y \geq sw_t} \Big\{ H_t(y - D_{t+1,t}) + (1 - P_{i_t})c_t(z - y) + c_t^f(y - w_t) + c_t^s(z - y) + \\ + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \Big[P_{i_t} U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + \\ + (1 - P_{i_t}) U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big] \Big\},$$

where $\boldsymbol{D}_t' = (D_{t,t+1}, \ldots, D_{t,N})$, $\boldsymbol{\epsilon}_t' = (\epsilon_{t,t+1}, \ldots, \epsilon_{t,N})$.

$$U_t(w_t, i_t, \boldsymbol{D}_t) = H_t(\tilde{f}_t - D_{t+1,t}) + (1 - P_{i_t})c_t(\tilde{s}_t - \tilde{f}_t) + c_t^f(\tilde{f}_t - w_t) + c_t^s(\tilde{s}_t - \tilde{f}_t) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \\ \Big[P_{i_t} U_{t+1}(\tilde{f}_t - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) U_{t+1}(\tilde{s}_t - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big], \\ t = 1, \ldots, N-1; \\ U_t(w_t, i_t, \boldsymbol{D}_t) = \inf_y \Big\{ c_t^f(y - w_t) + H_t(y - D_{t,t} - \epsilon_{t,t}) \Big\}, t = N$$

where $\tilde{f}_t = \tilde{f}_t(w_t, i_t, \boldsymbol{D}_t), \tilde{s}_t = \tilde{s}_t(w_t, i_t, \boldsymbol{D}_t)$ *are the optimal order up to levels which satisfy*

$$\tilde{f}_t(w_t, i_t, \boldsymbol{D}_t) = w_t + \hat{F}_t(w_t, i_t, \boldsymbol{D}_t), \qquad \tilde{s}_t(w_t, i_t, \boldsymbol{D}_t) - \tilde{f}_t(w_t, i_t, \boldsymbol{D}_t) = \hat{S}_t(w_t, i_t, \boldsymbol{D}_t)$$

and $\hat{F}_t(w_t, i_t, \boldsymbol{D}_t), \hat{S}_t(w_t, i_t, \boldsymbol{D}_t)$ *are the optimal fast and slow order quantities correspondingly in period* t *given* $(w_t, i_t, \boldsymbol{D}_t)$.

Proof of Lemma C.7. Let $\tilde{f}_t(w_t, i_t, \boldsymbol{D}_t), \tilde{s}_t(w_t, i_t, \boldsymbol{D}_t)$ satisfy

$$\tilde{f}_t(w_t, i_t, \boldsymbol{D}_t) = w_t + \hat{F}_t(w_t, i_t, \boldsymbol{D}_t), \qquad \tilde{s}_t(w_t, i_t, \boldsymbol{D}_t) - \tilde{f}_t(w_t, i_t, \boldsymbol{D}_t) = \hat{S}_t(w_t, i_t, \boldsymbol{D}_t)$$

and $\hat{F}_t(w_t, i_t, \boldsymbol{D}_t), \hat{S}_t(w_t, i_t, \boldsymbol{D}_t)$ are the optimal fast and slow order quantities correspondingly in period t given $(w_t, i_t, \boldsymbol{D}_t)$ for all t. Let $y = sw_t + q_{t-1}S_{t-1} + F_t$, $z = y + S_t$;
We prove this proposition by induction. When $t = N$, obviously this is satisfied. Suppose that

when $t=n+1$, this is satisfied. Then when $t=n$, we have

$$
\begin{aligned}
&U_n(w_n, i_n, \boldsymbol{D}_n)\\
=&\inf_{z\geq y\geq w_n}\Big\{H_{n+1}(y-D_{n,n}-\epsilon_{n,n})+(1-P_{i_n})c_n(z-y)+c_n^f(y-w_n)+c_n^s(z-y)+\\
&+\mathbb{E}_{\boldsymbol{\epsilon}_n}\sum_{i_{n+1}}P_{i_n,i_{n+1}}\Big[P_{i_n}U_{n+1}(y-D_{n,n}-\epsilon_{n,n},i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)+(1-P_{i_n})*\\
&U_{n+1}(z-D_{n,n}-\epsilon_{n,n},i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)\Big]\Big\}\\
=&\inf_{F_n,S_n\geq 0}\Big\{H_{n+1}(x_n+q_{n-1}S_{n-1}-D_{n,n}-\epsilon_{n,n})+(1-P_{i_n})c_n(S_n)+c_n^f(F_n)+\\
&+c_n^s(S_n)+\mathbb{E}_{\boldsymbol{\epsilon}_n}\sum_{i_{n+1}}P_{i_n,i_{n+1}}\Big[P_{i_n}V_{n+1}(x_n+q_{n-1}S_{n-1}-D_{n,n}-\epsilon_{n,n},S_n,0,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)+\\
&+(1-P_{i_n})\big(V_{n+1}(x_n+q_{n-1}S_{n-1}-D_{n,n}-\epsilon_{n,n},S_n,1,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)-c_n(S_n)\big)\Big]\Big\}\\
=&\inf_{F_n,S_n\geq 0}\Big\{c_n^f(F_n)+c_n^s(S_n)+\mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{n+1}}P_{i_n,i_{n+1}}\Big[P_{i_n}V_{n+1}(x_n+q_{n-1}S_{n-1}-D_{n,n}-\epsilon_{n,n},\\
&S_n,0,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)+(1-P_{i_n})V_{n+1}(x_n+q_{n-1}S_{n-1}-D_{n,n}-\epsilon_{n,n},S_n,1,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)\Big]\Big\}\\
=&V_n(x_n,S_{n-1},q_{n-1},i_n,\boldsymbol{D}_n)-q_{n-1}c_{n-1}(S_{n-1})
\end{aligned}
$$

We then prove the Bellman's equations for $U_t(w_t, i_t, \boldsymbol{D}_t)$. When $t=N$, obviously the Bellman's equations are satisfied. Assume when $t=n+1$, the Bellman's equations are satisfied. Then for $t=n$, we plug in order up to levels, $\tilde{f}_n, \tilde{s}_n$, into the following cost expressions.

$$
\begin{aligned}
&H_{n+1}(\tilde{f}_n-D_{n,n}-\epsilon_{n,n})+(1-P_{i_n})c_n(\tilde{s}_n-\tilde{f}_n)+c_n^f(\tilde{f}_n-w_n)+c_n^s(\tilde{s}_n-\tilde{f}_n)+\mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{n+1}}P_{i_n,i_{n+1}}\\
&\Big[P_{i_n}U_{n+1}(\tilde{f}_n-D_{n,n}-\epsilon_{n,n},i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)+(1-P_{i_n})U_{n+1}(\tilde{s}_n-D_{n,n}-\epsilon_{n,n},i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)\Big]\\
=&c_n^f(\hat{F}_n)+c_n^s(\hat{S}_n)+H_n(X_{n+1})+\mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{n+1}}P_{i_n,i_{n+1}}\big(P_{i_n}V_{n+1}(X_{n+1},\hat{S}_n,0,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)+\\
&+(1-P_{i_n})V_{n+1}(x_n+q_{n-1}S_{n-1}+\hat{F}_n-D_{n,n}-\epsilon_{n,n},\hat{S}_n,1,i_{n+1},\boldsymbol{D}'_n+\boldsymbol{\epsilon}'_n)\big)\\
=&V_n(x_n,S_{n-1},q_{n-1},i_n,\boldsymbol{D}_n)-q_{n-1}c_{n-1}(S_{n-1})=U_n(w_n,i_n,\boldsymbol{D}_n)
\end{aligned}
$$

Hence we prove the Bellman's equations for $U_t(w_t, i_t, \boldsymbol{D}_t)$. $\square$

Lemma C.8. *$U_t(w_t, i_t, \boldsymbol{D}_t)$ is convex in w_t.*

Proof of Lemma C.8. We know that $V_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t)$ is (jointly) convex in (x_t, S_{t-1}). Define

$$
\begin{aligned}
&B_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, F_t, S_t)\\
=&c_t^f(F_t) + c_t^s(S_t) + H_t(X_{t+1}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \Big(P_{i_t} V_{t+1}(x_t + q_{t-1}S_{t-1} + F_t - D_{t,t} - \epsilon_{t,t}, S_t, 0,\\
&i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) V_{t+1}(x_t + q_{t-1}S_{t-1} + F_t - D_{t,t} - \epsilon_{t,t}, S_t, 1, i_{t+1}, \boldsymbol{D}_{t+1}) \Big)
\end{aligned}
$$

Since $V_{t+1}(x_t + S_{t-1}q_{t-1} + F_t - D_{t,t} - \epsilon_{t,t}, S_t, i_t, \boldsymbol{D}_{t+1})$ is (jointly) convex in $(x_t + S_{t-1}q_{t-1} + F_t - D_{t,t} - \epsilon_{t,t}, S_t)$, $i_t \in 0, 1$, $t \geq 1$, $\forall \epsilon_{t,t}$, $B_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, F_t, S_t)$ is (jointly) convex in $(x_t + q_{t-1}S_{t-1} + F_t - D_{t,t}, S_t)$. Let $w_t = x_t + q_{t-1}S_{t-1} + F_t$, from the proof of Lemma C.7, we know

$$
U_t(w_t, i_t, \boldsymbol{D}_t) = \inf_{F_t, S_t} B_t(x_t, S_{t-1}, q_{t-1}, i_t, \boldsymbol{D}_t, F_t, S_t)
$$

Therefore $U_t(w_t, i_t, \boldsymbol{D}_t)$ is convex in w_t. In addition, by the definition of $U_N(w_N, i_N, \boldsymbol{D_N})$, $U_N(w_N, i_N, \boldsymbol{D_N})$ is convex in w_N. $\square$

Proof of Proposition 4.1. The results in Proposition 4.1 can be derived directly from Lemma C.7 and Lemma C.8. $\square$

Proof of Theorem 4.1. For period t, let $y = \bar{y}$, $z = \bar{z}$ be the minimum points of the following function on the region of $z \geq y \geq sw_t$.

$$
\begin{aligned}
&c_t^f(y - w_t) + c_t^s(z - y) + (1 - P_{i_t})c_t(z - y) + H_t(y - D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}}\\
&\Big[P_{i_t} U_{t+1}(y - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) U_{t+1}(z - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big]
\end{aligned}
$$

y_t^*, z_t^* are given by

$$
\begin{aligned}
y_t^* =& \arg\min_y \Big\{ c_t^f(y - w_t) - (c_t^s + (1 - P_{i_t})c_t)y + H_t(y - D_{t+1,t}) + E_{\boldsymbol{\epsilon}_t}\big[\sum_{i_{t+1}} P_{i_t,i_{t+1}} P_{i_t} * \\
& \quad U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})\big]\Big\} \\
:=& \arg\min_y L_t(y, w_t, i_t, \boldsymbol{D}_t) \\
y_N^* =& \arg\min_y \Big\{ c_N^f(y - w_N) + H_{N+1}(y - D_{N+1,N}) \Big\} \\
z_n^* =& \arg\min_z \Big\{ (c_t^s + (1 - P_{i_t})c_t) + E_{\boldsymbol{\epsilon}_t}\big[\sum_{i_{t+1}} P_{i_t,i_{t+1}}(1 - P_{i_t}) U_{t+1}(y - D_{t,t} - \epsilon t, t, i_{t+1}, \boldsymbol{D}_{t+1})\big] + \\
& \quad + \delta(y_t^* - z)\big[L_t(z, w_t, i_t, \boldsymbol{D}_t) - L_t(y_t^*, w_t, i_t, \boldsymbol{D}_t)\big]\Big\} \\
:=& \arg\min_z T_t(y_t^*, w_t, i_t, \boldsymbol{D}_t)
\end{aligned}
$$

where $\delta(a - t) = 1$ when $t \leq a$ and $\delta(a - t) = 0$ otherwise.

If $z_t^* \geq y_t^*$, based on the definition of y_t^*, z_t^*, we have $y_t^* = \bar{y}, z_t^* = \bar{z}$. So we only discuss the case where $y_t^* \geq z_t^*$ and try to show

$$
\begin{aligned}
& c_t^f(y - w_t) + c_t^s(z - y) + (1 - P_{i_t})c_t(z - y) + H_t(y - D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} * \\
& \quad * \Big[P_{i_t} U_{t+1}(y - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big] \\
\geq & c_t^f(z_t^* - s w_t) + H_t(z_t^* - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_n^* - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}).
\end{aligned}
$$

For z_t^*, we have

$$
\begin{aligned}
& c_t^f(z_t^* - w_t) + H_t(z_t^* - D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
=& c_t^f(z_t^* - w_t) - (c_t^s + (1-P_{i_t})c_t)z_t^* + H_t(z_t^* - D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(z_t^* - \\
& - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) - \Big[c_t^f(y_t^* - w_t) - (c_t^s + (1-P_{i_t})c_t)y_t^* + H_t(y_t^* - D_{t,t} - \epsilon_{t,t}) + \\
& + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y_t^* - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big] + \Big[c_t^f(y_t^* - w_t) - (c_t^s + (1-P_{i_t})c_t)y_t^* \\
& + H_t(y_t^* - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y_t^* - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big] + \\
& + (c_t^s + (1-P_{i_t})c_t)z_t^* + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} (1-P_{i_t}) U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
\leq& \Big\{ \big(L_t(z, w_t, i_t, \boldsymbol{D}_t) - L_t(y_t^*, w_t, i_t, \boldsymbol{D}_t) \big) + (c_t^s + (1-P_{i_t})c_t)z + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} (1-P_{i_t}) * \\
& U_{t+1}(z - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big\} + L_t(y_t^*, w_t, i_t, \boldsymbol{D}_t) \quad \forall z \\
=& c_t^f(z - w_t) + H_t(z - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}),
\end{aligned}
\tag{C.23}
$$

where

$$
\begin{aligned}
L(y, sw_t, i_t, \boldsymbol{D}_t) =& c_t^f(y - w_t) - (c_t^s + (1-P_{i_t})c_t)y + H_t(y - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} * \\
& * U_{t+1}(y - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}).
\end{aligned}
$$

From the definition of $\bar{y}, \bar{z}$, $(\bar{y}, \bar{z})$ minimizes

$$
\begin{aligned}
& c_t^f(y - w_t) + c_t^s(z - y) + (1-P_{i_t})c_t(z - y) + H_t(y - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} * \\
& * \Big[P_{i_t} U_{t+1}(y - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1-P_{i_t}) U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \Big].
\end{aligned}
$$

We know that the following function is convex in y,

$$c_t^f(y - w_t) - (c_t^s + (1 - P_{i_t})c_t)y + H_t(y - D_{t+1,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t}\big[\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y - D_{t,t} - \epsilon_{t,t},$$
$$i_{t+1}, \boldsymbol{D}_{t+1})\big].$$

If $\bar{y} \le \bar{z} \le y_n^*$, then based on the convexity of the above function in y, we know $(\bar{z}, \bar{z})$ minimizes the above expression, i.e.,

$$\begin{aligned}
& c_t^f(y - w_t) + c_t^s(z - y) + (1 - P_{i_t})c_t(z - y) + H_t(y - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \\
& \Big[P_{i_t}U_{t+1}(y - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t})U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1})\Big] \\
\ge & c_t^f(\bar{z} - w_t) + H_t(\bar{z} - D_{t,t} - \epsilon_{t,t}) + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(\bar{z} - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
\ge & c_t^f(z_t^* - w_t) + H_t(z_t^* - D_{t+1,t}) + E_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}).
\end{aligned}$$

If $\bar{z} > y_t^* > z_t^*$, based on the convexity of $(c_n^s + (1 - P_{i_t})c_t)z + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1})$ in z, $\bar{z}$ is the minimizer of the previous function.

$$\begin{aligned}
& (c_t^s + (1 - P_{i_t})c_t)\bar{z} + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(\bar{z} - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
\le & (c_t^s + (1 - P_{i_t})c_t)z + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}), \forall z \\
\le & (c_t^s + (1 - P_{i_t})c_t)z + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + \delta(y_n^* - z) * \\
& \Big\{L_t(z, sw_t, i_t, \boldsymbol{D}_t) - L_t(y_t^*, sw_t, i_t, \boldsymbol{D}_t)\Big\}, \forall z,
\end{aligned}$$

This contradicts to the assumption that $\bar{z} > z_n^*$, as z_n^* is the minimizer of

$$\begin{aligned}
& (c_t^s + (1 - P_{i_t})c_t)z + \mathbb{E}_{\boldsymbol{\epsilon}_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(z - D_{t,t} - \epsilon_{t,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + \\
& + \delta(y_n^* - z)\Big\{L_t(z, w_t, i_t, \boldsymbol{D}_t) - L_t(y_t^*, w_t, i_t, \boldsymbol{D}_t)\Big\}.
\end{aligned}$$

shown in Equation (C.23). □

Proof of Proposition 4.2. We first consider the case $y_t^* < z_t^*$. For optimal y_t^*, z_t^*, we have

$$
\begin{aligned}
& c^s + (1 - P_{i_t})c + \frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}}(1 - P_{i_t})U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) = 0 \\
\Leftrightarrow & \frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{j} \mathcal{P}_{i_t,i_{t+1}}U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) = -\frac{c^s + (1 - P_{i_t})c}{1 - P_{i_t}}, \\
& c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y}(H_t(y_t^* - D_{t+1,t}) + \\
& + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})) = 0 \\
\Leftrightarrow & \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{j} \mathcal{P}_{i_t,i_{t+1}}U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
& = -\frac{c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y}H_t(y_t^* - D_{t+1,t})}{P_{i_t}}.
\end{aligned}
$$

Note that $\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}}U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})$ is convex in y. We have

$$
\begin{aligned}
& y_t^* < z_t^* \\
\Leftrightarrow & -\frac{c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y}H_t(y_t^* - D_{t+1,t})}{P_{i_t}} < -\frac{c^s + (1 - P_{i_t})c}{1 - P_{i_t}} \\
\Leftrightarrow & (1 - P_{i_t})(c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y}H_t(y_t^* - D_{t+1,t})) > P_{i_t}(c^s + (1 - P_{i_t})c) \\
\Leftrightarrow & c^f - \frac{c^s}{1 - P_{i_t}} - c + \frac{\partial}{\partial y}H_t(y_t^* - D_{t+1,t}) > 0.
\end{aligned}
$$

As $\frac{\partial}{\partial y}U_{t+1}(y, i_{t+1}, \boldsymbol{D}_{t+1}) \geq -c_{t+1}^f$, at $y = y_t^*$, we have

$$
\begin{aligned}
& \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \geq -P_{i_t}c_{t+1}^f \\
\Leftrightarrow & c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y}H_t(y - D_{t+1,t}) \geq P_{i_t}c_{t+1}^f \\
\Leftrightarrow & (1 - P_{i_t})(c^f - \frac{c^s}{1 - P_{i_t}} - c) + \frac{\partial}{\partial y}H_t(y - D_{t+1,t}) \geq 0. \qquad \text{(C.24)}
\end{aligned}
$$

Therefore, to have $y_t^* < z_t^*$, the sufficient condition is $c^f - \frac{c^s}{1-P_{i_t}} - c > 0$.

We then analyze the case $y_t^* > z_t^*$. For optimal y_t^*, z_t^*, we have

$$
\begin{aligned}
&c^f + \frac{\partial}{\partial z} H_t(z_t^* - D_{t+1,t}) + \frac{\partial}{\partial z} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) = 0 \\
\Leftrightarrow &\frac{\partial}{\partial y} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_t' + \boldsymbol{\epsilon}_t) = -(c^f + \frac{\partial}{\partial z} H_t(z_t^* - D_{t+1,t})) \\
&c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) + \\
&+ \frac{\partial}{\partial y} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} P_{i_t} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) = 0 \\
\Leftrightarrow &\frac{\partial}{\partial y} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_t' + \boldsymbol{\epsilon}_t) \\
&= -\frac{c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t})}{P_{i_t}}.
\end{aligned}
$$

Rearranging the terms in the above equalities, we have

$$
\begin{aligned}
&\frac{\partial}{\partial y} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + \frac{\partial}{\partial z} H_t(z_t^* - D_{t+1,t}) = -c^f, \\
&\frac{\partial}{\partial y} \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) \\
= &- \frac{c^f + c^s + (1 - P_{i_t})c}{P_{i_t}} - (\frac{1}{P_{i_t}} - 1) \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}).
\end{aligned}
$$

As $\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_t' + \boldsymbol{\epsilon}_t') + H_t(y - D_{t+1,t})$ is convex in y, we have

$$
\begin{aligned}
&y_t^* > z_t^* \\
\Leftrightarrow &- \frac{c^f + c^s + (1 - P_t)c}{P_{i_t}} - (\frac{1}{P_{i_t}} - 1) \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) > -c^f \\
\Leftrightarrow &c^f - \frac{1}{1 - P_{i_t}} c^s - c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) < 0.
\end{aligned}
$$

Since $(1 - P_{i_t})(c^f - \frac{c^s}{1-P_{i_t}} - c) + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) \geq 0$, $c^f - \frac{c^s}{1-P_{i_t}} - c < 0$ is a necessary condition for $y_t^* > z_t^*$.

We then consider the case $y_t^* = z_t^*$. With $y_t^* < z_t^*$,

$$
c^f - \frac{c^s}{1 - P_{i_t}} - c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) \leq 0; \tag{C.25}
$$

with $y_t^* > z_t^*$,

$$c^f - \frac{c^s}{1-P_{i_t}} - c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) \geq 0. \tag{C.26}$$

Therefore in for $y_t^* = z_t^*$,

$$c^f - \frac{c^s}{1-P_{i_t}} - c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) = 0.$$

As $(1-P_{i_t})(c^f - \frac{c^s}{1-P_{i_t}} - c) + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) \geq 0$, the necessary condition is $c^f - \frac{c^s}{1-P_{i_t}} - c \leq 0$. To summarize, $c^f - \frac{c^s}{1-P_{i_t}} - c \leq 0$ is the necessary and sufficient condition for $y_t^* \geq z_t^*$.

Finally we show that $T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)$ is continuous and differentiable at $z = y_t^*$. For $z > y_t^*$, the partial derivative of $T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)$ with respect to z is

$$\frac{\partial T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)}{\partial z} = (1-P_{i_t}) \frac{\partial}{\partial z} E_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + c^s + (1-P_{i_t})c.$$

For $z < y_t^*$, the partial derivative of $T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)$ with respect to z is

$$\frac{\partial T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)}{\partial z} = \frac{\partial}{\partial z} E_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + c^f + \frac{\partial}{\partial y} H_t(z - D_{t+1,t}).$$

It is easy to verify that $T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)$ is continuous and differentiable on $(-\infty, y_t^*)$ and (y_t^*, ∞). Hence,

$$\begin{aligned}
\lim_{z \to (y_t^*)^+} &= (1-P_{i_t}) \frac{\partial}{\partial z} E_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + c^s + (1-P_{i_t})c \\
\lim_{z \to (y_t^*)^-} &= \frac{\partial}{\partial z} E_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) + c^f + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}).
\end{aligned}$$

Note that for y_t^*, we have

$$\begin{aligned}
&c^f - c^s - (1-P_{i_t})c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t}) + \\
&+ P_{i_t} \frac{\partial}{\partial z} E_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) = 0
\end{aligned} \tag{C.27}$$

Therefore,

$$\lim_{z\to(y_t^*)^+} T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t) = \lim_{z\to(y_t^*)^-} T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t),$$

i.e. $T_t(z, y_t^*, w_t, i_t, \boldsymbol{D}_t)$ is continuous and differentiable at y_t^*.

As $\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})$ is convex in y, for $y_t^* < z_t^*$ and $y_t^* > z_t^*$, $z_t^* - y_t^*$ increases with $\frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) - \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})$.

For $y_t^* < z_t^*$,

$$\begin{aligned}
&\frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) - \\
&- \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
=&\frac{c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t})}{P_{i_t}} - \frac{c^s + (1 - P_{i_t})c}{1 - P_{i_t}} \\
=&\frac{c^f - c - \frac{c^s}{1 - P_{i_t}}}{P_{i_t}} + \frac{\frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t})}{P_{i_t}}
\end{aligned}$$

For $y_t^* > z_t^*$,

$$\begin{aligned}
&\frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) - \\
&- \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) \\
=&\frac{c^f - c^s - (1 - P_{i_t})c + \frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t})}{P_{i_t}} - (c^f + \frac{\partial}{\partial z} H_t(z_t^* - D_{t+1,t})) \\
=&\frac{1 - P_{i_t}}{P_{i_t}}(c^f - c - \frac{c^s}{1 - P_{i_t}}) + \frac{\frac{\partial}{\partial y} H_t(y_t^* - D_{t+1,t})}{P_{i_t}} - \frac{\partial}{\partial z} H_t(z_t^* - D_{t+1,t})
\end{aligned}$$

In both cases,

$$\frac{\partial}{\partial z}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(z_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1}) - \frac{\partial}{\partial y}\mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_j \mathcal{P}_{i_t,i_{t+1}} U_{t+1}(y_t^* - D_{t+1,t}, i_{t+1}, \boldsymbol{D}_{t+1})$$

increases with the effective cost difference $c^f - c - \frac{c^s}{1-P_{i_t}}$. Therefore $z_t^* - y_t^*$ increases with the effective cost difference. □

Proof of Theorem 4.2. We prove the results by induction. Note that $\boldsymbol{F}_t, \boldsymbol{S}_t$ record the pipeline inventory in period t and $\boldsymbol{q}_{t-1}$ records the disruption of submitted slow orders up to period $t-1$.

Obviously when $t = N$, this equality holds. Assume that when $t = n+1$, this equality holds. Then when $t = n$, we have

$$
\begin{aligned}
&U_n(w_n, i_n, \boldsymbol{D}_n) \\
=&\inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + (1 - P_{i_n}) c_n(S_n) + H_{n+l_f-1}(X_{n+l_f}) + \mathbb{E}_{\boldsymbol{\epsilon}_n} \sum_{i_{n+1}} P_{i_n, i_{n+1}} \\
&\Big(P_{i_n} U_{n+1}(y - D_{n+1,n}, i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + (1 - P_{i_n}) U_{n+1}(z - D_{n+1,n}, i_n, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) \Big) \Big\} \\
=&\inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + \mathbb{E}_{\boldsymbol{\epsilon}_n} \sum_{i_{n+1}} P_{i_n, i_{n+1}} \Bigg(P_{i_n} \big(U_{n+1}(y - D_{n+1,n}, i_n, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + \\
&+ H_{n+l_f-1}(X_{n+l_f}) \big) + (1 - P_{i_n}) \big(U_{n+1}(z - D_{n+1,n}, i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + H_{n+l_f-1}(X_{n+l_f}) + \\
&+ c_n(S_n) \big) \Bigg) \Big\} \\
=&\inf_{F_n, S_n \geq 0} \Big\{ c_n^f(F_n) + c_n^s(S_n) + \mathbb{E}_{\boldsymbol{\epsilon}_n} \sum_{i_{n+1}} P_{i_n, i_{n+1}} \Bigg(P_{i_n} V_{n+1}(X_{n+1}, \boldsymbol{F}_n, \boldsymbol{S}_n, (\boldsymbol{q}_{n-1}, 0), i_{n+1}, \\
&\boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) + (1 - P_{i_n}) V_{n+1}(X_{n+1}, \boldsymbol{F}_n, \boldsymbol{S}_n, (\boldsymbol{q}_{n-1}, 1), i_{n+1}, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) \Bigg) \Big\} \\
=&V_n(x_n, \boldsymbol{F}_{n-1}, \boldsymbol{S}_{n-1}, \boldsymbol{q}_{n-1}, i_n, \boldsymbol{D}'_n + \boldsymbol{\epsilon}'_n) - q_{n-1} c_{n-1}(S_{n-1})
\end{aligned}
$$

We then show that $l_s - l_f = 1$ is the necessary and sufficient condition that the optimal policy is a two-threshold policy. If $l_s - l_f > 1$, we redefine $w_t = x_t + \sum_{k=t-l_f+1}^{t-1} F_k + \sum_{k=t-l_s+1}^{t+l_f-l_s} q_k S_k$, $n_t = \sum_{k=t+l_f-l_s+1}^{t-1} q_k S_k$ and let $y = F_t + w_t, z = y + S_t + n_t$. Therefore, we modify the value function as

$$
\begin{aligned}
&U_t(w_t, n_t, i_t, \boldsymbol{D}_t) = \\
&\inf_{z \geq y + n_t, y \geq s w_t} \Big\{ H_{t+l_f-1}(y - D_{t+1,t}) + (1 - P_{i_t}) c_t(z - y - n_t) + c_t^f(y - w_t) + c_t^s(z - y - n_t) + \\
&+ \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \big[P_{i_t} U_{t+1}(w_{t+1}, n_{t+1}, i_{t+1}, \boldsymbol{D}_{t+1}) + (1 - P_{i_t}) U_{t+1}(w_{t+1}, n'_{t+1}, i_{t+1}, \boldsymbol{D}_{t+1}) \big] \Big\},
\end{aligned}
$$

$$
\begin{aligned}
w_{t+1} &= x_{t+1} + \sum_{k=t-l_f+2}^{t} F_k + \sum_{k=t-l_s+2}^{t+l_f-l_s} q_k S_k \\
&= x_t + F_{t-l_f+1} - D_{t+1,t} + q_{t-l_s+1} S_{t-l_s+1} + \sum_{k=t-l_f+2}^{t} F_k + \sum_{k=t-l_s+2}^{t+l_f-l_s+1} q_k S_k \\
&= sw_t + F_t - D_{t+1,t} + q_{t+l_f-l_s+1} S_{t+l_f-l_s+1}.
\end{aligned}
$$

In order to keep w_{t+1} tractable only based on the decision in period t, we need $l_f - l_s + 1 = 0$. Otherwise, when $l_f - l_s + 1 < 0$, we have to keep track of the slow order of period $t + l_f - l_s + 1$ in period t, which means we need to keep track of all past slow orders. Then we check n_{t+1}, n'_{t+1},

$$
\begin{aligned}
n_{t+1} &= \sum_{k=t+l_f-l_s+1}^{t} q_k S_k = n_t - q_{t+l_f-l_s+1} S_{t+l_f-l_s+1}, \\
n'_{t+1} &= \sum_{k=t+l_f-l_s+1}^{t} q_k S_k = n_t - q_{t+l_f-l_s+1} S_{t+l_f-l_s+1} + S_t.
\end{aligned}
$$

When $l_s - l_f = 1$, n_{t+1} and n'_{t+1} are tractable only based on the decision in period t. So when we plug in this equality into the $U_t(w_t, n_t, i_t, \boldsymbol{D}_t)$ functions we can directly ignore these n_t terms, so that we have the forms stated in the proposition. □

Optimal Policy with Markovian Modulated Demand

Rather than MMFE, previous literature also assumes Markovian modulated demand to model demand forecast. With Markovian modulated demand, the current demand state provides information about the demand in the next period. We consider the discrete demand case of Markovian modulated demand. The firm observes the demand state k_t in the end of period t, which indicates the realized demand, and the demand state evolves following a Discrete Time Markov chain. Let $\mathcal{K}$ denote the set of demand states and $\mathcal{M}$ denote the transition matrix of demand states: $\mathcal{M}_{i,j} = Prob\{k_{t+1} = j | k_t = i\}, t = 1, \ldots, N; i, j \in \mathcal{K}$. With Markovian modulated demand, the realized demand in every period also infers information of possible demand in the following periods.

Theorem C.1. *Consider Markovian modulated demand. Assume $c^s_t(x) = c^s_t x, c_t(x) = c_t x$. The optimal policy is a state-dependent two-threshold base-stock policy with the structure stated in Theorem 4.1.*

We show that the optimality of the two-threshold type policy is preserved with Markovian modulated demand. Therefore we focus on the original model to generate insights.

Proof of Theorem C.1. Let d_k denote the demand value when demand state is k. Without loss of generality, we label the states such that for state $i < j$, $d_i < d_j$. Let $\mathcal{R}$ denote the transition probability matrix of the DTMC of demand.

Let $V_t^m(x_t, S_{t-1}, q_{t-1}, g_{t-1}, i_t)$ be the optimal cost function in period t, where g_{t-1} is the realized demand state in period t. With the same procedure in Lemma C.6, it is easy to show that the Bellman's equations satisfy

$$\begin{aligned}
&V_t^m(x_t, S_{t-1}, q_{t-1}, g_{t-1}, i_t) \\
&= \inf_{F_t, S_t} \Big\{ H_t(x_t) + c_t^f(F_t) + c_t^s(S_t) + q_{t-1}c_{t-1}(S_{t-1}) + \\
&\quad + \sum_{g_t} \mathcal{R}_{g_{t-1},g_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \big[P_{i_t} V_{t+1}^m(x_t + q_{t-1}S_{t-1} - d_{g_t}, S_t, 0, g_t, i_{t+1}) + \\
&\quad + (1 - P_{i_t}) V_{t+1}^m(x_t + q_{t-1}S_{t-1} - d_{g_t}, S_t, 1, g_t, i_{t+1}) \big] \Big\}.
\end{aligned}$$

Furthermore following the same procedure in the proof of Proposition 4.1 that, with $w_t = x_t + q_{t-1}S_{t-1}$,

$$\begin{aligned}
U_t^m(w_t, g_{t-1}, i_t) = &\inf_{z \geq y \geq sw_t} \Big\{ c_t^f(y - w_t) + (c_t^s + (1 - P_{i_t})c_t)(z - y) + \sum_{g_t} \mathcal{R}_{g_{t-1},g_t} H_t(y - d_{g_t}) + \\
&+ \sum_{g_t} \mathcal{R}_{g_{t-1},g_t} \sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \big[P_{i_t} * \\
&* U_{t+1}^m(y - g_t, g_t, i_{t+1}) + (1 - P_{i_t}) U_{t+1}^m(z - g_t, g_t, i_{t+1}) \big] \Big\} \\
U_N^m(w_N, g_{N-1}, i_N) = &\inf_{y \geq w_N} \Big\{ c_N^f(y - w_N) + \sum_{g_N} \mathcal{R}_{g_{N-1},g_N} H_{N+1}(y - d_{g_N}) \Big\}
\end{aligned}$$

with $U_t^m(w_t, g_{t-1}, i_t) = V_t^m(x_t, S_{t-1}, q_{t-1}, g_{t-1}, i_t) - q_{t-1}c_{t-1}(S_{t-1})$. Assume $c_t^s(x) = c_t^s x, c_t(x) =$

$c_t x$.

$$
\begin{aligned}
y_t^* = & \arg\min_y \Big\{ \sum_j \mathcal{R}_{g_{t-1},g_t} H_t(y - d_{g_t}) + c_t^f(y - w_t) - (c_t^s + (1 - P_{i_t})c_t)y + \sum_{g_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \\
& \mathcal{R}_{g_{t-1},g_t} P_{i_t} U_{t+1}^m(y - g_t, g_t, i_{t+1}) \Big\} \\
= & \arg\min_y L_t^m(y, g_{t-1}, i_t) \quad t = 1, 2, \ldots, N-1 \\
y_N^* = & \arg\min_y \Big\{ \sum_j \mathcal{R}_{g_{t-1},g_t} H_t(y - d_{g_t}) + c_t^f(y - w_t) \Big\} \\
z_t^* = & \min_z \Big\{ (c_t^s + (1 - P_{i_t})c_t)z + \sum_{g_t}\sum_{i_{t+1}} \mathcal{P}_{i_t,i_{t+1}} \mathcal{R}_{g_{t-1},g_t}(1 - P_{i_t}) U_{t+1}^m(z - g_t, g_t, i_{t+1}) \Big\} + \\
& + \delta(y_t^* - z)\big\{ L_t^m(z, g_{t-1}, i_t) - L_t^m(y_t^*, g_{t-1}, i_t) \big\} \\
= & \arg\min_z T_t^m(z, y_t^*, g_{t-1}, i_t) \quad t = 1, 2, \ldots, N-1
\end{aligned}
$$

Following the same argument in the proof of Theorem 4.1, we can prove the optimality of the state-dependent two-threshold base-stock policy. □

Optimal Policy with Random Yield

In practice firms also meet partial supply disruption, i.e., random yield or partial order availability. In this section we consider supply chain disruption in the form of random yield such that a probability matrix characterizes the proportion of order the firm would receive for a given disruption state. Let j_t denote the random yield state in period t, $t = 1, \ldots, N$ and $\mathcal{J}$ denote the set of random yield states. If the supplier's random yield state is j, he receives a proportion r_j of the total order. Let $\mathcal{Q}$ denote the probability matrix such that with disruption state $i_t = i$, the probability that the random yield state is $j_t = j$ is $\mathcal{Q}_{i,j} = Prob\{j_t = j | i_t = i\}, t = 1, \ldots, N; j \in \mathcal{J}$.

Theorem C.2. *Consider random yield of the slow supplier. Assume $c_t^s(x) = c_t^s x, c_t(x) = c_t x$. The optimal policy is a state-dependent two-threshold base-stock policy. The optimal base-stock levels $y_t^*, z_t^*, \bar{z}_t^*$ characterize the optimal ordering decisions as follows:*

$y_t^* \le z_t^*$

$$(F_t^*, S_t^*) = \begin{cases} (y_t^* - w_t, z_t^* - y_t^*) & sw_t < y_t^* \\ (0, z_t^* - w_t) & y_t^* \le sw_t < z_t^* \\ (0,0) & o.w. \end{cases}$$

$y_t^* > z_t^*$

$$(F_t^*, S_t^*) = \begin{cases} (\bar{z}_t^* - w_t, 0) & sw_t < \bar{z}_t^* \\ (0,0) & o.w. \end{cases}$$

With random yield, when $y_t^* > z_t^*$, the base-stock level is $\bar{z}_t^*$ instead of z_t^*, $t = 1, \ldots, N$. As the optimality of two-threshold policy still preserves with random yield rather than complete disruption, for the simplicity of the analysis we focus on the complete disruption case to generate insights.

Proof of Theorem C.2. Following exactly the same procedure with the complete disruption case, it is straight forward to show the optimal cost function satisfies

$$U_t^r(w_t, i_t, \boldsymbol{D}_t) = \inf_{z \ge y \ge sw_t} \Bigg\{ c_t^f(y - w_t) + c_t^s(z-y) + \sum_j Q_{i_n,j} r_j c_n(z-y) + H_t(y - D_{t+1,t}) + \\ + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \sum_j Q_{i_t,j} U_{t+1}^r(y - D_{t+1,t} + r_j(z-y), i_{t+1}, \boldsymbol{D}_{t+1}) \Bigg\}.$$

and $U_t^r(w_t, i_t, \boldsymbol{D}_t)$ is convex in w_t. Thus the function inside the inf operator is a (jointly) convex function of (y,z). Therefore with $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$, a unique pair of (y,z) minimizes $U_t^r(w_t, i_t, \boldsymbol{D}_t)$. The optimal order policy should follow a similar procedure based on y_n^*, z_n^* where y_n^*, z_n^* minimizes the following function,

$$W_t(y, z, sw_t, i_t, \boldsymbol{D}_t) = c_t^f(y - w_t) + c_t^s(z-y) + \sum_j Q_{i_t,j} r_j c_t(z-y) + H_t(y - D_{t+1,t}) + \\ + \mathbb{E}_{\boldsymbol{\epsilon}_t} \sum_{i_{t+1}} \mathcal{P}_{i_t, i_{t+1}} \sum_j Q_{i_t,j} U_{t+1}^r(y - D_{t+1,t} + r_j(z-y), i_{t+1}, \boldsymbol{D}_{t+1}).$$

With $\tilde{s}_t = (w_t, i_t, \boldsymbol{D}_t)$, if $y_t^* \le z_t^*$, the optimal order policy is the same with part (i) in Theorem 4.1, i.e., $(F_t^*, S_t^*) = (y_t^* - \min(w_t, y_t^*), z_t^* - \max(y_t^*, w_t))$. When $y_t^* \ge z_t^*$, we define $\bar{y}, \bar{z}$ to be the maximizer to the above expression with the constraints $\bar{z} \ge \bar{y} \ge sw_t$. However, since $W_t(y, z, w_t, i_t, \boldsymbol{D}_t)$

is (jointly) convex in (y, z). If we only add the constraint $\bar{z} \geq \bar{y}$, the optimal $(\bar{y}^*, \bar{z}^*)$ is on the intersecting line of the surfaces $(y, z, W_t(y, z, w_t, i_t, \boldsymbol{D}_t))$ and $y = z$. The optimal solution satisfies $\bar{y}^* = \bar{z}^*$. Hence the optimal ordering policy should be, ordering up to $\bar{z}^*$ from the fast supplier if possible. $\square$

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