REPRESENTATIONS OF THE HECKE GROUP G(2) FROM FERMIONIC MODULAR CATEGORIES

An Undergraduate Research Scholars Thesis

by

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ABSTRACT

Representations of the Hecke Group G(2) from Fermionic Modular Categories

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This project explores a conjecture which states that groups from the Fermionic Modular Category are finite; specifically representations of the Hecke group G(2) will be explored which are important in number theory. These representations are used for a mathematical model of Topological Quantum Computation (TQC) based on topological symmetries rather than geometric symmetries. The use of topological symmetries reduces the effects of outside interference on computations due to the nature of topological symmetries relying on the general shape instead of particular distances or angles. TQC would aid in the development of quantum computing by helping to solve the problem of interference in quantum particles. Magma algebraic software was used in order to generate these group representations and provide information on their resulting structure to aid in identification.

DEDICATION

To my grandmother, grandfather, and my family who have always been there to support me.

ACKNOWLEDGMENTS

Tremendous amount of thanks goes out to my advisor Dr. Eric C. Rowell for his assistance and guidance throughout the project, from performing the research to writing this thesis. His insight and monitoring provided me with an opportunity to grow and gain experience in ways that have shaped my interests and future plans. I am very thankful for getting the opportunity to be a part of his research in this exciting field.

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NOMENCLATURE

FMC	Fermionic Modular Category
MMC	Metaplectic Modular Category
TQC	Topological Quantum Computation

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1. INTRODUCTION

1.1 Background

Quantum computing offers benefits over traditional bit-arithmetic computing due to the use of Qubits, which provide more computational power by "using the two characteristic attributes of quantum mechanics – superposition and entanglement"[1] However, quantum particles are affected by the act of being observed, so Qubits suffer from large amounts of environmental interference. Topological Quantum Computing is a method of quantum computation which focuses on encoding information in topological invariants. By using topological symmetries, interference can be mitigated since the general structure, not distance and angle degree, decides equivalence in topological structures as in figure 1.1.

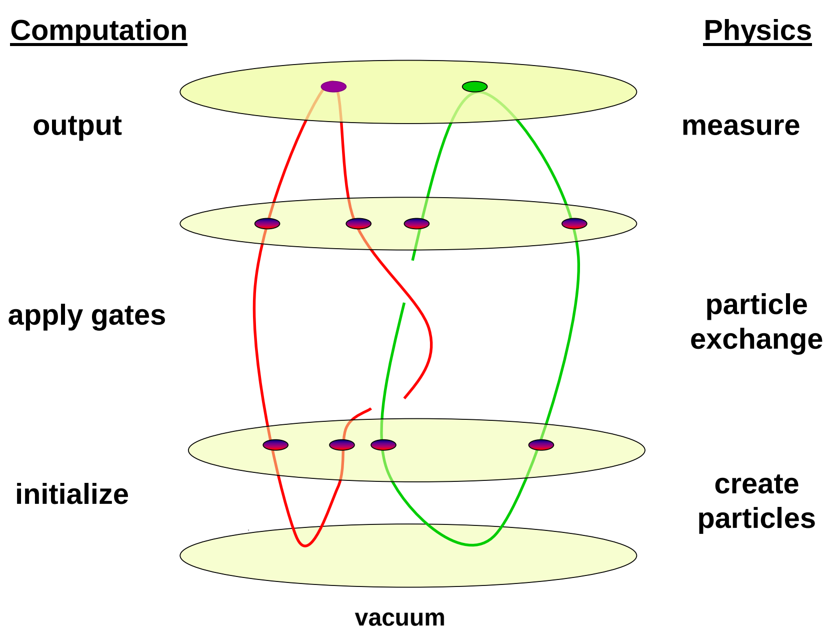

Figure 1.1: Computation model utilizing braid structures[1]

1.2 Project Description

For analysis of FMCs, smaller examples were generated in order to generalize the structure. Generation was accomplished by using cyclic groups with order $k = 8n$ where $n \in \mathbb{N}$ (Z_k). An S and T matrix, representing the spin and twist respectively, were constructed with size dependent on the order of the cyclic group where T is diagonal. From Z_8 and Z_{16} the method was able to be generalized such that generating S and T matrices for further Z_k could be automated. Magma algebraic software was then used to check the order of the group generated by the two matrices, as well as gather further information about the structure. The quotient group modulo the center was also analyzed until no longer possible and similar information was gathered for each subgroup.

Another example of FMCs were Metaplectic Modular Categories which were also studied in this project. The structure is based off of odd positive integers and covered in further detail in the methods section. Analysis was also carried out as with FMCs, using magma software to gather information on the resulting groups and the quotient groups. The data collected has been included and is discussed in the results section, and further conclusions are discussed in the summary. The magma code for FMCs and MMCs has also been included in the appendix.

[1] http://www.math.tamu.edu/ rowell/RowellTyler09nopause.pdf

2. METHODS

The matrix representation of the two categories of interest, FMCs and MMCs, are generated by the use of an S and T matrix which take on a unique form for each category. A thorough treatment along with further examples can be found in [2], which supplies the I_{sing} matrix containing the desired Fermion

$$I_{sing} := \frac{1}{2}\begin{bmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{bmatrix}$$

For the FMC and MMC cases, specific forms which contain a non-trivial boson to use with the I_{sing} matrix generate the S and T matrices as detailed in [3]. Using a script in magma, the S and T matrices can be generated at various sizes, and the resulting group analyzed using standard functions included in the software. The structural differences between the two categories required the creation of two separate scripts. This section will cover structural details of each category and motivations behind magma script development.

2.1 Fermionic Modular Categories

The FMC structure is based on the use of Z_N cyclic groups, an example provided in [2]. The needed matrices are derived from the Kronecker product of the 3x3 I_{sing} matrix with the objects of the FMC. The full matrix for a $Z_N \otimes I_{sing}$ system consisted of $9N^2$ entries, many of which were not needed, so a reduced form based on entries which commuted in (2.1) was found, which reduced to the even entries sharing positive sign, and odd entries sharing negative sign.

$$S_{(x,y)} = \sigma_{(x,y)} \tag{2.1}$$

It was found that Z_K where $K = 8n$ for $n \in \mathbb{Z}^+$, always contains a non-trivial Boson at index $S_{\frac{K}{2},\frac{K}{2}}$. Due to interest in non-trivial Bosons, the Z_N examples were restricted to Z_K. Thus, for a Z_K group the accompanying S matrix was generated using the description found in [2], resulting in a $\frac{3}{4}K$ square matrix of the form:

$$\frac{1}{\sqrt{K}}\begin{bmatrix} A & \sqrt{2}B \\ \sqrt{2}B^T & C \end{bmatrix} \tag{2.2}$$

where:

$$\mathbf{A}_{a,b} = e^{i\frac{4\pi}{2K}(2a*2b)} \qquad 0 \leq a, b < \frac{1}{2}K \tag{2.3}$$

$$\mathbf{B}_{c,d} = e^{i\frac{4\pi}{2K}(2c(2d+1))} \qquad \frac{1}{2}K \leq c, d < \frac{3}{4}K \tag{2.4}$$

and $\mathbf{C}$ a square zero matrix of dimension $\frac{1}{4}K$

The T matrix is a diagonal square matrix of dimension $\frac{3}{4}K$ with entries:

$$e^{\frac{\pi i}{8}}(\theta_0, \theta_2, \ldots, \theta_{2a}, \theta_1, \theta_3, \ldots, \theta_{2j+1}) \tag{2.5}$$

Where $\theta_a = e^{\frac{i\pi a^2}{K}}$ for $0 \leq a \leq K-1$ and $\theta_{2j+1} = e^{\frac{i\pi(2j+1)^2}{n}}$ for $0 \leq j \leq \frac{K}{2}$

Once the two matrices were generated, T was squared and the resulting group generated by T^2 and S was analyzed for structural detail. Magma's standard function for matrix groups revealed data about the group by giving data such as the order and whether the given group was extra-special, simple, or nilpotent for example. Magma also has the ability to find the center of a group, thus the quotient group modulo the center was analyzed using the same functions. From this, the most basic structure of the group could be found and linked back to the starting characteristics of the FMC. This entire process was captured into a script using algorithm 1 taking an input of a positive integer N to generate S, T, the resulting matrix group, and perform the desired analysis.

Algorithm 1 Fermionic Modular Category Group

procedure GENERATE FMC MATRIX GROUP
  Input: N a positive integer
  $K = 8 * N$
  ZField $\leftarrow$ Cyclotomic Field with Kth root of unity
  Set root to ZField's root element
A Matrix:
  A is Zero square matrix of dimension $\frac{3}{4}K$
  for i in $[0,\frac{K}{2})$ **do**
    for j in $[0,\frac{K}{2})$ **do**
      A[i,j] = root^{2i*2j}
B Matrix:
  B is Zero square matrix of dimension $\frac{3}{4}K$
  for i in $[0,\frac{K}{2})$ **do**
    for j in $[\frac{K}{2}, \frac{3}{4}K)$ **do**
      B[i,j] = $\sqrt{2}$ root$^{2i*(2j+1)}$
S Matrix:
  S is Zero square matrix of dimension $\frac{3}{4}K$
  S = A + B + Transpose of B
  Scale S by $\frac{1}{\sqrt{K}}$
T Matrix:
  T is Zero square matrix of dimension $\frac{3}{4}K$
  for i in $[0,\frac{K}{2})$ **do**
    T[i,i] = root$^{2*i^2}$
  for j in $[\frac{K}{2}, \frac{3}{4}K)$ **do**
    T[j,j] = root$^{(2j+1)^2+8}$
Generate Group:
  $A \leftarrow$ Matrix Group generated from S and T^2

2.2 Metaplectic Modular Categories

MMC structure is based on the SO$(m)_2$ categories as described in [4]. m was restricted to the odd numbers as defined for Type B categories in [5]. The process was similar to

FMCs in using the I_{sing} matrix although the entries for S and T differed. For SO$(m)_2$ where $m = 2r+1$ for $r \in \mathbb{Z}^+$, the S matrix is a square matrix of size $r+4$ of form:

$$\frac{1}{\sqrt{2m}}\begin{bmatrix} 1 & 1 & \gamma_1 & \gamma_2 & \dots & \gamma_r & \sqrt{m} & \sqrt{m} \\ 1 & 1 & \gamma_1 & \gamma_2 & \dots & \gamma_r & -\sqrt{m} & -\sqrt{m} \\ \gamma_1 & \gamma_1 & 4\cos(\frac{a_1 b_1 \pi}{m}) & 4\cos(\frac{a_1 b_2 \pi}{m}) & \dots & \cos(\frac{a_1 b_r \pi}{m}) & 0 & 0 \\ \gamma_2 & \gamma_2 & 4\cos(\frac{a_2 b_1 \pi}{m}) & 4\cos(\frac{a_2 b_2 \pi}{m}) & \dots & \cos(\frac{a_2 b_r \pi}{m}) & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ \gamma_r & \gamma_r & 4\cos(\frac{a_r b_1 \pi}{m}) & 4\cos(\frac{a_r b_2 \pi}{m}) & \dots & 4\cos(\frac{a_r b_r \pi}{m}) & 0 & 0 \\ \sqrt{m} & -\sqrt{m} & 0 & 0 & \dots & 0 & \sqrt{m} & -\sqrt{m} \\ \sqrt{m} & -\sqrt{m} & 0 & 0 & \dots & 0 & -\sqrt{m} & \sqrt{m} \end{bmatrix}$$

where $\gamma_1 = \cdots = \gamma_r = 2$ and $a_i = i, b_j = j$

The T matrix is diagonal with entries described by:

$$(1, 1, \theta_{\gamma_1}, \theta_{\gamma_2}, \dots, \theta_{\gamma_r}, e^{\frac{2\pi i}{8}}, e^{\frac{2\pi i}{8}}) \tag{2.6}$$

Where $\theta_{\gamma_j} = e^{-j^2 \frac{2\pi i}{m}}$

A matrix group was generated using T^2 and S. Magma functions were utilized in the same fashion as with FMCs, taking the quotient group modulo the center. Because of the restriction of m to odd positive integers ≥ 3, quadratic gaussian sums were used to express $\sqrt{m}$ in terms of the root element for any integers which were not perfect squares as required by the magma system. The full method is given in algorithm 2

Algorithm 2 Metaplectic Modular Category

procedure GENERATE MMC MATRIX GROUP
    Input: r a positive integer
    Set m to $2r + 1$
    KField $\leftarrow$ Cyclotomic Field with $8m$th root of unity
    Set root to KField's root element
S Matrix:
    Set S to a Zero square matrix of dimension $r + 4$
    Set first two columns in first and second row of S to 1
    for i in [1,r] **do**
        Set 2+ith column in first two rows to 2
        Set 2+ith rows in first two columns to 2
    Set last two columns in first row of S to $\sqrt{m}$
    Set last two rows in first column of S to $\sqrt{m}$
    Set last two columns in second row of S to $-\sqrt{m}$
    Set last two rows in second column of S to $-\sqrt{m}$
    for i in [1,r] **do**
        for j in [1,r] **do**
            S[2+i,2+j] = $4 * \cos(\frac{2\pi ij}{m})$
    Set bottom right 4x4 elements of S to $\sqrt{m}$ on diagonal and $-\sqrt{m}$ off diagonal
T Matrix:
    Set T to a Zero square diagonal matrix of dimension $r + 4$
    Set first two elements of T to 1
    for i in [1,r] **do**
        T[i,i] = root$^{-8i^2}$
    Set last two elements of T to 8th roots of unity
Generate Group:
    $A \leftarrow$ Matrix Group generated from S and T^2

3. RESULTS

The data included are that which were output from magma. The quotient chain column lists the orders of the subgroups that resulted from taking the quotient group modulo the center until it no longer resulted in a proper subgroup. Due to computational limits some data were not able to be collected but have been included as projected data. The projected data have been appropriately noted. Finding the center for groups of orders larger than $2 \cdot 10^5$ seemed to be the computational limit, and any results for characteristics such as solvability were most likely implemented to use other methods such as Burnside's theorem. This is consistent between both sets of data where the quotient chain could not be generated for groups with order greater than $2 \cdot 10^5$.

3.1 Fermionic Modular Categories

The order of the T matrix was also investigated and the resulting data included in table 3.2. A trend was spotted and was utilized to complete the data. This is further discussed in the next section.

3.2 Metaplectic Modular Categories

The MMC matrix group was generated using a cyclotomic field with a $Z = 8n$ root of unity as shown in the table. This was done in order to generate an 8th root as needed for the T matrix and ensure compatibility with the magma system. Whether the resulting final quotient group was simple was also recorded and included in table 3.3.

Table 3.1: FMC Matrix Group Data

N	Z	Quotient Chain	Order(Factored)	Order
1	8	$32, 8, 4, 1$	2^5	32
2	16	$256, 16, 8, 4, 1$	2^8	256
3	24	$18432, 1152, 576, 288$	$2^{11} \cdot 3^2$	18432
4	32	$2048, 64, 16, 8, 4, 1$	2^{11}	2048
5	40	921600	$2^{12} \cdot 3^2 \cdot 5^2$	921600
6	48	$147456, 2304, 1152, 576, 288$	$2^{14} \cdot 3^2$	147456
7	56	7225344*	$2^{14} \cdot 3^2 \cdot 7^2$	7225344
8	64	65536*	2^{16}	65536
9	72	13436928*	$2^{11} \cdot 3^8$	13436928
10	80	7372800*	$2^{15} \cdot 3^2 \cdot 5^2$	7372800
11	88	111513600*	$2^{12} \cdot 3^2 \cdot 5^2 \cdot 11^2$	111513600
12	96	1179648*	$2^{17} \cdot 3^2$	1179648
13	104	305270784*	$2^{12} \cdot 3^2 \cdot 7^2 \cdot 13^2$	305270784
14	112	57802752*	$2^{17} \cdot 3^2 \cdot 7^2$	57802752
15	120	1592524800*	$2^{18} \cdot 3^5 \cdot 5^2$	1592524800

*computational limits prevented data acquisition

Table 3.2: FMC Matrix Group Data (cont.)

N	Z	Solvable	Nilpotent	Order of T
1	8	True	True	4
2	16	True	True	16
3	24	True	False	12
4	32	True	True	32
5	40	—-*	False	20
6	48	True	False	48
7	56	—-*	—-*	28
8	64	True	True	64
9	72	True	False	36
10	80	—-*	—-*	80
11	88	—-*	—-*	44^b
12	96	True	False	96
13	104	—-*	—-*	52^b
14	112	—-*	—-*	112^b
15	120	—-*	—-*	60^b

*computational limits prevented data acquisition
^{b} projected value

Table 3.3: MMC Matrix Group Data

r	SO(n)	Z	Quotient Chain	Order(Factored)	Order	Final Simple	Solvable
1	3	24	$768, 96, 48, 12$	$2^8 \cdot 3^1$	768	True	False
2	5	40	$1920, 480, 240, 60$	$2^7 \cdot 3^1 \cdot 5^1$	1920	True	False
3	7	56	$10752, 1344, 672, 168$	$2^9 \cdot 3^1 \cdot 7^1$	10752	True	False
4	9	72	$10368, 2592, 1296, 324$	$2^7 \cdot 3^4$	10368	False	True
5	11	88	$42240, 5280, 2640, 660$	$2^8 \cdot 3^1 \cdot 5^1 \cdot 11^1$	42240	True	False
6	13	104	$34944, 8736, 4368, 1092$	$2^{11} \cdot 3^2 \cdot 5^1$	34944	True	False
7	15	120	$92160, 5760, 2880, 720$	$2^{11} \cdot 3^2 \cdot 5^1$	92160	False	False
8	17	136	$78336, 19584, 9792, 2448$	$2^9 \cdot 3^2 \cdot 17^1$	78336	True	False
9	19	152	218880*	$2^8 \cdot 3^2 \cdot 5^1 \cdot 19^1$	218880	—-*	False
10	21	168	129024*	$2^{11} \cdot 3^2 \cdot 7^1$	129024	—-*	—-*

*computational limits prevented data acquisition

4. SUMMARY AND CONCLUSIONS

In cases where FMCs were based off of Z_{2^n} groups, taking the quotient group modulo the center resulted in the standard cyclic group with extra elements. Of the p-groups, only Z_8 was a special group, while the remaining groups were solvable. The T matrix was also found to have an order dependent on n for Z_{8n}; an order of $8n$ for even n, and an order of $4n$ for odd n.

From taking the quotient group, the MMC groups for SO$(m)_2$ were found to have a relation to the special linear group SL(2,m). Ending up with an order of $\frac{1}{2}$ the order of the corresponding SL(2,m) group.

4.1 Further Study

It should be noted that during this project the conjecture under investigation has been proven. This should encourage further exploration as to the structure of the resulting groups since it is now known for certain that these supermodular categories result in finite groups. The scope of this project only explored two examples in Z_n and SO$(m)_2$. Further work could be done in analyzing the other examples given in [2]. The I_{sing} theory also has other possible candidates which could be analyzed to see the resulting groups.

The code itself could also be improved upon to become more efficient by making use of the generalized nature of the S and T matrices studied, as well as in-built features of the magma system. By doing this, in areas where computational limits prevented acquisition of data, further data could be collected.

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APPENDIX

The magma code used has been included for both MMCs and FMCs.

Fermionic Modular Categories

The alpha term to scale S must be explicitly expressed for some N values, specifically for $N < 3$. if $\sqrt{k}$ can be expressed in terms of $\sqrt{2}$ or $\sqrt{3}$ it would be desirable to use that instead since it uses less memory and processing power than the gaussian sum. However, this must be done manually. Specifically for $N = 4$ the gaussian sum gives an order which is twice as large as using $\frac{1}{4\sqrt{2}}$.

```
N := 5;
k := 8*N;
dim:= Floor((3/4)*k);
K<w>:= CyclotomicField(k);
sq:= w^N + w^(-N);
if (IsSquare(k)) then
tempsqr:= Floor(Sqrt(k));
else
//Gaussian sum
tempsqr:= 0;
for j:= 0 to k-1 do
tempExponent:= (j)^2;
tempsqr:= tempsqr + w^tempExponent;
end for;
end if;
alpha:= (tempsqr)^-1;
```

```
S:=ZeroMatrix(K,dim,dim);
for i:=1 to (Floor(k/2)) do
for j:=1 to (Floor(k/2)) do
xp := ((2*(i-1))*(2*(j-1)));
S[i,j]:= w^xp;
end for;
end for;

for i:=1 to (Floor(k/2)) do
for j:=1 to (Floor(k/4)) do
xp:= ((2*(i-1))*(2*(j-1)+1));
S[i,j+Floor(k/2)]:= w^xp * sq;
end for;
end for;

for i:=1 to (Floor(k/4)) do
for j:=1 to (Floor(k/2)) do
xp:= ((2*(j-1))*(2*(i-1)+1));
S[i+Floor(k/2),j]:= w^xp * sq;
end for;
end for;

S:= S * alpha;

T:=ZeroMatrix(K,dim,dim);
```

```
for j:=1 to Floor(k/2)+1 do
xp:= 2*(j-1)^2;
T[j,j]:=w^xp;
end for;

for j:= 1 to Floor(k/4) do
xp:= ((((2*(j-1))+1)^2) mod (2*k)) + N;
T[j+Floor(k/2),j+Floor(k/2)]:= w^xp;
end for;

T2:= T^2;

A:=MatrixGroup<dim,K|S,T2>;
#A
```

Metaplectic Modular Categories

The code has been defaulted to $r = 1$ which can be changed to generate different examples. The loops at the end can be modified to print out the desired information on the quotient groups.

```
r:= 1;
N:=(2*r)+1;
K<w>:= CyclotomicField(8*N);
dim:=r+4;
GL:=GeneralLinearGroup(dim,K);
sq:= w^N + w^(-N);
```

```
if (IsSquare(N)) then
sqr:= Floor(Sqrt(N));
else
tempsqr:= 0;
for j:= 0 to (N-1) do
tempExponent:= (j)^2;
tempsqr:= tempsqr + w^(8*tempExponent);
end for;
sqr:=tempsqr;
end if;


alpha:= (2*sqr)^-1;
S:= ZeroMatrix(K,dim,dim);
//constructing S
S[1,1]:= alpha*1; S[1,2]:=alpha*1; S[2,1]:=alpha*1; S[2,2]:= alpha*1;
for i:=1 to r do
S[1,2+i]:= alpha*2; S[2+i,1]:= alpha*2;
S[2,2+i]:= alpha*2; S[2+i,2]:= alpha*2;
end for;
S[1,dim-1]:= alpha*sqr; S[1,dim]:= alpha*sqr;
S[dim-1,1]:= alpha*sqr; S[dim,1]:= alpha*sqr;
S[2,dim-1]:= -alpha*sqr; S[2,dim]:= -alpha*sqr;
S[dim-1,2]:= -alpha*sqr; S[dim,2]:= -alpha*sqr;

//B
```

```
for i:=1 to r do
for j:=1 to r do
S[2+i,2+j]:= 2*alpha*(w^(8*i*j) + w^(-8*i*j));
end for;
end for;

//C
S[dim-1,dim-1]:= alpha*sqr;
S[dim,dim]:= S[dim-1,dim-1];
S[dim-1,dim]:= -alpha*sqr;
S[dim,dim-1]:= S[dim-1,dim];



//Constructing T
T:= ZeroMatrix(K,dim,dim);
T[1,1]:= 1;
T[2,2]:= 1;
for i:=1 to r+1 do
T[2+i,2+i]:= w^(-8*i*i);
end for;
T[dim-1,dim-1]:= w^N;
T[dim,dim]:= w^N;

T2:=T^2;
As:=MatrixGroup<dim,K|S,T2>;
```

```
#As;
FactoredOrder(As);
IsSolvable(As);
IsNilpotent(As);
IsSimple(As);

if (IsSolvable(As)) then
print("Solvable Loop");
while(Order(As) ne Order((As/Center(As)))) do
As:= As/Center(As);
#As;
FactoredOrder(As);
IsSolvable(As);
IsNilpotent(As);
IsSimple(As);
end while;

else
print("Simple Loop");
while((not IsSimple(As)) and (Order(As) ne Order((As/Center(As))))) do
As:= As/Center(As);
#As;
FactoredOrder(As);
IsSolvable(As);
IsNilpotent(As);
```

```
IsSimple(As);
end while;

end if;
```