Rating firms and sensitivity analysis

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Abstract. This paper introduces a model for rating a firm's default risk based on fuzzy logic and expert system and an associated model of sensitivity analysis (SA) for managerial purposes.

The rating model automatically replicates the evaluation process of default risk performed by human experts. It makes use of a modular approach based on rules blocks and conditional implications. The SA model investigates the change in the firm's default risk under changes in the model inputs and employs recent results in the engineering literature of Sensitivity Analysis. In particular, it (i) allows the decomposition of the historical variation of default risk, (ii) identifies the most relevant parameters for the risk variation, and (iii) suggests managerial actions to be undertaken for improving the firm's rating.

Keywords. Credit rating, default risk, fuzzy logic, fuzzy expert system, sensitivity analysis.

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1 Introduction

This paper presents a model for rating firms combined with a model for accomplishing sensitivity analysis (SA). The rating model is based on a fuzzy expert system, the SA model is based on recent results aiming at quantifying the impact of the model's input factors on the model's output change.

Several recent works analyze credit rating under managerial and financial perspectives (Bonsall IV et al. 2017, Griffin, Hong and Ryou 2018, Kouvelis and Zhao 2017, Kisgen 2006, Karampatsas et al. 2014, An and Chan 2008, Lim et al. 2017).

The evaluation of credit rating may be performed via several different quantitative methods (Hwang 2013a, 2013b, Pfeuffer et al. 2019, Doumpos et al. 2015, Doumpos and Zopounidis 2011, Angilella and Mazzù 2017). However, financial (quantitative) data are often insufficient or even unreliable for measuring the credit rating of an enterprise where judgmental, qualitative information is to be considered (Angilella and Mazzù 2015). Fuzzy logic is suited for providing financial analyses and for building rating models whose functioning is influenced by human judgment and whose parameters are vague and difficult to express into precise real numbers (Chen and Chiou 1999, Syau et al. 2001, Jiao et al. 2007. See also Levy et al. 1991, Peña et al. 2018, Bai et al. 2019).

Fuzzy logic is often employed along with techniques of artificial intelligence. Typically, expert systems, artificial neural networks, machine learning, and hybrid intelligence systems are applied to almost every area of management (see Ignizio 1990 for an overview of expert systems). Several studies show that artificial intelligence achieves high performance in predicting credit rating, in terms of explanatory power and stability (e.g., Lee 2007, Kim and Ahn 2012, Huang et al. 2004). As for finance, applications of artificial intelligence are numerous (Brown et al. 1990, Matsatsinis et al. 1997, Bahrammirzaee 2010, Dirks et al. 1995, Ferreira et al. 2019, Dawood 1996, Volberda and Rutges 1999, Lincy and John 2016, Chen and Li 2014).

Fuzzy expert systems have been advanced as well in several areas of finance and management (Magni et al. 2004, Marzouk and Aboushady 2018, Magni et al. 2006, Malagoli et al. 2007, Cheng et al. 2013, Doumpos and Figueira 2018, Vassiliou 2013, Agliardi and Agliardi 2009).

We present a rating model which is an input-output model formally represented by a fuzzy expert system: It automatically provides a firm's default risk (model output) and its associated credit rating on the basis of 18 selected key drivers (model inputs). The latter are aggregated in a modular approach via "if-then" implications applied to fuzzy numbers. As such, it is capable of taking into account both quantitative and qualitative financial and managerial variables. The proposed rating model is a judgmental expert-based system for credit risk assessment, differing from widely adopted statistical and machine learning approaches. Statistical models are based on mathematical descriptions aiming at representing the patterns in the economic data via selecting an optimal method a priori; machine learning techniques are computational-based, data-driven algorithms, less relying on assumptions about data (Galindo and Tamayo 2000). In contrast, judgmental expert-based systems reproduce the evaluation and decision processes performed by human experts, through logical inference, knowledge base, and heuristics. More specifically, in comparing machine learning with expert systems, both belong to the artificial intelligence techniques class, but machine learning is an adaptive information processing system using learning and generalization capabilities whereas an expert system is a computer system containing a well-structured, static body of knowledge imitating expert skills, capable to solve difficult problems requiring significant human expertise (Bahrammirzaee 2010).

In addition, we associate the fuzzy expert system with a sensitivity analysis (SA) model which enables performing a detailed financial and managerial analysis, proposing a combination method which has been analogously applied to other research areas of management and policy making, such as the assessment of ecological and human sustainability of countries (Grigoroudis et al. 2014, Andriantiatsaholiniaina et al. 2004). In particular, given a change in the output of a model and given two associated sequences of input parameters, a SA technique enables measuring the impact of each input parameter on the output change. Also, it enables ranking the parameters according to their importance. In such a way, it is possible to understand the reasons why the output change has occurred and the appropriate actions that may lead the decision maker toward an improvement in the output change by a proper management of the key drivers.

SA techniques are widely employed in various areas of finance and management (Huefner 1972, Luo et al. 2015, Donders et al. 2018, Madu 1988, Borgonovo and Peccati 2004, 2006, Borgonovo et al. 2010, Délèze and Korkeamäki 2018, Talavera et al. 2010, Percoco and Borgonovo 2012, Marchioni and Magni 2018, Chapman et al. 1984, Vázquez-Abad and LeQuoc 2001, Parnes 2010).

Among the various SA techniques, a recent approach is based on the notion of Finite Change Sensitivity Index (FCSI) (Borgonovo 2010a, 2010b), which we employ in our model. The FCSI represents a powerful analytical tool, which is used for studying a finite change in the model output. We aim at applying this SA technique to the rating model in order to identify the causes of variation in the default risk and then analyze the effects of different financial and managerial actions on the prospective rating.

However, while the FCSIs provide the correct ranking of the input factors in terms of their impact on the output change, they are not aimed at providing an exact decomposition of the output change, in the sense that the sum of the contributions of the input factors to the output change is not equal to the output change, owing to some double-counting of interactions among variables. In other words, given a change in the default risk and given a set of n economic parameters that affect the model output, the FCSI provides the parameter's contribution to an output change which includes individual contribution and joint interactions with the other model inputs. However, the sum of all the FCFIs does not equate the output change.[1] For this reason, we fine-tune the FCSI notion via a duplication-free procedure and supply a "clean FCSI". We apply it to the rating model for managerial and financial analysis for exactly decomposing the contributions of the input factors to the output change. We

[1] For example, suppose the selected inputs are $n = 3$. It might turn out that 45% of the output change has been generated by the change of parameter 1, 35% has been generated by the change of parameter 2, and 30% has been generated by the change of parameter 3. The sum of the contributions is $0.45 + 0.35 + 0.30 = 1.1 \neq 1$.

call the combined model (fuzzy expert system + SA model) the "Default Risk & Sensitivity Model" (DRSM): It rates the firm and, at the same time, ranks the parameters affecting the risk change in terms of their importance. We show how the DRSM may be applied for (i) rating a firm automatically, based on a given set of input parameters, (ii) identifying the causes of the change of the default risk in two different years, (iii) decomposing the change in the default risk and ranking the key drivers in terms of impact on such a change. For illustrative purposes, we also apply the DRSM to an Italian-controlled industrial company. We provide its rating in various years and analyze the change of the default risk and the change in rating in different years. Furthermore, while DRSM merges a fuzzy expert system with a model of sensitivity analysis, we stress that the proposed SA application for credit rating can be usefully combined to any approach for rating firms such as statistical and machine learning techniques adopting analogous fuzzy-logic models; in particular, SA may be applied as a tool enhancing the interpretability and comprehensibility of fuzzy models, whose comprehension is often hard because of the adoption of complex rule bases. Furthermore, SA is helpful for testing and validating the representativeness of the underlying credit scoring model: Additional simulation runs which measure the sensitivity of the output under changes in the various inputs may corroborate the model or reveal the need for revising some of the choices made in the model setup (Pianosi et al. 2016).

The remainder of the paper is structured as follows. Section 2 presents the fuzzy expert system. Section 3 illustrates the basic notions of sensitivity analysis and defines the FCSI and its use. Section 4 fine-tunes the FCSI via a duplication-free procedure and provides an exact decomposition of the output change of a model. Section 5 applies the DRSM (rating model + clean FCSI) to an Italian-controlled industrial company and shows some possible uses of it. Some remarks conclude the paper.

2 Fuzzy-logic expert system for credit rating

The current work introduces a credit rating model based on fuzzy logic and expert system, which derives the default risk and the rating class of a corporation according to rules blocks based on conditional implications. Our fuzzy-logic rating model considers a set of 18 economic and financial variables (the model inputs), both quantitative (such as Leverage, OCF-to-Debt, EBITDA on Sales) and qualitative (as Product Positioning and Industry Prospects), which are grouped under a managerial and financial perspective in first-level intermediate variables which are in turn gathered to form second-level intermediate variables which are in turn grouped to form a third level of intermediate variables. Finally, the latter determine the firm's default risk (model output). Figure 1 represents the conceptual map of variables aggregation from the model inputs to the Default Risk through the various intermediate steps (see also descriptions of input and intermediate variables in the Appendix).[2]

[2]The inputs may themselves be considered 0-level intermediate variables, determined by lower-level basic parameters. For example, the Return On Investment (ROA) is a function of three parameters: NOPAT, R&D and invested capital (see Appendix).

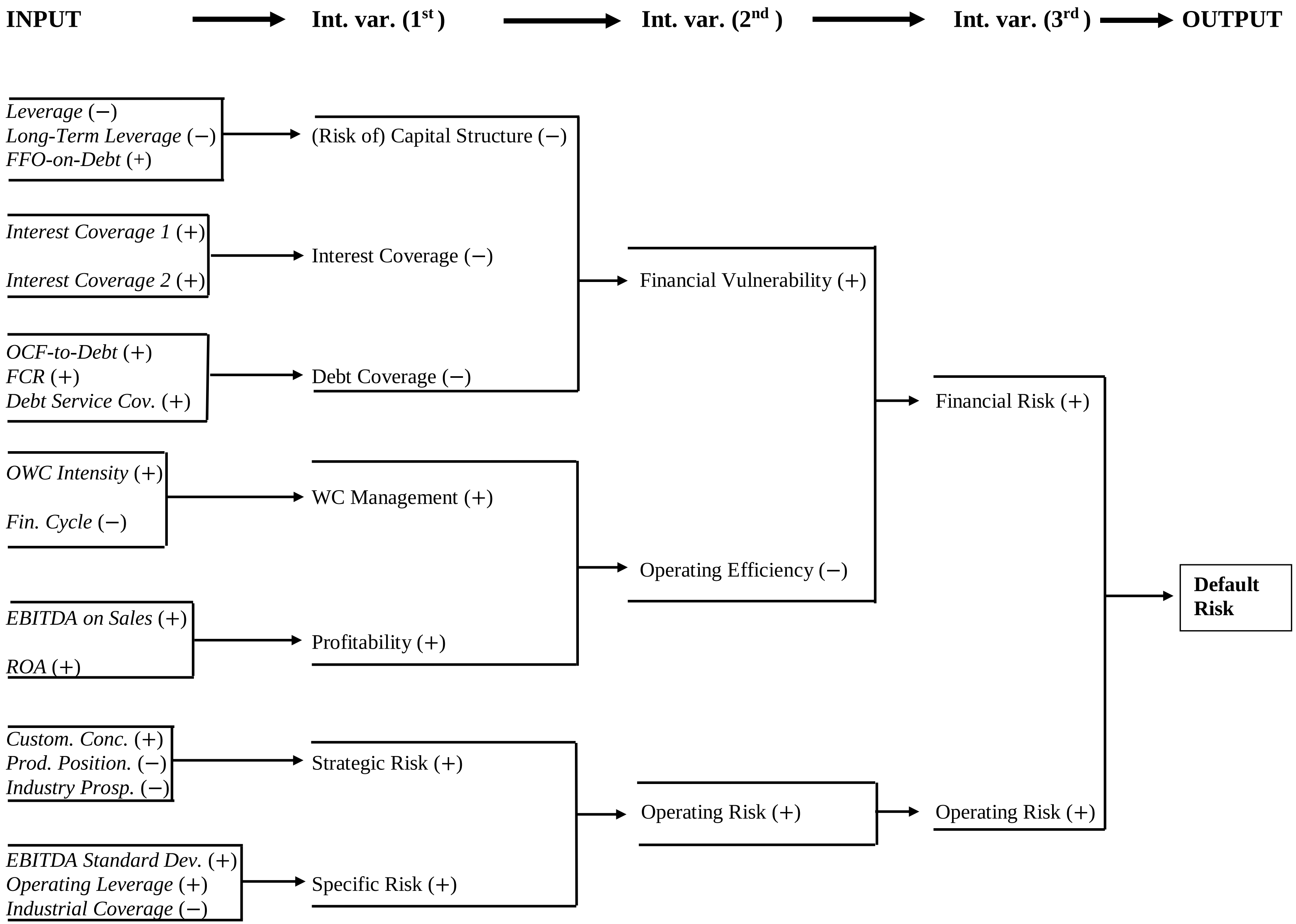

Figure 1: Conceptual map of variables aggregation

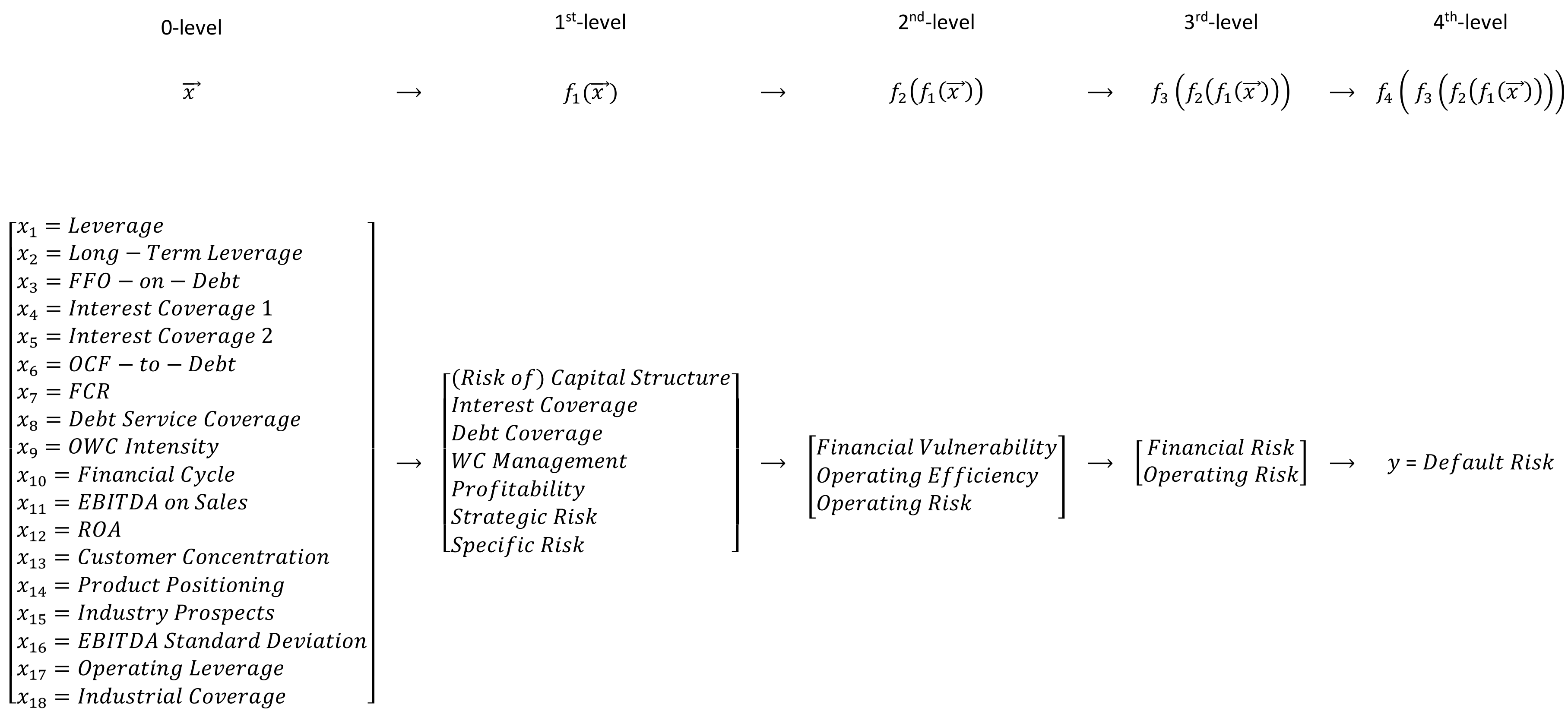

Figure 2: The expert system as a composed function

The approach is then modular and gives rise to an evaluation tree that is run from branches to trunk. The link between the set of the input parameters and the output may be represented as a function of the 18 variables, $x_i, i = 1, 2, \ldots, 18$ affecting the dependent variable, y (Default risk), so that $y = f(\vec{x})$, where $\vec{x} = (x_1, x_2, \ldots, x_{18})$. For any given value of $\vec{x}$, the model automatically provides the default risk. Mathematically, the model is a composed function. There are 4 composing functions, whose values represent the four steps through which the inputs are processed and the output is fleshed out:

$$\vec{x} \to f_1(\vec{x}) \to f_2(f_1(\vec{x})) \to f_3(f_2(f_1(\vec{x}))) \to f_4(f_3(f_2(f_1(\vec{x})))) = y.$$

As shown in Figure 2, starting from 18 parameters, one gets a vector of 7 components (via f_1), then a vector of 3 components (via f_2), then a vector of 2 components (via f_3) and, finally one single component, the model's output (via f_4). Figure 1 is the representation of the fuzzy expert system as a conceptual map, Figure 2 is the same expert system described as a composed function.

Each composing function is either monotonically increasing or monotonically decreasing with respect to prior-level intermediate variables. Figure 1 indicates monotonicity via plus (+) or minus (−) sign. Specifically, a given variable z may affect the next-level variable q positively (+) or negatively (−). Variable z affects variable q positively if q increases (decreases) whenever z increases (decreases); it affects q negatively if q decreases (increases) whenever z increases (decreases).

Each variable of the model (inputs, intermediate variables, model output) can be associated with several attributes, which are represented graphically by fuzzy numbers and a membership function. For instance, the input factor Fixed Charge Ratio (FCR, defined in the Appendix) is characterized by the membership function reported in Figure 3. The horizontal axis collects the numerical values of FCR, while the vertical axis reports the membership degrees (or activation levels) of each linguistic attribute. For each value of FCR, all the attributes are activated at a certain degree, ranging from 0 to 1. For example, a FCR equal to 1.05 is at the same time:

- low at degree 0;
- medium low at degree 0.22;
- medium high at degree 0.78;
- high at degree 0.

The intermediate variables at any level and the model output Default Risk are evaluated using rules blocks built upon conditional ("if-then") implications which map the variables attributes at the previous level onto the attributes of the next level through a modular approach. For instance, in Table 1 we report the rules block for determining (the risk of) the Capital Structure (first-level intermediate variable), depending on the input variables Leverage, Long-Term Leverage, and FFO-on-Debt. For example, the first four-rule block informs about the

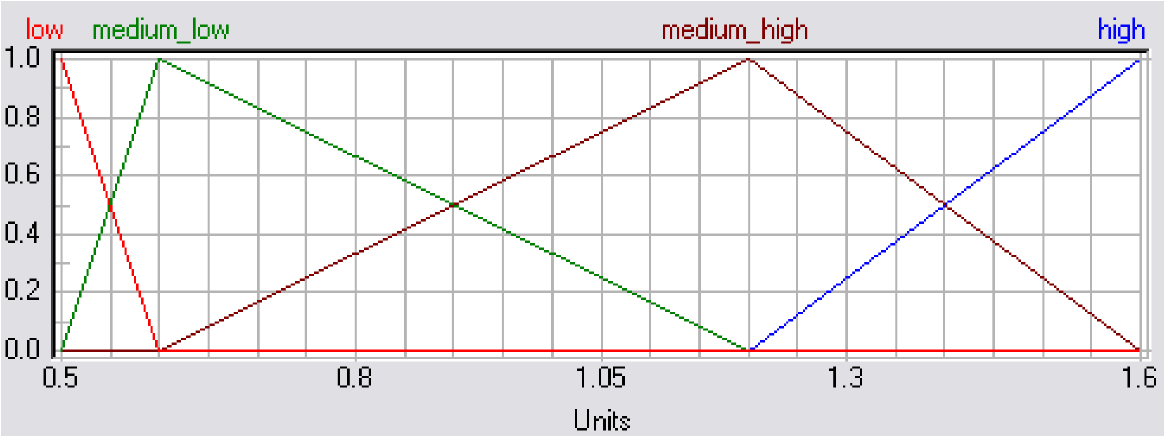

Figure 3: Membership function of FCR

degree of risk of the capital structure under changes in the FFO-on-Debt while Leverage and Long-Term Leverage are kept at low levels. Note that, for increasing value of FFO-on-Debt, the risk level of the Capital Structure increases (i.e., the capital structure becomes riskier), meaning that the weight of debt becomes higher. For example, the fourth rule may be read as follows:

IF	
Leverage	is Low
Long-Term Leverage	is Low
FFO-on-Debt	is High
THEN	
(Risk of) Capital Structure	is Very High

Each rule of any block is activated simultaneously, at a certain degree, precisely because each variable has a certain membership degree for each attribute. All variables (including the output, Default Risk) are fuzzy numbers which are associated with membership degrees.

Reading Figure 1 backward from output to inputs, one can see that the Default Risk depends on two variables, Financial Risk and Operating Risk. Financial Risk depends in turn by Financial Vulnerability and Operating Efficiency, each of which in turn depends on other variables; specifically, Financial Vulnerability depends on (Risk of) Capital Structure, Interest Coverage, Debt Coverage while Operating Efficiency depends on WC Managament and Profitability. In turn, each of the latter depends on some group of inputs. Likewise, Operating Risk depends on Strategic Risk and Specific Risk,[3] which in turn depend on different groups of inputs.

Whenever the input vector is selected, the output (Default Risk) is automatically provided. Table 2 reports the rules block for Default Risk, conditionally to Financial Risk and Operating Risk (e.g., focusing on the fourth rule, if Financial Risk is AAA and Operating Risk is BBB, then the Default Risk is evaluated at AA). Note that the Financial Risk, the Operating Risk, and the Default Risk are described in terms of eight rating classes, from the safest one, AAA,

[3]Operating Risk is associated to Financial Risk to determine the Default Risk, so it is repeated as a 2^{nd}-level and 3^{rd}-level intermediate variable. In terms of composing function, one may interpret it as an identity function.

Table 1: Rules block for (Risk of) Capital Structure

IF			THEN
Leverage	**Long-Term Leverage**	**FFO-on-Debt**	**(Risk of) Capital Structure**
low	low	low	high
low	low	medium-low	high
low	low	medium-high	very-high
low	low	high	very-high
medium-low	low	low	medium-high
medium-low	low	medium-low	high
medium-low	low	medium-high	high
medium-low	low	high	very-high
medium-high	low	low	medium-low
medium-high	low	medium-low	medium-high
medium-high	low	medium-high	high
medium-high	low	high	high
high	low	low	medium-low
high	low	medium-low	medium-low
high	low	medium-high	medium-high
high	low	high	high
low	medium-low	low	medium-high
low	medium-low	medium-low	high
low	medium-low	medium-high	high
low	medium-low	high	very-high
medium-low	medium-low	low	medium-low
medium-low	medium-low	medium-low	medium-high
medium-low	medium-low	medium-high	high
medium-low	medium-low	high	high
medium-high	medium-low	low	medium-low
medium-high	medium-low	medium-low	medium-low
medium-high	medium-low	medium-high	medium-high
medium-high	medium-low	high	high
high	medium-low	low	low
high	medium-low	medium-low	medium-low
high	medium-low	medium-high	medium-low
high	medium-low	high	medium-high
low	medium-high	low	medium-low
low	medium-high	medium-low	medium-high
low	medium-high	medium-high	high
low	medium-high	high	high
medium-low	medium-high	low	medium-low
medium-low	medium-high	medium-low	medium-low
medium-low	medium-high	medium-high	medium-high
medium-low	medium-high	high	high
medium-high	medium-high	low	low
medium-high	medium-high	medium-low	medium-low
medium-high	medium-high	medium-high	medium-low
medium-high	medium-high	high	medium-high
high	medium-high	low	very-low
high	medium-high	medium-low	low
high	medium-high	medium-high	medium-low
high	medium-high	high	medium-low
low	high	low	medium-low
low	high	medium-low	medium-low
low	high	medium-high	medium-high
low	high	high	high
medium-low	high	low	low
medium-low	high	medium-low	medium-low
medium-low	high	medium-high	medium-low
medium-low	high	high	medium-high
medium-high	high	low	very-low
medium-high	high	medium-low	low
medium-high	high	medium-high	medium-low
medium-high	high	high	medium-low
high	high	low	very-low
high	high	medium-low	very-low
high	high	medium-high	low
high	high	high	medium-low

Table 2: Rules block for Default Risk

IF		THEN
Financial Risk	**Operating Risk**	**Default Risk**
AAA	AAA	AAA
AAA	AA	AAA
AAA	A	AAA
AAA	BBB	AA
AAA	BB	AA
AAA	B	AA
AAA	CCC	AA
AAA	CC	AA
AA	AAA	AA
AA	AA	AA
AA	A	AA
AA	BBB	A
AA	BB	A
AA	B	A
AA	CCC	A
AA	CC	A
A	AAA	A
A	AA	A
A	A	A
A	BBB	A
A	BB	BBB
A	B	BBB
A	CCC	BBB
A	CC	BBB
BBB	AAA	A
BBB	AA	BBB
BBB	A	BBB
BBB	BBB	BBB
BBB	BB	BBB
BBB	B	BB
BBB	CCC	BB
BBB	CC	BB
BB	AAA	BBB
BB	AA	BB
BB	A	BB
BB	BBB	BB
BB	BB	BB
BB	B	BB
BB	CCC	B
BB	CC	B
B	AAA	BB
B	AA	BB
B	A	BB
B	BBB	BB
B	BB	BB
B	B	BB
B	CCC	BB
B	CC	B
CCC	AAA	B
CCC	AA	B
CCC	A	B
CCC	BBB	CCC
CCC	BB	CCC
CCC	B	CCC
CCC	CCC	CCC
CCC	CC	CCC
CC	AAA	CCC
CC	AA	CCC
CC	A	CCC
CC	BBB	CC
CC	BB	CC
CC	B	CC
CC	CCC	CC
CC	CC	CC

to the riskiest one, CC. The output provided by the rule block, the Default Risk, is a fuzzy number. Through a defuzzification procedure, the default risk is automatically converted into a crisp (real) number in the normalized interval $[0, 1]$.[4]

Finally, a conversion table (Table 3) converts the (crisp) default risk into a rating class. Given a sequence of inputs, there automatically corresponds a firm's default risk and, hence, a class of rating. The logical chain is then as follows:

Inputs (fuzzy numbers)	$\vec{x}$
$\Longrightarrow$ first-level intermediate variables (fuzzy numbers)	f_1
$\Longrightarrow$ second-level intermediate variables (fuzzy numbers)	f_2
$\Longrightarrow$ third-level intermediate variables (fuzzy numbers)	f_3
$\Longrightarrow$ Default Risk (fuzzy number)	$f_4 = y$
$\Longrightarrow$ Default Risk (crisp number)	defuzzification
$\Longrightarrow$ Rating class (letter)	conversion

Table 3: Conversion table from default risk to rating class

Default risk	**Rating class**
[0, 0.125)	AAA
[0.125, 0.25)	AA
[0.25, 0.375)	A
[0.375, 0.5)	BBB
[0.5, 0.625)	BB
[0.625, 0.75)	B
[0.75, 0.875)	CCC
[0.875, 1]	CC

The 18 attributes selected represent a minimum set of meaningful risk components and profiles. The choice depends on our operational experience in corporate finance practice (debt restructuring in particular) and on the fact that they are commonly used by rating agencies' models. Therefore, the choice of this minimum set reflects the knowledge base of the experts. However, the fuzzy expert system is flexible enough for customization: It may be augmented with other appropriate input factors, which may be aggregated via if-then rules in a modular approach, as previously seen.

In general, statistical data might be collected and processed to determine and tune memberships degrees and decisions rules. Industry prospects, for example, might be based on data available from accredited sources; product positioning might be based on data from interviews to a statistically significant sample of customers; accounting data such as ROA might be compared with a sample of comparable firms of the same sector and membership degrees might be evaluated on the basis of the sample mean. Even in our model, study sectors and comparisons with industry means as well as our expertise have been relevant for determining the

[4]The defuzzification procedure applied to the Default Risk uses the Center of Maximum method (CoM) (von Altrock 1995).

membership degrees. Decision rules in our model are based on our expertise as advisors and academics, but automatic extensions may be conceived in several ways, with the purpose of automatically infer the fuzzy rules based on large samples of historic data. Large amounts of historical data make it possible to use different types of approaches, based on the knowledge or technology or types of analysis software available; for example, neuro-fuzzy models, used to model the membership functions as well as to create the blocks of rules, or genetic algorithms or the widely employed fuzzy-clustering methods. This is particularly important if the model is enriched with a high number of inputs, which would make the work of the experts extremely burdensome and characterized by a significant degree of inaccuracy. In this respect, there may be a trade-off between interpretability and automatic learning methods and several authors have dealt with the problem of rule generation (see Guillaume 2001, Gómez-Skarmeta et al. 1999, Zhang et al. 2009, Xiao and Liu 2005). In Guillaume and Charnomordic (2011) a free software is proposed, available on the web, which allows the interpretation of systems built automatically from the data, in all phases of design.

3 Sensitivity Model and FCSI

In this section, we associate the fuzzy expert system described above with a model of sensitivity analysis (SA). The expert system and the SA model form what we call the Default Risk & Sensitivity Model (DRSM).

SA is the "study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the model input" (Saltelli et al. 2004, p. 45). Given a model and a set of inputs (parameters), SA measures the parameters' influence in terms of variability of the model output. Specifically, SA models aim to investigate the variation of the objective function (in our case, the firm's default risk) under changes in the model inputs, also aiming at identifying the most influential risk factors affecting the model output.

Many SA techniques are defined in the literature (see Borgonovo and Plischke 2016, Pianosi et al. 2016, for review of SA methods) and the choice of technique depends on several factors, among which the purpose of the analysis and the size of the variation of the parameters.

In our case, the default risk variation caused by changes in key drivers or groups of key drivers, is analyzed in both chronological and managerial perspectives:

- DRSM decomposes the historical realized variation of the enterprise default risk into the effects of key parameters and identifies the main reasons of rating variation across time
- DRSM suggests managerial actions which should be undertaken for improving the rating, especially for increasing the success of complex financial operations such as bond issues, mergers and acquisitions, and debt restructuring.

The scope of DRSM is multiple and concerns several dimensions of analysis:

- it supports the evaluator in identifying the effects of each parameter on the rating variation

- it enables accomplishing a selective analysis in terms of groups of parameters. For example, it measures the impact of the following main groups on the default risk profile: (i) Financial Vulnerability, (ii) Operating Efficiency, (iii) Operating Risk
- it enables ranking any group of variables according to their relevance on the default risk variation
- it enables identifying the maximum effect of a variable or group of variables on the default risk
- it supports the financial manager in her/his activities of financial planning and optimization, and in functions of programming, control and capital structuring
- it offers managerial actions for improving and controlling the credit risk profile of the enterprise.

It is worth noting that

- the DRSM can be performed even starting from primitive, 0-level economic and financial variables as they result from the operations (such as revenues, COGS, long-term debt), not just from worked drivers such as indices and ratios (e.g. Leverage, Long-Term Leverage, FFO-on-Debt. See also footnote 2)
- the application of SA is independent of the adopted rating model: While we present it in conjunction with the fuzzy expert system illustrated in the previous sections, the SA model is readily available for any algorithm and any set of parameters defining any possible rating model (i.e., the SA model does not depend on the credit rating model).

Finite Change Sensitivity Indices (FCSIs; Borgonovo 2010a, 2010b) represent a Sensitivity Analysis technique focusing on the output change due to a finite variation of the inputs. The FCSI technique is applicable for whatever parameters variation; it does not require any peculiar variation scheme or sufficiently small parameters changes.[5]

Let f be the objective function, defined on the *parameter space* X, which maps the vector of inputs (or parameters or key drivers) $x = (x_1, x_2, \ldots, x_n) \in X$ onto the model output $y(x)$:

$$f : X \subset \mathbb{R}^n \to \mathbb{R}, \quad y = f(x), \quad x = (x_1, x_2, \ldots, x_n) . \tag{1}$$

Let $x^0 = (x_1^0, \ldots, x_n^0)$ be the base (or initial) value of the parameters and $f(x^0)$ be the corresponding model output. The parameters vary from x^0 to $x^1 = (x_1^1, x_2^1, \ldots, x_n^1) \in X$, the so-called realized value, and the related output is $f(x^1)$. The output variation is $\Delta f = f(x^1) - f(x^0)$.

Let $(x_i^1, x_{(-i)}^0) = (x_1^0, x_2^0, \ldots, x_{i-1}^0, x_i^1, x_{i+1}^0, \ldots, x_n^0)$ be obtained by varying the parameter x_i to the new value x_i^1, while the remaining $n - 1$ parameters are fixed at x^0. Similarly,

[5] FCSIs are based on the properties of functional ANOVA decomposition for finite changes (Rabitz and Alis 1999, Borgonovo 2010b).

$(x_i^1, x_j^1, x^0_{(-i,j)}) = (x_1^0, x_2^0, \dots, x_{i-1}^0, x_i^1, x_{i+1}^0, \dots, x_{j-1}^0, x_j^1, x_{j+1}^0, \dots, x_n^0)$ is the vector of inputs assuming that x_i and x_j are set to the new values, while the remaining $n-2$ are unvaried, and so forth for all j-tuples of inputs, $j = 1, 2, \dots, n$.

Two viable definitions of Finite Change Sensitivity Indices are First Order FCSI and Total Order FCSI. The First Order FCSI of parameter x_i considers the individual effect of x_i on the variation of f (Borgonovo 2010b):

$$\Delta_i^1 f = f(x_i^1, x^0_{(-i)}) - f(x^0) \tag{2}$$

and, in normalized version, $\Phi_i^{1,f} = \frac{\Delta_i f}{\Delta f}$.

The Total Order FCSI of a parameter, instead, measures the total effect of the input on f, including both the individual contribution and the interactions between the parameter and the other parameters. The interaction between x_i and x_j, denoted as $\Delta_{i,j} f$, is the portion of $f(x_i^1, x_j^1, x^0_{(-i,j)}) - f(x^0)$ that is not explained by the individual effects $\Delta_i^1 f$ and $\Delta_j^1 f$: $\Delta_{i,j} f = f(x_i^1, x_j^1, x^0_{(-i,j)}) - f(x^0) - \Delta_i^1 f - \Delta_j^1 f$. Likewise, the interaction between the triplet of inputs x_i, x_j and x_h, identified as $\Delta_{i,j,h} f$, is the portion of $f(x_i^1, x_j^1, x_h^1, x^0_{(-i,j,h)}) - f(x^0)$ that is not explained by the individual effects and by the interactions between any pair of inputs x_i, x_j and x_h:

$$\Delta_{i,j,h} f = f(x_i^1, x_j^1, x_h^1, x^0_{(-i,j,h)}) - f(x^0) - \Delta_i^1 f - \Delta_j^1 f - \Delta_h^1 f - \Delta_{i,j} f - \Delta_{i,h} f - \Delta_{j,h} f$$

(analogously for a group of $s > 3$ parameters). The variation of f between the base and the realized case, Δf, can be written as the sum of individual effects and interactions between parameters and groups of parameters (Borgonovo 2010b):[6]

$$\Delta f = \overbrace{\sum_{i=1}^{n} \Delta_i^1 f}^{\text{individual effects}} + \underbrace{\overbrace{\sum_{i_1<i_2} \Delta_{i_1,i_2} f}^{\text{pairs}} + \overbrace{\sum_{i_1<i_2<i_3} \Delta_{i_1,i_2,i_3} f}^{\text{triplets}} + \dots + \overbrace{\sum_{i_1<i_2\dots<i_s} \Delta_{i_1,i_2,\dots,i_s} f}^{s\text{-tuples}} + \dots + \overbrace{\Delta_{i_1,i_2,\dots,i_n} f}^{n\text{-tuple}}}_{\text{interactions}}, \tag{3}$$

where the general term $\sum_{i_1<i_2\dots<i_s} \Delta_{i_1,i_2,\dots,i_s} f$ is the sum of the interactions between groups of s parameters.

The Total Order FCSI of x_i, denoted as $\Delta_i^T f$, is defined as the sum of the individual effect of x_i and the interaction effect of x_i, which is the sum of any interaction involving x_i, identified as $\Delta_i^I f$:

$$\Delta_i^T f = \Delta_i^1 f + \Delta_i^I f = \Delta_i^1 f + \sum_{\substack{i_1<i_2 \\ i\in\{i_1,i_2\}}} \Delta_{i_1,i_2} f + \dots + \sum_{\substack{i_1<i_2\dots<i_s \\ i\in\{i_1,i_2,\dots,i_s\}}} \Delta_{i_1,i_2,\dots,i_s} f + \dots + \Delta_{i_1,i_2,\dots,i_n} f \tag{4}$$

and the normalized Total Order FCSI is $\Phi_i^T = \frac{\Delta_i^T f}{\Delta f}$.

[6]Each interaction between parameters and group of parameters is counted only once in this formula.

Borgonovo (2010b, Proposition 1) showed that $\Delta_i^T f$ is also obtained as

$$\Delta_i^T f = f(x^1) - f(x_i^0, x_{(-i)}^1), \; \forall i = 1, 2, \ldots, n, \tag{5}$$

where $(x_i^0, x_{(-i)}^1)$ is the point with each parameter equal to the realized value x^1, except for the parameter x_i which is equal to x_i^0.

Considering a subset of parameters $S_k = \{x_{i_1}, x_{i_2}, \ldots, x_{i_s}\}$, the relevance of the subset is defined from the notion of importance measures of a single parameter in (2) and (5). The First Order FCSI of S_k is $\Delta^1_{S_k} f = f(x^1_{(i_1,i_2,\ldots,i_s)}, x^0_{(-(i_1,i_2,\ldots,i_s))}) - f(x^0)$, that we denote as $f(x^1_{S_k}, x^0_{(-S_k)}) - f(x^0)$, and the Total Order FCSI is $\Delta^T_{S_k} f = f(x^1) - f(x^0_{(i_1,i_2,\ldots,i_s)}, x^1_{(-(i_1,i_2,\ldots,i_s))})$, which can be denoted as $f(x^1) - f(x^0_{S_k}, x^1_{(-S_k)})$.
Furthermore, given a pair of disjoint subsets of parameters (i.e. whose intersection is the empty set), here denoted as S_k and S_l, the interaction between S_k and S_l is $\Delta_{S_k,S_l} f = f(x^1_{S_k}, x^1_{S_l}, x^0_{(-S_k,S_l)}) - f(x^0) - \Delta^1_{S_k} f - \Delta^1_{S_l} f$; the interaction between an increasing group of disjoint subsets (e.g., a triplet of subsets) can be calculated similarly to an increasing group of parameters.

Finally, consider a group of d disjoint subsets whose union is the whole set of parameters; Δf can be decomposed in the sum of individual effects of any subset and the interactions between any group of subsets, similarly to (3). The Total Order FCSI of the subset S_k, $\Delta^T_{S_k} f$, can be calculated as the sum of its individual effect $\Delta^1_{S_k} f$ and its interaction effect $\Delta^I_{S_k} f$, defined as the sum of any interaction involving S_k, consistently with equation (4):

$$\Delta^T_{S_k} f = \Delta^1_{S_k} f + \Delta^I_{S_k} f = \Delta^1_{S_k} f + \sum_{\substack{k_1 < k_2 \\ k \in \{k_1, k_2\}}} \Delta_{S_{k_1}, S_{k_2}} f + \ldots + \sum_{\substack{k_1 < k_2 \cdots < k_s \\ k \in \{k_1, k_2, \ldots, k_s\}}} \Delta_{S_{k_1}, S_{k_2}, \ldots, S_{k_s}} f + \cdots + \Delta_{S_{k_1}, S_{k_2}, \ldots, S_{k_d}} f.$$

Despite its usefulness, the definition of Total Order FCSI does not provide a clean decomposition of the output change in terms of Total FCSIs. In other words, the sum of the parameters' effects is not equal to the function variation.

The reason is that (4) includes duplications of the interactions between pairs, triplets, s-tuples. More precisely, the summand $\sum_{i_1<i_2} \Delta_{i_1,i_2} f$ includes twice the interaction between any pair of parameters, the summand $\sum_{i_1<i_2<i_3} \Delta_{i_1,i_2,i_3} f$ contains three times the interaction between any triplet of parameters, and, in general, $\sum_{i_1<i_2\cdots<i_s} \Delta_{i_1,i_2,\ldots,i_s} f$ contains s times the interactions between any s-tuple of parameters. Conversely, in (3), the interaction terms only appears once. As a result:

$$\Delta_1^T f + \Delta_2^T f + \ldots + \Delta_n^T f \neq \Delta f$$

or, dividing by Δf,

$$\Phi_1^T + \Phi_2^T + \ldots + \Phi_n^T \neq 1.$$

This means that the Total FCSIs do not sum up to 100% of the output change: It either explains less or more than 100%.

Example 1. Let f be the market value of the equity of a firm, depending on the share price p and the number of shares q. The vector of inputs is $x = (p, q)$ and the equity market value is $f(p, q) = p \cdot q$. We assume that the initial state is $x^0 = (p^0, q^0) = (10,\ 200)$, which implies that the equity value is $f(p^0, q^0) = p^0 \cdot q^0 = 10 \cdot 200 = 2,000$; we also assume that, after one year, price and number of share have changed to $x^1 = (p^1, q^1) = (13,\ 300)$, so that the market value of equity is $f(p^1, q^1) = 13 \cdot 300 = 3,900$. The change in the equity value is then $\Delta f = f(x^1) - f(x^0) = 3,900 - 2,000 = 1,900$. We aim at identifying the relevance of the share price and the number of share in terms of the variation of the market value of equity. From eq. (2), the First Order FCSI of share price is $\Delta_p^1 f = f(p^1, q^0) - f(p^0, q^0) = 13 \cdot 200 - 10 \cdot 200 = 600$ and the First Order FCSI of q is $\Delta_q^1 f = f(p^0, q^1) - f(p^0, q^0) = 10 \cdot 300 - 10 \cdot 200 = 1,000$. The interaction between p and q, $\Delta_{p,q} f$ is equal to the interaction effect of both the parameters:

$$\begin{aligned}
\Delta_{p,q} f &= f(p^1, q^1) - f(p^0, q^0) - \Delta_p^1 f - \Delta_q^1 f \\
&= 13 \cdot 300 - 10 \cdot 200 - 600 - 1,000 \\
&= 300 \\
&= \Delta_p^I f = \Delta_q^I f.
\end{aligned}$$

However, from (4), the Total Order FCSI of the share price is $\Delta_p^T f = \Delta_p^1 f + \Delta_p^I f = 600 + 300 = 900$ and the Total Order FCSI of the number of shares is $\Delta_q^T f = \Delta_q^1 f + \Delta_q^I f = 1,000 + 300 = 1,300$.[7] Therefore, the sum of the Total Order FCSIs is different from Δf:

$$\Delta_p^T f + \Delta_q^T f = 900 + 1,300 = 2,200 \neq 1,900 = \Delta f.$$

The reason is that the interaction term between price and number of shares is included in both $\Delta_p^T f$ and $\Delta_q^T f$, so there is double-counting that prevents the correct decomposition of the output change. Equivalently, one may write

$$\Phi_p^T + \Phi_q^T = (900/1,900) + (1,300/1,900) = 0.4737 + 0.6842 = 1.1579 \neq 1.$$

In this case, the Total FCSI explains too much. $\Diamond$

We now solve the problem by introducing a duplication-cleaning procedure which eliminates the redundant, multiple interactions and allows a complete and exact decomposition of the output change through the Clean Total Order FCSIs.

4 Cleaning the Total Order FCSI

We fine-tune the FCSI by defining the *clean* interaction effect of parameter x_i, as the interaction effect $\Delta_i^I f$ multiplied for a special corrective factor α. Denoting as $\Delta_i^{I*} f$ the clean interaction

[7] The Total Order FCSIs can also be determined from (5): $\Delta_p^T f = f(p^1, q^1) - f(p^0, q^1) = 13 \cdot 300 - 10 \cdot 300 = 900$ and $\Delta_q^T f = f(p^1, q^1) - f(p^1, q^0) = 13 \cdot 300 - 13 \cdot 200 = 1,300$.

effect:

$$\Delta_i^{I*} f = \Delta_i^I f \cdot \alpha, \tag{6}$$

where we define α as

$$\alpha = \frac{\sum_{j_1<j_2} \Delta_{j_1,j_2} f + \cdots + \sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f + \cdots + \Delta_{j_1,j_2,\ldots,j_n} f}{\sum_{j=1}^n \Delta_j^I f}. \tag{7}$$

Since α is the ratio of the sum of the true interaction effects over the total imputed interaction effect, it measures the degree of redundancy (if it is smaller than 1) or deficiency (if it is greater than 1) of the Total Order FCSI. From (3), α can be rewritten as

$$\alpha = \frac{\Delta f - \sum_{j=1}^n \Delta_j^1 f}{\sum_{j=1}^n \Delta_j^I f} \tag{8}$$

whence

$$\Delta_i^{I*} f = \Delta_i^I f \cdot \frac{\Delta f - \sum_{j=1}^n \Delta_j^1 f}{\sum_{j=1}^n \Delta_j^I f} = \overbrace{\frac{\Delta_i^I f}{\sum_{j=1}^n \Delta_j^I f}}^{\text{interaction imputed to parameter } i} \cdot \overbrace{\left(\Delta f - \sum_{j=1}^n \Delta_j^1 f\right)}^{\text{overall interaction}} \tag{9}$$

The clean interaction effect $\Delta_i^{I*} f$ can then be interpreted as the component of $\Delta f - \sum_{j=1}^n \Delta_j^1 f$ according to the proportion of $\Delta_i^I f$ over the sum of $\Delta_j^I f$ for any parameter.

We can now define the Clean Total Order FCSI of parameter x_i, $\Delta_i^{T*} f$, as the sum of individual contribution and clean interaction effect of x_i:

$$\Delta_i^{T*} f = \Delta_i^1 f + \Delta_i^{I*} f \tag{10}$$

and, in normalized version, $\Phi_i^{T*} = \frac{\Delta_i^{T*} f}{\Delta f}$. We now show that the clean indeces perfectly decompose the output change, explaining the 100% of the variation.

Proposition 1. *The sum of Clean Total Order FCSIs is equal to the variation of the model output f: $\sum_{i=1}^n \Delta_i^{T*} f = \Delta f$. In normalized version, $\sum_{i=1}^n \Phi_i^{T*} = 1$.*

Proof. From (9),

$$\sum_{i=1}^n \Delta_i^{I*} f = \sum_{i=1}^n \frac{\Delta_i^I f}{\sum_{j=1}^n \Delta_j^I f} \cdot \left(\Delta f - \sum_{j=1}^n \Delta_j^1 f\right) = \Delta f - \sum_{j=1}^n \Delta_j^1 f. \tag{11}$$

From (10) and (11),

$$\sum_{i=1}^n \Delta_i^{T*} f = \sum_{i=1}^n \Delta_i^1 f + \sum_{i=1}^n \Delta_i^{I*} f = \sum_{i=1}^n \Delta_i^1 f + \Delta f - \sum_{i=1}^n \Delta_i^1 f = \Delta f. \tag{12}$$

Diving both terms of the equality by Δf, one gets $\sum_{i=1}^n \Phi_i^{T*} = 1$. □

The duplication-cleaning procedure is applicable not also for measuring the relevance of single drivers but also for determining the importance of disjoint subsets of parameters. The clean interaction effect of a subset S_k, denoted as $\Delta^{I*}_{S_k} f$, can be obtained from (6) and (9) just considering interactions between subsets, interaction effect and individual effect of the subset, instead of the effects of single parameters:

$$\begin{aligned} \Delta^{I*}_{S_k} f &= \Delta^{I}_{S_k} f \cdot \frac{\sum_{l_1<l_2} \Delta_{S_{l1},S_{l2}} f + \cdots + \sum_{l_1<l_2\cdots<l_s} \Delta_{S_{l1},S_{l2},\ldots,S_{ls}} f + \cdots + \Delta_{S_{l1},S_{l2},\ldots,S_{ld}} f}{\sum_{l=1}^{d} \Delta^{I}_{S_l} f} \\ &= \frac{\Delta^{I}_{S_k} f}{\sum_{l=1}^{d} \Delta^{I}_{S_l} f} \cdot \Big(\Delta f - \sum_{l=1}^{d} \Delta^{1}_{S_l} f\Big). \end{aligned} \tag{13}$$

Similarly, the Clean Total Order FCSI of S_k, represented as $\Delta^{T*}_{S_k} f$, can be determined from (10) by summing up the individual effect and the clean interaction effect of the subset:

$$\Delta^{T*}_{S_k} f = \Delta^{1}_{S_k} f + \Delta^{I*}_{S_k} f. \tag{14}$$

Example 2. Consider Example 1. From (9), the clean interaction effect attributable to the price, p, is

$$\begin{aligned} \Delta^{I*}_{p} f &= \frac{\Delta^{I}_{p} f}{\Delta^{I}_{p} f + \Delta^{I}_{q} f} \cdot \Big(\Delta f - \Delta^{1}_{p} f - \Delta^{1}_{q} f\Big) \\ &= \frac{300}{300+300} \cdot \Big((13 \cdot 300 - 10 \cdot 200) - 600 - 1,000\Big) \\ &= 150 \end{aligned}$$

and is equal to the clean interaction effect of q: $\Delta^{I*}_{q} f = \Delta^{I*}_{p} f = 150$. From (10), the clean Total Order FCSI of p is $\Delta^{T*}_{p} f = \Delta^{1}_{p} f + \Delta^{I*}_{p} f = 600 + 150 = 750$ and the clean Total Order FCSI of q is $\Delta^{T*}_{q} f = \Delta^{1}_{q} f + \Delta^{I*}_{q} f = 1,000 + 150 = 1,150$.[8] The sum of the clean Total Order FCSIs is equal to the variation of f: $\Delta^{T*}_{p} f + \Delta^{T*}_{q} f = 750 + 1,150 = 1,900 = \Delta f$.

5 A case study

We apply DRSM to an Italian-controlled industrial company, mainly operating in the automotive business. We have used real, publicly available, consolidated financial statements of the company in recent years. We denote as 0 the base year, and rating has been determined for four years: 0, 3, 5, and 6. The vector of inputs $x = (x_1, x_2, \ldots, x_n) \in X$ consists of the 18 economic and financial variables (rating model inputs) which we have described in Section 2. The model output $y(x)$ is the default risk. We calculate the default risk and the credit rating of the company in the four periods via the application of the fuzzy-logic expert rating model introduced in this work and determine the changes in default risk from period to period. The evolution of the default risk, rating and risk variation across time is summarized in the following table:

[8] In this trivial case, interaction effect is split up in half, but this is not so in general.

Year	Default Risk	Rating	Risk variation
0	0.8185	CCC	–
3	0.7143	B	−0.1042
5	0.5079	BB	−0.2064
6	0.5714	BB	+0.0635

In the first two intervals (0, 3) and (3, 5) the company has reduced its default risk and improved the credit rating from class CCC (in 0) to B (in 3) and from class B (in 3) to BB (in 5); in the last interval (5, 6) the default risk has increased, but the rating class has not varied.

Decomposition of the change in default risk and ranking of parameters. We focus on the decrease in default risk from year 0 to year 3. Specifically, the change in default risk in this time interval has been $\Delta f = -0.1042$ (see Table 4). The (clean) importance measures of the 18 key parameters are reported in Table 5. Note that

- many variables have no individual effect whatsoever nor interaction effects (e.g. FFO-on-Debt and Interest Coverage 2): Their influence on the change in default risk is zero
- for all inputs (except Interest Coverage 1), First Order FCSI and interaction effect have opposite sign, which means that they tend to offset each other
- one input (Interest Coverage 1) has no First Order effect but (slightly) affects the change in default risk via the interaction effect.

As now evident, the sum of Clean Total Order FCSIs is equal to the variation of the default risk ($\Delta f = -0.1042$). Table 5 ranks the input variables according to their relevance on the change in default risk. It turns out that

- the decrease in default risk (and the related rating improvement) is mainly determined by the increase of OFC-to-Debt (rank 2), which improves the financial vulnerability profile, by the increase of OWC Intensity (rank 3), which determines an efficiency enhancement, and by reduction of Leverage and Long-Term Leverage (ranks 4 and 5), which contribute to decrease the financial vulnerability of the firm
- the improvement in rating is smoothed by the increase of EBITDA Standard Deviation, which increases the Operating Risk via the Specific Risk. The standard deviation of EBITDA is the most relevant variable of the set of parameters (rank 1). The improvement in rating is also negatively affected by the decrease of Operating Leverage and Interest Coverage 1 (however, their effect on the output change is very mild)
- all the remaining variables have no influence on the default risk variation.

Figure 4 is the graphical representation of Table 5. The parameters are reported on the horizontal axis, sorted by decreasing influence on rating variation (hence, rank of parameters decreases from left to right); as for the vertical dimension, the Clean Total Order FCSIs ($\Delta_i^{T*} f$) are reported: A bar above the axis informs that the parameter has increased the default risk, while a bar below the axis informs that the parameter has decreased the default risk.

Table 4: Values of the parameters in 0 and 3

	Variable	**0**	**3**	**Variation**
1	Leverage	0.8589	0.7249	−0.1340
2	Long-Term Leverage	0.8126	0.5295	−0.2831
3	FFO-on-Debt	0.0959	0.1143	0.0184
4	Interest Coverage 1	0.7292	2.0779	1.3487
5	Interest Coverage 2	−0.2502	0.5485	0.7987
6	OCF-to-Debt	−0.0518	0.0919	0.1437
7	FCR	0.2790	0.3118	0.0328
8	Debt Service Coverage	−0.2915	0.2869	0.5784
9	OWC Intensity	0.0404	0.3073	0.2669
10	Financial Cycle	0.6250	0.6438	0.0188
11	EBITDA on Sales	0.0399	0.0182	−0.0217
12	ROA	−0.0122	0.0256	0.0378
13	Customer Concentration	0.6063	0.6250	0.0187
14	Product Positioning	0.6438	0.6250	−0.0188
15	Industry Prospects	0.5188	0.6063	0.0875
16	EDITDA Standard Deviation	0.3550	0.7313	0.3763
17	Operating Leverage	0.3550	0.3750	0.0200
18	Industrial Coverage	2.0836	2.2674	0.1838
	Output			
	Default Risk	0.8185	0.7143	−0.1042

Impact on output change of one key driver as opposed to the residual drivers. As the most determinant parameter for risk reduction in $(0, 3)$ is OCF-to-Debt, a viable application of DRSM is to investigate the role of OCF-to-Debt as compared with the residual input factors. To this end, we divide the set of parameters into OCF-to-Debt, on one side, and the subset of the residual 17 drivers, on the other side. We determine (i) the individual contribution of OCF-to-Debt, (ii) the individual effect of the above mentioned subset, and (iii) the interaction between OCF-to-Debt and the subset. It is worth noting that the individual effect of the subset consisting of the residual drivers quantifies the change in the default risk in case all variables except OCF-to-Debt vary from the initial value at time 0 to the realized value at time 3 (with OCF-to-Debt kept constant at its initial value at 0). Table 6 shows that the individual variation of OCF-to-Debt explains the 62.76% of the change in default risk in the interval $(0, 3)$, while the individual effect of the other 17 variables, taken together, determines the 89.44% of the default risk variation. Therefore, the OCF-to-Debt has a relative impact equal to $70.17\% = 62.76\%/89.44\%$ of the impact of the other 17 parameters considered together, thereby confirming the crucial influence of OCF-to-Debt on risk variation.

Table 5: (Clean) importance measures and ranks of the parameters

	Variable	First Order FCSI	Interaction	Total Order FCSI	Normalized Total Order FCSI	Rank
16	EBITDA Standard Deviation	0.0782	−0.0202	0.0580	−55.62%	1
6	OCF-to-Debt	−0.0654	0.0177	−0.0476	45.71%	2
9	OWC Intensity	−0.0535	0.0145	−0.0390	37.44%	3
1	Leverage	−0.0531	0.0144	−0.0387	37.13%	4
2	Long-Term Leverage	−0.0531	0.0144	−0.0387	37.13%	5
17	Operating Leverage	0.0023	−0.0006	0.0017	−1.61%	6
4	Interest Coverage 1	0	0.0002	0.0002	−0.19%	7
3	FFO-on-Debt	0	0	0	0%	8
5	Interest Coverage 2	0	0	0	0%	8
7	FCR	0	0	0	0%	8
8	Debt Service Coverage	0	0	0	0%	8
10	Financial Cycle	0	0	0	0%	8
11	EBITDA on Sales	0	0	0	0%	8
12	ROA	0	0	0	0%	8
8	Customer Concentration	0	0	0	0%	8
14	Product Positioning	0	0	0	0%	8
15	Industry Prospects	0	0	0	0%	8
18	Industrial Coverage	0	0	0	0%	8
	Sum	−0.1446	0.0404	−0.1042	100.00%	

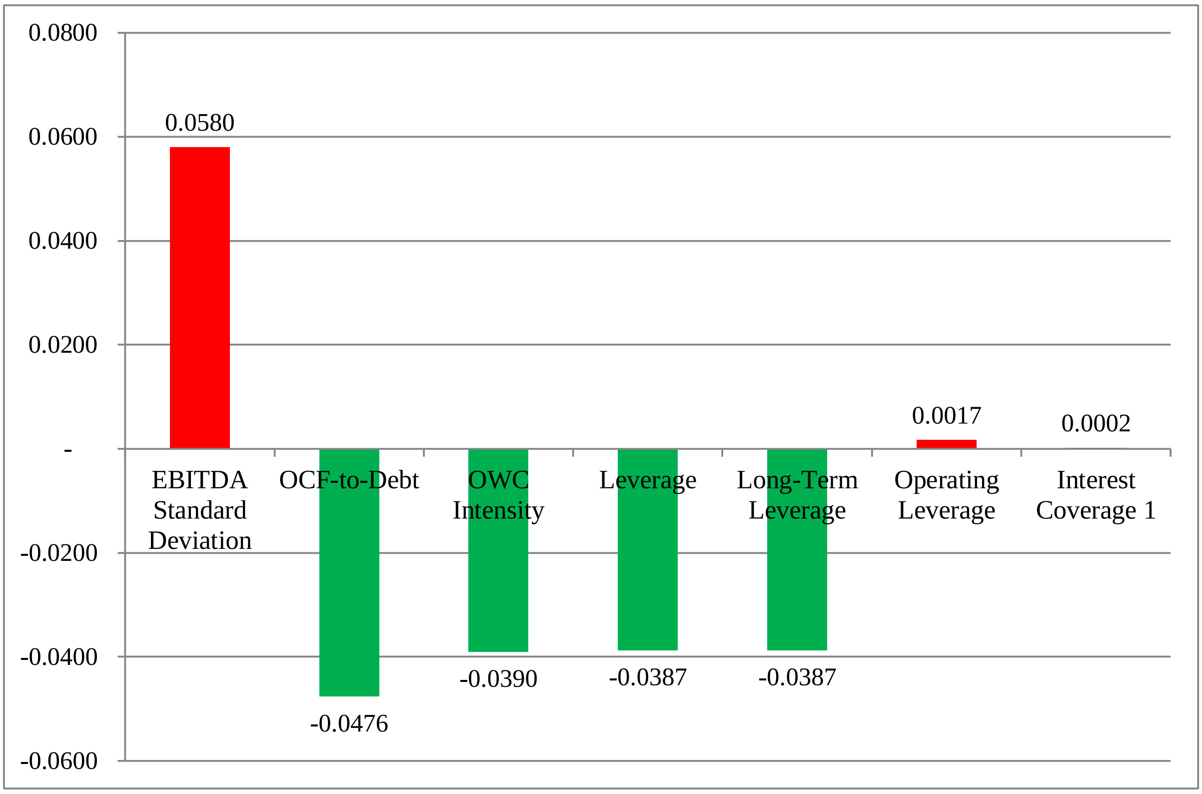

Figure 4: (Clean) Total Order FCSIs of the parameters ($\Delta_i^{T*} f$)

Table 6: The role of OCF-to-Debt

Effect	Description	Change in risk	%
Individual effect of OCF-to-Debt	OCF-to-Debt varies, residual drivers are constant	-0.0654	$+62.76\%$
Individual effect of residual drivers	OCF-to-Debt is constant, residual drivers vary	-0.0932	$+89.44\%$
Interaction effect	Interaction between OCF-to-Debt and residual drivers	$+0.0544$	-52.20%
	Sum	-0.1042	100.00%

Analysis of groups of variables. A further useful application of the DRSM consists of analyzing the role of selective groups of variables bearing special importance, aiming at identifying the influence of different areas of financial management on default risk variation. This analysis aims at pointing out the most effective managerial actions and policies for the evolution of the enterprise credit risk across time. For example, referring to Figure 1, consider the following areas pinpointed by the second-level intermediate variables, namely, Financial Vulnerability, Operating Efficiency, and Operating Risk:

- Financial Vulnerability represents the degree at which the firm is exposed to risk owing to an excessive debt. It is a second-level intermediate variable and is affected by 8 input factors. We denote it as V;
- Operating Efficiency represents the degree at which the firm is able to manage the operations in an efficient way. It is a second-level intermediate variable which has to do with the economic profitability (EBITDA, ROA) and the ability of collecting cash from customers early and delaying payments to suppliers (operating cycle, cash cycle). It is affected by 4 input factors. We denote this group as E;[9]
- Operating Risk joins two kinds of risk: The strategic risk, related to such drivers as the customer concentration, the product positioning, the industry prospects, and the specific risk, referred to specific features of the firm under analysis (standard deviation of EBITDA, operating leverage, industrial coverage). It is a second-level intermediate variable which is affected by 6 key drivers. We denote it as R.

For each subset S_k we determine the First Order FCSI ($\Delta^1_{S_k} f$) and any interaction involving S_k. For instance, the individual effect of the Financial Vulnerability on the risk change from 0 to 3 is $\Delta^1_V f = f(x^1_V, x^0_{(-V)}) - f(x^0) = -0.1078$, meaning that it has played a positive role. As for the pairwise interaction, the interaction of this group with the Operating Efficiency is $\Delta_{V,E} f = f(x^1_V, x^1_E, x^0_R) - f(x^0) - \Delta^1_V f - \Delta^1_E f = 0.0535$, meaning that it has negatively (albeit very slightly) affected the rating; the interaction with the Operating Risk has acted positively, since $\Delta_{V,R} f = f(x^1_V, x^1_R, x^0_E) - f(x^0) - \Delta^1_V f - \Delta^1_R f = -0.0798$. The interaction between the three groups is $\Delta_{V,E,R} f = f(x^1) - f(x^0) - \Delta^1_V f - \Delta^1_E f - \Delta^1_R f - \Delta_{V,E} f - \Delta_{V,R} f - \Delta_{E,R} f = 0.0388$. Individual effects and interactions are collected in Table 7. Using the duplication-cleaning procedure, we perfectly decompose the change in default risk. Indeed, the sum of any contribution (individual effect and interaction), counted just once, is equal to the change in risk, $\Delta f = -0.1042$. The ranking is shown in Table 8 and in Figure 5. As can be gleaned from inspection of table and figure,

- the better rating at time 3 is primarily driven by the reduction in the Financial Vulnerability (V), which is the most influential area of financial management in the analysis, and by the decrease of the Operating Risk (R), which represents the second most relevant subset of parameters

[9]Note that Financial Vulnerability and Operating Efficiency are the antecedents of the (third-level intermediate variable) Financial Risk.

- the better rating is curbed by the worsening of the Operating Efficiency (E), which is, however, the least influential management area.

Table 7: First Order FCSIs and interactions of the subsets of parameters

First Order FCSIs	
$\Delta^1_V f$	-0.1078
$\Delta^1_E f$	-0.0535
$\Delta^1_R f$	0.0834
Interactions	
$\Delta_{V,E} f$	0.0535
$\Delta_{V,R} f$	-0.0798
$\Delta_{E,R} f$	-0.0388
$\Delta_{V,E,R} f$	0.0388
Sum $= \Delta f$	-0.1042

Table 8: Ranking of the subsets of parameters

	Subset	**First Order FCSI**	**Interaction**	**Total Order FCSI**	**Normalized Total Order FCSI**	**Rank**
V	Financial Vulnerability	−0.1078	0.0242	−0.0836	80.25%	1
R	Operating Risk	0.0834	−0.1533	−0.0700	67.13%	2
E	Operating Efficiency	−0.0535	0.1029	0.0494	−47.38%	3
	Sum of contributions	−0.0780	−0.0262	−0.1042	100.00%	

Maximum effect of a variable. Another possible use of DRSM is the study of the maximum effect of a variable or a subset of variables on the default risk. For example, consider the parameter Fixed Charge Ratio (FCR), which represents a significant synthetic ratio of the firm's capacity to service the debt and, probably, the most informative index of financial stability. We analyze the effects of the improvement in FCR, compared to the base-case year 0, while all the residual parameters are fixed at their initial value in 0. Table 9 collects the levels of default risk and credit rating corresponding to increasing values of FCR. The first line of the table describes the base-case and reports the values of FCR, default risk and credit rating in year 0; in the second line FCR is evaluated in 3, while all the residual parameters are equal to their initial value; the following lines are obtained by increasing FCR by 0.06 starting from the base case, with all the other variables evaluated in 0. From inspection of the table,

- other things equal, the improvement in FCR is able to decrease the default risk to a minimum of 0.57142 and increase the rating to a maximum of BB, corresponding to a 30.19% risk reduction and a two-classes rating improvement

- values of FCR greater than 1.659 are uninfluential, so that a further improvement in rating must be accomplished by improving some other input factors.

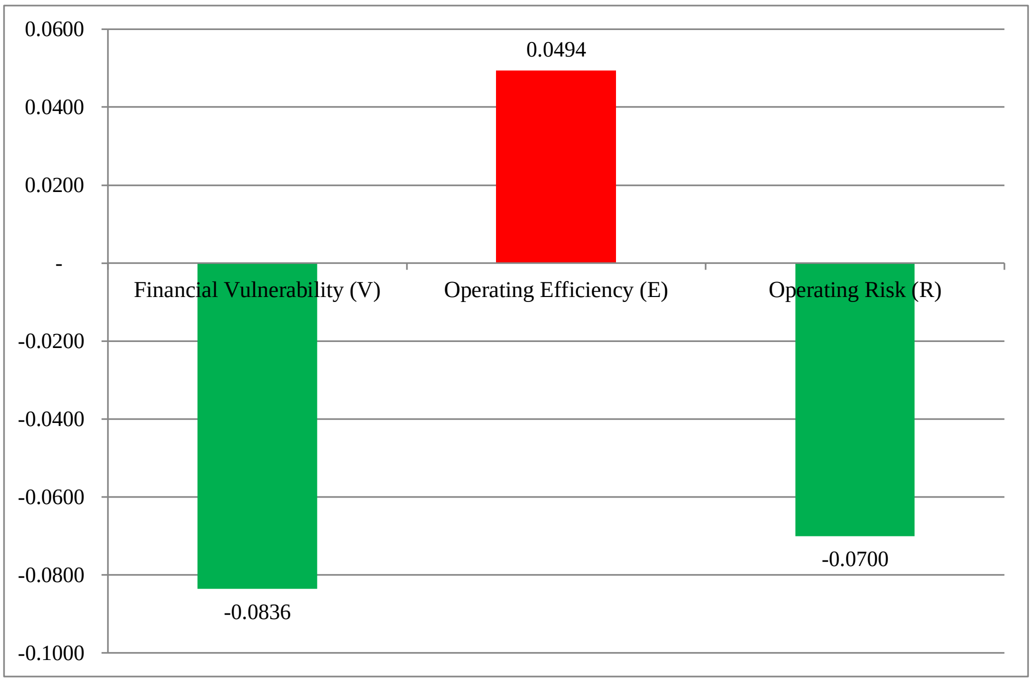

Figure 5: Group Analysis: (Clean) Total Order FCSIs ($\Delta^{T*}_{S_k} f$)

The relationship between the increase of FCR and the decrease in default risk is represented in Figure 6. The relatively high impact of FCR on credit risk resulted from this analysis (assuming other things equal) is not surprising if one considers that FCR is an important measure of financial stability. While we have shown the impact of a single key driver, the analysis may be extended to considering the maximum effect of a subset of the parameters. This is accomplished by improving each variable belonging to the subset while all the parameters outside the relevant subset are kept fixed at their initial value in 0. The analysis becomes less trivial (because interaction effects among the group's variables occur) but the DRSM easily manages any such case and the change in risk may be exactly decomposed.

6 Concluding remarks

This paper introduces a credit rating model based on fuzzy logic and expert system, able to replicate and attribute logical consistency to the evaluation process of default risk and credit rating which is usually performed by human experts on the basis of available data. The expert system uses available data (knowledge base) and an inferential engine to produce the output. We consider a set of 18 economic and financial variables, both quantitative and qualitative. The system determines the default risk and the rating class in various years through a modular approach which aggregates the variables under a managerial and financial perspective.

We associate the rating system with the Finite Change Sensitivity Indices (Borgonovo 2010a, 2010b), a recent addition to the techniques of Sensitivity Analysis (SA) which aims at measuring

Table 9: Maximum effect of FCR on default risk and rating

	FCR	Default risk	Rating
(year 0)	0.279	0.8185	CCC
(year 3)	0.312	0.8185	CCC
	0.339	0.8185	CCC
	0.399	0.8185	CCC
	0.459	0.8185	CCC
	0.519	0.7784	CCC
	0.579	0.75574	CCC
	0.639	0.73256	B
	0.699	0.73256	B
	0.759	0.73256	B
	0.819	0.73256	B
	0.879	0.73256	B
	0.939	0.73256	B
	0.999	0.73256	B
	1.059	0.73256	B
	1.119	0.73256	B
	1.179	0.72640	B
	1.239	0.71428	B
	1.299	0.71428	B
	1.359	0.71428	B
	1.419	0.71428	B
	1.479	0.70386	B
	1.539	0.67882	B
	1.599	0.57834	BB
	1.659	0.57142	BB
	> 1.659	0.57142	BB

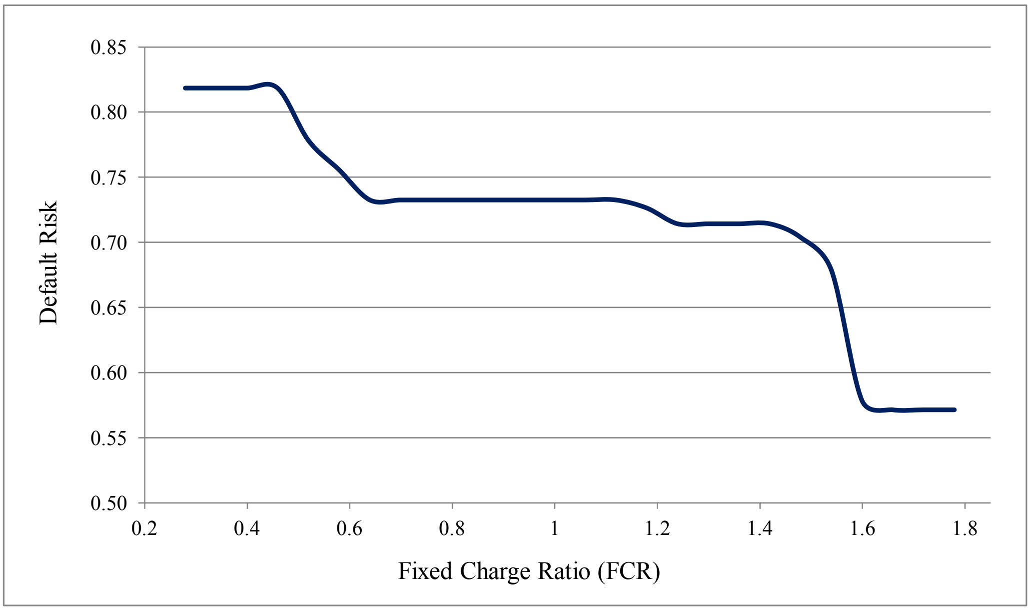

Figure 6: Increase of FCR and decrease in default risk

the impact of the model inputs on the output change occurred passing from a base value (e.g., the output value at a given date) to a realized value (the output value at a subsequent date). We fine-tune FCSIs by eliminating some duplication effects and provide a clean, exact decomposition of the output change.

We use the results for giving rise to the Default Risk Sensitivity Model (DRSM) which investigates the variation of the enterprise default risk under changes in the model inputs for ex post analysis and for managerial decision-making. As for the former perspective, the DRSM allows the decomposition of the historic change of default risk and identifies the most relevant parameters which generated the change; as for the latter perspective, it suggests suitable managerial actions to be undertaken for improving the prospective rating and/or increasing the success of complex financial operations that are to be taken. Overall, a sensitivity analysis module, such as the one presented in this work, is a valuable tool to enhance the understanding of a complex fuzzy-logic model by providing insights into how the inputs affect the outputs of such models, thus strengthening the confidence of credit analysts in using such method in practice. From this point of view, sensitivity analysis is also crucial in model testing/validation: Additional simulation runs may be used for corroborating and, when necessary, calibrating the model.

Several categories of companies may benefit from the application of DRSM: Firms aiming at controlling and/or reducing their credit risk profile, enterprises needing a dynamic and mindful debt management, and public companies which are willing to inform the financial markets about the firm's present economic results and future prospects.

We have applied the DRSM to an Italian-controlled industrial company. We have identified the effects of parameters on the default risk and the rating change through time, then have determined the aggregate effects of groups of variables (specifically, Financial Vulnerability, Operating Efficiency, Operating Risk), have analyzed the impact of the ratio of the operating cash flow to the debt amount as opposed to the impact of the other variables taken together, and have calculated the maximum effect of a variable (FCR) on default risk.

Finally, it is worth noting that the sensitivity model is detached from the expert system: They are reciprocally autonomous in that each of them may be used independently. In particular, the sensitivity model presented does not depend on the fuzzy expert system: It is suitable for applications with any possible rating model and, therefore, any set of parameters (symmetrically, the rating model may also be adopted in association with other SA techniques). A potential scenario of future development is the combination of sensitivity analysis with automatic machine-learning algorithms for rating firms, aiming at melting the high learning and generalization capabilities of adaptive, computational-based, data-driven system with the promising feature of increasing the comprehensibility of complex models via the application of sensitivity analysis. Future researches may also be conducted for formal testing/validation of machine-learning approaches using data-driven schemes.

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Appendix

This Appendix reports a legend of accounting and financial terminology, a list of primary relations involving the main dimensions of the analysis, the description of the 18 model inputs, and the structure of the intermediate variables of the rating system.

Legend

COGS	= Cost of Goods Sold
D&A	= Depreciation and Amortization Expenditures
EBIT	= Earnings Before Interest and Taxes
EBITDA	= Earnings Before Interest, Taxes, Depreciation and Amortization
FCR	= Fixed Charge Ratio
FE	= Financial Expenses
FFO	= Funds from Operations
NI	= Net Income
NOPAT	= Net Operating Profit After Taxes
OCF	= Operating Cash Flow
OWC	= Operating Working Capital
PBT	= Profit Before Taxes
R&D	= Research and Development expenses
ROA	= Return on Assets
SG&A	= Selling, General and Administrative Expenses
T	= Taxes

Primary relations

Gross Profit	= Revenues − COGS
EBITDA	= Gross Profit − SG&A
EBIT	= EBITDA − D&A
NOPAT	= EBIT·(1−tax rate)
PBT	= EBIT − FE + Interest Income ± Extraordinary Items
NI	= PBT − T
OCF	= EBIT + D&A − investments + disposals − ΔOWC

Model inputs

The definition of the model inputs is reported in the following table:

	INPUT	DESCRIPTION
1	Leverage	Total Debt/(Total Debt + Equity Value)
2	Long-Term Leverage	Long-Term Debt/(Long-Term Debt + Equity Value)
3	FFO-on-Debt	(NOPAT + D&A + other noncash items)/Total Debt
4	Interest Coverage 1	(EBITDA − R&D)/ Financial Expenses
5	Interest Coverage 2	(EBIT − R&D)/ Financial Expenses
6	OCF-to-Debt	OCF/Total Debt
7	Fixed Charge Ratio (FCR)	(EBITDA − R&D)/(Debt Service + Taxes + Capital Expenditures)
8	Debt Service Coverage	OCF/Debt Service
9	OWC Intensity	*qualitative*
10	Financial Cycle	*qualitative*
11	EBITDA on Sales (Adjusted)	(EBITDA − R&D)/Revenues
12	ROA (Adjusted)	(EBIT − R&D)/Total Assets
13	Customer Concentration	*qualitative*
14	Product Positioninig	*qualitative*
15	Industry Prospects	*qualitative*
16	EBITDA Standard Deviation	Standard deviation of EBITDA (last five years)
17	Operating Leverage	Fixed Costs/Total Costs
18	Industrial Coverage	Gross Profit/Capital Expenditure

Notes

1. R&D are deducted from EBIT and EBITDA only if they are capitalized.
2. Total assets in ROA is net of minority participations.
3. Industry prospects is related to the market risk.
4. OWC intensity is the average amount of accounts receivable as opposed to the accounts payable.
5. Product positioning is an indicator of the product quality as perceived by the customers.
6. Customer concentration refers to the percentage of revenues of the 4 most important buyers (the higher the concentration, the higher the risk). Depending on the sector and on the case at hand, an index of geographic concentration may also be used in place or along with this metric.
7. Financial cycle is the lapse of time between when cash is paid to suppliers and cash is received from customers.

Intermediate variables

Figure 7 represents the sequence of the intermediate variables of the rating system until reaching the model output Default Risk (see also Figure 1).

Intermediate variables (1-st level)	(Risk of) Capital Structure
	Interest Coverage
	Debt Coverage
	WC Management
	Profitability
	Strategic Risk
	Specific Risk
Intermediate variables (2-nd level)	Financial Vulnerability
	Operating Efficiency
	Operating Risk
Intermediate variables (3-rd level)	Financial Risk
	Operating Risk
Model output (4-th level)	Default Risk

Figure 7: Intermediate variables