



## Multiple regimes in operation of the Swiss Railway System and potential influences on power load

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# Multiple regimes in operation of the Swiss Railway System and potential influences on power load

J. Bosch, R. M. Fuchsli, C. Zaugg

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**Index Terms**—load forecasting, load modeling, punctuality, rail traffic control, rail transportation power systems, statistics

## I. NOMENCLATURE

SBB: Swiss Federal Railways

Punctuality: The ratio of trains that runs not more than three minutes late in the whole railway grid.

## II. INTRODUCTION

THE 16.7 Hz electric railway power supply grid in Central Europe exhibits many features expected to be present in the public grid, once an exit from nuclear and fossil-energy-carriers will be realized. Among others, these are: High share in renewable energies, decentralized and non-orchestrated production (in the railway system from recuperation of the vehicles) and a high fluctuation of the residual load. Several publications discuss demand side management for 50 or 60 Hz public power supply grids [1] [2]. Methods for the system design of railway power grid exist [3] [4], but focus on the average demand and not on the management of short-term fluctuations. The Swiss Federal Railways (SBB) operates an electric power supply grid that is separated from the public grid. According to its energy strategy, the electric power supply of SBB in 2025 will rely completely on renewable energy sources and exploit the possibilities of information technology for a maximally efficient operation in various manners [5]. It is a prototypical example for a smart grid, according to the definition given in the US Energy Independence and Security Act of 2007 (EISA-2007). SBB is

implementing as first railway grid operator a demand side management system [6] to reduce the global load peaks.

An example of the total load profile of the electric railway power supply of the SBB that exhibits large load peaks and steep gradients is presented in Fig. 1 (further information in [7]). Load increases of 240 % in 70 s are challenging but daily business. In order to put Fig. 1 into perspective, we give some background to the presented data: The basis of studies such as [7] is a measurement of the total load of the SBB electric railway power grid with a sample resolution of one second. The power grid has an total installed production power of 1 300 MW, the maximum load peaks are about 740 MW. The Swiss railway power grid is connected with those of Germany and Austria. The three grids are operated in a common primary control area [8]. The Swiss railways accomplished in 2015 a traffic performance of 18 560 million passenger kilometers and 15 065 million net ton kilometers of freight traffic by use of total 1 844 GWh of electrical traction energy [9]. The operational maximum speed in the Swiss railway grid is 200 km/h, the punctuality of the passengers was 2015 88 % (ratio of the passengers that arrived early, punctually or with less than 3 minutes delay) [9]. For the future, an increase of the load peaks, at least in size but probably also in frequency, is expected because of the planned increase of capacity and more powerful rolling stock.

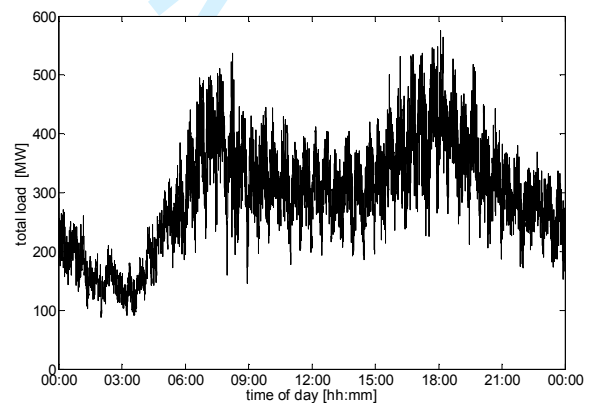


Fig. 1. Typical daily total load profile of the SBB electric railway power supply on a workday

The high volatilities in the power load of railway grids are a challenge for the demand side management. Comparable challenges are expected to emerge in the public power supply

1 grid, once the shift towards renewable energy carriers has been  
2 implemented, because power production based on renewable  
3 energy carriers like photovoltaic or wind turbines is more  
4 volatile than power production based on fossil or nuclear  
5 energy carriers [10]. The railway power grid therefore may  
6 serve as a laboratory for operating public power supply grids  
7 with high volatilities.  
8

9 There is a variety of tools one can use in order to  
10 implement demand side management. We pursue the following  
11 approach: The load curve is split into a deterministic and a  
12 stochastic part (we will define precisely what we mean by  
13 “deterministic” below). The stochastic part is analyzed further;  
14 firstly, with the goal to get insight into the distribution of the  
15 noise, which allows predicting the probabilities for the  
16 appearance of load peaks of given size. Secondly, we analyze  
17 the relation between noise in the power load and the statistics  
18 of observable quantities of the operation of the railway grid.  
19 This with the goal to reduce/control peaks in the power load  
20 by finding potential mechanisms that generate the peaks.  
21

22 In brief, we are guided by the idea that the appearance of a  
23 correlation (in form of a clustering) between noise in the  
24 power load and fluctuations in another operational observable  
25 indicate a non – trivial behavior of the latter. In this paper, we  
26 focus on punctuality, and in fact, we observed such a  
27 clustering and could demonstrate that local punctuality (means  
28 the punctuality with respect to a major hub) exhibits a  
29 multimodal distribution. Multimodality in probability  
30 distributions is a strong indicator for an underlying dynamics  
31 governed by multiple dynamical regimes (see subsequent  
32 explanations and, for a more mathematical presentation [11]).  
33 The information that a system is governed by multiple regimes  
34 is in turn of high relevance for the design of an appropriate  
35 control strategy (which is not part of this paper).

36 In more detail, there are several reasons why one seeks  
37 knowledge about noise and noise reduction (more general: the  
38 stochastics) of global system variables, especially about the  
39 power load. A first one is purely economic: Knowing the  
40 statistics of noise is of value for short term contracting in  
41 controlling power. A second is given by the general  
42 requirement of being as economical as possible with respect to  
43 energy consumption. This requirement can be answered in  
44 various ways. Further efficiency gains in the rolling stock may  
45 well be possible; however, the potential is probably limited.  
46 Another way to lower overall energy consumption is to reduce  
47 the necessary controlling power range one needs to guarantee a  
48 (at least with respect to energy) smooth operation of the  
49 railway system. In order to achieve this, a better understanding  
50 of the nature of noise in electric power load is necessary. This  
51 includes a deepened insight into the statistics of noise as well  
52 as into the mechanisms generating it. Thirdly, as we will  
53 demonstrate, the analysis of the distribution of noise gives  
54 evidence that the railway system switches between different  
55 regimes or modes of operation. More precisely, the system  
56 seems to run either in a punctual mode with comparable low  
57 noise or in an unpunctual mode with higher noise. These two  
58 modes of operation seem to be clearly distinct and there is no  
59  
60

detailed knowledge about the precise way how transitions  
between them take part. Knowing about such modes and  
transitions is of interest first because one gets a better  
understanding of dynamics underlying the railway system.  
Second, such knowledge can be crucial for the design of an  
appropriate control strategy for the whole system. In such a  
strategy, one has to be aware of the actual mode of the system  
and to apply a control that is optimal for the specific mode and  
third (if possible and desired) one has to take appropriate  
means in order to switch as smoothly and quickly as possible  
from one into another mode.

Many natural and technical systems can be described by a  
single default state (a single regime) and some fluctuations  
around it. This default state is often a stationary or even  
equilibrium state, but may also be a limit cycle or a more  
complex form of attractor. We intend to use the term  
“attractor” in a rather colloquial sense which does not refer to  
mathematical details but captures the idea of a system behavior  
that remains upon small perturbations in some limited region  
of the parameter space. Therefore, we will speak of “regimes”,  
which do not only refer to the attractor, but also its respective  
basin of attraction.

Large technical systems often exhibit several distinct  
regimes. This multitude of regimes may be an intentional  
feature of the design of the system or a (maybe not desired)  
consequence of the system dynamics. In the former case, the  
switch between the regimes may happen according to some  
deterministic dynamics. If one models the system behavior, it  
is often appropriate to use two different models, one for each  
regime. The latter case is more demanding; usually, one of the  
regimes is the intended default regime. Since the system  
fluctuates, it may eventually leave this default regime and  
settle down in one of the other basins of attraction. Modeling  
this situation requires methods used in the study of stochastic  
processes (see [11]). Controlling a system with several regimes  
requires more elaborate control strategies because counter –  
intuitive system behavior may occur. If a system has only one  
attractor, reduction of noise is sufficient to bring the system  
closer to the default state. If, however, there are multiple  
regimes, a reduction of noise may lock the system into a non –  
desired state, with no chance whatsoever to switch back to the  
desired state. A proper control would require first to bring the  
system close to a desired (self – stabilizing) state and only then  
to reduce the noise (for a detailed discussion of this class of  
phenomena which includes stochastic resonance, see e.g. [12],  
and for a very accessible discussion using examples from  
biology, [13]).

A typical signature for the presence of multiple regimes  
consists of multimodal (i.e. multi – peaked) probability  
distributions with respect to system observables. A word of  
caution is necessary: multimodal distributions can also indicate  
a strongly non – uniform usage of the system with two  
effectively separate regimes. “Effectively separate” thereby  
means that despite the fact that the system in consideration  
could in principle be run in a continuous range of modes of  
operation, the effective modes of operation can be categorized

in a number of distinct regimes. In order to run the system in an optimal manner, one may want to distinguish these regimes and apply respective optimal operation procedures.

The paper is structured as follows. In Sec. III, we define our notion of noise and discuss how we split the power load curve into a deterministic and a stochastic part. In Sec. IV, we analyze the relation between the variance of noise in the load and the distribution of punctuality of the railway system. This motivates the claim that the dynamics of the Swiss railway system exhibits stochastic switching between multiple regimes. In Sec. V, we discuss these findings. The paper closes with an outlook.

### III. DETERMINISTIC AND STOCHASTIC PART OF THE LOAD PROFILE

#### A. Definition of Noise

Splitting a signal into a deterministic and a stochastic part is, at least conceptually, comparably easy as long as the signal can be understood as arising from of a well – defined system with known internal deterministic dynamics and a connection to / an embedding into a stochastically varying environment. In case of the SBB power grid, the distinction into deterministic and stochastic behavior is more involved, because the dynamics of the system under consideration (the trains on the railway network) and their mutual interactions are not known in all detail. Consequently, the split of the power load curve into a deterministic and a stochastic part follows a different paradigm: The regular interval time table suggests a corresponding regular part in power demand. The regular part of the power demand is determined by frequency analysis and regarded as deterministic, because it can be mapped onto features of the operation of the railway grid. Consequently, the difference between the actual load and the deterministic part is deemed to be noise. This separation of the total load profile in deterministic regular frequency components and noise by a Fourier transformation has been described in [14]. This noise is, at least in part, related to the heavy use of the Swiss railway grid. With 160 trains per track and day [9] the Swiss railway grid is one of the most heavily used railway grids in the world [15].

We make two remarks concerning the concept of noise, as we presented it here. Firstly, the way how we define noise is not immanent to the system but reflects our knowledge about the system. As will be discussed in the next section, we can map certain frequencies in the Fourier spectrum of the power load onto aspects of the operation. This mapping defines what we understand by “deterministic”. Secondly, we emphasize that the term “noise” is mathematically adequate (since the fluctuations are stochastic in nature) but may lead to a misunderstanding. The noise we analyze in this paper is not white noise, at least not in general because the noise is not necessarily independent (i.e. exhibits non – trivial auto – correlation). This for (at least) two reasons: First, the contingencies mentioned above are not isolated events and may take effect over a longer period. Second, even if these

contingencies are highly localized in time, they may lead to avalanche effects in the sense that one contingency may promote other events that disturb the traffic. In fact, as will be discussed in Sec. IV, a closer analysis of the noise suggests a non – trivial structure, i.e. a characteristics of the distribution of noise that indicates an operation of the railway system in multiple, separated regimes.

#### B. Frequency Analysis

Based on time series of the load profile with a resolution of one second, the respective Fourier spectrum is presented in Fig. 2 and 3. We identify those characteristic frequencies which can be related to different aspects of the operation of an integrated regular interval timetable.

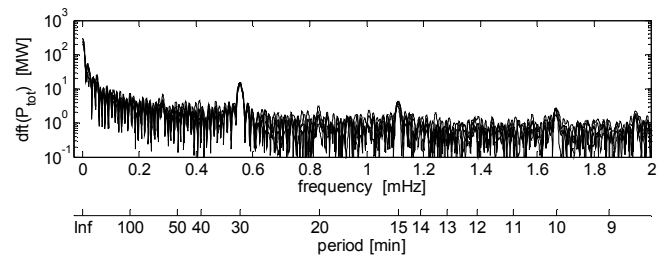


Fig. 2. Absolute value of the Fourier spectrum of the SBB total load. Frequency domain from 0 to 2 mHz [14]

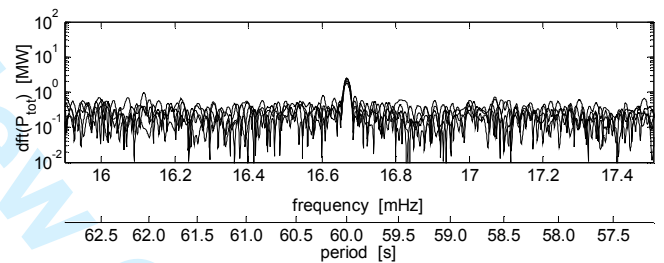


Fig. 3. Absolute value of the Fourier spectrum of the SBB total load. Frequency domain from 15.9 to 17.5 mHz [14]

The low frequency components under 0.5 mHz, describe the ground pattern of the load profile, caused by the traffic rush hours and environmental effects, such as the temperature influence, on the load. The half hour periodicity and its harmonics (i.e. 15 min, 10 min, 7.5 min periodicities) reflect the symmetrical regular interval timetable. Furthermore, one finds a periodicity of one minute that is due to the fact that punctual trains departure always happens some seconds after the full minutes (hh:mm:00). These components are predictable and are well explored. The other parts of the spectrum are regarded as noise and related to contingencies in the railway operation like door blockages with following delays or phenomena that are not yet adequately described like individual driving style of train operators.

We analyze the power load with respect to one – hour time intervals. This choice is justified, because it is bigger than 30 minutes, which is the natural time window of the Swiss regular interval time table. Furthermore, we expect the hour from, say, 7:00 to 8:00 to be comparable for all workdays, whereas the frequency spectrum of a rush hour differs from the spectrum



observed during the night, when traffic is reduced. In what follows, we regard the deterministic part of the power load as composed of the Fourier components with periods bigger or equal to 30 min, and the components with periods 15, 10 and 1 minute. As explained above and in [14], this choice is justified by consideration of the railway system operation; from a mathematical perspective other choices are conceivable.

#### IV. CORRELATION BETWEEN LOAD NOISE AND PUNCTUALITY: EVIDENCE FOR REGIME SWITCHING

We relate the punctuality (i.e. the ratio of the number of trains that runs not more than three minutes late to the total number of trains) in the whole Swiss railway system to the variance of the load noise (i.e. the variance of that part of the load profile regarded as noise, according to the definition given in Sec. III. A). We focus on the time window from 6:00 to 10:00; this is motivated by Fig. 4. Starting from a relatively high level of average punctuality between 6:00 – 7:00, we observe a decrease and only between 9:00 – 10:00 the average punctuality recovers. The time intervals of decreased punctuality are those of interest for detailed analysis of the inner dynamics of the railway system, because these are the times which bear the potential for a direct benefit of the passengers and with the highest load peaks in the power demand.

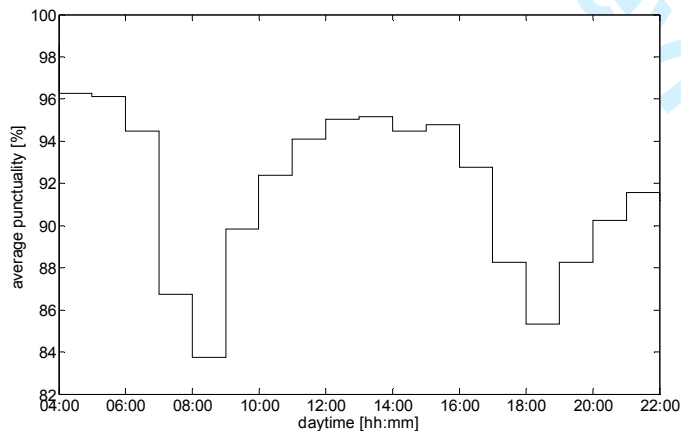


Fig. 4. Average punctuality of the SBB railway grid for the hours from 4:00 to 18:00

We compute the variance of the noise for time intervals covering full hours from 6:00 to 10:00 and plot it versus the punctuality in the according time interval. The result is given in Fig. 5. A closer analysis of the data reveals that the samples can be grouped into two clusters. In the lower right, one finds data points belonging to the hours from 6:00 to 7:00 with high punctuality (> 88%) and low variance of the noise (cluster 1) and a cluster (cluster 2) of hours from 7:00 to 10:00 with high variance of noise in a wide range of punctuality.

The question arises whether the two clusters are truly distinct or result from a simple scaling effect. If it were the case that the system usage (i.e. the power demand) is higher during the hours from 7:00 to 10:00 compared to the interval from 6:00 to 7:00, an upscaling of the noise would be no surprise. But as shown in Fig. 6 the hourly average of the total load

consumption of the Swiss electric power grid during the hours from 6:00 to 9:00 is similar and during the hour from 9:00 to 10:00 even lower than in the previous three hours. Therefore, simple scaling can be ruled out as explanation for the appearance of two clusters.

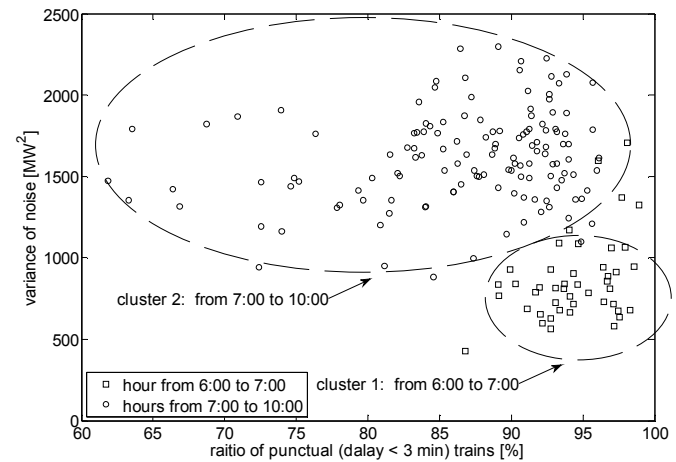


Fig. 5. Variance of the noise vs. punctuality on an hourly base, the ratio of punctual trains in the SBB railway grid to the total number of trains in a given hour. Each plotted sample represent data from days within a period from 19.01.2015 to 20.03.2015, the data are selected to workdays and the hours are from 6:00 to 10:00.

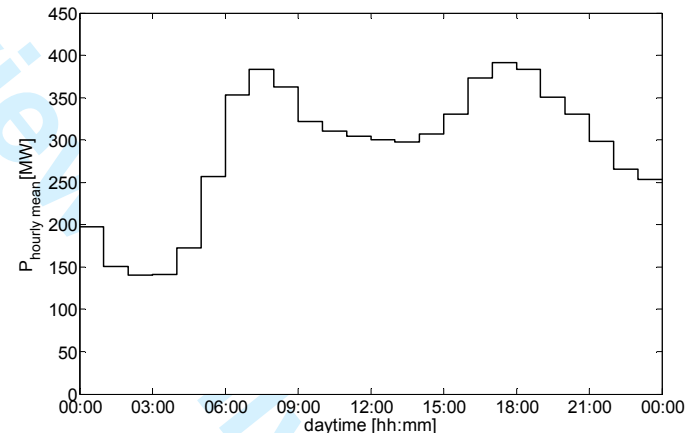


Fig. 6. Hourly average of the SBB total load on workdays

In the introduction, we mentioned that our investigation is guided by the idea that the appearance of clusters in a plot of variance of power load vs. a system variable such as punctuality is an indicator for a non-trivial distribution of the latter. The analysis of this distribution could in turn reveal relevant information useful for the design of a control strategy. This is particularly true if the distribution under consideration is multimodal. However, from Fig. 5 it is not obvious that the distribution of punctualities exhibits particular features. But we point out an issue to be considered. Analyzing the statistics of delays by aggregating data from the whole Swiss railway system may be misleading. As we will demonstrate (see later in Figs. 7 – 10), there are clear indications for multiple regimes if one analyzes the delay statistics of individual (major) train stations. The existence of local multiple regimes is obscured if the distribution of delays is analyzed aggregating

1 data from whole over Switzerland. This is no surprise;  
 2 dynamic formation of regimes by self – organization (or self –  
 3 stabilization of undesired states) may occur in a part of the  
 4 railway system, but rarely affects the whole railway traffic. In  
 5 order to overcome this problem, we work with the local delay  
 6 statistics of Olten, a major hub in the Swiss railway system, for  
 7 which according data is available.

8 In order to study the internal structure of the clusters in Fig. 5,  
 9 we take time intervals with a duration of one hour of different  
 10 workdays as samples and compute the relative punctuality of  
 11 passenger trains for each sample and day. The histograms use  
 12 a bin width of 2.7 %. The samples are taken from the local  
 13 delay statistics of the hub of Olten, spanning the period from  
 14 19.01.2015 to 20.03.2015, selected to be workdays. The  
 15 histograms represent the absolute frequency of punctuality on  
 16 different workdays (Figs. 7 - 10). Visual inspection suggests  
 17 multimodality for the distributions in Figs. 7, 8 and 10. Since  
 18 visual inspection alone can be misleading, we apply a  
 19 statistical test for multimodality, Hartigan's dip test (Details  
 20 about the test and references for the underlying theory and  
 21 numerical implementations are given in the appendix). The  
 22 major assumption to satisfied for the application of Hartigan's  
 23 dip test is independence of the samples. Since we analyze the  
 24 punctuality of samples for identical time intervals of different  
 25 days but always covering the same hour, this independence  
 26 may safely be assumed (It is clear that if there are delays in,  
 27 say, the interval from 6:00 to 7:00 of a given day, the  
 28 probability to have also delay at a later hour of the same day is  
 29 enhanced. But yesterday's delay usually doesn't affect the  
 30 delays of today).

31 The results of Hartigan's dip test are in the captions of Figs. 7-  
 32 10 (p-value). The p-value is the probability of obtaining a test  
 33 statistics, which is equal or more extreme than the observed  
 34 value (cf. Appendix, Sec. VII. A), assuming the null  
 35 hypothesis of unimodality. Any p-value less than a significance  
 36 level of  $\alpha = 0.05$  suggests that observed data is inconsistent  
 37 with the null hypothesis of unimodality.

38 From these results, we gain evidence that there are (at least)  
 39 two modes of operation: a punctual mode (with punctuality  
 40 higher than 92%) and a mode of low punctuality. A local  
 41 switch between these modes in a central hub suffices to  
 42 establish a mutual hindrance of many trains. Acceleration  
 43 processes of these trains are coupled and will produce peaks in  
 44 the total load of the Swiss railway grid. A local decrease in  
 45 punctuality in a major hub of the railway grid amounts in an  
 46 increase of variance of noise in the whole power grid.

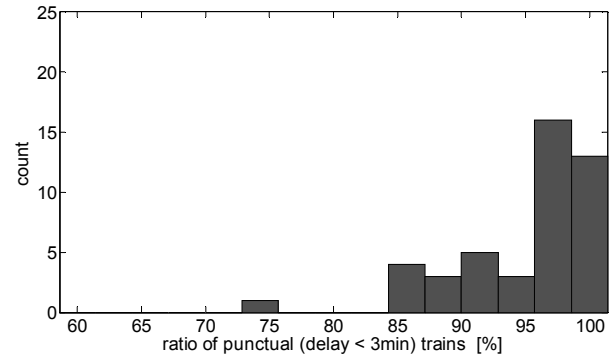


Fig. 7. Histogram of the punctuality of the trains for the hour 6:00-7:00; shown are delay statistics for Olten, a central hub in the Swiss railway system; p-value of the Hartigan's dip test  $< 2.2e-16$

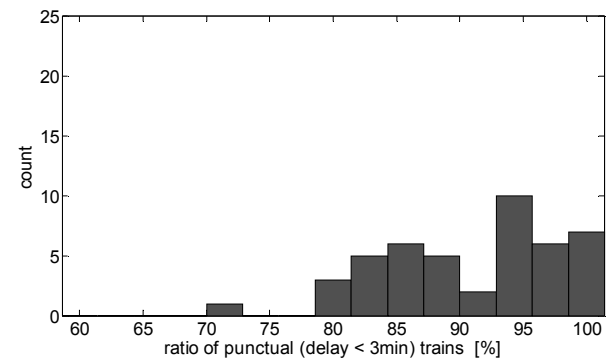


Fig. 8. Histogram of the punctuality of the trains for the hour 7:00 – 8:00; shown are delay statistics for Olten, a central hub in the Swiss railway system; p-value of the Hartigan's dip test: 0.0311

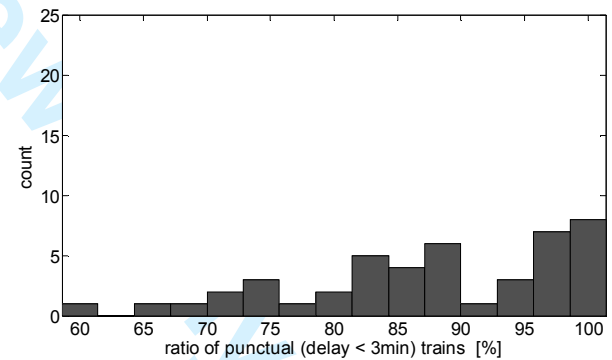


Fig. 9. Histogram of the punctuality of the trains for the hour 8:00 – 9:00; shown are delay statistics for Olten, a central hub in the Swiss railway system; p-value of the Hartigan's dip test: 0.0795

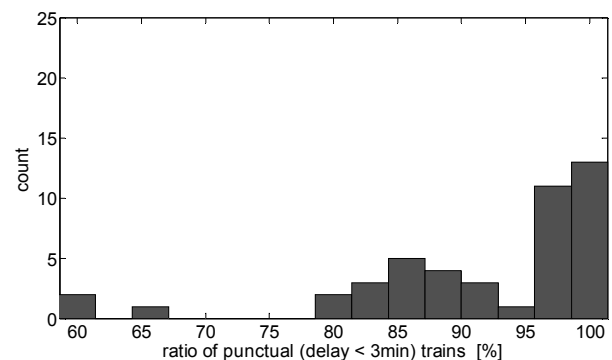


Fig. 10. Histogram of the punctuality of the trains for the hour 9:00 – 10:00; shown are delay statistics for Olten, a central hub in the Swiss railway system; p-value of the Hartigan's dip test: 0.000217

## V. SUMMARY AND DISCUSSION

We analyzed the variance of noise from the load profile and the punctuality of the Swiss railway system. Motivated by a visual inspection of the diagram of variance of noise versus punctuality, we hypothesized multimodality in the distribution of local punctualities and consequently (and for the operation of the railway system more relevant) evidence for the appearance of multiple regimes in the operation of the railway system. A statistical scrutiny, based on Hartigan's dip test, corroborated the visual impression of multimodality insofar as the null hypothesis of unimodality can be rejected for several time intervals. We have to express a word of caution that has always to be applied when statistical tests are used; statistical tests may justify the rejection of a null hypothesis, but not more. What we have demonstrated is that the data is not compatible with the assumption of unimodality, this must not be confused with an actual proof for multimodality, although in practice, the rejection of unimodality constitutes strong evidence for multimodality. Furthermore, the appearance of multimodality does not necessarily imply the existence of multiple regimes, although multiple regimes are a very natural explanation for the observation multimodality. Moreover, multimodality is a strong sign for a complex underlying dynamics of the system under consideration. Lastly, even if the observed dynamics exhibits multiple regimes, the question remains whether these multiple regimes are caused by intrinsic mechanisms (self-organized regimes) or extrinsic causes (e.g. multimodality in the numbers of trains, which in turn leads to multimodality in other system observables). But in any case, whatever causes multiple regimes, awareness of their existence and a characterization of their stochastics is useful in order to determine operation procedures optimal for specific regimes.

Our analysis suggests a correlation between punctuality (P) and noise (N) of the electric load profile. We discuss possible causations. First, the decrease in P and the accompanying increase in N could have a common source. Second, the decrease in P could cause an increase in N and finally, third, the increase in N could result in a decrease of P. This latter possibility seems not to be plausible, since a more noisy consumption of energy may increase costs, but has no direct influence on the operation of the railway grid. Although we cannot rule out the first possibility, we favor the hypothesis that decreased punctuality causes the emergence of larger noise which is in accordance with the findings in [16]. There, it has been shown that a decreased punctuality is correlated with an increased power consumption of the railway system.

Even if one accepts a lowered P as a cause for increased N, one has not justified multiple regimes. A potential mechanism that could establish a "tipping point", i.e. a switching between a punctual and an unpunctual mode of operation, is given by the following consideration. The regular interval timetable in the Swiss railway system exhibits a specific feature: the connections are organized in such a manner that trains depart from main hubs around the same minutes of the hour. This leads to a peak of the density of trains and in consequence to time windows with increased probability for track conflicts on

main lines. Comparably small perturbations can therefore have a large impact which is readily transferred over the whole network. Track conflicts (and their resolution) in term lead to increased variations in the speed of trains and consequently to noise in the electric load profile.

At the present state, the available data allows the identification of multimodal distribution but not the underlying dynamics. Concerning the analysis of the relation between noise in the power load and punctuality, a main obstacle for obtaining a deeper insights into the system behavior lies in the fact that we demonstrated that punctuality has to be analyzed with respect to local hubs. But analyzing power load on a local level doesn't make sense, because local acceleration effects are swapping the influence of punctuality.

## VI. CONCLUSION AND OUTLOOK

Even if the detailed mechanisms leading to the multimodality observed are not known, optimal operation protocols should take into account the detailed stochastics of system variables. A further investigation should focus on the question whether such multimodality is caused by a multimodal usage profile or whether it is the result of some internal systems dynamics. If the latter turns out to be true, optimal operation could profit from the detection of early warning signals announcing the transition from one regime into the other.

## VII. APPENDIX

### A. Hartigan's Dip Test

A first step towards the determination of an appropriate control strategy is to analyze whether or not system variables exhibit uni- or multimodal distributions. According statistical tests exist; we use a test developed by Hartigan and Hartigan [17]. Since this test is not generally known, we provide a brief description. Given a sample of  $n$  data points, Hartigan's dip test statistics  $D_n$  measures the departure of the empirical distribution function  $F_n$  from unimodality. In their paper (cf. [17], theorem 6) the authors prove  $D_n = d$  based on an interval  $(x_L, x_U)$  and a non - decreasing function  $G$  satisfying:

- (i)  $G$  is the greatest convex minorant (GCM) of  $F_n + d$  in  $(-\infty, x_L)$
- (ii)  $G$  has constant maximum slope in  $(x_L, x_U)$
- (iii)  $G$  is the least concave majorant (LCM) of  $F_n + d$  in  $[x_U, \infty)$
- (iv)  $d \geq \sup \{ |F_n(x) - G(x)| : x \in (x_U, x_L) \}$

Maechler's R-package (cf. [18]) illustrates Hartigan's method. We present an example, see Fig. 11. Samples of fifty points are drawn from two Gaussians with  $N(0,1)$  and  $N(5,1)$ . Both samples are merged and its density is shown in Fig. 11.

Maechler's package also computes the dip statistics value and its p-value for the test of unimodality by interpolating tabulated quantiles of  $\sqrt{n}D_n$  (for visualization see Fig. 12). In our example the p-value is  $p = 0.00015$ . On a significance level  $\alpha = 0.05$  the null hypothesis of unimodality is rejected. The dip test statistics is computed in order  $n$  operations for  $n$  observations. Any unimodal distribution can be tested against any multimodal distribution.

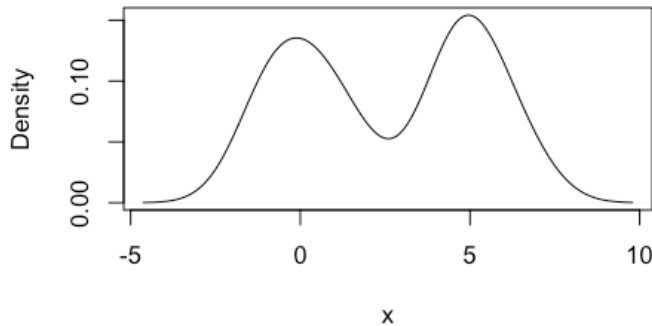


Fig. 11. Density plot of a mixture of two Gaussians

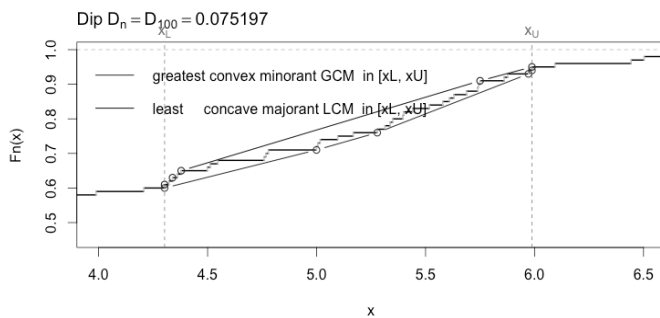


Fig. 12. Depicts the empirical distribution function together with its GCM and LCM on the interval of interest  $(x_L, x_U)$

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