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Prepared for submission to JHEP

Prospects for the determination of the top-quark Yukawa coupling at future e^+e^- colliders

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Abstract: We estimate the sensitivity to the top-quark Yukawa coupling y_t at future e^+e^- colliders. We go beyond the standard approach that focuses on $t\bar{t}h$ production and consider final states with a Higgs boson but not top quarks. The sensitivity to y_t in such processes comes from the coupling of the Higgs boson to top quarks in loops. Such final states can be produced in significant numbers at center-of-mass energies that will be accessible by all proposed e^+e^- colliders. In a simplified theoretical framework to parametrise deviations from the Standard Model, we find that at FCC-*ee* and CEPC operating at $\sqrt{s} = 240$ GeV, y_t could potentially be measured with precision better than 1%. For CLIC and ILC the extraction of y_t could be improved by a factor of about 2 and 7 respectively, compared to its extraction from just $t\bar{t}h$ final states.

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[1] Preprint: Cavendish-HEP-18/07

Contents

1 Introduction

The high-precision determination of the Higgs boson couplings is one of the major tasks facing High Energy Physics [1, 2]. Despite the wealth of information already delivered by the LHC [3], and expected from its future high-luminosity operation, a truly detailed study of the Higgs sector will likely be possible only at a future e^+e^- collider.

The top-quark Yukawa coupling y_t dominates the renormalization group evolution of the Higgs potential at high energy scales (see, for instance, refs. [4–6]). Therefore y_t is among the main drivers of SM predictions at very high energies and often dominates in, amongst others, studies of the self-consistency of the SM at GUT-scale energies and in searches for physics beyond the SM.

The ideal way for measuring y_t would be through $t\bar{t}h$ final states since they allow cleaner interpretation of the measurement in terms of y_t. However, to produce such a final state a center-of-mass (c.m.) energy of at least 500 GeV is required. From all proposed e^+e^- colliders - which we detail in sec. 2 below - only the Compact Linear Collider (CLIC) and the International Linear Collider (ILC) will be capable of achieving such c.m. energies. Preliminary studies concerning $t\bar{t}h$ final states suggest [7–9] that CLIC and ILC will be able to measure y_t with a precision of about 4–5%. Given the importance of the top-quark Yukawa coupling such ultimate precision is not entirely satisfactory. Indeed, it can be contrasted to the expected 10% precision [10] from the HL-LHC and the 1% precision expected at a future 100 TeV hadron collider [11].

Given the central importance of the top-quark Yukawa coupling as well as the seemingly puzzling fact that future e^+e^- colliders may not be able to measure it better than a future hadron collider, in this work we set ourselves the goal of addressing the following question: *what is the ultimate precision with which y_t can be measured at future e^+e^- colliders*?

To answer this question we explore a new approach for the determination of the top-quark Yukawa coupling, utilizing loop-induced Higgs production and decay processes, in a simple version of the so-called κ-framework [12]. Loop-induced processes have an advantage in that they potentially allow for a precise determination of y_t at colliders such as FCC-*ee* or CEPC which are designed to operate at c.m. energies below the $t\bar{t}h$ threshold. Furthermore, even at colliders that can produce $t\bar{t}h$ final states, measurements at different c.m. energies could be combined in order to derive more precise determination of y_t than from $t\bar{t}h$ final states alone. Such indirect approaches are already being pursued in, for example, the determination of the Higgs self-interaction at the LHC [13–18].

This work is organized as follows: in sec. 2 we provide a general overview of future e^+e^- colliders and the Higgs processes considered in this paper. In sec. 3 we describe our approach for determining y_t and detail the fitting procedure adopted in our analysis. Our numerical results are presented in sec. 4. In sec. 5 we discuss the limitations of our study and possible future extensions. Our conclusions are summarized in sec. 6.

2 Higgs production and decay processes at e^+e^- colliders

A number of future lepton colliders have been proposed over the years: [1]

- The Circular Electron Positron Collider (CEPC) in China would collect 5 ab^{-1} of integrated luminosity at 240 GeV [19–21]. A run at 350 GeV could also be envisioned;

- The Future Circular Collider with e^+e^- (FCC-*ee*) is a high-luminosity, high-precision circular collider envisioned in a new 80-100 km tunnel at CERN [22, 23]. The FCC-*ee* aims at collecting multi-ab^{-1} integrated luminosity at $\sqrt{s} = 90$, 160, 240, and 350 GeV. In particular, 10 ab^{-1} of data would be collected at 240 GeV and 2.6 ab^{-1} at 350 GeV;

- The Compact Linear Collider (CLIC) at CERN would collect 100 fb^{-1}at the top threshold, 500 fb^{-1} at 380 GeV, 1.5 ab^{-1} at 1.5 TeV, and 3 ab^{-1} at 3 TeV [24]. The study of Higgs measurements at CLIC [7] assumes a different scenario: 500 fb^{-1} at 350 GeV, 1.5 ab^{-1} at 1.4 TeV and 2 ab^{-1} at 3 TeV. In the following we assume this scenario, for which an accurate estimate of the relevant experimental uncertainties is available;

- The International Linear Collider (ILC) is a proposed linear e^+e^- collider to run in the energy range between 200 GeV and 500 GeV [8, 9]. Here we follow a scenario with three energy stages: 250, 350, and 500 GeV, and accumulated luminosity of, respectively, 2 ab^{-1}, 200 fb^{-1} and 4 ab^{-1}. The 350 GeV stage of the ILC is mainly intended for the determination of the top quark mass from the $t\bar{t}$ threshold. This stage will not be considered in our study due to its low luminosity.

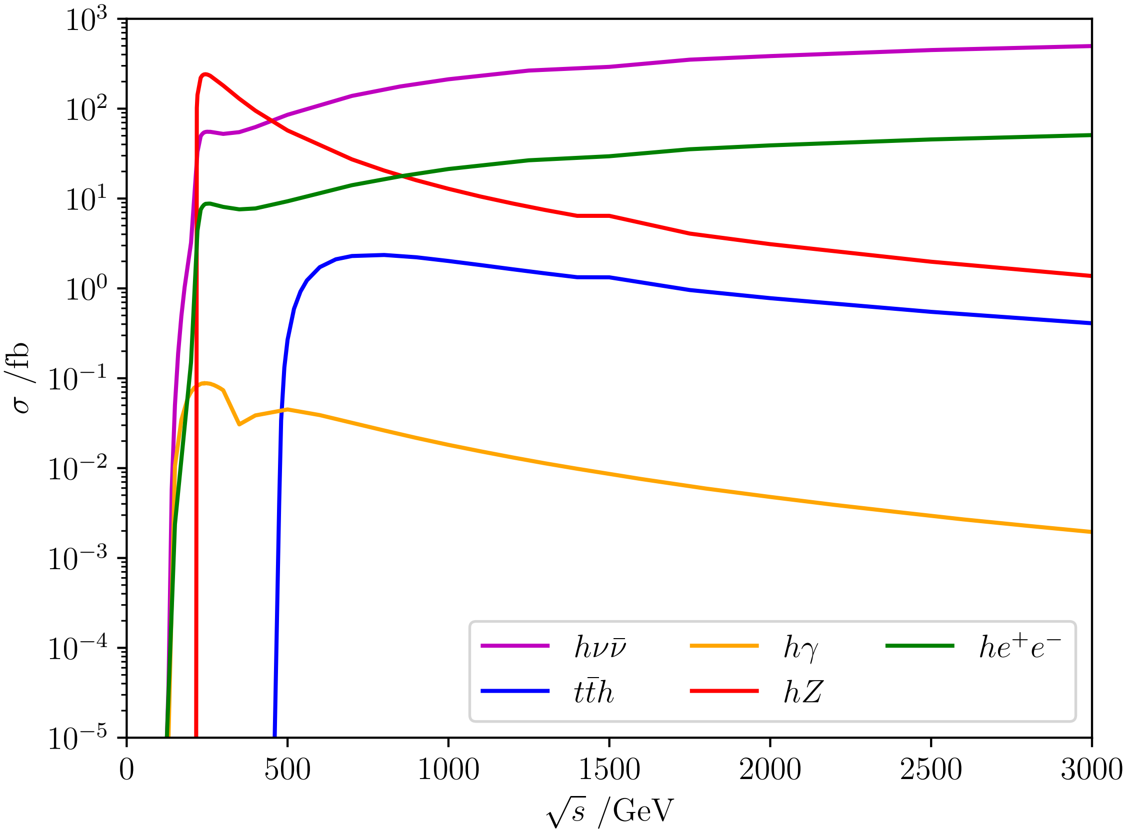

Figure 1. LO cross-section for the single Higgs production channels at future lepton colliders described in Sec. 2. The process labeled hZ includes all decays of the Z boson.

The dominant single-Higgs production mechanisms at the above future colliders are the s-channel Higgstrahlung process $e^+e^- \to hZ$ and the t-channel charged vector-boson fusion (VBF) process resulting in $h\nu\bar{\nu}$ final states. The relative importance of these two processes depends on the c.m. energy; the Higgstrahlung process dominates around 240-250 GeV while the VBF cross section takes over around 500 GeV. At even higher energies, the neutral VBF process $e^+e^- \to he^-e^+$ also becomes significant. One should keep in mind, however, that the separation of the various processes is not unambiguous once the Z decays have been taken into account. In particular, Higgstrahlung with Z decaying to neutrinos (electrons) yields the same final-state as the charged (neutral) vector boson fusion processes. This contamination is particularly relevant at 240 GeV where the inclusive $h\nu\bar{\nu}$ rate is dominated by Higgstrahlung.

For this reason in fig. 1 we show the c.m. dependence of the computed at leading order

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[1] Since this study was conducted and this publication prepared, FCC-ee, CEPC and CLIC have all produced updates to their operational baselines. The potential consequences for our analysis are discussed in sec. 7.

	FCC-ee		CEPC
$\sqrt{s}$ (GeV)	240	350	240
$\mathcal{L}_{\text{int.}}$ (fb^{-1})	$1.0 \cdot 10^4$	$2.6 \cdot 10^3$	$5.0 \cdot 10^3$
σ_{hZ} (fb)	240	130	240
$\mathcal{N}_{hZ}$	$2.4 \cdot 10^6$	$3.38 \cdot 10^5$	$1.2 \cdot 10^6$
$\sigma_{\nu\bar{\nu}h}$ (fb)	54.4	54.7	54.4
$\mathcal{N}_{\nu\bar{\nu}h}$	$5.44 \cdot 10^5$	$1.42 \cdot 10^5$	$2.72 \cdot 10^5$
σ_{eeh} (fb)	7.9	7.13	7.9
$\mathcal{N}_{eeh}$	$7.9 \cdot 10^4$	$1.85 \cdot 10^4$	$3.95 \cdot 10^4$
$\sigma_{h\gamma}$ (fb)	$8.96 \cdot 10^{-2}$	$3.18 \cdot 10^{-2}$	$8.96 \cdot 10^{-2}$
$\mathcal{N}_{h\gamma}$	896	82	448

Table 1. Inclusive LO cross-sections and numbers of expected events for the main Higgs production modes at FCC-*ee* and CEPC. The process labeled hZ includes all decays of the Z boson.

(LO) inclusive cross-section for the final states described above. The $h\nu\bar{\nu}$ and he^+e^- channels include the Higgsstrahlung contribution; the process labeled hZ on the other hand includes all Z decay modes. The same applies to the results in tables 1,2 below. We note, however, that in table 3 the process labeled hZ is not inclusive in the Z decay and includes only certain Z decay modes which are specific to the analyses referenced in that table.

In this study we are interested in processes which are sensitive to a non-vanishing anomalous contribution to y_t. For this reason, in fig. 1 we show two more Higgs production processes: $t\bar{t}h$, which is usually considered as the only available channel for the extraction of y_t, and the loop-induced process $h\gamma$ which is one of the inputs to our analysis.

To get an overall impression about the potential of the various Higgs production modes, in tables 1 and 2 we show the expected number of events for each run of the future colliders described above. The expected numbers of events are derived by multiplying the inclusive cross-sections with the corresponding luminosities shown in tables 1 and 2.

As far as Higgs decays are concerned, the loop-induced processes $h \to gg$ and $h \to \gamma\gamma$ are both sensitive to y_t and will be considered in the following. The Higgs decay to gluons is generated by massive quarks in the loops with the top-quark being the dominant contribution. In the $m_t \to \infty$ limit this coupling is known with next-to-next-to-next to leading order (N^{3}LO) QCD accuracy [25]. In contrast, the Higgs decay to photons (as well as $h\gamma$ production) has a dominant contributions from loops involving gauge bosons which results in a reduced sensitivity to y_t compared to $h \to gg$.

	CLIC			ILC	
$\sqrt{s}$ (GeV)	350	1400	3000	250	500
$\mathcal{L}_{\text{int.}}$ (fb^{-1})	$5.0 \cdot 10^2$	$1.5 \cdot 10^3$	$2.0 \cdot 10^3$	$2.0 \cdot 10^3$	$4.0 \cdot 10^3$
σ_{hZ} (fb)	130	6.42	1.37	240	57.2
$\mathcal{N}_{hZ}$	$6.50 \cdot 10^4$	$9.6 \cdot 10^3$	$2.74 \cdot 10^3$	$4.80 \cdot 10^5$	$2.29 \cdot 10^5$
$\sigma_{\nu\bar{\nu}h}$ (fb)	54.4	293	498	55.0	85.2
$\mathcal{N}_{\nu\bar{\nu}h}$	$2.73 \cdot 10^4$	$4.39 \cdot 10^5$	$9.96 \cdot 10^5$	$1.10 \cdot 10^5$	$3.41 \cdot 10^5$
σ_{eeh} (fb)	7.13	28.3	49.1	8.2	8.7
$\mathcal{N}_{eeh}$	$3.56 \cdot 10^3$	$4.24 \cdot 10^4$	$9.82 \cdot 10^4$	$1.64 \cdot 10^4$	$3.48 \cdot 10^4$
$\sigma_{t\bar{t}h}$ (fb)	-	1.33	0.41	-	0.27
$\mathcal{N}_{t\bar{t}h}$	-	1995	820	-	$1.08 \cdot 10^3$
$\sigma_{h\gamma}$ (fb)	$3.18 \cdot 10^{-2}$	$1.20 \cdot 10^{-2}$	$3.08 \cdot 10^{-3}$	$8.97 \cdot 10^{-2}$	$4.74 \cdot 10^{-2}$
$\mathcal{N}_{h\gamma}$	16	18	6	179	189

Table 2. As in table 1 but for CLIC and ILC.

3 Our approach for determining y_t

The problem of determining y_t at a given run of a hypothetical future collider is formulated in terms of the new physics contribution Δy_t to the top-quark Yukawa coupling y_t

$$y_t = y_t^{\text{SM}} + \Delta y_t \,, \tag{3.1}$$

where we assume that this is the only source of deviation from the SM. This assumption is discussed in sec. 5.

The main limitation of an analysis restricted to $t\bar{t}h$ data is that it requires a c.m. energy of at least 500 GeV. To circumvent this limitation, we also consider e^+e^- observables which are *indirectly* sensitive to y_t and, at the same time, have sufficiently large number of expected events. The y_t dependence in such processes originates in the Higgs boson coupling to top quarks in loops, either in the production or in the decay of the Higgs boson [2]. Thus, in addition to $t\bar{t}h$ production, we consider the Higgs decays to gluons and photons as well as Higgs production in association with a hard photon. The idea is to exploit the y_t dependence of single Higgs processes with the added benefit that these processes are accessible at all c.m energies.

In order to constrain Δy_t we define a global χ^2 for each run of the future colliders described in the previous section

$$\chi^2(\Delta y_t) = \sum_{i=1}^{N_p} \sum_{j=1}^{N_d} \frac{\left[\mu_{ij}\left(\Delta y_t\right) - 1\right]^2}{\delta_{ij}^2} \,, \tag{3.2}$$

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[2] We will not consider final states with top quarks but no Higgs. These final states are included in the $t\bar{t}$ threshold scan studies [26–28].

Collider	$\sqrt{s}$ (GeV)	$\mathcal{L}$ (fb^{-1})	$h \to gg$		$h \to \gamma\gamma$		$h \to b\bar{b}$	
			hZ	$\nu\bar{\nu}h$	hZ	$\nu\bar{\nu}h$	$h\gamma$	$t\bar{t}h$
FCC-*ee*	240	$1.0 \cdot 10^4$	1.4%	-	3.0%	-	4.4%	-
	350	$2.6 \cdot 10^3$	3.1%	4.7%	14%	21%	14%	-
CEPC	240	$5.0 \cdot 10^3$	1.2%	-	9.0%	-	6.2%	-
CLIC	350	$5.0 \cdot 10^2$	6.1%	10%	-	-	-	-
	1400	$1.5 \cdot 10^3$	-	5.0%	-	15%	-	8.0%
	3000	$2.0 \cdot 10^3$	-	4.3%	-	10%	-	12.5%
ILC	250	$2.0 \cdot 10^3$	2.5%	-	12%	-	10%	-
	500	$4.0 \cdot 10^3$	3.9%	1.4%	12%	6.7%	9.8%	9.9%

Table 3. The estimated one-sigma uncertainties δ_{ij} used in eq. (3.2). The ones for CLIC, CEPC and FCC-*ee* at $\sqrt{s} = 240$ GeV are taken from refs. [7, 19, 22], respectively. The ones for ILC and FCC-*ee* at $\sqrt{s} = 350$ GeV are from ref. [29]. The $\sqrt{s} = 3$ TeV CLIC $t\bar{t}h$ result is derived by extrapolating the 1.4 TeV one with the corresponding number of events. The statistical uncertainties for $h\gamma$ production are derived from the expected number of events reported in tables 1 and 2. The process labeled hZ includes selected Z decays, and their content is specific to each analysis referenced in this table.

with N_p and N_d being, respectively, the number of available production and decay channels. The sums in eq. (3.2) include only the processes for which δ_{ij} values are explicitly shown in table 3.

The degrees of freedom of the χ^2 in eq. (3.2) are the signal-strengths μ_{ij} of all Higgs boson processes which are sensitive to a non-vanishing value of Δy_t and which can be measured with a sufficient precision. The signal-strength μ_{ij} for a generic Higgs production mode i and decay channel j can be written in the narrow-width approximation as

$$\mu_{ij} = \left(\frac{\sigma_i}{\sigma_i^{\text{SM}}}\right)\left(\frac{\Gamma_j}{\Gamma_j^{\text{SM}}}\right)\left(\frac{\Gamma_h}{\Gamma_h^{\text{SM}}}\right)^{-1} . \tag{3.3}$$

In eq. (3.3) Γ_h is the total Higgs width and σ_i and Γ_j are the corresponding production cross-section and partial decay width. Due to the small number of expected $t\bar{t}h$ and $h\gamma$ events, these two production channels are included with only the dominant $h \to b\bar{b}$ decay mode.

The one-sigma uncertainties δ_{ij} appearing in eq. (3.2) and listed in table 3 are taken from the literature [7, 19, 22, 29]. Their values have been derived in a realistic framework that accounts for acceptance cuts, background contributions and detector simulation for the reconstruction of the final state. To the best of our knowledge such studies are not available for the $h\gamma$ process. For this reason the one-sigma uncertainties for this channel have been derived by calculating the Poissonian error and have to be considered as optimistic estimates of the total uncertainties.

The analytic expressions for the $h\gamma$ and $t\bar{t}h$ signal-strengths, as functions of Δy_t, read

$$\mu_{h\gamma}\begin{pmatrix}\sqrt{s}=240\,\mathrm{GeV}\\ \sqrt{s}=250\,\mathrm{GeV}\\ \sqrt{s}=350\,\mathrm{GeV}\\ \sqrt{s}=500\,\mathrm{GeV}\end{pmatrix}=\frac{\sigma_{h\gamma}}{\sigma_{h\gamma}^{\mathrm{SM}}}=1-\begin{pmatrix}0.43\\ 0.45\\ 0.73\\ 0.13\end{pmatrix}\Delta y_t \tag{3.4}$$

$$\mu_{t\bar{t}h}\begin{pmatrix}\sqrt{s}=500\,\mathrm{GeV}\\ \sqrt{s}=1400\,\mathrm{GeV}\\ \sqrt{s}=3000\,\mathrm{GeV}\end{pmatrix}=\frac{\sigma_{t\bar{t}h}}{\sigma_{t\bar{t}h}^{\mathrm{SM}}}=1+\begin{pmatrix}1.99\\ 1.83\\ 1.71\end{pmatrix}\Delta y_t, \tag{3.5}$$

In the calculation of the above expressions we do not include corrections beyond LO. Such higher-order effects have been studied in ref. [7] for the 1.4 TeV run of CLIC. These corrections result in a relatively small shift in the corresponding coefficient in eq. (3.5) from 1.83 to 1.89. In turn, this slightly increases the y_t precision in the $t\bar{t}h$ channel.

For the two loop-induced Higgs decay processes we get

$$\mu_{h\to gg}=\frac{\Gamma_{h\to gg}}{\Gamma_{h\to gg}^{\mathrm{SM}}}=1+2\Delta y_t\,, \tag{3.6}$$

$$\mu_{h\to\gamma\gamma}=\frac{\Gamma_{h\to\gamma\gamma}}{\Gamma_{h\to\gamma\gamma}^{\mathrm{SM}}}=1-0.56\Delta y_t\,. \tag{3.7}$$

All computations in this work have been carried out in the G_μ input scheme with the help of the `Madgraph5_aMC@NLO_v2.6.1` code [30]. Eqs. (3.4–3.7) have been derived in the following way: we first compute the corresponding cross-sections and decay widths for a number of different values of Δy_t and then fit the resulting expressions for μ_{ij} with a parabola. Finally, we take its linear approximation for small values of Δy_t. In deriving $\mu_{h\to gg}$ the bottom quark contribution in the loop has been neglected.

4 Results

Our main results, namely, the 68% CL constraints following from eq. (3.2), are displayed in table 4 and in fig. 2. We report results for the following scenarios: the FCC-*ee* runs at c.m. energies of 240 GeV and 350 GeV, the CEPC run at 240 GeV, the three CLIC runs at 350 GeV, 1.4 TeV and 3 TeV and the 250 GeV and 500 GeV runs of the ILC.

From table 4 and fig. 2 we conclude that the decay process $h\to gg$ is a strong potential candidate for precise determination of y_t. Combining the high y_t sensitivity of $h\to gg$ seen in eq. (3.6) with the high luminosities at the 240 GeV FCC-*ee* and CEPC runs and at the 500 GeV ILC run, one may potentially be able to determine Δy_t with uncertainty of 0.6–0.7%.

The potential of $h\to\gamma\gamma$ for a precise determination of y_t is much smaller than $h\to gg$. Despite the low cross-section of $e^+e^-\to h\gamma$ (see fig. 1), this loop-induced process allows access to y_t at both FCC-*ee* and CEPC. While not directly competitive with $h\to gg$, this additional y_t sensitivity is on par with the one expected at HL-LHC and may be useful for

Collider	$\sqrt{s}$ (GeV)	$\mathcal{L}$ $\left(\text{fb}^{-1}\right)$	$h \to gg$	$h \to \gamma\gamma$	$h\gamma$	$t\bar{t}h$	total
FCC-*ee*	240	$1.0 \cdot 10^4$	0.7%	5.3%	10%	-	0.7%
	350	$2.6 \cdot 10^3$	1.3%	21%	19%	-	1.3%
CEPC	240	$5.0 \cdot 10^3$	0.6%	16%	14%	-	0.6%
CLIC	350	$5.0 \cdot 10^2$	2.6%	-	-	-	2.6%
	1400	$1.5 \cdot 10^3$	2.5%	27%	-	4.4%	2.2%
	3000	$2.0 \cdot 10^3$	2.2%	18%	-	7.3%	2.1%
ILC	250	$2.0 \cdot 10^3$	1.2%	21%	23%	-	1.2%
	500	$4.0 \cdot 10^3$	0.7%	10%	75%	5.0%	0.7%

Table 4. 68% CL boundaries on Δy_t for different runs and processes. In the last column we report the results of the global χ^2 analysis described in sec. 4.

disentangling Wilson coefficients in a more refined Effective Field Theory (EFT) approach (see sec. 5 for further details and ref. [31] for a recent review).

As far as CLIC is concerned, its 350 GeV run allows y_t to be determined from purely loop-induced processes with precision of about 2.6%. At higher CLIC energies the precision in the y_t determination from loop-induced processes is significantly larger than the one expected from the standard $t\bar{t}h$-based approach. Our estimates show that by combining the extraction of y_t from $t\bar{t}h$ with that from loop-induced final states one can reach y_t-precision of about 2.1–2.2% at both the $\sqrt{s} = 1.4$ TeV and $\sqrt{s} = 3.0$ TeV CLIC runs. This is 2-3 times better than the precision expected from purely $t\bar{t}h$ final states.

From table 4 and fig. 2 we also conclude that for all collider runs considered by us, loop-induced processes (mostly $h \to gg$) could potentially lead to significantly more precise determination of y_t compared to $t\bar{t}h$ final states.

5 Limitations of the present study and possible further improvements

The precision in the various y_t determinations estimated in the previous section are based on a number of assumptions and approximations. We discuss them in turn.

For most processes and colliders we have used existing studies for Higgs production and decay. An exception is the process $e^+e^- \to h\gamma$ [32–35]. In this work we have computed this process at LO in the SM with the help of `Madgraph5_aMC@NLO_v2.6.1`, i.e. accounting fully for the loop in the Higgs production process [3]. No detector simulation, efficiency estimates or realistic estimate of backgrounds and systematic effects have been performed by us for this process. For this reason the purely statistical errors derived by us are, likely, optimistic.

In all our estimates we assume that the top-quark mass m_t is perfectly known. While the future lepton colliders capable of reaching the $t\bar{t}$ threshold of $\sqrt{s} = 350$ GeV should be

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[3] The rational terms in the relevant `Madgraph5_aMC@NLO_v2.6.1` model file are written in terms of the top-quark mass. They need to be recast in terms of y_t in order to obtain the correct y_t dependence of the $h\gamma$ cross-section.

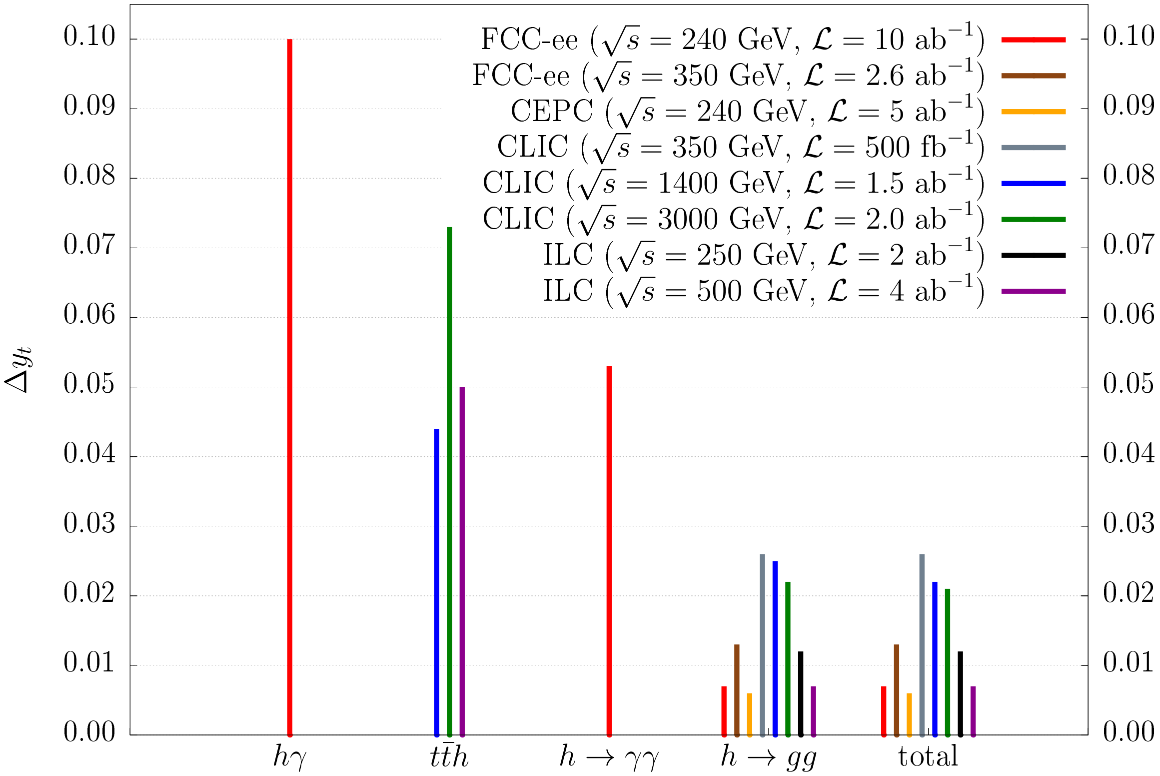

Figure 2. 68% CL boundaries on Δy_t for different runs and processes. In the last column we report the results of the global χ^2 analysis described in sec. 3.

able to measure m_t with excellent precision of about 50 MeV [26–28, 36–38], the timing of such measurements may be an issue. For example, a $t\bar{t}$ threshold scan may come only after the measurements at 240 GeV have been performed. In any case one can either reanalyze the lower energy measurement in light of new m_t measurements or utilize then-available HL-LHC measurements. Indeed, the $\mathcal{O}(300$ MeV$)$ uncertainty on m_t expected from HL-LHC will already be sufficiently precise to be a subdominant effect for even the most precise expected determination of y_t.

An important caveat for our study is the interpretation of the possible measurements in terms of uncertainty on y_t alone. As we already mentioned in sec. 3, we work in a simple extension of the SM where Δy_t is the only possible anomalous coupling. In effect this allows us to simply trade any uncertainty in the measurement for an uncertainty in y_t. In reality such an assumption is not well motivated since the assumption of non-vanishing Δy_t implies physics beyond the SM, and once such an assumption is made then there is no good justification for assuming a single source of deviation from SM. In this sense, ideally, one would like to treat the problem of the y_t determination within the κ-framework or a full-blown EFT approach (see refs. [39–52] for global analyses in the LHC framework and refs. [29, 53, 54] for EFT analyses in the context of future lepton colliders).

While the introduction of many EFT couplings will reduce the sensitivity to y_t from the measurements we discuss, one should bear in mind that we have not assumed any prior knowledge on y_t or any other EFT coupling. In reality the LHC, especially after its high luminosity phase, is expected to produce a significant set of constraints on both y_t and the EFT couplings that will contribute to the extraction of y_t at future lepton colliders (a recent LHC update with HL-LHC projections for relevant operators can be found in ref. [55]). This will benefit the proposed extraction of y_t.

Our indirect approach for the extraction of y_t could also be applied/contrasted with the HL-LHC. A naive translation of the latest projections from a κ-framework study on Higgs physics at the HL-LHC [56] into our framework using $\kappa_g^2 = \Gamma_{h\to gg}/\Gamma^{\text{SM}}_{h\to gg}$ and eq. (3.6), yields $\Delta y_t \approx 2.5\%$. This is better than the expected direct y_t determination from $t\bar{t}h$ final states, but still around a factor of four worse than what some future e^+e^- colliders can achieve.

Another tacit assumption made in our analysis is the perfect (or near perfect) knowledge of the SM predictions for the processes under consideration. Full NLO SM accuracy is now easily achievable for non-loop-induced process thanks to automated tools like `Madgraph5_aMC@NLO_v2.6.1`, `Sherpa` [57] and `Whizard` [58, 59]. Full NLO SM accuracy is also desired for the loop-induced processes discussed here. This requires the calculation of two-loop multiscale amplitudes. A lot of progress in this direction has recently been achieved at the LHC [60–68] making this a doable, albeit non-trivial, problem.

6 Conclusions

We estimate the ultimate precision with which the top-Yukawa coupling y_t can be extracted at the various proposed high-energy e^+e^- colliders, utilising a simplified version of the κ-framework. Our motivation for embarking on this study stems from the recognition that the traditional approach for extracting y_t from $t\bar{t}h$ final states may be too restrictive: such an approach can only be realised at a couple of proposed colliders (CLIC and ILC) and in both cases results in somewhat limited precision of about 4–5%. Such precision has to be viewed in the context of the high-luminosity LHC, which will precede any future e^+e^- machine, and where precision on y_t of about 10%, or even better, is expected.

To increase the scope for precise extraction of y_t at future e^+e^- colliders, in this work we consider an alternative set of final states that are *indirectly* sensitive to y_t. The main advantage in considering loop-induced processes is that due to their large expected event yields and sensitivity to y_t, such processes can significantly increase the range of lepton colliders at which y_t can be precisely determined.

We find that potentially one could measure y_t with precision of about 0.6% at the 240 GeV CEPC run. Similarly, the 240 GeV FCC-*ee* and 500 GeV ILC runs have the potential for determining y_t with precision of 0.7%. Such high y_t precision is driven mainly by the Higgs decay to gluons. Furthermore, the inclusion of $h \to gg$ data significantly increases the sensitivity to y_t at higher c.m. energies. For example, at the 1.4 TeV CLIC run one can get an improvement by a factor of about two compared to $t\bar{t}h$-only data.

The loop-induced Higgs decay $h \to \gamma\gamma$ is not as sensitive to y_t but still offers a decent, better than 6%, precision at the FCC-*ee* collider. While not directly competitive with the $h \to gg$ decay, $h \to \gamma\gamma$ data could be useful for disentangling contributions from effective couplings and in some cases offers precision better than the one expected form the HL-LHC.

Finally, we have identified the loop-induced associated process $e^+e^- \to h\gamma$, with $h \to b\bar{b}$, which does not rely on loop-induced Higgs decays and could allow y_t precision of about 10%. Such a precision is comparable to the one expected from the HL-LHC.

7 Note added

As mentioned in section 2, after our work was completed and made available on arXiv, updates to the baselines of FCC-*ee*, CEPC and CLIC appeared. The CEPC CDR [69] reports a modest $\sim$ 10% increase in integrated luminosity. In contrast, CLIC now plans to take more than double the integrated luminosity than quoted here, whereas the FCC-*ee* data-taking expectations [70] have been roughly halved. Both have also made small changes to their planned centre-of-mass energies. Whilst these are large operational changes, none are predicted to make a significant effect on the $h \to gg$ process. The $h \to gg$ signal strength (eq. (3.6)) is not affected by $\sqrt{s}$, while the experimental error on extracting $\sigma \times \mathrm{BR}(h \to gg)$ - as listed in table 3 - is not predicted by FCC-*ee* or CEPC to significantly change with their new baselines. Since we find that this process dominates the extraction of y_t at all future lepton colliders, the broad conclusions we have drawn here remain unchanged. This is illustrated by an approximate scaling of our results performed in light of CLIC's new baseline, which modestly improves CLIC's reach to below 2% on Δy_t [71].

Acknowledgments

The authors are grateful to Ambresh Shivaji for useful discussions and for providing some independent checks. A.M. thanks the Department of Physics at Princeton University for hospitality during the completion of this work. The work of S.B. and A.M. is supported by the European Research Council Consolidator Grant NNLOforLHC2. The work of A.M. is also supported by the UK STFC grants ST/L002760/1 and ST/K004883/1.

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