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Computational testing algorithmic procedure of assessment for lifetime performance index of products with Weibull distribution under progressive type I interval censoring

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ABSTRACT

The assessing of the lifetime performance is a very important topic in manufacturing or service industries. Process capability indices had been widely used to evaluate the process performance to the continuous improvement of quality and productivity. The lifetimes of products are assumed to have Weibull distribution with a known shape parameter and the larger-the-better lifetime performance index is considered. The maximum likelihood estimator is used to estimate the lifetime performance index based on the progressive type I interval censored sample. The asymptotic distribution of this estimator is also investigated. We use this estimator to develop the new hypothesis testing algorithmic procedure with respect to a lower specification limit. Finally, two practical examples are given to illustrate the use of this testing algorithmic procedure to determine whether the process is capable.

1. Introduction

In the competitive market, the quality of products is the main concern for most consumers. For measuring the quality of products, process capability indices (PCIs) had been widely used such as C_p, C_{pk}, C_{pm} and C_{pmk}. See more examples and details in Montgomery [1]. Since the lifetime of products is a larger-the-better type quality characteristic, the PCI of unilateral tolerance, C_L is consider to assess the performance of lifetime with Weibull distribution. For example, Tong et al. [2] constructed the uniformly minimum variance unbiased estimator (UMVUE) of C_L and built a hypothesis testing procedure under the assumption of one-parameter exponential distribution for the complete sample.

However, in practice, the experimenter may not always be in a position to observe the lifetimes of all the items on test. This may be because of time limitation and/or other restrictions (such as money, material resources, negligence of typist or recorder, mechanical or experimental difficulties, etc.) on data collection. Therefore, some incomplete data could be collected, such as progressive censoring data (see Balakrishnan and Aggarwala [3], Aggarwala [4], Wu et al. [5], Wu et al. [6], Sanjel and Balakrishnan [7], Lee et al. [8], and Wu [9]). Ahmadi et al. [10] estimated the lifetime performance index with Weibull distribution based on progressive first-failure censoring scheme.

Wu and Lin [11] constructed the maximum likelihood estimator (MLE) for C_L and provided a hypothesis testing procedure for the one-parameter exponential distribution based on a progressive type I interval sample. The progressive type I interval censoring scheme is described as follows: Suppose that there are n products put on a life test at time 0. Let $(t_1, \ldots, t_m)$ be the predetermined inspection times, where t_m is scheduled to terminate the experiment. At time t_i, p_i is the pre-specified removal percentage of the remaining survival units at time t_i, $i = 1, \ldots, m$, where $p_m = 1$. During the first time interval

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$(0, t_1]$., the number of failure units X_1 is observed, and then $R_1 = [(n - X_1)\, p_1]$ units are randomly removed from the remaining $n - X_1$ survival units, where $[w]$ denotes the largest integer which is smaller than or equal to w. During the second time interval $(t_1, t_2]$, the number of failure units X_2 is observed and then $R_2 = [(n - X_1 - R_1 - X_2)\, p_2]$ units are randomly removed from the remaining $n - X_1 - R_1 - X_2$ survival units. Repeat the same process until the last time interval $(t_{m-1}, t_m]$, the number of failure units $X_m$is observed during and the remaining $R_m = n - \sum_{i=1}^{m} X_i - \sum_{i=1}^{m-1} R_i$ units are all removed. Then the life test is terminated at time t_m. The Weibull distribution with known shape parameter has been considered in literature and applied in practice (see for example [12–14]). Note that the widely used lifetime distributions, exponential and Rayleigh distributions, are special cases of this distribution. In this paper, we consider to investigate the estimation of C_L and build a hypothesis testing procedure to assess the lifetime performance of Weibull distribution, where the shape parameter is pre-determined by the maximum p-value method proposed by Lee [15] and hence supposed that the shape parameter is known.

The rest of this paper is organized as follows: In Section 2, some properties of the lifetime performance index for products with Weibull lifetimes are introduced and the relationship between the lifetime performance index and conforming rate is discussed. The MLE of the lifetime performance index and its related statistical properties based on the progressive type I interval censored sample from Weibull distribution are obtained in Section 3. In Section 4, we develop an algorithmic of hypothesis testing procedure for the lifetime performance index. Power analysis of the proposed testing procedure is also conducted in this section. Two numerical examples are given to illustrate our proposed testing procedure in Section 5. Finally, the conclusion is made in Section 6.

2. The lifetime performance index and the conforming rate

Suppose that the lifetime (U) of products follows a Weibull distribution with the probability density function (pdf) and the cumulative distribution function (cdf) as follows:

$$f(u) = \frac{\beta}{\lambda}\left(\frac{u}{\lambda}\right)^{\beta-1} e^{-(\frac{u}{\lambda})^{\beta}}, \quad u > 0, \beta > 0, \lambda > 0 \tag{1}$$

and

$$F(u) = 1 - \exp\left\{-\left(\frac{u}{\lambda}\right)^{\beta}\right\}, \quad u > 0, \beta > 0, \lambda > 0 \tag{2}$$

where λ is the scale parameter and β is shape parameter and the failure rate function is defined as

$$h(u) = \frac{f_U(u)}{1 - F_U(u)} = \frac{\beta}{\lambda}\left(\frac{u}{\lambda}\right)^{\beta-1}. \tag{3}$$

Montgomery [1] developed a process capability index C_L to measure the larger-the-better type quality characteristics as follows:

$$C_L = \frac{\mu - L}{\sigma}, \tag{4}$$

where μ denotes the process mean, σ represents the process standard deviation, and L is the known lower specification limit. This lifetime performance index can be used to assess the performance of lifetime of products. Using the transformation $Y = U^{\beta}$, then the new lifetime variable Y follows an exponential distribution with scale parameter $\tilde{k}$, where $\tilde{k} = k^{-1}$ and $k = \frac{1}{\lambda^{\beta}}$. The pdf and cdf of the new lifetime variable Y are given as follows:

$$f(y) = k\exp\{-ky\}, \quad y > 0, k > 0 \tag{5}$$

and

$$F(y) = 1 - \exp\{-ky\}, \quad y > 0, k > 0. \tag{6}$$

The mean and standard deviation of new lifetime variable Y are given by $\mu = E(Y) = \frac{1}{k}$ and $\sigma = \sqrt{Var(Y)} = \frac{1}{k}$. If L_U is the lower specification limit for lifetime variable U, then $L = L_U^{\beta}$ is the corresponding lower specification limit for new lifetime variable Y. The lifetime performance index C_L can be rewritten as

$$C_L = \frac{\mu - L}{\sigma} = \frac{1/k - L}{1/k} = 1 - kL. \tag{7}$$

We can see that the index $C_L > 0$ when $\lambda > L$ and $C_L < 0$ when $\lambda < L$. We can also see that the smaller the failure rate k the larger the lifetime performance index C_L. Therefore, the lifetime performance index C_L can accurately assess the lifetime performance of products.

An item of product is identified as a conforming one if its lifetime exceeds the lower specification limit (i.e. $Y \geq L$) and hence the conforming rate is defined as

$$P_r = P(Y \geq L) = \exp\{-kL\} = \exp\{C_L - 1\}, \quad -\infty < C_L < 1. \tag{8}$$

Table 1
The lifetime performance index C_L and its corresponding conforming rates P_r.

C_L	P_r	C_L	P_r	C_L	P_r
$-\infty$	0.000000	−0.125	0.324652	0.550	0.637628
−3.000	0.018316	0.000	0.367879	0.575	0.653770
−2.750	0.023518	0.125	0.416862	0.600	0.670320
−2.500	0.030197	0.150	0.427415	0.625	0.687289
−2.250	0.038774	0.175	0.438235	0.650	0.704688
−2.125	0.043937	0.200	0.449329	0.675	0.722527
−2.000	0.049787	0.225	0.460704	0.700	0.740818
−1.750	0.063928	0.250	0.472367	0.725	0.759572
−1.500	0.082085	0.275	0.484325	0.750	0.778801
−1.250	0.105399	0.300	0.496585	0.775	0.798516
−1.125	0.119433	0.325	0.509156	0.800	0.818731
−1.000	0.135335	0.350	0.522046	0.825	0.839457
−0.750	0.173774	0.375	0.535261	0.850	0.860708
−0.500	0.223130	0.400	0.548812	0.875	0.882497
−0.250	0.286505	0.425	0.562705	0.900	0.904837
−0.225	0.293758	0.450	0.576950	0.925	0.927743
−0.200	0.301194	0.475	0.591555	0.950	0.951229
−0.175	0.308819	0.500	0.606531	0.975	0.975310
−0.150	0.316637	0.525	0.621885	1.000	1.000000

Apparently, there is a strictly increasing relationship existing between the conforming rate P_r and the lifetime performance index C_L. The values of C_L and its corresponding conforming rates P_r using Eq. (8) are computed and listed in Table 1. From Table 1, we can see that if the manager desires P_r to exceed 0.818731, then C_L is supposed to be set up to exceed 0.8.

3. Maximum likelihood estimator of the lifetime performance index

Suppose that $X_1, \ldots, X_m$ is the progressive type I interval censored sample observed at pre-set times $t_1, \ldots, t_m$ with $R_1, \ldots, R_m$ survival units randomly removed from the remaining survival units based on the removal percentages $p_1, \ldots, p_m$ at times $t_1, \ldots, t_m$. The likelihood function of the progressive type I interval censored sample $X_1, \ldots, X_m$ is

$$\begin{aligned} L(k) &\propto \prod_{i=1}^{m} (F(t_i) - F(t_{i-1}))^{X_i} (1 - F(t_i))^{R_i} \\ &= \prod_{i=1}^{m} (1 - e^{-k(y_i - y_{i-1})})^{x_i} e^{-k(y_{i-1}x_i + y_i R_i)}, \end{aligned} \tag{9}$$

and the log-likelihood function is

$$\ln L(k) = \sum_{i=1}^{m} (x_i \ln(1 - \exp\{-k(y_i - y_{i-1})\}) - k(y_{i-1}x_i + y_i R_i)). \tag{10}$$

Taking derivative with respect to parameter λ and equating to zero, we have the log-likelihood equation

$$\frac{d}{dk} \ln L(k) = \sum_{i=1}^{m} \left(x_i \frac{(y_i - y_{i-1}) e^{-k(y_i - y_{i-1})}}{1 - \exp\{-k(y_i - y_{i-1})\}} \right) - (y_{i-1}x_i + y_i R_i) \equiv 0. \tag{11}$$

The MLE of k can be obtained by solving Eq. (11) numerically and denoted by $\hat{k}$. From Eq. (11), we have

$$\begin{aligned} \frac{d^2}{dk^2} \ln L(k) &= \sum_{i=1}^{m} x_i \frac{-(y_i - y_{i-1})^2 e^{-k(y_i - y_{i-1})} (1 - e^{-k(y_i - y_{i-1})}) - (y_i - y_{i-1})^2 e^{-2k(y_i - y_{i-1})}}{(1 - e^{-k(y_i - y_{i-1})})^2} \\ &= \sum_{i=1}^{m} x_i \frac{-(y_i - y_{i-1})^2 e^{-k(y_i - y_{i-1})}}{(1 - e^{-k(y_i - y_{i-1})})^2}. \end{aligned}$$

The Fisher's information is defined as $I(\lambda) = -\frac{Ed^2 \log L(k)}{dk^2}$.

It is observed that

$$X_i | X_{i-1}, \ldots, X_1, R_{i-1}, \ldots, R_1 \sim \text{Binomial} \left(n - \sum_{i=1}^{i-1} X_i - \sum_{i=1}^{i-1} R_i, q_i \right), \tag{12}$$

where $q_i = \frac{F(t_i) - F(t_{i-1})}{1 - F(t_{i-1})} = 1 - e^{-k(y_i - y_{i-1})}$, $i = 1, \ldots, m$.

Hence, we have

$$E(X_1) = nq_1,$$
$$E(R_1) = EE(R_1|X_1) = E((n - X_1)p_1|X_1) = n(1-q_1)p_1,$$
$$E(X_2) = EE(X_2|X_1, R_1) = E((n - X_1 - R_1)q_2|X_1, R_1) = n(1-q_1)(1-p_1)q_2.$$

Repeating the similar computation procedure, we have

$$\begin{aligned} E(X_i) &= EE(X_i|X_{i-1}, \ldots, X_1, R_{i-1}, \ldots, R_1) \\ &= nq_i \prod_{j=1}^{i-1} (1-p_j)(1-q_j), \quad i = 1, \ldots, m. \end{aligned}$$

Observed that $\frac{1}{1-q_i} = e^{k(y_i - y_{i-1})}$ and $-\ln(1-q_i) = k(y_i - y_{i-1})$.

The Fisher's information can be obtained as

$$\begin{aligned} I(k) &= -E\left[\frac{d^2}{dk^2}\ln L(k)\right] \\ &= -E\left[\sum_{i=1}^{m} X_i \frac{-\frac{1}{k^2}(1-q_i)\ln^2(1-q_i)}{q_i^2}\right] \\ &= \frac{1}{k^2}\sum_{i=1}^{m}\frac{(1-q_i)\ln^2(1-q_i)}{q_i^2}E(\mathrm{X_i}) \\ &= \frac{n}{k^2}\sum_{i=1}^{m}\frac{\ln^2(1-q_i)}{q_i}\prod_{j=1}^{i-1}(1-p_j)\prod_{j=1}^{i}(1-q_j). \end{aligned} \tag{13}$$

From Serfling [16], it is known that the asymptotic distribution for the MLE $\hat{k}$ has a normal distribution with mean k and variance of $I^{-1}(k)$ denoted as

$$\hat{k} \xrightarrow[m\to\infty]{d} \mathrm{N}\left(k, I^{-1}(k)\right). \tag{14}$$

If the monitoring and censoring occur periodically with equal time interval $t_i - t_{i-1} = t$ is considered, we have

$$q_i = \frac{F(t_i) - F(t_{i-1})}{1 - F(t_{i-1})} = 1 - e^{-k(y_i - y_{i-1})} = 1 - e^{-k[(it)^\beta - ((i-1)t)^\beta]}, \quad \text{and}$$

$1 - q_i = e^{-k[(it)^\beta - ((i-1)t)^\beta]}$ and $-\ln(1-q_i) = k[(it)^\beta - ((i-1)t)^\beta]$, then Eq. (11) can be simplified as

$$\begin{aligned} \frac{d\ln L(k)}{dk} &= \sum_{i=1}^{m}\left[x_i\frac{(y_i - y_{i-1})e^{-k(y_i - y_{i-1})}}{1 - e^{-k(y_i - y_{i-1})}} - (y_iR_i + y_{i-1}x_i)\right] \\ &= \sum_{i=1}^{m}\left[x_i\frac{[(it)^\beta - ((i-1)t)^\beta]e^{-k[(it)^\beta - ((i-1)t)^\beta]}}{1 - e^{-k[(it)^\beta - ((i-1)t)^\beta]}} - \left((it)^\beta R_i + [(i-1)t]^\beta x_i\right)\right] = 0. \end{aligned} \tag{15}$$

Solving the above equation for k, the MLE of k can be obtained numerically.

Furthermore, when the percentages of removals are the same, i.e. $p_1 = \cdots = p_{m-1} = p$, the asymptotic variance of $\hat{k}$ can be expressed as

$$I(k) = -E\left[\frac{d^2\ln L(k)}{dk^2}\right] = n\sum_{i=1}^{m}\frac{[(it)^\beta - ((i-1)t)^\beta]^2}{1 - e^{-k[(it)^\beta - ((i-1)t)^\beta]}}(1-p)^{i-1}\prod_{j=1}^{i}e^{-k[(it)^\beta - ((i-1)t)^\beta]}. \tag{16}$$

By the property of the invariance of MLE, the MLE of C_L can be obtained as

$$\hat{C}_L = 1 - \hat{k}L. \tag{17}$$

Since we have $\hat{k} \xrightarrow[m\to\infty]{d} \mathrm{N}\left(k, I^{-1}(\hat{k})\right)$, then we have

$$\hat{C}_L \xrightarrow[m\to\infty]{d} \mathrm{N}\left(C_L, V\left(\hat{C}_L\right)\right), \tag{18}$$

where $V\left(\hat{C}_L\right) = L^2 I^{-1}(\hat{k})$. Hence, the MLE $\hat{C}_L$ is an asymptotic unbiased estimator of C_L. The estimate of the asymptotic variance of $\hat{C}_L$ can be calculated by $V\left(\hat{C}_L\right) = L^2 I^{-1}(\hat{k})$.

4. Testing procedure algorithm for the lifetime performance index

In this section, a hypothesis testing procedure is developed by using the ML estimate of C_L given by (17) to assess whether the lifetime performance index reaches the required level. Let the required level to be c_0. The process is capable if the lifetime performance index is larger than c_0. The statistical hypothesis is set up as follows:

$H_0 : C_L \le c_0$ (the process is not capable) v.s. $H_a : C_L > c_0$ (the process is capable). The ML estimate of C_L is used as the test statistic and the critical value C_L^0 for the one-sided hypothesis testing is determined as follows:

$$P\left(\hat{C}_L > C_L^0 \middle| C_L \le c_0\right) = P(1-\hat{k}L > C_L^0 | 1-kL \le c_0) = P\left(\hat{k} < \frac{1-C_L^0}{L} | k \ge \frac{1-c_0}{L}\right)$$
$$= P\left(Z < \left(\frac{1-C_L^0}{L} - k\right) \Big/ \sqrt{I^{-1}(k)} | k \ge \frac{1-c_0}{L}\right) \le \alpha,$$

where $Z = \frac{\hat{k}-k}{\sqrt{I^{-1}(k)}} \xrightarrow[m\to\infty]{d} N(0,1)$ by the asymptotic distribution in (14).

$$\Rightarrow \sup_{k \ge \frac{1-c_0}{L}} P\left(Z < \left(\frac{1-C_L^0}{L} - k\right) \Big/ \sqrt{I^{-1}(k)}\right) = \alpha.$$

The supremum of $P\left(Z < \left(\frac{1-C_L^0}{L} - k\right) / \sqrt{I^{-1}(k)}\right)$ occurred at $k_0 = \frac{1-c_0}{L}$ since $\left(\frac{1-C_L^0}{L} - k\right) / \sqrt{I^{-1}(k)}$ is a decreasing function of k when $k \ge \frac{1-c_0}{L}$. Thus, we have

$$P\left(Z < \left(\frac{1-C_L^0}{L} - k_0\right) \Big/ \sqrt{I^{-1}(k_0)} \middle| k_0 = \frac{1-c_0}{L}\right) = \alpha$$
$$\Rightarrow Z_{1-\alpha} = \left(\frac{1-C_L^0}{L} - k_0\right) \Big/ \sqrt{I^{-1}(k_0)}$$

where $k_0 = \frac{1-c_0}{L}$ and Z_α represents the lower $100(1-\alpha)$th percentile of a standard normal distribution.

Thus, the critical values can be derived as

$$C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right) \tag{19}$$

where $k_0 = \frac{1-c_0}{L}$ The rejection region for this test is $\left\{\hat{C}_L \middle| \hat{C}_L > C_L^0\right\}$.

The proposed testing algorithmic procedure about C_L can be constructed as follows:

Algorithm:

Step 1: Given a known lower specification L_U, then $L = L_U^\beta$ is the corresponding lower specification limit for new lifetime variable Y. Observe the progressive type I interval censored sample $X_1, \ldots, X_m$ at the pre-set times $t_1, \ldots, t_m$ with censoring schemes of $R_1, \ldots, R_m$ from the one-parameter exponential distribution.

Step 2: Determine the required level c to achieve a pre-assigned conforming rate P_r from Table 1. Then the testing null hypothesis $H_0 : C_L \le c_0$ and the alternative hypothesis $H_a : C_L > c_0$ are constructed, and c_0 is the target value.

Step 3: Obtain the value of test statistic $\hat{C}_L = 1 - \hat{k}L$, where $\hat{k}$ is the solution of (11).

Step 4: For level of significance of α, we can calculate the critical value $C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right)$ where $k_0 = \frac{1-c_0}{L}$ and $I^{-1}(k_0)$ is defined in Eq. (13).

Step 5: The decision rule is to conclude that the lifetime performance index of the products meets the required level if $\hat{C}_L > C_L^0$.

Moreover, the power of the proposed statistical test can be obtained as follows:

The power $h(c_1)$ at the point of $C_L = c_1 > c_0$ is

$$h(c_1) = P\left(\hat{C}_L > C_L^0 | c_1 = 1 - k_1 L\right)$$
$$= P\left(1 - \hat{k}L > 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right) \middle| k_1 = \frac{1-c_1}{L}\right)$$
$$= P\left(\hat{k} < \left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right) \middle| k_1 = \frac{1-c_1}{L}\right)$$

$$= P\left(\frac{\hat{k}-k_1}{\sqrt{I^{-1}(k_1)}} < \frac{\left(k_0+Z_{1-\alpha}\sqrt{I^{-1}(k_0)}-k_1\right)}{\sqrt{I^{-1}(k_1)}}\middle| k_1=\frac{1-c_1}{L}\right)$$

$$= \Phi\left(\frac{k_0-k_1+Z_{1-\alpha}\sqrt{I^{-1}(k_0)}}{\sqrt{I^{-1}(k_1)}}\right) \tag{20}$$

where $\Phi(\cdot)$ is the cdf for the standard normal distribution, $k_0 = \frac{1-c_0}{L}$ and $k_1 = \frac{1-c_1}{L}$.

In addition, the $100(1-\alpha)\%$ lower confidence bound for C_L can be derived as follows:

From Eq. (18), we have $Z = \frac{\hat{C}_L - C_L}{\sqrt{\hat{V}(\hat{C}_L)}} \xrightarrow[m\to\infty]{d} N(0,1)$, where $V\left(\hat{C}_L\right) = L^2 I^{-1}(\hat{k})$. Using $Z = \frac{\hat{C}_L - C_L}{\sqrt{\hat{V}(\hat{C}_L)}}$ as the pivotal quantity, then we can obtain the lower confidence bound for C_L as $\hat{C}_L - Z_\alpha\sqrt{\hat{V}\left(\hat{C}_L\right)}$.

The powers $h(c_1)$ for testing $H_0 : C_L \le 0.8$ are tabulated in Tables 2–4 at $\alpha = 0.01, 0.05, 0.1$ respectively for the values of $h(c_1)$ at $\alpha = 0.01$ for $c_1 = 0.75, 0.80(0.125)0.90$, $m = 5(1)8$, $n = 30(10)60$ and $p = 0.05\,(0.025)\,0.1$ under $L = 0.05$, $t = 0.1$. The powers are also displayed in Figs. 1–3 for some typical cases. From Tables 2–4, we have the following findings: (1) the power $h(c_1)$ is a non-decreasing function of n for fixed $m = 5$, $p = 0.05$ and $\alpha = 0.1$ which is shown in Fig. 1 (Other combinations of m, p and α also have the same patterns); (2) the power $h(c_1)$ is a non-decreasing function of m for fixed $n = 60$, $p = 0.05$ and $\alpha = 0.1$ which is shown in Fig. 2 (Other combinations of n, p and α also have the same pattern) (3) the power $h(c_1)$ is a non-increasing function of the pre-specified removal percentage p for fixed $n = 60$, $m = 5$ and $\alpha = 0.1$ which is shown in Fig. 3 (Other combinations of n, m and α also have the same pattern); (4) from Figs. 1–3, the power $h(c_1)$ increases when the value of c_1 increases for any combinations of n, m, p and α.

5. Two numerical examples

To illustrate the above testing algorithmic procedure, two data sets are considered as follows.

Example 1. The data in Caroni [17] representing the failure times (number of cycles) of $n = 25$ ball bearing in an automatic life test is listed as follows:

0.1788, 0.2892, 0.3300, 0.4152, 0.4212, 0.4560, 0.4848, 0.5184, 0.5196, 0.5412, 0.5556, 0.6780, 0.6780, 0.6780, 0.6864, 0.6864, 0.6888, 0.8412, 0.9312, 0.9864, 1.0512, 1.0584, 1.2792, 1.2804, 1.7340.

The G test based on Gini statistic (see Gill and Gastwirth [18]) is 0.4991 and the corresponding P-value is 0.9882. It indicated that the Weibull distribution fits the data well. To determine the shape parameter β, the p-values for different β values are plotted as follows:

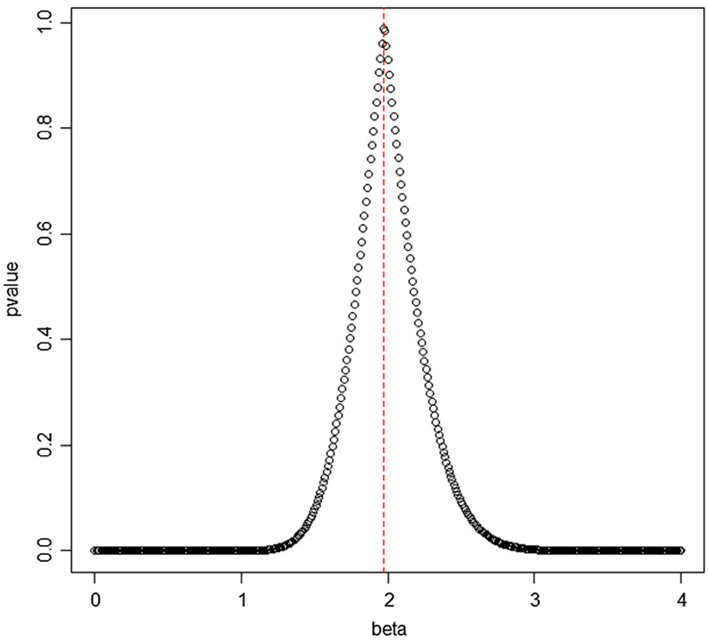

Table 2
The values of $h(c_1)$ at $\alpha = 0.01$ for $c_1 = 0.7, 0.75(0.025)0.95$, $m = 5(1)8$, $n = 60(20)100$ and $p = 0.05\,(0.025)\,0.1$ under $L = 0.05$, $t = 0.1$ and $c_0 = 0.8$.

m	n	p	c_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
5	60	0.050	0.000000	0.000001	0.000196	0.010000	0.126809	0.495982	0.850709	0.975181	0.996425	0.999010
		0.075	0.000000	0.000001	0.000204	0.010000	0.124403	0.486069	0.841432	0.971906	0.995649	0.998733
		0.100	0.000000	0.000001	0.000211	0.010000	0.122097	0.476447	0.832062	0.968385	0.994753	0.998396
	80	0.050	0.000000	0.000000	0.000116	0.010000	0.156115	0.596331	0.919854	0.992240	0.999338	0.999859
		0.075	0.000000	0.000000	0.000122	0.010000	0.152839	0.584963	0.912807	0.990802	0.999137	0.999804
		0.100	0.000000	0.000000	0.000127	0.010000	0.149700	0.573847	0.905523	0.989185	0.998888	0.999732
	100	0.050	0.000000	0.000000	0.000072	0.010000	0.185367	0.679957	0.957765	0.997618	0.999879	0.999980
		0.075	0.000000	0.000000	0.000076	0.010000	0.181213	0.668020	0.952891	0.997040	0.999830	0.999970
		0.100	0.000000	0.000000	0.000080	0.010000	0.177232	0.656248	0.947728	0.996360	0.999766	0.999955
6	60	0.050	0.000000	0.000001	0.000192	0.010000	0.129540	0.509836	0.865509	0.980741	0.997760	0.999496
		0.075	0.000000	0.000001	0.000200	0.010000	0.126756	0.498270	0.855144	0.977469	0.997106	0.999298
		0.100	0.000000	0.000001	0.000208	0.010000	0.124116	0.487140	0.844674	0.973888	0.996319	0.999041
	80	0.050	0.000000	0.000000	0.000112	0.010000	0.160064	0.612684	0.931001	0.994552	0.999650	0.999943
		0.075	0.000000	0.000000	0.000118	0.010000	0.156253	0.599515	0.923425	0.993239	0.999505	0.999911
		0.100	0.000000	0.000000	0.000124	0.010000	0.152639	0.586727	0.915548	0.991719	0.999315	0.999866
	100	0.050	0.000000	0.000000	0.000069	0.010000	0.190566	0.697370	0.965331	0.998492	0.999946	0.999994
		0.075	0.000000	0.000000	0.000073	0.010000	0.185717	0.683679	0.960305	0.998011	0.999916	0.999989
		0.100	0.000000	0.000000	0.000078	0.010000	0.181118	0.670249	0.954920	0.997422	0.999874	0.999981
7	60	0.050	0.000000	0.000001	0.000191	0.010000	0.131136	0.518665	0.875076	0.984102	0.998461	0.999716
		0.075	0.000000	0.000001	0.000199	0.010000	0.128085	0.505834	0.863866	0.980843	0.997896	0.999569
		0.100	0.000000	0.000001	0.000207	0.010000	0.125217	0.493581	0.852550	0.977223	0.997189	0.999366
	80	0.050	0.000000	0.000000	0.000111	0.010000	0.162438	0.623153	0.937991	0.995836	0.999791	0.999974
		0.075	0.000000	0.000000	0.000117	0.010000	0.158240	0.608600	0.930014	0.994615	0.999680	0.999954
		0.100	0.000000	0.000000	0.000123	0.010000	0.154295	0.594559	0.921688	0.993164	0.999526	0.999923
	100	0.050	0.000000	0.000000	0.000068	0.010000	0.193743	0.708489	0.969907	0.998935	0.999972	0.999998
		0.075	0.000000	0.000000	0.000072	0.010000	0.188386	0.693453	0.964771	0.998519	0.999952	0.999995
		0.100	0.000000	0.000000	0.000077	0.010000	0.183349	0.678776	0.959218	0.997990	0.999921	0.999991
8	60	0.050	0.000000	0.000001	0.000190	0.010000	0.132052	0.524317	0.881431	0.986252	0.998862	0.999825
		0.075	0.000000	0.000001	0.000199	0.010000	0.128820	0.510540	0.869555	0.983002	0.998361	0.999712
		0.100	0.000000	0.000001	0.000207	0.010000	0.125803	0.497472	0.857583	0.979348	0.997711	0.999546
	80	0.050	0.000000	0.000000	0.000110	0.010000	0.163851	0.629917	0.942542	0.996607	0.999862	0.999986
		0.075	0.000000	0.000000	0.000117	0.010000	0.159384	0.614318	0.934249	0.995451	0.999773	0.999973
		0.100	0.000000	0.000000	0.000123	0.010000	0.155215	0.599353	0.925571	0.994046	0.999643	0.999951
	100	0.050	0.000000	0.000000	0.000067	0.010000	0.195672	0.715686	0.972814	0.999184	0.999984	0.999999
		0.075	0.000000	0.000000	0.000072	0.010000	0.189956	0.699627	0.967586	0.998811	0.999969	0.999998
		0.100	0.000000	0.000000	0.000076	0.010000	0.184619	0.684026	0.961896	0.998320	0.999945	0.999995

Therefore, the scale parameter is determined as $\beta = 1.97$ based on the maximum p-value method. Furthermore, we create a progressive type I censored sample based on this data. Let the number of inspections $m = 5$ with the equal length of inspection interval $t = 0.4$ (thousand cycles) and the pre-specified removal percentages of the remaining survival units given by $(p_1, p_2, p_3, p_4, p_5) = (0.2, 0.2, 0.2, 0.2, 1.0)$. Now we can start to do the above testing procedure about C_L as follows:

Step 1: Suppose that the lower lifetime limit for the lifetimes for ball bearing is $L_U = 0.1876$ cycles and then $L = L_U^{\beta} = 0.1876^{1.97} = 0.037$. Observe the progressive type I interval censored sample $(X_1, X_2, X_3, X_4, X_5) = (3, 11, 5, 1, 0)$ at the pre-set times $(t_1, t_2, t_3, t_4, t_5) = (0.4, 0.8, 1.2, 1.6, 2)$ with censoring schemes of $(R_1, R_2, R_3, R_4, R_5) = (4, 0, 0, 1, 0)$

Step 2: If the conforming rate P_r of products is required to exceed 0.818731, then the lifetime performance index target value c_0 should be taken as 0.8 from Table 1. Thus, the testing null hypothesis $H_0 : C_L \leq 0.8$ and the alternative hypothesis $H_a : C_L > 0.8$ is constructed.

Step 3: Obtain the value of test statistic $\hat{C}_L = 1 - \hat{k}L = 0.9480$

Step 4: For level of significance $\alpha = 0.05$, we can calculate the critical value $C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right) = 0.8293$

Step 5: Since $\hat{C}_L = 0.9480 > C_L^0 = 0.8293$ we concluded to reject the null hypothesis $H_0 : C_L \leq 0.8$. Thus, we can conclude that the lifetime performance index of product does meet the required level.

Example 2. The data is simulating from Weibull distribution with $\beta = 1.97$ and $\lambda = 8$ representing the lifetimes (in years) of $n = 30$ electrical appliances listed below:

0.4026076, 0.4634817, 0.5865632, 0.5872504, 0.5990099, 0.7807500, 1.0558488, 1.0674860, 1.0739254, 1.2243494, 1.2355577, 1.3435948, 1.4445313, 1.4739490, 1.5362624, 1.5519690, 1.7333865, 1.9558721, 1.9712404, 2.0374946, 2.1261545, 2.1967276, 2.3175976, 2.3882075, 2.5068328, 2.6014563, 3.1099016, 3.2159070, 3.9335707, 4.4029930

Table 3
The values of $h(c_1)$ at $\alpha = 0.05$ for $c_1 = 0.7, 0.75(0.025)0.95$, $m = 5(1)8$, $n = 60(20)100$ and $p = 0.05\,(0.025)\,0.1$ under $L = 0.05$, $t = 0.1$ and $c_0 = 0.8$.

m	n	p	c_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
5	60	0.050	0.000000	0.000047	0.002848	0.050000	0.288074	0.682155	0.922981	0.987879	0.998063	0.999330
		0.075	0.000000	0.000051	0.002942	0.050000	0.283904	0.672863	0.916976	0.986042	0.997606	0.999133
		0.100	0.000000	0.000055	0.003036	0.050000	0.279876	0.663723	0.910805	0.984030	0.997068	0.998891
	80	0.050	0.000000	0.000014	0.001858	0.050000	0.334336	0.766590	0.963275	0.996638	0.999674	0.999910
		0.075	0.000000	0.000016	0.001934	0.050000	0.329114	0.757214	0.959325	0.995933	0.999567	0.999874
		0.100	0.000000	0.000018	0.002011	0.050000	0.324065	0.747900	0.955158	0.995123	0.999433	0.999825
	100	0.050	0.000000	0.000005	0.001258	0.050000	0.377324	0.829295	0.982625	0.999073	0.999945	0.999988
		0.075	0.000000	0.000005	0.001318	0.050000	0.371161	0.820485	0.980215	0.998821	0.999922	0.999981
		0.100	0.000000	0.000006	0.001380	0.050000	0.365195	0.811642	0.977604	0.998518	0.999890	0.999972
6	60	0.050	0.000000	0.000045	0.002774	0.050000	0.293390	0.695999	0.932823	0.991005	0.998843	0.999671
		0.075	0.000000	0.000049	0.002876	0.050000	0.288551	0.685281	0.926295	0.989241	0.998473	0.999535
		0.100	0.000000	0.000053	0.002978	0.050000	0.283923	0.674803	0.919559	0.987264	0.998019	0.999356
	80	0.050	0.000000	0.000013	0.001794	0.050000	0.341219	0.780709	0.969606	0.997762	0.999837	0.999965
		0.075	0.000000	0.000015	0.001876	0.050000	0.335148	0.770031	0.965504	0.997149	0.999764	0.999945
		0.100	0.000000	0.000017	0.001959	0.050000	0.329334	0.759468	0.961133	0.996419	0.999666	0.999915
	100	0.050	0.000000	0.000004	0.001205	0.050000	0.385619	0.842580	0.986376	0.999448	0.999977	0.999996
		0.075	0.000000	0.000005	0.001270	0.050000	0.378451	0.832694	0.983992	0.999250	0.999964	0.999993
		0.100	0.000000	0.000006	0.001336	0.050000	0.371575	0.822792	0.981365	0.998999	0.999944	0.999989
7	60	0.050	0.000000	0.000044	0.002739	0.050000	0.296650	0.704930	0.939095	0.992834	0.999236	0.999821
		0.075	0.000000	0.000048	0.002846	0.050000	0.291320	0.693105	0.932165	0.991129	0.998928	0.999723
		0.100	0.000000	0.000053	0.002952	0.050000	0.286262	0.681613	0.924995	0.989182	0.998532	0.999585
	80	0.050	0.000000	0.000013	0.001763	0.050000	0.345496	0.789764	0.973482	0.998361	0.999907	0.999985
		0.075	0.000000	0.000015	0.001849	0.050000	0.338793	0.778079	0.969269	0.997811	0.999854	0.999972
		0.100	0.000000	0.000016	0.001935	0.050000	0.332425	0.766568	0.964742	0.997136	0.999777	0.999953
	100	0.050	0.000000	0.000004	0.001178	0.050000	0.390811	0.851018	0.988575	0.999629	0.999989	0.999999
		0.075	0.000000	0.000005	0.001246	0.050000	0.382889	0.840304	0.986210	0.999465	0.999980	0.999997
		0.100	0.000000	0.000006	0.001315	0.050000	0.375350	0.829599	0.983568	0.999248	0.999966	0.999995
8	60	0.050	0.000000	0.000044	0.002723	0.050000	0.298647	0.710765	0.943235	0.993976	0.999454	0.999893
		0.075	0.000000	0.000048	0.002833	0.050000	0.292964	0.698092	0.935986	0.992314	0.999188	0.999819
		0.100	0.000000	0.000053	0.002942	0.050000	0.287609	0.685842	0.928476	0.990385	0.998832	0.999709
	80	0.050	0.000000	0.000013	0.001747	0.050000	0.348155	0.795672	0.975969	0.998709	0.999941	0.999992
		0.075	0.000000	0.000015	0.001836	0.050000	0.340994	0.783212	0.971664	0.998203	0.999900	0.999984
		0.100	0.000000	0.000016	0.001924	0.050000	0.334236	0.770989	0.967012	0.997564	0.999836	0.999971
	100	0.050	0.000000	0.000004	0.001164	0.050000	0.394066	0.856494	0.989943	0.999726	0.999994	0.999999
		0.075	0.000000	0.000005	0.001234	0.050000	0.385595	0.845144	0.987586	0.999584	0.999988	0.999999
		0.100	0.000000	0.000006	0.001305	0.050000	0.377586	0.833832	0.984926	0.999388	0.999977	0.999997

we create a progressive type I censored sample based on this data. Let the number of inspections $m = 5$ with the equal length of inspection interval $t = 0.5$ year and the pre-specified removal percentages of the remaining survival units given by $(p_1, p_2, p_3, p_4, p_5) = (0.2, 0.2, 0.2, 0.2, 1)$. Now we can start to do the above testing procedure about C_L as follows:

Step 1: Suppose that the lower lifetime limit for the electrical appliance is $L_U = 0.543$ and then $L = L_U^{\beta} = 0.543^{1.97} = 0.3$ and observe the progressive type I interval censored sample $(X_1, X_2, X_3, X_4, X_5) = (2, 4, 6, 2, 2)$ at the pre-set times $(t_1, t_2, t_3, t_4, t_5) = (0.5, 1, 1.5, 2, 2.5)$ with censoring schemes of $(R_1, R_2, R_3, R_4, R_5) = (5, 4, 2, 0, 3)$.

Step 2: If the conforming rate P_r of products is required to exceed 0.904837, then the lifetime performance index target value c_0 should be taken as 0.9 from Table 1. Thus, the testing null hypothesis $H_0 : C_L \le 0.9$ and the alternative hypothesis $H_a : C_L > 0.9$ is constructed.

Step 3: Obtain the value of test statistic $\hat{C}_L = 1 - \hat{k}L = 0.9141$

Step 4: For level of significance $\alpha = 0.05$, we can calculate the critical value $C_L^0 = 1 - L\left(k_0 + Z_{1-\alpha}\sqrt{I^{-1}(k_0)}\right) = 0.8452$

Step 5: Since $\hat{C}_L = 0.9141 > C_L^0 = 0.8452$, we concluded to reject the null hypothesis $H_0 : C_L \le 0.9$. Thus, we can conclude that the lifetime performance index of product does meet the required level.

6. Conclusion

The lifetime performance index C_L was used to assess the lifetime performance of a product with Weibull distribution with lower specification limit LU. For many practical cases, the progressive type I interval censored sample is collected instead of complete sample. The ML estimates of the unknown parameter was obtained by solving the log-likelihood equation and the ML estimate of C_L is utilized to construct a testing procedure algorithm to test if C_L reaches the target value based on the progressive type I interval censored sample. We also give power curves to discuss the impacts of various values of n, m and p. Two practical examples are given to illustrate the proposed testing procedure to determine whether the process is capable or not under progressive type I interval censoring.

Table 4
The values of $h(c_1)$ at $\alpha = 0.1$ for $c_1 = 0.7, 0.75(0.025)0.95$, $m = 5(1)8$, $n = 60(20)100$ and $p = 0.05\ (0.025)\ 0.1$ under $L = 0.05$, $t = 0.1$ and $c_0 = 0.8$.

m	n	p	c_1									
			0.700	0.750	0.775	0.800	0.825	0.850	0.875	0.900	0.925	0.950
5	60	0.050	0.000000	0.000293	0.009412	0.100000	0.401895	0.767808	0.948571	0.991979	0.998624	0.999459
		0.075	0.000000	0.000316	0.009696	0.100000	0.397032	0.759654	0.944091	0.990675	0.998284	0.999296
		0.100	0.000000	0.000340	0.009978	0.100000	0.392315	0.751573	0.939442	0.989232	0.997881	0.999094
	80	0.050	0.000000	0.000102	0.006469	0.100000	0.453273	0.837830	0.977020	0.997914	0.999780	0.999930
		0.075	0.000000	0.000112	0.006709	0.100000	0.447447	0.830142	0.974290	0.997448	0.999705	0.999900
		0.100	0.000000	0.000123	0.006951	0.100000	0.441786	0.822438	0.971378	0.996907	0.999610	0.999861
	100	0.050	0.000000	0.000038	0.004584	0.100000	0.499225	0.886725	0.989748	0.999457	0.999965	0.999991
		0.075	0.000000	0.000042	0.004785	0.100000	0.492607	0.879917	0.988190	0.999301	0.999949	0.999986
		0.100	0.000000	0.000047	0.004988	0.100000	0.486165	0.873017	0.986481	0.999110	0.999928	0.999978
6	60	0.050	0.000000	0.000278	0.009161	0.100000	0.408418	0.780304	0.955995	0.994197	0.999199	0.999740
		0.075	0.000000	0.000302	0.009470	0.100000	0.402776	0.770974	0.951204	0.992973	0.998931	0.999629
		0.100	0.000000	0.000327	0.009777	0.100000	0.397353	0.761771	0.946201	0.991583	0.998598	0.999482
	80	0.050	0.000000	0.000095	0.006247	0.100000	0.461265	0.849639	0.981414	0.998653	0.999894	0.999973
		0.075	0.000000	0.000105	0.006508	0.100000	0.454505	0.840979	0.978637	0.998258	0.999844	0.999957
		0.100	0.000000	0.000116	0.006769	0.100000	0.447994	0.832323	0.975636	0.997782	0.999775	0.999934
	100	0.050	0.000000	0.000034	0.004394	0.100000	0.508431	0.897138	0.992171	0.999688	0.999986	0.999997
		0.075	0.000000	0.000039	0.004611	0.100000	0.500759	0.889601	0.990667	0.999569	0.999977	0.999995
		0.100	0.000000	0.000044	0.004829	0.100000	0.493353	0.881962	0.988983	0.999415	0.999964	0.999991
7	60	0.050	0.000000	0.000271	0.009032	0.100000	0.412491	0.788384	0.960677	0.995469	0.999483	0.999860
		0.075	0.000000	0.000296	0.009358	0.100000	0.406266	0.778136	0.955650	0.994307	0.999264	0.999782
		0.100	0.000000	0.000321	0.009680	0.100000	0.400327	0.768077	0.950376	0.992958	0.998978	0.999670
	80	0.050	0.000000	0.000091	0.006131	0.100000	0.466292	0.857194	0.984064	0.999037	0.999941	0.999988
		0.075	0.000000	0.000102	0.006405	0.100000	0.458828	0.847781	0.981253	0.998690	0.999905	0.999979
		0.100	0.000000	0.000114	0.006679	0.100000	0.451692	0.838400	0.978183	0.998258	0.999853	0.999964
	100	0.050	0.000000	0.000033	0.004293	0.100000	0.514243	0.903713	0.993564	0.999796	0.999993	0.999999
		0.075	0.000000	0.000038	0.004521	0.100000	0.505773	0.895615	0.992098	0.999699	0.999988	0.999998
		0.100	0.000000	0.000043	0.004750	0.100000	0.497656	0.887415	0.990429	0.999569	0.999979	0.999996
8	60	0.050	0.000000	0.000268	0.008968	0.100000	0.415040	0.793694	0.963749	0.996252	0.999637	0.999918
		0.075	0.000000	0.000293	0.009303	0.100000	0.408390	0.782736	0.958532	0.995134	0.999451	0.999860
		0.100	0.000000	0.000320	0.009634	0.100000	0.402086	0.772029	0.953043	0.993813	0.999198	0.999771
	80	0.050	0.000000	0.000090	0.006070	0.100000	0.469467	0.862132	0.985747	0.999255	0.999963	0.999994
		0.075	0.000000	0.000101	0.006353	0.100000	0.461484	0.852136	0.982905	0.998942	0.999936	0.999988
		0.100	0.000000	0.000112	0.006635	0.100000	0.453901	0.842204	0.979777	0.998538	0.999894	0.999978
	100	0.050	0.000000	0.000032	0.004240	0.100000	0.517931	0.907975	0.994420	0.999853	0.999996	1.000000
		0.075	0.000000	0.000037	0.004475	0.100000	0.508871	0.899441	0.992976	0.999770	0.999993	0.999999
		0.100	0.000000	0.000042	0.004710	0.100000	0.500242	0.890814	0.991313	0.999655	0.999986	0.999998

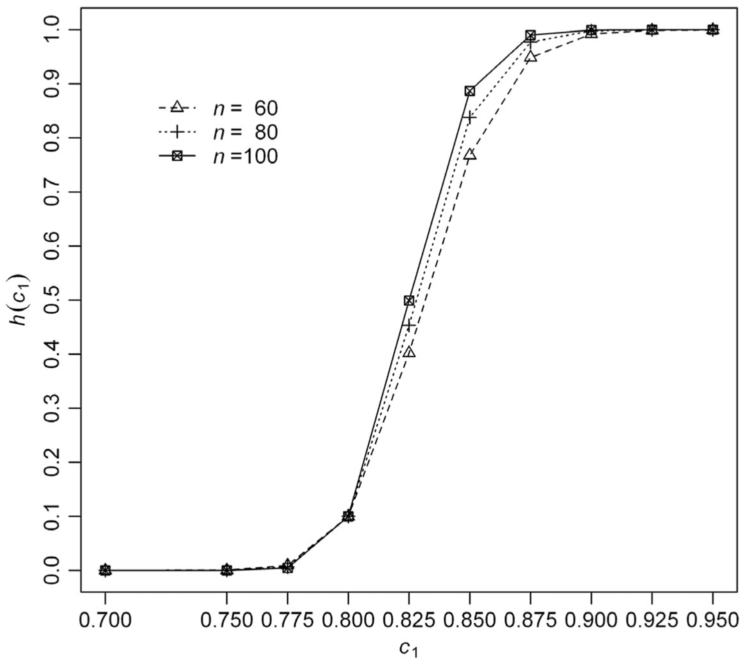

Fig. 1. Power function for the test at $\alpha = 0.1$ under $m = 5$ and $p = 0.05$.

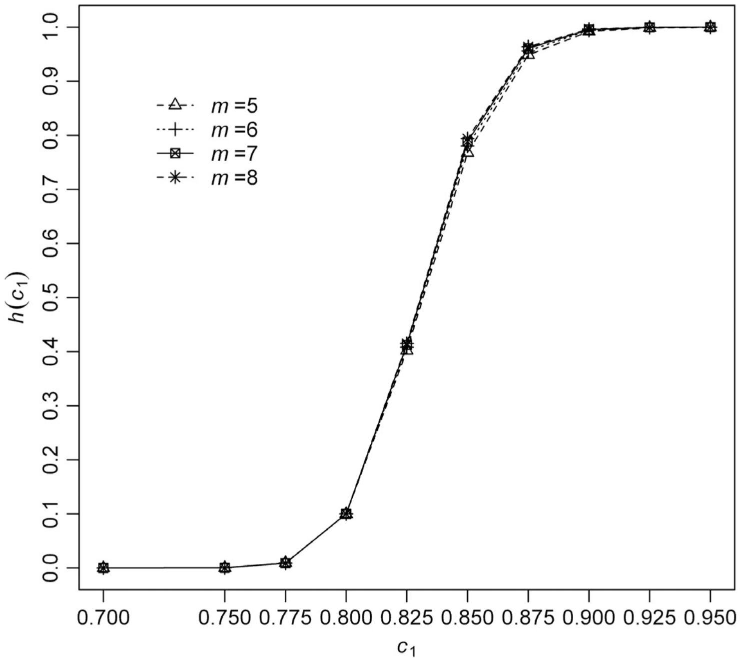

Fig. 2. Power function for the test at $\alpha = 0.1$ under $n = 60$ and $p = 0.05$.

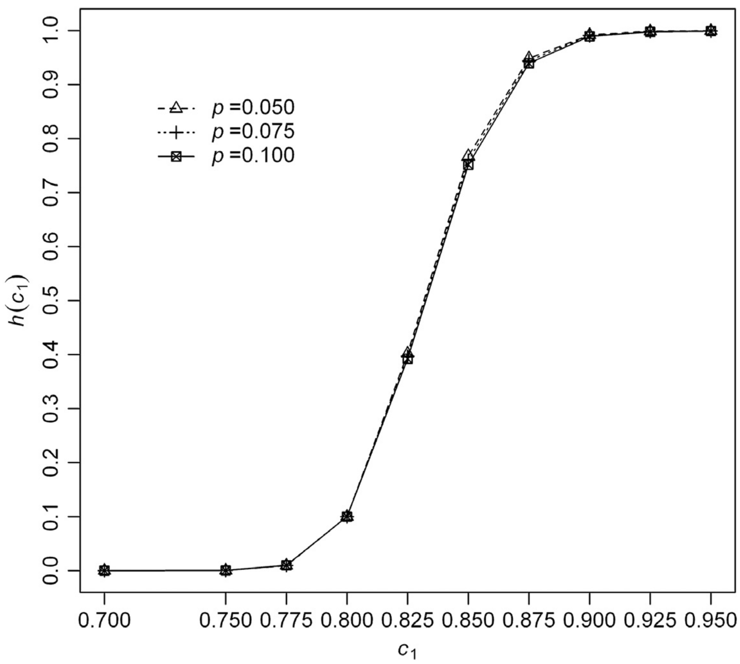

Fig. 3. Power function for the test at $\alpha = 0.1$ under $n = 60$ and $m = 5$.

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