An integrated inventory model with capacity constraint and order-size dependent trade credit

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ABSTRACT

Trade credit has many forms in today's business practice. The most common form of trade credit policy that is used to encourage retailers to buy larger quantities is order-size dependent. When the number of ordered units exceeds the capacity of the own warehouse, an additional rented warehouse is required to store the excess units. Therefore, to incorporate the concept of order-size dependent trade credit and limited storage capacity, we proposed an integrated inventory model with capacity constraint and a permissible delay payment period that is order-size dependent. In addition, the unit production cost, which is a function of the production rate, is considered. Three theorems and an algorithm are developed to determine the optimal production and replenishment policies for both the supplier and the retailer. Finally, numerical examples are presented to illustrate the solution procedure and the sensitivity analyses of some key parameters are provided to demonstrate the proposed model.

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1. Introduction

A traditional EOQ model makes a basic assumption: the retailer needs to pay the full amount to the supplier when the products are received. However, in real market transactions, retailers usually do not need to pay the total amount at the time the product is received; they are allowed delayed payments by suppliers instead. This type of trade credit is very common in today's business world. For suppliers, offering delayed payments may attract more retailers and thus increase sales. For retailers, not only does this lower the opportunity cost of capital, but it also allows them to earn interest on the revenue of goods sold within the permissible delay period. Hence, trade credit policy is beneficial to both suppliers and retailers. Goyal (1985) was the first to relax the assumption of the traditional EOQ model in which payment is made once products are received, and established an EOQ model based on a fixed delay payment period. Afterwards, many researchers proposed inventory models relating to the permissible delay in payments such as Aggarwal and Jaggi (1995), Shin (1997), Jamal, Saker, and Wang (2000), Salameh, Abboud, El-Kassar, and Ghattas (2003), Teng, Chang, and Goyal (2005), Cheng, Lou, Ouyang, and Chiang (2010),

and Chung (2012). The above mentioned papers regarded the delayed payment period as a fixed value.

In real life situations, the trade credit policy that the supplier offers could have many variations. Some researchers assumed that the length of the delayed payment period is related to the retailer's order quantity, e.g., Chang, Ouyang, and Teng (2003), Shinn and Hwang (2003), Chang (2004), Chung, Goyal, and Huang (2005), Huang (2007a), Chung, Lin, and Srivastava (2012, 2013). In addition, some researchers assumed that both retailers and customers can buy with trade credit which is called a two-level trade credit policy. There were several relevant papers related to the two-level trade credit policy, such as Huang (2003, 2007b), Biskup, Simons, and Jahnke (2003), Jaggi, Aggarwal, and Goel (2007), Jaggi, Goyal, and Goel (2008), Teng and Goyal (2007), Liao (2008), Teng and Chang (2009), Min, Zhou, and Zhao (2010), Sharma, Goel, and Dua (2012), and Teng, Yang, and Chern (2013). Moreover, some researchers assumed that the supplier provides the delay payment and cash discount simultaneously, i.e., the supplier offers a cash discount to the retailer to encourage him to settle the account earlier. The available results adopting this assumption can be found in the work of Ouyang, Chen, and Chuang (2002), Ouyang, Teng, Chuang, and Chuang (2005), Chang (2002), Goyal, Teng, and Chang (2007), Sana and Chaudhuri (2008), and Yang (2010). All of the above mentioned papers discussed the issue from the

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Table 1
The comparison between this paper and previous studies with trade credit and limited storage space.

Papers	Integrated/EOQ	Trade credit policy	Limited storage space
Huang (2006)	EOQ	Two-level trade credit	Yes
Ouyang et al. (2007)	EOQ	Fixed credit period and cash discount for early payment	Yes
Liang and Zhou (2011)	EOQ	Fixed credit period	Yes
Zhou et al. (2012)	Two-echelon decentralized supply chain	fixed credit period	Yes
Liao et al. (2012)	EOQ	Conditional trade credit	Yes
Liao et al. (2013)	Integrated	Fixed credit period	Yes
This paper	Integrated	Order-size dependent trade credit	Yes

perspective of the supplier or the retailer, and just focused on one-sided optimal strategies.

With the impact of market globalization, businesses must integrate their supply chains to enhance their operational efficiency, respond to customers' needs more efficiently, and lower inventory costs. In a non-totalitarian supply chain system, since there is a latent difference in motives, the objectives of the supplier and the retailer may conflict with the objective of the entire supply chain. This will cause the optimal decision of the supplier or the retailer to not match that of the supply chain. When the activities and the decisions in a supply chain are inconsistent or not coordinated, the supply chain will lose its competitive advantages. Goyal (1976) first developed an integrated inventory model to determine the optimal joint inventory policy for a single supplier and a single retailer. Abad and Jaggi (2003) combined the concept of an integrated inventory model and trade credit policy, and established a supplier–retailer integrated system in which the supplier offers trade credit to the retailer. Afterward, several models concerning the integrated inventory model with various trade credit policies can be found in Jaber and Osman (2006), Yang and Wee (2006), Chen and Kang (2007, 2010), Su, Ouyang, Ho, and Chang (2007), Ho, Ouyang, and Su (2008), Ouyang, Ho, and Su (2008, 2009), Chang, Ho, Ouyang, and Su (2009), Thangam and Uthayakumar (2009), Huang, Tsai, Wu, and Chung (2010), Lin, Ouyang, and Dang (2012), Ouyang and Chang (2013), Chern, Pan, Teng, Chan, and Chen (2013), Chung, Lin, and Srivastava (2014), Chung, Lin, and Srivastava (2015), and their references. The above papers assumed that the warehouse owned by the retailer is adequate to store the entire procured inventory.

In practice, retailers might order more items when faced with an attractive price discount or when the ordering cost is relatively higher than the holding cost. If the space in the own warehouse (OW) is not sufficient to store all the purchased units, an external rented warehouse (RW) is used to hold the excess units. Usually, the RW has a higher unit holding cost than the OW; hence, purchased units fill up in the OW first and the excess units are kept in the RW. In addition, the units at the RW should be exhausted before units from the OW are retrieved. Hartley (1976) first established an inventory model with two levels of storage. This research topic attracted the attention of many researchers. Some related studies in this area include Pakkala and Achary (1992), Goyal and Giri (2001), Zhou and Yang (2005), Yang (2006), Dye, Ouyang, and Hsieh (2007), Hsieh, Dye, and Ouyang (2008), Rong, Mahapatra, and Maiti (2008), Lee and Hsu (2009), Panda, Maiti, and Maiti (2010), and their references. All of the above papers discuss a two-warehouse inventory problem with or without deterioration. However, the impact of trade credit policy on the optimal solution is not considered in these papers. Huang (2006) first established an EOQ model with two warehouses and a two-level trade credit policy. Ouyang, Wu, and Yang (2007) proposed an EOQ model with limited storage capacity and trade credit. The trade credit policy discussed in their paper is that the supplier not only provides a delayed payment period to the retailer, but also offers a cash discount if the retailer pays earlier. Later, Liang and Zhou (2011) developed a two-warehouse inventory model with

trade credit policy for deteriorating items, which assumed that the RW has a higher deterioration rate than the OW. Recently, Liao, Huang, and Chung (2012) established an EOQ model for deteriorating items with two warehouses and the delay payment is permitted only when the ordering quantity meets a given threshold. These papers discussing two warehouses determined the optimal inventory policy only from the retailer's perspective.

Since trade credit policy is a common business practice nowadays, it is important to explore the impact of trade credit policy on an integrated inventory problem. Zhou, Zhong, and Li (2012) proposed a supplier-Stackelberg game inventory model with trade credit and limited storage space where the permissible delay period is a fixed given parameter. Liao, Chung, and Huang (2013) established an integrated inventory model for deteriorating items with trade credit and two warehouses. Likewise, the delay payment period in this study is a fixed given parameter. The studies that addressed the issues of limited storage space and trade credit policy simultaneously are shown in Table 1. When the length of the delayed payment is linked to order quantity rather than a given parameter, the retailer is encouraged to order large quantities with a longer delayed payment period so that the order quantity might exceed the OW capacity. As shown in Table 1, previous studies did not consider the impact of various delay payment length on order quantity. Therefore, this paper deals with an integrated inventory problem with an order-size dependent trade credit and limited storage capacity. In addition, the unit production cost in this study is considered as a function of the production rate. From theoretical analysis, three theorems are developed to determine the optimal policy for the supplier and the retailer. Finally, numerical examples are given to illustrate the solution procedure and a sensitivity analysis of the optimal solution with respect to the major parameters is performed.

2. Notation and assumptions

The following notation and assumptions are made in deriving the model:

Notation:

D	Retailer's demand rate
R	Supplier's production rate, $R > D$
A_R	Retailer's ordering cost per order
A_S	Supplier's setup cost per setup
r_{R1}	Retailer's holding cost rate for items in the OW, excluding interest charges
r_{R2}	Retailer's holding cost rate for items in the RW, excluding interest charges, $r_{R2} > r_{R1}$
r_S	Supplier's holding cost rate, excluding interest charged
F_0	Fixed transportation cost per shipment
F_1	Unit transportation cost
$c(R)$	Supplier's unit production cost which is a convex function of R
v	Retailer's unit purchasing cost, $v > c$

p	Retailer's unit retail price, $p > v$
ρ	Supplier's capacity utilization, i.e., $\rho = D/R$, which is a constant where $\rho < 1$
W	Maximum storage capacity of the retailer's OW
I_{Sp}	Supplier's capital opportunity cost per dollar per unit time
I_{Rp}	Retailer's capital opportunity cost per dollar per unit time
I_{Re}	Retailer's interest earned per dollar per unit time
Q	Retailer's order quantity
T	Retailer's replenishment cycle length, where $T = Q/D$
m	Number of shipment from the supplier to the retailer
T_W	Length of depletion time for the maximum storage capacity of OW, i.e., $T_W = W/D$
M	Number of shipments from the supplier to the retailer per production run, a positive integer

Assumptions:

1. There are a single retailer and a single supplier in the inventory system. The retailer orders Q units in each order. The supplier manufactures mQ units in each production run to reduce the setup cost, and delivers Q units to the retailer in each shipment.
2. The unit production cost $c(R)$ is a convex function of the production rate R, and is given by $c(R) = c_0 + c_1/R + c_2 R$, where c_0, c_1 and c_2 are non-negative real numbers. The fixed cost c_0 can be regarded as the material cost. The cost component c_1/R decreases as the production rate increases, representing costs such as labor cost or energy cost. The third term $c_2 R$ denotes a cost component that increases with the production rate such as additional tool or die wear at high production rate. For notational simplicity, $c(R)$ and c are used interchangeably in this paper. (This assumption has been used by Khouja (1995), Ouyang et al. (2008) and others).
3. To encourage the retailer to order more quantities, the credit period $M_i, \ i = 1, 2, \ldots, k$, offered by the supplier is related to the retailer's order quantity and is given as

i	Q	M
1	$q_1 \leqslant Q < q_2$	M_1
2	$q_2 \leqslant Q < q_3$	M_2
$\vdots$	$\vdots$	$\vdots$
k	$q_k \leqslant Q < q_{k+1} = \infty$	M_k

where $0 \leqslant M_1 < M_2 < \cdots < M_k$ and $0 = q_1 < q_2 < \cdots < q_k < \infty$ is the sequence of quantities at which a specific credit period is offered. That is, M_i denotes the trade credit applicable to lot size falling in the interval $[q_i, q_{i+1})$.
4. During the credit period, the retailer sells the items and uses the sales revenue to earn interest at a rate of I_{Re}. At the end of the permissible delay period, the retailer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_{Rp} for the items in stock.
5. In offering trade credit to the retailer, the supplier endures a capital opportunity cost at rate I_{Sp} with the time gap between the product being shipped and paid for.
6. To obtain a longer delay payment period, the retailer might order more items than can be stored in an owned warehouse (OW). These excess quantities are stored in a rented warehouse (RW). The OW has a limited capacity, and the RW has an unlimited capacity. Due to the holding cost of RW is larger than that of OW, the items in RW are sold first and then the items in OW.

3. Model formulation

In this section, we first establish the total profit functions for the supplier and the retailer respectively, and then make some appropriate combination to obtain the supplier–retailer integrated total profit function.

3.1. Supplier's total profit per unit time

The supplier produces mQ units in each production run, hence, the production cycle length for the supplier is $mQ/D = mT$. The supplier's total profit per unit time is the total sales revenue minus the total relevant cost (which consists of the production cost, setup cost, inventory holding cost, and opportunity cost for offering trade credit). These components are evaluated as follows:

(a) *Sales revenue:* The sales revenue per unit time is given by vD.
(b) *Production cost:* The production cost per unit time is given by cD.
(c) *Setup cost:* The setup cost per unit time is $A_S/(mT)$.
(d) *Holding cost:* The holding cost per unit time is $c(r_S + I_{Sp}) DT[(m-1)(1-\rho) + \rho]/2$, where $\rho = D/R$. (a detailed explanation can be found in Joglekar (1988), Ouyang et al. (2009))
(e) *Opportunity cost:* Offering a credit period M_i to the retailer, the opportunity cost per unit time is $vI_{Sp}mQM_i/(mT) = vI_{Sp}DM_i$.

Consequently, when the supplier provides a given credit period $M_i, i = 1, 2, \ldots, k$, to the retailer, the total profit per unit time (denoted by $STP_i(m)$) is a function of m and can be expressed as

$$STP_i(m) = \text{sales revenue} - \text{production cost} - \text{setup cost}$$
$$- \text{holding cost} - \text{opportunity cost}$$
$$= vD - cD - \frac{A_S}{mT} - \frac{c(r_S + I_{Sp})DT}{2}[(m-1)(1-\rho) + \rho] - vI_{Sp}DM_i. \tag{1}$$

3.2. Retailer's total profit per unit time

As to the retailer, due to he/she orders Q units each time, the replenishment cycle length for the retailer is $Q/D = T$. The total profit per unit time is composed of sales revenue, purchasing cost, ordering cost, transportation cost, holding cost, interest earned and opportunity cost. These components are evaluated as follows:

(a) *Sales revenue:* The sales revenue per unit time is given by pD.
(b) *Purchasing cost:* The purchasing cost per unit time is given by vD.
(c) *Ordering cost:* The ordering cost per unit time is A_R/T.
(d) *Transportation cost:* The transportation cost per unit time is $(F_0 + F_1 Q)/T = F_0/T + F_1 D$.
(e) *Holding cost:* If $Q \leqslant W$ (i.e., $T \leqslant T_W$), the retailer does not need a rented warehouse. Otherwise, he/she must rent an extra warehouse to hold inventories. Hence, the holding cost excluding the interest charge per unit time is as follows: (the detailed explanation can be found in Hartley (1976))

$$\begin{cases} \frac{r_{R1}vDT}{2}, & T \leqslant T_W, \quad (2a) \\ \frac{r_{R2}v(DT-W)^2}{2DT} + \frac{r_{R1}v(2DT-W)W}{2DT}, & T \geqslant T_W. \quad (2b) \end{cases} \tag{2}$$

(f) *Opportunity cost and interest earned:* The credit period $M_i, \ i = 1, 2, \ldots, k$, offered by the supplier is related to the retailer's order quantity. Based on the values of M_i and T, we have the following two cases: (1) $T \leqslant M_i$ and (ii) $T \geqslant M_i$. These two cases are depicted in Fig. 1. Now, let us discuss the detailed formulation in each case.
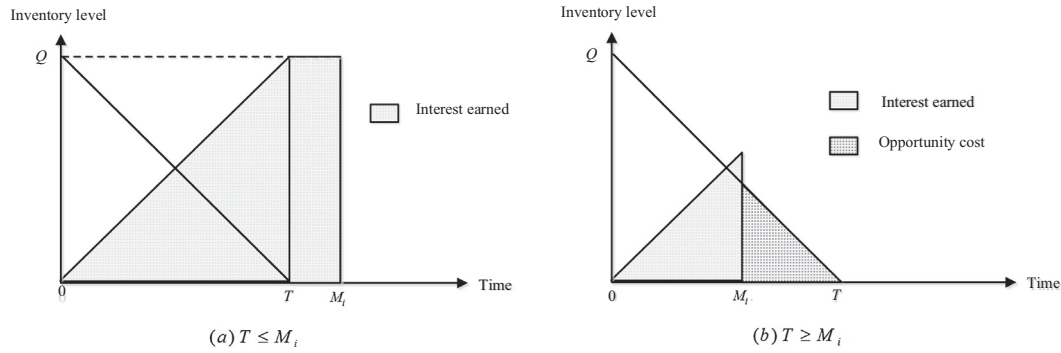
Fig. 1. Interest earned and opportunity cost under various situations.

Case 1. $T \leqslant M_i$ $(i = 1, 2, \ldots, k)$.

In this case, the retailer's delay payment period M_i is longer than or equal to the ordering cycle length T(see Fig. 1(a)). Therefore, the retailer pays no opportunity cost for the items kept in stock. At the same time, the retailer uses the sales revenue to earn interest at a rate of I_{Re}, and hence the interest earned per unit time is

$$\left[pI_{Re} \int_0^T Dt\,dt + pI_{Re}DT(M_i - T) \right] \Big/ T = pI_{Re}D\left(M_i - \frac{T}{2} \right). \tag{3}$$

Case 2. $T \geqslant M_i$ $(i = 1, 2, \ldots, k)$.

In this case, the retailer has some inventory available after the due date M_i (see Fig. 1(b)); therefore, the capital opportunity cost per unit time is

$$\left(vI_{Rp} \int_{M_i}^T D(T - t)\,dt \right) \Big/ T = vI_{Rp} \frac{D(T - M_i)^2}{2T}. \tag{4}$$

Also, the retailer can use the sales revenue during the credit period to earn interest at a rate of I_{Re}. Hence, the interest earned per unit time is

$$\left(pI_{Re} \int_0^{M_i} Dt\,dt \right) \Big/ T = \frac{pI_{Re}DM_i^2}{2T}. \tag{5}$$

For given M_i, $i = 1, 2, \ldots, k$, based on the length of T_W and M_i, the total profit per unit time for the retailer (denoted by $RTP_i(T)$) under various situations which is a function of T can be expressed as follows:

$RTP_i(T) = $ sales revenue − purchasing cost
 − ordering cost − transportation cost − holding cost
 − opportunity cost + interest earned

$$= \begin{cases} RTP_i^{(1)}(T), & T_W \leqslant M_i, \\ RTP_i^{(2)}(T), & T_W \geqslant M_i, \end{cases} \tag{6}$$

where

$$RTP_i^{(1)}(T) = \begin{cases} RTP_{1i}(T), & 0 < T \leqslant T_W \leqslant M_i, \\ RTP_{2i}(T), & T_W \leqslant T \leqslant M_i, \\ RTP_{3i}(T), & T_W \leqslant M_i \leqslant T, \end{cases} \tag{7}$$

$$RTP_i^{(2)}(T) = \begin{cases} RTP_{4i}(T), & 0 < T \leqslant M_i \leqslant T_W, \\ RTP_{5i}(T), & M_i \leqslant T \leqslant T_W, \\ RTP_{6i}(T), & M_i \leqslant T_W \leqslant T. \end{cases} \tag{8}$$

The detailed explanations for Eqs. (7) and (8) are as follows. For $0 < T \leqslant T_W \leqslant M_i$, it indicates that the retailer pays no opportunity cost and using the sales revenue to earn interest (refer to Eq. (3)).

Also, the retailer does not need to rent an additional warehouse and the holding cost is referred to Eq. (2a). As previously mentioned, the total profit per unit time for the retailer is composed of sales revenue, purchasing cost, ordering cost, transportation cost, holding cost and interest earned and is given by

$$RTP_{1i}(T) = pD + pI_{Re}D\left(M_i - \frac{T}{2} \right) - vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1 D - \frac{r_{R1} vDT}{2}. \tag{9}$$

Similarly, for $T_W \leqslant T \leqslant M_i$, the retailer pays no opportunity cost and using the sales revenue to earn interest. But, he/she needs to rent an additional warehouse (the holding cost is referred to Eq. (2b)). Hence, we have

$$RTP_{2i}(T) = pD + pI_{Re}D\left(M_i - \frac{T}{2} \right) - vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1 D$$
$$- \frac{r_{R2} v(DT - W)^2}{2DT} - \frac{r_{R1} v(2DT - W)W}{2DT}. \tag{10}$$

For $T_W \leqslant M_i \leqslant T$, it indicates that the retailer uses the sales revenue to earn interest during the credit period (refer to Eq. (5)) and pays a capital opportunity cost at the end of the permissible delay period (refer to Eq. (4)). Due to $T \geqslant T_W$, it needs to rent an additional warehouse. Hence, the total profit per unit time for the retailer is

$$RTP_{3i}(T) = pD + pI_{Re}\frac{DM_i^2}{2T} - vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1 D - \frac{r_{R2} v(DT - W)^2}{2DT}$$
$$- \frac{r_{R1} v(2DT - W)W}{2DT} - vI_{Rp}\frac{D(T - M_i)^2}{2T}. \tag{11}$$

For $0 < T \leqslant M_i \leqslant T_W$, the result is the same as case $0 < T \leqslant T_W \leqslant M_i$. Hence, we have

$$RTP_{4i}(T) = RTP_{1i}(T) = pD + pI_{Re}D\left(M_i - \frac{T}{2} \right)$$
$$- vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1 D - \frac{r_{R1} vDT}{2}. \tag{12}$$

For $M_i \leqslant T \leqslant T_W$, it indicates that the retailer uses the sales revenue to earn interest during the credit period (refer to Eq. (5)) and pays a capital opportunity cost at the end of the permissible delay period (refer to Eq. (4)). Due to $T \leqslant T_W$, it does not need to rent an additional warehouse. Hence, we get

$$RTP_{5i}(T) = pD + \frac{pI_{Re}DM_i^2}{2T} - vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1 D - \frac{r_{R1} vDT}{2}$$
$$- \frac{vI_{Rp}D(T - M_i)^2}{2T}. \tag{13}$$

For $M_i \leqslant T_W \leqslant T$, the result is the same as case $T_W \leqslant M_i \leqslant T$. Hence, we have

$$RTP_{6i}(T) = RTP_{3i}(T) = pD + pI_{Re}\frac{DM_i^2}{2T} - vD - \frac{A_R}{T} - \frac{F_0}{T} - F_1D$$
$$- \frac{r_{R2}v(DT-W)^2}{2DT} - \frac{r_{R1}v(2DT-W)W}{2DT} - vI_{Rp}\frac{D(T-M_i)^2}{2T}. \tag{14}$$

3.3. The joint total profit per unit time

When supplier and retailer have decided to share resources to undertake mutually beneficial cooperation, the joint total profit per unit time which is a function of m and T can be obtained as the sum of the supplier's and the retailer's total profit per unit time. From the above arguments, for given $M_i, i = 1, 2, \ldots, k$, the joint total profit per unit time can be obtained by combining Eq. (1) with Eqs. (11)–(14), and is given by

$$\Pi_i(m,T) = \begin{cases} \Pi_i^{(1)}(m,T) = STP_i(m) + RTP_i^{(1)}(T), & T_W \leqslant M_i, \\ \Pi_i^{(2)}(m,T) = STP_i(m) + RTP_i^{(2)}(T), & T_W \geqslant M_i, \end{cases} \tag{15}$$

where

$$\Pi_i^{(1)}(m,T) = \begin{cases} \Pi_{1i}(m,T) = STP_i(m) + RTP_{1i}(T), & 0 < T \leqslant T_W \leqslant M_i, \\ \Pi_{2i}(m,T) = STP_i(m) + RTP_{2i}(T), & T_W \leqslant T \leqslant M_i, \\ \Pi_{3i}(m,T) = STP_i(m) + RTP_{3i}(T), & T_W \leqslant M_i \leqslant T, \end{cases} \tag{16}$$

$$\Pi_i^{(2)}(m,T) = \begin{cases} \Pi_{4i}(m,T) = STP_i(m) + RTP_{4i}(T), & 0 < T \leqslant M_i \leqslant T_W, \\ \Pi_{5i}(m,T) = STP_i(m) + RTP_{5i}(T), & M_i \leqslant T \leqslant T_W, \\ \Pi_{6i}(m,T) = STP_i(m) + RTP_{6i}(T), & M_i \leqslant T_W \leqslant T, \end{cases} \tag{17}$$

and

$$\Pi_{1i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ DM_i(pI_{Re} - vI_{Sp}) - \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right), \tag{18}$$

$$\Pi_{2i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{pI_{Re} + r_{R2}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ vDT_W(r_{R2} - r_{R1})\left(1 - \frac{T_W}{2T}\right) + DM_i(pI_{Re} - vI_{Sp})$$
$$- \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right), \tag{19}$$

$$\Pi_{3i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{v(r_{R2} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ vDT_W(r_{R2} - r_{R1})\left(1 - \frac{T_W}{2T}\right) + DM_iv(I_{Rp} - I_{Sp})$$
$$+ \frac{DM_i^2}{2T}(pI_{Re} - vI_{Rp}) - \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right), \tag{20}$$

$$\Pi_{4i}(m,T) = \Pi_{1i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ DM_i(pI_{Re} - vI_{Sp}) - \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right), \tag{21}$$

$$\Pi_{5i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{v(r_{R1} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ DM_iv(I_{Rp} - I_{Sp}) + \frac{DM_i^2}{2T}(pI_{Re} - vI_{Rp})$$
$$- \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right), \tag{22}$$

$$\Pi_{6i}(m,T) = \Pi_{3i}(m,T) = D(p - F_1 - c)$$
$$- \frac{DT}{2}\{v(r_{R2} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$$
$$+ vDT_W(r_{R2} - r_{R1})\left(1 - \frac{T_W}{2T}\right) + DM_iv(I_{Rp} - I_{Sp})$$
$$+ \frac{DM_i^2}{2T}(pI_{Re} - vI_{Rp}) - \frac{1}{T}\left(A_R + F_0 + \frac{A_S}{m}\right). \tag{23}$$

Note that $\Pi_{1i}(m, T_W) = \Pi_{2i}(m, T_W)$ and $\Pi_{2i}(m, M_i) = \Pi_{3i}(m, M_i)$ for fixed m; therefore, $\Pi_i^{(1)}(m, T)$ is a continuous function of T. Similarly, $\Pi_{4i}(m, M_i) = \Pi_{5i}(m, M_i)$ and $\Pi_{5i}(m, T_W) = \Pi_{6i}(m, T_W)$ for fixed m; hence $\Pi_i^{(2)}(m, T)$ is a continuous function of T.

Our problem is to determine the retailer's optimal replenishment cycle length, T^*, and the optimal number of shipments per production run from the supplier to the retailer, m^*, which maximizes the joint total profit per unit time, $\Pi_i(m, T)$, for given $M_i, i = 1, 2, \ldots, k$. Once the optimal solution (m^*, T^*) is obtained, the buyer's optimal order quantity per order $Q^* = DT^*$ follows.

4. Solution procedure

First, for given $M_i, i = 1, 2, \ldots, k$, and fixed T, we examine the effect of m on the joint total profit per unit time. Taking the second-order partial derivatives of $\Pi_i^{(1)}(m, T)$ and $\Pi_i^{(2)}(m, T)$ with respect to m, we obtain

$$\frac{\partial^2 \Pi_i(m,T)}{\partial m^2} = \frac{\partial^2 \Pi_{ji}(m,T)}{\partial m^2} = \frac{2A_S}{m^3 T} < 0, \quad j = 1, 2, \ldots, 6. \tag{24}$$

Eq. (24) shows that $\Pi_i(m, T)$ is a concave function of m for a fixed T. Hence, the search for the optimal shipment number, m^*, is reduced to finding a local optimal solution. Next, for given $M_i, i = 1, 2, \ldots, k$, and T_W, we show how to determine the optimal value of T for a fixed m. We consider the following two situations: (1) $T_W \leqslant M_i$, and (2) $T_W \geqslant M_i$.

Situation 1. When $T_W \leqslant M_i$.

Situation 1.1. When $0 < T \leqslant T_W \leqslant M_i$.

For a fixed m, $\Pi_{1i}(m, T)$ is a concave function of T because $\partial^2 \Pi_{1i}(m, T)/\partial T^2 < 0$. Thus, there exists a unique value of T (denoted by T_{1i}) which maximizes $\Pi_{1i}(m, T)$. T_{1i} can be obtained by solving the equation $\partial \Pi_{1i}(m, T)/\partial T = 0$, and is given by

$$T_{1i} = \sqrt{\frac{2\left(A_R + F_0 + \frac{A_S}{m}\right)}{D\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}} > 0. \tag{25}$$

To ensure $T_{1i} \leqslant T_W$, we substitute Eq. (25) into the inequality $T_{1i} \leqslant T_W$, and obtain

$$T_{1i} \leqslant T_W \text{ if and only if } A_R + F_0 + A_S/m \leqslant \Delta_1, \tag{26}$$

where $\Delta_1 \equiv (DT_W^2/2)\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}$.

On the other hand, when $A_R + F_0 + A_S/m > \Delta_1$ and $0 < T \leqslant T_W$, it can be shown that

$$\frac{\partial \Pi_{1i}(m,T)}{\partial T} > \frac{D\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}{2T^2}$$
$$\times (T_W^2 - T^2) \geqslant 0. \tag{27}$$

Eq. (27) implies that $\Pi_{1i}(m,T)$ is a strictly increasing function of $T \in (0, T_W]$. Therefore, for a fixed m, $\Pi_{1i}(m,T)$ has a maximum value at $T = T_W$. Combining Eqs. (26) and (27), we obtain the following corollary.

Corollary 1. *For any given m and $T_W \leqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m \leqslant \varDelta_1$, then $\Pi_{1i}(m,T)$ has maximum value at $T = T_{1i}$.*
(b) *If $A_R + F_0 + A_S/m > \varDelta_1$, then $\Pi_{1i}(m,T)$ has maximum value at $T = T_W$.*

Situation 1.2. When $T_W \leqslant T \leqslant M_i$

For a fixed m, $\Pi_{2i}(m,T)$ is a concave function of T because $\partial^2 \Pi_{2i}(m,T)/\partial T^2 < 0$. Thus, there exists a unique value of T(denoted by T_{2i}) which maximizes $\Pi_{2i}(m,T)$. T_{2i} can be obtained by solving the equation $\partial \Pi_{2i}(m,T)/\partial T = 0$, and is given by

$$T_{2i} = \sqrt{\frac{2\left(A_R + F_0 + \frac{A_S}{m}\right) + vDT_W^2(r_{R2} - r_{R1})}{D\{pI_{Re} + r_{R2}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}} > 0. \quad (28)$$

To ensure $T_W \leqslant T_{2i} \leqslant M_i$, we substitute Eq. (28) into the inequality $T_W \leqslant T_{2i} \leqslant M_i$, and obtain

$$T_W \leqslant T_{2i} \leqslant M_i \text{ if and only if } \varDelta_1 \leqslant A_R + F_0 + A_S/m \leqslant \varDelta_{2i}, \quad (29)$$

where $\varDelta_1$ is defined as above, and

$$\varDelta_{2i} \equiv \Big\{ DM_i^2\{pI_{Re} + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\} \\ + r_{R2}vD(M_i^2 - T_W^2) + r_{R1}vDT_W^2 \Big\}/2.$$

Conversely, when $A_R + F_0 + A_S/m < \varDelta_1$ and $T_W \leqslant T \leqslant M_i$, it can be shown that

$$\frac{\partial \Pi_{2i}(m,T)}{\partial T} < \frac{D\{pI_{Re} + r_{R2}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}{2T^2} \\ \times (T_W^2 - T^2) \leqslant 0. \quad (30)$$

Eq. (30) implies that $\Pi_{2i}(m,T)$ is a strictly decreasing function of $T \in [T_W, M_i]$. Therefore, for a fixed m, $\Pi_{2i}(m,T)$ has a maximum value at $T = T_W$. On the other hand, if $A_R + F_0 + A_S/m > \varDelta_{2i}$ and $T_W \leqslant T \leqslant M_i$, it can be shown that

$$\frac{\partial \Pi_{2i}(m,T)}{\partial T} > \frac{D\{pI_{Re} + r_{R2}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}{2T^2} \\ \times (M_i^2 - T^2) \geqslant 0. \quad (31)$$

Eq. (31) implies that $\Pi_{2i}(m,T)$ is an strictly increasing function of $T \in [T_W, M_i]$. Therefore, for a fixed m, $\Pi_{2i}(m,T)$ has a maximum value at $T = M_i$. Combining Eqs. (29)–(31), we obtain the following corollary.

Corollary 2. *For any given m and $T_W \leqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m < \varDelta_1$, then $\Pi_{2i}(m,T)$ has maximum value at $T = T_W$*
(b) *If $\varDelta_1 \leqslant A_R + F_0 + A_S/m \leqslant \varDelta_{2i}$, then $\Pi_{2i}(m,T)$ has maximum value at $T = T_{2i}$.*
(c) *If $A_R + F_0 + A_S/m > \varDelta_{2i}$, then $\Pi_{2i}(m,T)$ has maximum value at $T = M_i$.*

Situation 1.3. When $T_W \leqslant M_i \leqslant T$

For a fixed m, taking the first-order and second-order partial derivatives of $\Pi_{3i}(m,T)$ with respect to T, we obtain

$$\frac{\partial \Pi_{3i}(m,T)}{\partial T} = -\frac{D}{2}\Big\{v(r_{R2} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\Big\} \\ + \frac{2\left(A_R + F_0 + \frac{A_S}{m}\right) + vDT_W^2(r_{R2} - r_{R1}) - DM_i^2(pI_{Re} - vI_{Rp})}{2T^2}, \quad (32)$$

$$\frac{\partial^2 \Pi_{3i}(m,T)}{\partial T^2} = \frac{-2\left(A_R + F_0 + \frac{A_S}{m}\right) - vDT_W^2(r_{R2} - r_{R1}) + DM_i^2(pI_{Re} - vI_{Rp})}{T^3}. \quad (33)$$

By solving $\partial \Pi_{3i}(m,T)/\partial T = 0$, we can get the value of T (denoted by T_{3i}) as

$$T_{3i} = \sqrt{\frac{2\left(A_R + F_0 + \frac{A_S}{m}\right) + vDT_W^2(r_{R2} - r_{R1}) - DM_i^2(pI_{Re} - vI_{Rp})}{D\{v(r_{R2} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}}. \quad (34)$$

To ensure $T_{3i} \geqslant M_i$, we substitute Eq. (34) into the inequality $T_{3i} \geqslant M_i$, and obtain

$$T_{3i} \geqslant M_i \text{ if and only if } A_R + F_0 + A_S/m \geqslant \varDelta_{2i}, \quad (35)$$

where $\varDelta_{2i}$ is defined as above. Note that when $A_R + F_0 + A_S/m \geqslant \varDelta_{2i}$, we can obtain

$$2(A_R + F_0 + A_S/m) + vDT_W^2(r_{R2} - r_{R1}) - DM_i^2(pI_{Re} - vI_{Rp}) \\ \geqslant DM_i^2\{v(r_{R2} + I_{Rp}) + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\} > 0. \quad (36)$$

Hence, T_{3i} is well-defined and $\frac{\partial^2 \Pi_{3i}(m,T)}{\partial T^2} < 0$. Therefore, T_{3i} is a unique value of T which maximizes $\Pi_{3i}(m,T)$.

On the other hand, when $A_R + F_0 + A_S/m < \varDelta_{2i}$ and $T_W \leqslant M_i \leqslant T$, it can be shown that

$$\frac{\partial \Pi_{3i}(m,T)}{\partial T} < \frac{D\{vI_{Rp} + r_{R2}v + c(r_S + I_{Sp})[(m-1)(1-\rho) + \rho]\}}{2T^2} \\ \times (M_i^2 - T^2) \leqslant 0. \quad (37)$$

Eq. (37) implies that $\Pi_{3i}(m,T)$ is a strictly decreasing function of $T \in [M_i, \infty)$. Therefore, for a fixed m, $\Pi_{3i}(m,T)$ has a maximum value at $T = M_i$. Combining Eqs. (35) and (37), we obtain the following corollary.

Corollary 3. *For any given m and $T_W \leqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m \geqslant \varDelta_{2i}$, then $\Pi_{3i}(m,T)$ has maximum value at $T = T_{3i}$.*
(b) *If $A_R + F_0 + A_S/m < \varDelta_{2i}$, then $\Pi_{3i}(m,T)$ has maximum value at $T = M_i$.*

Let $T_i^{(1)}$ denote the optimal replenishment cycle length for Situation 1. From Corollaries 1–3, and the facts that $\Pi_{1i}(m,T_W) = \Pi_{2i}(m,T_W)$, and $\Pi_{2i}(m,M_i) = \Pi_{3i}(m,M_i)$, we can obtain Theorem 1 as follows.

Theorem 1. *For any given m and $T_W \leqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m < \varDelta_1$, the optimal replenishment cycle length is $T_i^{(1)} = T_{1i}$.*
(b) *If $\varDelta_1 \leqslant A_R + F_0 + A_S/m \leqslant \varDelta_{2i}$, the optimal replenishment cycle length is $T_i^{(1)} = T_{2i}$.*
(c) *If $A_R + F_0 + A_S/m > \varDelta_{2i}$, the optimal replenishment cycle length is $T_i^{(1)} = T_{3i}$.*

Situation 2. When $T_W \geqslant M_i$

Situation 2.1. When $0 < T \leqslant M_i \leqslant T_W$.

For a fixed m, $\Pi_{4i}(m,T) = \Pi_{1i}(m,T)$; hence, $\Pi_{4i}(m,T)$ is a concave function of T. Let T_{4i} denote the value of T which maximizes $\Pi_{4i}(m,T)$. By solving $\partial\Pi_{4i}(m,T)/\partial T = 0$, we have

$$T_{4i} = T_{1i} = \sqrt{\frac{2(A_R + F_0 + A_S/m)}{D\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho)+\rho]\}}} > 0. \quad (38)$$

To ensure $T_{4i} \leqslant M_i$, we substitute Eq. (38) into the inequality $T_{4i} \leqslant M_i$, and obtain

$$T_{4i} \leqslant M_i \text{ if and only if } A_R + F_0 + A_S/m \leqslant \Delta_{3i}, \quad (39)$$

where $\Delta_{3i} \equiv (DM_i^2/2)\{pI_{Re} + r_{R1}v + c(r_S + I_{Sp})[(m-1)(1-\rho)+\rho]\}$.

Similar to the discussion in Situation 1.1, we obtain the following corollary.

Corollary 4. *For any given m and $T_W \geqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m \leqslant \Delta_{3i}$, then $\Pi_{4i}(m,T)$ has maximum value at $T = T_{4i}$.*
(b) *If $A_R + F_0 + A_S/m > \Delta_{3i}$, then $\Pi_{4i}(m,T)$ has maximum value at $T = M_i$.*

Situation 2.2. When $M_i \leqslant T \leqslant T_W$.

For a fixed m, the value of T (denoted by T_{5i}) which satisfies the equation $\partial\Pi_{5i}(m,T)/\partial T = 0$ is as follows:

$$T_{5i} = \sqrt{\frac{2(A_R + F_0 + A_S/m) - DM_i^2(pI_{Re} - vI_{Rp})}{D\{v(I_{Rp} + r_{R1}) + c(r_S + I_{Sp})[(m-1)(1-\rho)+\rho]\}}}. \quad (40)$$

To ensure $M_i \leqslant T_{5i} \leqslant T_W$, we substitute Eq. (40) into the inequality $M_i \leqslant T_{5i} \leqslant T_W$, and obtain

$$M_i \leqslant T_{5i} \leqslant T_W \text{ if and only if } \Delta_{3i} \leqslant A_R + F_0 + A_S/m \leqslant \Delta_{4i}, \quad (41)$$

where Δ_{3i} is defined as above, and

$$\Delta_{4i} \equiv \Big\{ DT_W^2\{vr_{R1} + c(r_S + I_{Sp})[(m-1)(1-\rho)+\rho]\} \\ + vI_{Rp}D(T_W^2 - M_i^2) + I_{Re}pDM_i^2 \Big\}/2.$$

Using an approach similar to that in Situation 1.3, we can see that when $A_R + F_0 + A_S/m \geqslant \Delta_{3i}$, T_{5i} is well-defined and is the unique value of T that maximizes $\Pi_{5i}(m,T)$. Therefore, we can obtain the following corollary.

Corollary 5. *For any given m and $T_W \geqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m < \Delta_{3i}$, then $\Pi_{5i}(m,T)$ has maximum value at $T = M_i$.*
(b) *If $\Delta_{3i} \leqslant A_R + F_0 + A_S/m \leqslant \Delta_{4i}$, then $\Pi_{5i}(m,T)$ has maximum value at $T = T_{5i}$.*
(c) *If $A_R + F_0 + A_S/m > \Delta_{4i}$, then $\Pi_{5i}(m,T)$ has maximum value at $T = T_W$.*

Situation 2.3. When $M_i \leqslant T_W \leqslant T$.

For a fixed m, $\Pi_{6i}(m,T) = \Pi_{3i}(m,T)$, the value of T (denoted by T_{6i}), which satisfies the equation $\partial\Pi_{6i}(m,T)/\partial T = 0$, is as follows:

$$T_{6i} = T_{3i} \\ = \sqrt{\frac{2(A_R + F_0 + A_S/m) + vDT_W^2(r_{R2} - r_{R1}) - DM_i^2(pI_{Re} - vI_{Rp})}{D\{v(I_{Rp} + r_{R2}) + c(r_S + I_{Sp})[(m-1)(1-\rho)+\rho]\}}}. \quad (42)$$

To ensure $M_i \leqslant T_W \leqslant T_{6i}$, we substitute Eq. (42) into the inequality $M_i \leqslant T_W \leqslant T_{6i}$, and obtain

$$M_i \leqslant T_W \leqslant T_{6i} \text{ if and only if } A_R + F_0 + A_S/m \geqslant \Delta_{4i}, \quad (43)$$

where Δ_{4i} be defined as above.

Similar to the discussion in Situation 1.3, we obtain the following corollary.

Corollary 6. *For any given m and $T_W \geqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m \geqslant \Delta_{4i}$, then $\Pi_{6i}(m,T)$ has maximum value at $T = T_{6i}$.*
(b) *If $A_R + F_0 + A_S/m < \Delta_{4i}$, then $\Pi_{6i}(m,T)$ has maximum value at $T = T_W$.*

Let $T_i^{(2)}$ denote the optimal replenishment cycle length for Situation 2. From Corollaries 4–6, and the fact that $\Pi_{4i}(m,M_i) = \Pi_{5i}(m,M_i)$, and $\Pi_{5i}(m,T_W) = \Pi_{6i}(m,T_W)$, we can obtain Theorem 2 as follows.

Theorem 2. *For any given m and $T_W \geqslant M_i$, we have:*

(a) *If $A_R + F_0 + A_S/m < \Delta_{3i}$, the optimal replenishment cycle length is $T_i^{(2)} = T_{4i}$.*
(b) *If $\Delta_{3i} \leqslant A_R + F_0 + A_S/m \leqslant \Delta_{4i}$, the optimal replenishment cycle length is $T_i^{(2)} = T_{5i}$.*
(c) *If $A_R + F_0 + A_S/m > \Delta_{4i}$, the optimal replenishment cycle length is $T_i^{(2)} = T_{6i}$.*

Following by Theorems 1, 2, Assumption 3, and $\Pi_i^{(n)}(m,T)$, $n = 1, 2$, being a continuous function of T, we obtain the following theorem.

Theorem 3. *For any given m and M_i, $i = 1, 2, \ldots, k$, let $Q_i^{(n)} = DT_i^{(n)}$, $n = 1, 2$, we have:*

(a) *If $q_i \leqslant Q_i^{(n)} < q_{i+1}$, then $\Pi_i^{(n)}(m,T)$ at point $T = T_i^{(n)}$ has a maximum value.*
(b) *If $Q_i^{(n)} \geqslant q_{i+1}$, then $T_i^{(n)}$ is not a feasible solution.*
(c) *If $Q_i^{(n)} < q_i$, then $T_i^{(n)}$ is not a feasible solution. However, $\Pi_i^{(n)}(m,T)$ is a strictly decreasing function in $T \in [q_i/D, q_{i+1}/D)$, hence, $\Pi_i^{(n)}(m,T)$ at point $T_i^{(n)} = q_i/D$ has a maximum value.*

In summary, we design an algorithm to obtain the optimal solution (m^*, T^*) (see Appendix A).

5. Numerical examples

Example 1. To illustrate the solution procedure, we consider an inventory system with the following data: $D = 30{,}000$ units/year, $R = 45{,}000$ units/year, $W = 2000$ units (hence, $T_W = 2000/30{,}000 = 0.067$ years), $A_S = \$1500$/setup, $A_R = \$800$/order, $r_S = 0.01$, $r_{R1} = 0.03$, $r_{R2} = 0.05$, $I_{Sp} = 0.1$, $I_{Rp} = 0.15$, $I_{Re} = 0.2$, $p = \$40$/unit, $c_0 = \$10$/unit, $c_1 = \$2.5 \times 10^4$, $c_2 = \$2.5 \times 10^{-5}$, $v = \$35$/unit, $F_0 = \$75$/shipment, $F_1 = \$0.5$/unit. The trade credit terms offered by the supplier are listed in Table 2.

Applying the algorithm in Appendix A, the solution procedure in Table 3 shows that the optimal solution is $(m^*, T^*) = (6, 0.0857)$. Hence, the retailer's optimal order quantity is $Q^* = DT^* = 2572$ units, the supplier's optimal production quantity is $m^*Q^* = 15{,}432$ units, and the maximal joint total annual profit is $\Pi^* = \$812{,}430$. Because the optimal order quantity is less than

Table 2
Credit terms for Example 1.

Order quantity (units)	Credit period (years)
$q_1 = 0 \leqslant Q < 5000 = q_2$	$M_1 = 15/365 = 0.0411$
$q_2 = 5000 \leqslant Q < 7500 = q_3$	$M_2 = 30/365 = 0.0822$
$q_3 = 7500 \leqslant Q$	$M_3 = 45/365 = 0.1233$

5000 units, the retailer may delay payment until 15 days after delivery. Furthermore, the capacity of the OW ($W = 2000$) is insufficient to store the ordered quantity; hence, an RW is necessary.

Example 2. This example concerns the impact of the OW capacity and credit period on the optimal solution. The parameter values are identical to those used in Example 1 except for W and (M_1, M_2, M_3). Table 4 shows the optimal solutions for each value of $W \in \{1{,}500,\ 2{,}000,\ 2{,}500,\ 3{,}000,\ 3{,}500\}$ and $(M_1, M_2, M_3) \in \{(15, 30, 45), (20, 40, 60), (30, 60, 90)\}$. The table shows that when the OW capacity is relatively small (i.e., $W \leqslant 2500$), the retailer must rent a warehouse. However, when the value of credit period is relatively long (i.e., $(M_1, M_2, M_3) = (20, 40, 60)$ or $(30, 60, 90)$), retailers typically order larger quantities to benefit from a longer credit period, such that a rented warehouse is necessary. Moreover, the supplier's optimal shipment number decreases for longer credit periods, and the joint total profit per unit time increases with the increase in capacity of OW or the length of credit period.

Based on the above results, we have several managerial implications. First, trade credit linked to the order quantity is an effective tool or mean if the purpose of the supply chain system is to encourage the retailer to order more units or to lower the supplier's shipment number. Second, the longer credit period set the supplier provided or the larger capacity of OW, the more benefit for the supply chain system will be.

Table 3
Solution procedure of Example 1.

m	i	T	Q	$T_i^{(1)}/T_i^{(2)}$	$\Pi_i^{(1)}/\Pi_i^{(2)}$	
1	1	$T_{61} = 0.1413$	4238	$T_1^{(2)} = T_{61} = 0.1413$	$\Pi_1^{(2)} = 804{,}843$	
	2	$T_{32} = 0.1348$	4045	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 807{,}799$	←
	3	$T_{33} = 0.1234$	3702	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 805{,}815$	
2	1	$T_{61} = 0.1135$	3406	$T_1^{(2)} = T_{61} = 0.1135$	$\Pi_1^{(2)} = 809{,}921$	
	2	$T_{32} = 0.1059$	3176	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 811{,}228$	←
	3	$T_{23} = 0.1005$	3014	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 807{,}209$	
3	1	$T_{61} = 0.1017$	3051	$T_1^{(2)} = T_{61} = 0.1017$	$\Pi_1^{(2)} = 811{,}555$	
	2	$T_{32} = 0.0352$	2806	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 811{,}658$	←
	3	$T_{23} = 0.0909$	2728	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 806{,}603$	
4	1	$T_{61} = 0.0946$	2838	$T_1^{(2)} = T_{61} = 0.0946$	$\Pi_1^{(2)} = 812{,}199$	←
	2	$T_{32} = 0.0862$	2585	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 811{,}337$	
	3	$T_{23} = 0.0853$	2558	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 805{,}497$	
5	1	$T_{61} = 0.0896$	2688	$T_1^{(2)} = T_{61} = 0.0896$	$\Pi_1^{(2)} = 812{,}422$	←
	2	$T_{22} = 0.0813$	2439	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 810{,}716$	
	3	$T_{23} = 0.0813$	2439	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 804{,}191$	
6	1	$T_{61} = 0.0857$	2572	$T_1^{(2)} = T_{61} = 0.0857$	$\Pi_1^{(2)} = 812{,}430$	←[a]
	2	$T_{22} = 0.0783$	2348	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 809{,}945$	
	3	$T_{23} = 0.0783$	2348	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 802{,}785$	
7	1	$T_{61} = 0.0826$	2477	$T_1^{(2)} = T_{61} = 0.0896$	$\Pi_1^{(2)} = 812{,}314$	←
	2	$T_{22} = 0.0758$	2247	$T_2^{(1)} = q_2/D = 0.1667$	$\Pi_2^{(1)} = 809{,}089$	
	3	$T_{23} = 0.0758$	2247	$T_3^{(1)} = q_3/D = 0.2500$	$\Pi_3^{(1)} = 801{,}321$	

Note: "←" denotes the optimal solution for given m.
[a] Denotes the optimal solution for the supply chain system.

Table 4
Sensitivity analysis on (M_1, M_2, M_3) and W.

(M_1, M_2, M_3)	W	m^*	T^*	Q^*	Π^*	Rented warehouse	Credit period
(15, 30, 45)	1500	6	0.0849	2548	$812,319	Yes	M_1
	2000	6	0.0857	2572	$812,430	Yes	M_1
	2500	5	0.0906	2718	$812,481	Yes	M_1
	3000	5	0.0911	2734	$812,487	No	M_1
	3500	5	0.0911	2734	$812,487	No	M_1
(20, 40, 60)	1500	3	0.1667	5000	$814,169	Yes	M_2
	2000	3	0.1667	5000	$814,396	Yes	M_2
	2500	3	0.1667	5000	$814,589	Yes	M_2
	3000	3	0.1667	5000	$814,746	Yes	M_2
	3500	3	0.1667	5000	$814,869	Yes	M_2
(30, 60, 90)	1500	2	0.2500	7500	$820,937	Yes	M_3
	2000	2	0.2500	7500	$821,206	Yes	M_3
	2500	2	0.2500	7500	$821,451	Yes	M_3
	3000	2	0.2500	7500	$821,672	Yes	M_3
	3500	2	0.2500	7500	$821,871	Yes	M_3

Example 3. This example outlines the effects of changes in the retailer's major parameters A_R, r_{R1}, r_{R2}, I_{Rp}, I_{Re}, F_0, and F_1 on the optimal solution. The results in Table 5 reveal the following managerial insights:

(1) As the value of A_R or F_0 increases, T^* and Q^* increase while Π^* decreases. The results show that the optimal replenishment cycle time and ordering quantity increases, whereas the joint total profit decreases with higher ordering and fixed transportation costs.

Table 5
Sensitivity analysis on the retailer's major parameters.

		m^*	T^*	Q^*	Π^*
A_R	600	6	0.0776	2327	$814,879
	700	6	0.0817	2452	$813,624
	800	6	0.0857	2572	$812,430
	900	5	0.0934	2802	$811,329
	1000	3	0.1167	5000	$810,458
r_{R1}	0.020	5	0.0945	2715	$812,863
	0.025	5	0.0862	2585	$812,644
	0.030	6	0.0857	2572	$812,430
	0.035	6	0.0853	2558	$812,216
	0.040	6	0.0848	2544	$812,003
r_{R2}	0.040	5	0.0903	2710	$812,454
	0.045	6	0.0860	2580	$812,441
	0.050	6	0.0857	2572	$812,430
	0.055	6	0.0854	2563	$812,419
	0.060	6	0.0851	2554	$812,408
I_{Rp}	0.13	5	0.0923	2770	$812,709
	0.14	5	0.0909	2728	$812,563
	0.15	6	0.0857	2572	$812,430
	0.16	6	0.0846	2538	$812,310
	0.17	6	0.0835	2506	$812,195
I_{Re}	0.18	5	0.0904	2711	$812,197
	0.19	6	0.0900	2583	$812,312
	0.20	6	0.0857	2572	$812,430
	0.21	6	0.0853	2560	$812,548
	0.22	6	0.0849	2548	$812,667
F_0	55	6	0.0849	2548	$812,664
	65	6	0.0853	2560	$812,547
	75	6	0.0857	2572	$812,430
	85	6	0.0861	2583	$812,313
	95	6	0.0865	2595	$812,197
F_1	0.3	6	0.0857	2572	$818,430
	0.4	6	0.0857	2572	$815,430
	0.5	6	0.0857	2572	$812,430
	0.6	6	0.0857	2572	$809,430
	0.7	6	0.0857	2572	$806,430

Table 6
Sensitivity analysis on the supplier's major parameters.

		m^*	T^*	Q^*	m^*T^* (production cycle length)	m^*Q^* (production quantity)	Π^*
A_S	1300	5	0.0880	2640	0.4400	13,200	$812,872
	1400	5	0.0888	2664	0.4440	13,320	$812,646
	1500	6	0.0857	2572	0.5142	15,432	$812,430
	1600	6	0.0864	2591	0.5184	45,546	$812,236
	1700	6	0.0870	2610	0.5220	45,660	$812,044
r_S	0.008	6	0.0860	2579	0.5160	15,474	$812,500
	0.009	6	0.0858	2575	0.5148	15,450	$812,465
	0.010	6	0.0857	2572	0.5142	15,432	$812,430
	0.011	6	0.0856	2568	0.5136	15,408	$812,395
	0.012	6	0.0855	2565	0.5130	15,390	$812,360
I_{Sp}	0.08	6	0.0882	2645	0.5292	15,870	$814,003
	0.09	6	0.0869	2607	0.5214	15,642	$813,214
	0.10	6	0.0857	2572	0.5142	15,432	$812,430
	0.11	5	0.0885	2655	0.4425	13,275	$811,678
	0.12	5	0.0875	2624	0.4375	13,120	$810,939

(2) The larger the value of r_{R1}, r_{R2}, or I_{Rp}, the lower the value of T^*, Q^* and Π^*. In other words, the retailer's holding cost or capital opportunity cost have negative influences on the optimal replenishment cycle time, ordering quantity, and joint total profit.

(3) When the value of I_{Re} increases, T^* and Q^* decrease while Π^* increases. This shows that when the retailer's interest earned increases, the retailer must reduce the optimal replenishment cycle time and ordering quantity to benefit from trade credit, which results in an increase in the optimal joint total profit.

(4) Although T^* and Q^* remain constant, Π^* decreases as F_1 increases. From Eqs. (25), (28), (34), (38), (40) and (42), it can be seen that the unit transportation cost has no inference on the optimal replenishment and production policies.

Example 4. This example highlights the effects of changes in the supplier's major parameters A_S, r_S, and I_{Sp} on the optimal solution. The results in Table 6 show that Π^* (m^*T^* and m^*Q^*) decreases (increase) as A_S increases. The results show that the supplier has a longer production cycle length and higher production quantity at higher setup cost. Furthermore, Π^*, m^*T^*, and m^*Q^* decrease as r_S or I_{Sp} increase. It is clear that the supplier has a shorter production cycle length and smaller production quantity at higher holding cost and capital opportunity cost.

6. Conclusion

This paper developed an integrated inventory model with a capacity constraint and an order-size-dependent trade credit. Three theorems and an algorithm were developed to determine the retailer's optimal order quantity and number of shipments per production run from the supplier to the retailer. Subsequently, numerical examples were provided and several managerial insights reveal as follows. (1) Trade credit linked to the order quantity is an effective tool or mean if the purpose of the supply chain system is to encourage the retailer to order more units or to lower the supplier's shipment number. (2) To improve the supply chain profit, the supplier may consider extending the length of trade credit or the retailer may consider expanding warehouse capacity by trade off the benefit increase and warehouse building costs. Furthermore, the following managerial insights were obtained through studying the effects of changes in the retailer's and the

supplier's major parameters: (1) the retailer may raise the order quantity when its fixed costs of the system (such as A_R, F_0) increase and the supplier may raise production quantity when its fixed cost of the system (A_S) increases. And then the increases in fixed cost of the system reduce the joint total profit of supply chain system; (2) increased holding cost or capital opportunity cost have a negative influence on the optimal replenishment cycle time, ordering quantity, production quantity, and joint total profit; (3) as the retailer's interest earned increases, he/her may reduce the optimal replenishment cycle time and ordering quantity to frequently benefit from trade credit, which results in a higher optimal joint total profit; and (4) although the change of unit transportation cost has a negative effect on the joint total profit, the optimal replenishment and production policies remain constant, regardless of any changes in unit transportation cost.

Future research on this subject should consider extending the proposed model to cases involving deteriorating products, imperfect production processes, and probabilistic demand. Additionally, it may be worthwhile considering an integrated inventory model for multiple suppliers and retailers.

Appendix A. Algorithm

Step 0. Let $m = 0, \Pi(m, T_{(m)}) = \Pi(0, T_{(0)}) = 0$
Step 1. Set $m = m + 1$
Step 2. For each $i, i = 1, 2, \ldots, k$ perform Step 3–Step 4
Step 3. If $T_W < M_i$ then   ($*$ Situation 1 $*$)
  Step 3.1. Evaluate Δ_1, Δ_{2i}
    If $A_R + F_0 + A_S/m < \Delta_1$ then $T_i^{(1)} = T_{1i}$.
    If $\Delta_1 \leqslant A_R + F_0 + A_S/m \leqslant \Delta_{2i}$ then $T_i^{(1)} = T_{2i}$.
    If $A_R + F_0 + A_S/m > \Delta_{2i}$ then $T_i^{(1)} = T_{3i}$.
  Step 3.2. Set $Q_i = DT_i^{(1)}$
    If $Q_i \geqslant q_{i+1}$ then set $\Pi_i^{(1)}(m, T) = 0$.
    If $q_i \leqslant Q_i < q_{i+1}$ then $\Pi_i^{(1)}(m, T) \equiv \Pi_{1i}^{(}(m, T_i^{(1)})$.
    If $Q_i < q_i$ then $\Pi_i^{(1)}(m, T) \equiv \Pi_{1i}(m, \frac{q_i}{D})$.
  Step 3.3. Set $\Pi(m, T_{(m)}) = \underset{i=1,2,\ldots,k}{Max} \left\{ \Pi_i^{(1)}(m, T) \right\}$
Step 4. If $T_W \geqslant M_i$ then   ($*$ Situation 2 $*$)
  Step 4.1. Evaluate Δ_{3i}, Δ_{4i}
    If $A_R + F_0 + A_S/m < \Delta_{3i}$ then $T_i^{(2)} = T_{4i}$.
    If $\Delta_{3i} \leqslant A_R + F_0 + A_S/m \leqslant \Delta_{4i}$ then $T_i^{(2)} = T_{5i}$.
    If $A_R + F_0 + A_S/m > \Delta_{4i}$ then $T_i^{(2)} = T_{6i}$.
  Step 4.2. Set $Q_i = DT_i^{(2)}$
    If $Q_i \geqslant q_{i+1}$ then set $\Pi_i^{(2)}(m, T) = 0$.
    If $q_i \leqslant Q_i < q_{i+1}$ then $\Pi_i^{(2)}(m, T) \equiv \Pi_{2i}^{(}(m, T_i^{(2)})$.
    If $Q_i < q_i$ then $\Pi_i^{(1)}(m, T) \equiv \Pi_{1i}(m, \frac{q_i}{D})$.
  Step 4.3. Set $\Pi(m, T_{(m)}) = \underset{i=1,2,\ldots,k}{Max} \left\{ \Pi_i^{(2)}(m, T) \right\}$
Step 5. If $\Pi(m, T_{(m)}) \geqslant \Pi(m - 1, T_{(m-1)})$ then return to Step 1; otherwise, perform Step 6
Step 6. Let $(m^*, T^*) = (m - 1, T_{(m-1)})$. (m^*, T^*) is the optimal solution and $Q^* = DT^*, \Pi^* = \Pi(m^*, T^*)$.

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