# Advance Sales System with Price-Dependent Demand and an Appreciation Period Under Trade Credit 

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#### Abstract

With globalization, companies are facing fierce competition. Offering an appreciation period has become a commonly adopted method by retailers to sustain competitive advantage. During the appreciation period, customers can request to return products for any reason. In addition, retailers provide advance sales to attract additional customers. The supplier usually provides the retailer with a trade credit, which they can use as a type of price reduction to attract additional customers. Price is viewed as an important vehicle to sell products and enhance revenues. Therefore, in this article, we establish an inventory model with price-dependent demand for a retailer who simultaneously receives trade credit from its supplier, and offers advance sales and an appreciation period to its customers. We first establish a proper model and then provide an easy-to-use method to obtain an ordering policy for the retailer to achieve its maximum total profit. Finally, numerical examples are given to illustrate the solution procedure.


Keywords: Inventory, trade credit, advance sales, return products, finance.

## 1. Introduction

With advances in technology and information, companies face fierce competition. In the real market, almost all items are price sensitive. For example, in the retail industry, retailers may dynamically adjust their prices to boost demand and enhance revenues. Cohen [11] determined both the optimal replenishment cycle and price for inventory that was subject to continuous decay over time at a constant rate. Wee [39] studied a pricing and replenishment policy for a deteriorating inventory with a price elastic demand rate that declined over time. Wee [40, 41] extended Cohen's [11] model to develop a replenishment policy for deteriorating items with price-dependent demand, with Weibull distribution deterioration, and separately considered the addition of a quantity discount. Wee and Law [42] developed an inventory model for deteriorating items with price-dependent demand in which the time value of money was taken into account. Abad [1] presented a model of pricing and lot sizing under conditions of perishability, finite production, and partial backlogging. Mukhopadhyay et al. [24, 25] re-established Cohen's
[11] model by taking a price elastic demand rate and considering a time-proportional and two-parameter Weibull distribution deterioration rate separately. Chang et al. [6] introduced a deteriorating inventory model with price-time dependent demand and partial backlogging. In Yang et al. [46], a model with price-dependent demand and partial backlogging is developed. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Begum et al. [2] developed the inventory model with deteriorating items and price-dependent demand and proposed an optimal ordering policy. Soni and Patel [26] developed an inventory model for non-instantaneous deteriorating items with imprecise deterioration free time and credibility constraints. The model assumes price sensitive demand when the product has no deterioration, and price and time dependent demand when the product has deterioration.

Many surveys reveal that more than $70 \%$ of customers will first consider the return conditions prior to making a purchase. Retailers offer consumers return guarantees to reduce consumer risk because customers cannot fully evaluate a product before purchase. Petersen and Kumar [30] empirically demonstrated the role of product returns in the exchange process by determining the exchange process factors that help explain product return behavior and the consequences of product returns on future customer and firm behavior. Davis et al. [12] developed a simple model that allows the retailer to determine conditions in which money-back guarantees (MBGs) work best to enhance profits and social welfare. Davis et al. [13] employed an analytical model to help identify potential causes for variation among retailers' return policies. Hess et al. [16] suggested that retailers provide a return guarantee to increase market demand and reduce consumer risk. Shieh [32] studied the informational role and optimality of the common business practice of money-back guarantees in a signaling model with quality uncertainty and risk-neutral buyers. Yalabik et al. [45] developed an integrated approach for analyzing logistics and marketing decisions within the context of designing an optimal returns system for a retailer servicing two distinct market segments. Li et al. [19] studied the direct distributor's pricing strategy, the return policy, and the quality policy in four scenarios. The scenarios include situations where customer's demand is sensitive to either price or the return policy, and where the return is sensitive to either the return policy or the quality.

Advance sales policies are widely used by retailers, for example, G-music.com.tw, Amazon.com, and Eslitebooks.com. Xie and Shugan [44] showed that advance-selling profits are not gained from buyer surplus, but from an increased number of buyers being able to purchase. Moe and Fader [23] demonstrated the ability to forecast new album sales before the actual launch of the album, based only on the pattern of advance orders. You [47] addressed a service inventory control problem in which a firm sells products through an advance booking system, with the aim of optimizing product price to maximize the total expected profit. You [48] developed an advance sales system in which a firm sells perishable inventory using a reservation system during the sales season over a limited planning time interval. You and Wu [49] investigated the problem of ordering and pricing over a finite time planning horizon for an inventory system with advance sales and spot sales. They sought to develop a solution procedure that determines the optimal
advance sales price, spot sales price, order size, and replenishment frequency. Tsao [37] considered retailer's promotion and replenishment policies with an advance sales discount under the supplier's and retailer's trade credits and presented an algorithm to simultaneously determine the optimal promotion effort and replenishment cycle time. Mesak et al. [22] employed the techniques of calculus of variations and optimal control theory to derive 10 main propositions and provide the optimal service capacity allocation policy in an advance selling environment in continuous time. Chen and Cheng [10] established an inventory model for retailers who simultaneously receive a permissible delay in payments from suppliers while offering advance sales to customers. Many related papers can be found in Weng and Parlar [43], Tang et al. [33], and McCardle et al. [21].

Furthermore, providing trade credit is the largest source of financing for small and medium enterprises. Goyal [14] used the average cost approach to establish an economic ordering quantity (EOQ) model and analyze the effect of trade credit on the optimal inventory policy. Teng [34] amended Goyal's [14] model by considering the difference between the unit price and unit cost and found that it makes economic sense for a wellestablished buyer to order less quantity and take the benefits of the permissible delay more frequently. Ouyang et al. [29] developed a general EOQ model with trade credit for a retailer to determine the optimal shortage interval and replenishment cycle. Huang et al. [18] proposed a retailer's EOQ model with limited storage space under partially permissible delay in payments. Chang et al. [5] established a mathematical model to determine the optimal payment period and replenishment cycle. In the developed model, the effect of the inflation rate, deterioration rate, and delay in payment are discussed simultaneously. Balkhi [3] provided a general finite horizon trade credit economic ordering policy for an inventory model with deteriorating items under inflation and time value of money when shortages are not allowed. Other papers related to trade credit can be found in works by Chang and Teng [4], Chung and Liao [7], Chung et al. [8], Huang [17], Sarker et al. [31], Ouyang et al. [27], Ouyang and Cheng [28], Teng et al. [35, 36], Goyal et al. [15], Chen and Kang [9], Urban [38], and Lou and Wang [20].

From the above discussion, we propose the inventory issues including price-dependent demand, return products, advance sales, and trade credit. In this paper, we establish an inventory model with price-dependent demand for a retailer who simultaneously receives trade credit from its supplier, and offers advance sales and an appreciation period to its customers. The paper is organized as follows. The related literature is reviewed in Section 1, and the notation and assumptions are introduced in Section 2. We develop the models in Section 3, and the theoretical results are presented in Section 4. In Section 5, a numerical example is given to illustrate the solution procedure. Finally, the conclusions and directions for future research are given in Section 6.

## 2. Notation and Assumptions

The mathematical model in this paper is developed on the following notation and assumptions.

## Notation:

$p \quad$ unit selling price
$p^{*} \quad$ optimal selling price
$c \quad$ unit purchase cost, $c<p$
$s \quad$ ordering cost per order
$k \quad$ cost of implementing advance sales
$h \quad$ unit holding cost per unit of time excluding interest charges
$I_{c} \quad$ interest charges per $\$$ investment in stocks per unit of time
$I_{e} \quad$ interest earned per $\$$ per unit of time
$M$ permissible delay in settling account / trade credit period
$N \quad$ appreciation period
$t_{p} \quad$ advance selling period
$\delta \quad$ advance sales discount rate (i.e., all products are $\delta \%$ off during the advance sale period ) with $\delta \leq 1-c / p$, a decision variable
$\gamma \quad$ sales discount rate for return products
$\beta \quad$ the prepaid deposit rate, where $0<\beta \leq 1$
$\theta \quad$ product return rate, where $0 \leq \theta<1$
$T$ sales period
$T^{*} \quad$ optimal sales period
$Q \quad$ order quantity
$Q^{*} \quad$ optimal order quantity
$Z(p, T)$ total profit
$Z^{*} \quad$ maximum total profit per unit of time, i.e., $Z^{*}=Z\left(p^{*}, T^{*}\right)$.
Assumptions:

1. The inventory system here is for a single item in a single season.
2. The replenishment occurs instantaneously at an infinite rate.
3. Shortages are not allowed.
4. Customers who accept advance sales offers must prepay a deposit for the pre-committed orders.
5. The demand rate decreases exponentially. It is an assumed function of the selling price $p$ and the relationship is described by the following formula. $D(p)=a e^{-b p}$, where $a(>0)$ is initial demand and $b$ is a constant, governing the decreasing rate of the demand.
6. In the advance selling period $\left[0, t_{p}\right]$, all customers are offered a unit advance sales price $(1-\delta) p$ for their purchases and required to pay a deposit $\beta(1-\delta) p$ for a precommitted order. At the end of the advance selling period, the customer receives the item he/she pre-ordered and has to pay the retailer the remaining balance.
7. Retailers offer consumers return guarantees during the appreciation period. Customers can make a request to return products for any kind of reason during the appreciation period.
8. At time $T+N$, all the return items are sold at a discounted price $p(1-\gamma)$.

## 3. Mathematical Formulation

This article discusses the inventory problem with appreciation period and trade credit under advance sales. The retailer simultaneously receives trade credit from its supplier, and offers advance sales and an appreciation period to its customers. Figure 1 displays the behavior of inventory level. In the advance selling period $\left[0, t_{p}\right]$, all customers are offered a unit advance sales price $(1-\delta) p$ for their purchases and required to pay a prepaid deposit with the rate $\beta$. In addition, product return rate $\theta$ is given.


Figure 1: The retailer's inventory level with advance sales.

The objective here is to maximize the retailer's total profit. The total profit consists of the following elements:
(a) the sales revenue $=$

$$
p(1-\delta)(1-\theta) D_{a}(p) t_{p}+p(1-\theta) D_{s}(p)\left(T-t_{p}\right)+p(1-\gamma)\left[\theta D_{a}(p) t_{p}+\theta D_{s}(p)\left(T-t_{p}\right)\right]
$$

(b) cost of implementing advance sales $=k$,
(c) cost of placing an order $=s$,
(d) cost of purchasing $=c\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]$,
(e) cost of carrying inventory (excluding interest payable) $=$

$$
h\left[\frac{D_{s}(p)\left(T-t_{p}\right)^{2}(1+\theta)}{2}+\theta D_{a}(p) t_{p}\left(T-t_{p}\right)+\frac{\theta D_{a}(p) t_{p} N}{2}\right]
$$

(f) interest payable and interest earned.

To calculate the interest payable and interest earned, based on whether the payment is made before or after the end of the spot selling period, we have the following two cases: (i) $T-t_{p} \leq M\left(T \leq M+t_{p}\right)$ and (ii) $T-t_{p} \geq M\left(T \geq M+t_{p}\right)$. Figure 2 displays the cumulative quantity to earn interest and to incur interest charges in these two cases.



Figure 2: The retailer's cumulated quantity to earn interest and to incur interest charges.

Case 1: $T \leq M+t_{p}$
In this case, the permissible payment time expires on or after the end of the spot selling period. Thus, the retailer pays no interest for the items kept in stock. In addition, the retailer uses the sales revenue to earn interest at the rate of $I_{e}$ during the period $\left[0, t_{p}+M\right]$. In the advance selling period $\left[0, t_{p}\right]$, all customers are offered a unit advance sales price $(1-\delta) p$ for their purchases and required to pay a deposit $\beta(1-\delta) p$ for a pre-committed order. At the end of the advance selling period (i.e., the beginning of the spot selling period), the customer receives the item he/she pre-ordered and has to pay the retailer the remaining balance. By using the deposit income in the advance selling period $\left[0, t_{p}\right]$, the retailer can earn interest $\frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2}$. Further, at time $t_{p}$, the customers will receive the item he/she pre-ordered and pay the retailer the remaining balance. Therefore, during the trade credit period $\left[t_{p}, t_{p}+M\right]$, the retailer uses advance sales income to earn interest $(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right)$. In addition, during the spot selling period $\left[t_{p}, T\right]$, the retailer sells the products and uses the sales revenue to earn interest. Therefore, the interest earned during $\left[t_{p}, t_{p}+M\right]$ is

$$
\begin{aligned}
& \frac{I_{e} p D_{s}(p)\left(T-t_{p}\right)^{2}}{2}+I_{e} p D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T\right) \\
& -\frac{I_{e} p \theta D_{s}(p)\left(T+N-t_{p}\right)\left(T-t_{p}\right)}{2}-I_{e} p \theta D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T-N\right)
\end{aligned}
$$

In addition, the amount of return product is $\theta\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]$. At time $T+N$, the retailer sells all the return items at a discounted price $p(1-\gamma)$ and obtains a return product sales income $p(1-\gamma) \theta\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]$. Using the amount, the retailer gains interest income $I_{e} p(1-\gamma) \theta\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]\left(t_{p}+M-T-N\right)$. Thus, the interest earned during this sales season, including advance sales and spot sales, is as follows:

$$
\begin{aligned}
& \frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2}+(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right) \\
& +\frac{I_{e} p D_{s}(p)\left(T-t_{p}\right)^{2}}{2}+I_{e} p D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T\right) \\
& -\frac{I_{e} p \theta D_{s}(p)\left(T+N-t_{p}\right)\left(T-t_{p}\right)}{2}-I_{e} p \theta D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T-N\right) \\
& +I_{e} p(1-\gamma) \theta\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]\left(t_{p}+M-T-N\right)
\end{aligned}
$$

Case 2: $T \geq M+t_{p}$
In this case, the permissible payment time expires on or before the end of the spot selling period. The interest payable is $\frac{c I_{c} D_{s}(p)\left(T-t_{p}-M\right)^{2}}{2}$. Similar with the situation that in Case 1, by using the deposit income, the retailer can earn interest $\frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2}$. During the trade credit period $\left[t_{p}, t_{p}+M\right]$, the retailer uses advance sales income to earn interest $(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right)$. In addition, during the spot selling period $\left[t_{p}, t_{p}+M\right]$, the retailer sells the products and uses the sales revenue to earn interest
$\frac{(1-\theta) p I_{e} D_{s}(p) M^{2}}{2}$. Thus, the interest earned during this sales season, including advance sales and spot sales, is as follows:

$$
\frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2}+(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right)+\frac{(1-\theta) p I_{e} D_{s}(p) M^{2}}{2}
$$

Therefore, the retailer's total profit is

$$
\left\{\begin{array}{l}
Z_{1}(p, T), T \leq M+t_{p}  \tag{3.1}\\
Z_{2}(p, T), T \geq M+t_{p}
\end{array}\right.
$$

where

$$
\begin{align*}
Z_{1}(p, T)= & p(1-\delta)(1-\theta) D_{a}(p) t_{p}+p(1-\theta) D_{s}(p)\left(T-t_{p}\right) \\
& +p(1-\gamma)\left[\theta D_{a}(p) t_{p}+\theta D_{s}(p)\left(T-t_{p}\right)\right]-c\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]-k-s \\
& -h\left[\frac{D_{s}(p)\left(T-t_{p}\right)^{2}(1+\theta)}{2}+\theta D_{a}(p) t_{p}\left(T-t_{p}\right)+\frac{\theta D_{a}(p) t_{p} N}{2}\right] \\
& +\frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2}+(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right) \\
& +\frac{I_{e} p D_{s}(p)\left(T-t_{p}\right)^{2}}{2}+I_{e} p D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T\right) \\
& -\frac{I_{e} p \theta D_{s}(p)\left(T+N-t_{p}\right)\left(T-t_{p}\right)}{2}-I_{e} p \theta D_{s}(p)\left(T-t_{p}\right)\left(M+t_{p}-T-N\right) \\
& +I_{e} p(1-\gamma) \theta\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]\left(t_{p}+M-T-N\right) \tag{3.2}
\end{align*}
$$

and

$$
\begin{align*}
Z_{2}(p, T)= & p(1-\delta)(1-\theta) D_{a}(p) t_{p}+p(1-\theta) D_{s}(p)\left(T-t_{p}\right) \\
& +p(1-\gamma)\left[\theta D_{a}(p) t_{p}+\theta D_{s}(p)\left(T-t_{p}\right)\right]-c\left[D_{a}(p) t_{p}+D_{s}(p)\left(T-t_{p}\right)\right]-k-s \\
& -h\left[\frac{D_{s}(p)\left(T-t_{p}\right)^{2}(1+\theta)}{2}+\theta D_{a}(p) t_{p}\left(T-t_{p}\right)+\frac{\theta D_{a}(p) t_{p} N}{2}\right] \\
& -\frac{c I_{c} D_{s}(p)\left(T-t_{p}-M\right)^{2}}{2}+\frac{I_{e} \beta p(1-\delta) D_{a}(p) t_{p}^{2}}{2} \\
& +(1-\delta) p I_{e} D_{a}(p) t_{p}\left(M-\theta M+\frac{\theta N}{2}\right)+\frac{(1-\theta) p I_{e} D_{s}(p) M^{2}}{2} \tag{3.3}
\end{align*}
$$

## 4. Theoretical Results

In this section, we present the solution procedure and find the optimal solution to the aforementioned two cases. Our purpose is to determine $p^{*}$ and $T^{*}$ which maximize the total profit $Z\left(p^{*}, T^{*}\right)$. The optimal solutions $P^{*}$ and $T^{*}$ (we denote them as $p_{1}^{*}$ and $T_{1}^{*}$ ), for case 1 , need to satisfy equations $\frac{\partial Z_{1}(p, T)}{\partial p}=0$ and $\frac{\partial Z_{1}(p, T)}{\partial T}=0$. Furthermore, to make sure that the total profit per unit time $Z_{1}(p, T)$ is concave and reaches its global maximum at point $\left(p^{*}, T^{*}\right)$, the following conditions have to be satisfied.

$$
\begin{equation*}
\left.\frac{\partial^{2} Z_{1}(p, T)}{\partial p^{2}}\right|_{p^{*}, T^{*}}<0 \tag{4.1}
\end{equation*}
$$

$$
\begin{equation*}
\left.\frac{\partial^{2} Z_{1}(p, T)}{\partial T^{2}}\right|_{p^{*}, T^{*}}<0, \tag{4.2}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{\partial^{2} Z_{1}(p, T)}{\partial p^{2}}\right|_{p^{*}, T^{*}} \times\left.\frac{\partial^{2} Z_{1}(p, T)}{\partial T^{2}}\right|_{p^{*}, T^{*}}-\left[\left.\frac{\partial^{2} Z_{1}(p, T)}{\partial p \partial T}\right|_{p^{*}, T^{*}}\right]^{2}<0 \tag{4.3}
\end{equation*}
$$

Consequently, the optimal solution occurs at point $\left(p^{*}, T^{*}\right)$, which satisfies $\frac{\partial Z_{1}(p, T)}{\partial p}=$ 0 and $\frac{\partial Z_{1}(p, T)}{\partial T}=0$, simultaneously.

Similarly, the optimal solutions $p^{*}$ and $T^{*}$ (we denote them as $p_{2}^{*}$ and $T_{2}^{*}$ ), for case 2, can be found by solving equations $\frac{\partial Z_{2}(p, T)}{\partial p}=0$ and $\frac{\partial Z_{2}(p, T)}{\partial T}=0$.

From the above results, we develop the following algorithm to obtain the optimal ordering policy.

## Algorithm

Step 1. By solving $\frac{\partial Z_{1}(p, T)}{\partial p}=0$ and $\frac{\partial Z_{1}(p, T)}{\partial T}=0$, we obtain $p_{1}^{*}$ and $T_{1}^{*}$, then substituting ( $p_{1}^{*}, T_{1}^{*}$ ) into equation (3.2). By solving $\frac{\partial Z_{2}(p, T)}{\partial p}=0$ and $\frac{\partial Z_{2}(p, T)}{\partial T}=0$, we obtain $p_{2}^{*}$ and $T_{2}^{*}$, then substituting ( $p_{2}^{*}, T_{2}^{*}$ ) into equation (3.3).
Step 2. (a) if $T_{1}^{*} \leq M+t_{p}$, we obtain $Z_{1}\left(p_{1}^{*}, T_{1}^{*}\right)$, otherwise, we let $Z_{1}\left(p_{1}^{*}, T_{1}^{*}\right)=0$.
(b) if $T_{2}^{*} \geq M+t_{p}$, we obtain $Z_{2}\left(p_{2}^{*}, T_{2}^{*}\right)$, otherwise, we let $Z_{2}\left(p_{2}^{*}, T_{2}^{*}\right)=0$.

Step 3. Set $Z\left(p^{*}, T^{*}\right)=\operatorname{Max}\left\{Z_{1}\left(p_{1}^{*}, T_{1}^{*}\right), Z_{2}\left(p_{2}^{*}, T_{2}^{*}\right)\right\}$ then $\left(p^{*}, T^{*}\right)$ is the optimal solution.

Once we obtain $\left(p^{*}, T^{*}\right)$, the optimal ordering quantity $Q^{*}=D_{a}\left(p^{*}\right) t_{p}+D_{s}\left(p^{*}\right)\left(T^{*}-\right.$ $t_{p}$ ) follows.

## 5. Numerical Examples

In this section, we give one numerical example to illustrate the above solution procedure. The supplier offers a permissible delay if the payment is made within 30 days (i.e., $M=1$ month). The retailer offers the customers a 45 -day advance selling period (i.e., $M=45 / 30$ month) and a 10 -day appreciation period (i.e., $M=10 / 30$ month). During the appreciation period, customers can make a request to return their products for any reason. The interest earned per $\$$ per year is $5 \%$ and the interest charges per $\$$ investment in stocks per year is $3 \%$. In addition, $h=\$ 1 /$ unit/month, $c=\$ 5 /$ unit, $\theta=0.03, a=10^{7}, b=1.5, \beta=0.3, \delta=0.3, \gamma=0.4, k=50$, and $s=\$ 50$.

Under the above-given parameter values, applying the solution procedure and algorithm, we obtain the optimal solution $\left(p^{*}, T^{*}\right)=\left(p_{2}^{*}, T_{2}^{*}\right)=(8.07497,2.73467)$ and $Z_{2}\left(p_{2}^{*}, T_{2}^{*}\right)=1975.4$. The optimal order quantity $Q^{*}=3185.46$ units.

## 6. Conclusions

In this article, we discussed the inventory issues including advance sales, return guarantees, and trade credit. We established an inventory model with price-dependent demand for a retailer who simultaneously receives trade credit from its supplier, and offers advance sales and an appreciation period to its customers. Offering an appreciation period is a commonly adopted method by retailers. During the appreciation period, customers can request to return products for any reason. We provide an easy and useful algorithm to find the optimal advance selling period and optimal sales period. Finally, a numerical example is given to illustrate the solution procedure. In future research, our model can be extended in several ways. It might be worth considering the situation in which retailers incorporate some hidden inventory costs, such as transportation costs.

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