Application of Polynomial Translation Method to Prediction of Annual Maximum Wind Speeds

Jun Kanda[a], Yuan-Lung Lo[b], Takayuki Wada[c]

[a] *The University of Tokyo, Kashiwa, Chiba, Japan, kandaj@k.u-tokyo.ac.jp*
[b] *The University of Tokyo, Kashiwa, Chiba, Japan, ohlalaloren@hotmail.com*
[c] *The University of Tokyo, Kashiwa, Chiba, Japan, takayuki0120w@hotmail.com*

1 INTRODUCTION

In order to provide an appropriate design wind speed for the wind resistant design of a structure, it is essential and important to estimate the probability distribution of annual maximum wind speeds. The annual maximum value of 10 minute mean wind speeds is only one value among 52,560 samples in one year. It seems reasonable to assume that the statistical nature of 10 minute mean wind speeds, such as four moments, provide sufficient information for the characteristics of annual maximum wind speeds.

In this research, samples of 10 minute mean wind speeds at 155 meteorological sites are utilized to investigate the general relationships between four moments and the probability distributions of annual maxima. Then, the application of polynomial translation method is verified by utilizing these four moments and a simulation procedure based on polynomial translation method is developed. Simulation results of annual maximum wind speeds of several meteorological sites show that such examinations of probabilistic characteristics of 10 minute mean wind speeds may provide good information for wind hazard models for individual sites.

2 CHARACTERISTICS OF FOUR MOMENTS OF PARENT DISTRIBUTION

2.1 *Yearly variation of four moments*

10 minute mean wind speeds from 1961 to 2002 at 155 meteorological sites in Japan are utilized to calculate four moments (mean, standard deviation, skewness, and unbiased kurtosis) to represent the yearly statistical nature. Figure 1 shows the parent distribution and its probability distribution at Tokyo in 2002. Four moments calculated from these samples are 3.192 for mean, 1.528 for the standard deviation, 0.916 for the skewness and 1.255 for the unbiased kurtosis ("kurtosis" hereafter), which can be indicated as the statistical nature of the probability distribution. The annual maximum sample is pointed as the tail point of the probability distribution. Once the statistical nature is properly estimated, the simulation of annual maxima can be expected by estimating the four moments.

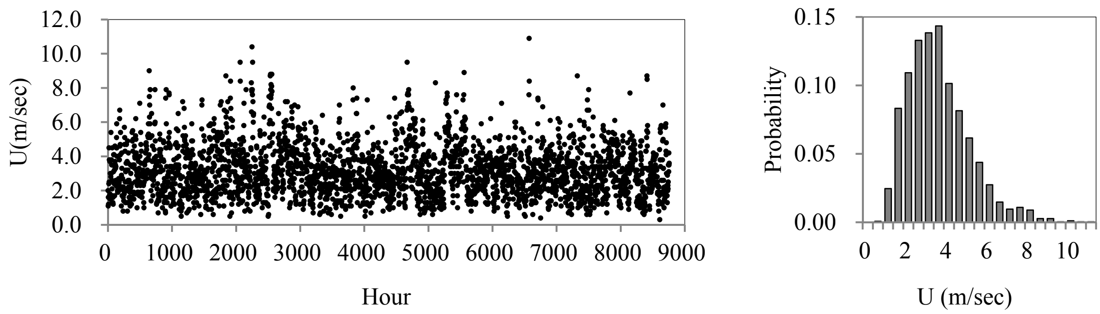


Figure 1 Parent distribution and probability distribution of 10 minute mean wind speeds at Tokyo in 2002

Yearly variations of four moments for one specific site, such as Tokyo in Figure 2, can then be obtained to observe the long term characteristics. It is found that in Figure 2 the variation of skewness and kurtosis are highly correlated. Among 155 meteorological sites, correlation coefficients between four moments are calculated and taken mean and standard deviation values as Table 1. A high mean value and a relatively low standard deviation value of 155 correlation coefficients indicate that the high correlation exists between yearly skewness and kurtosis in most sites.

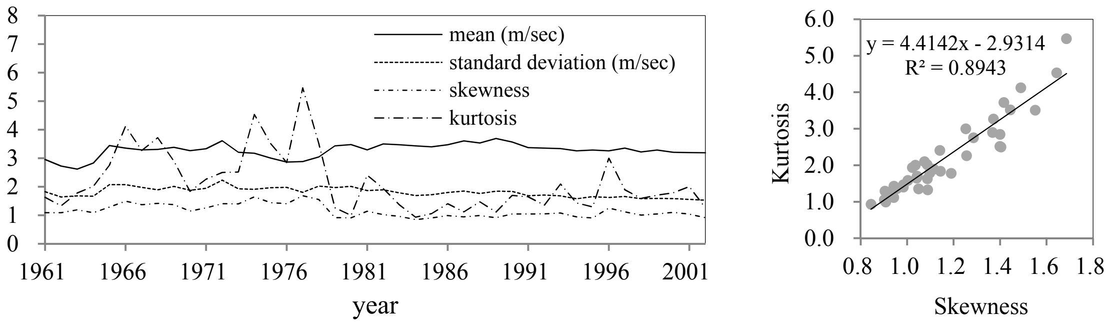


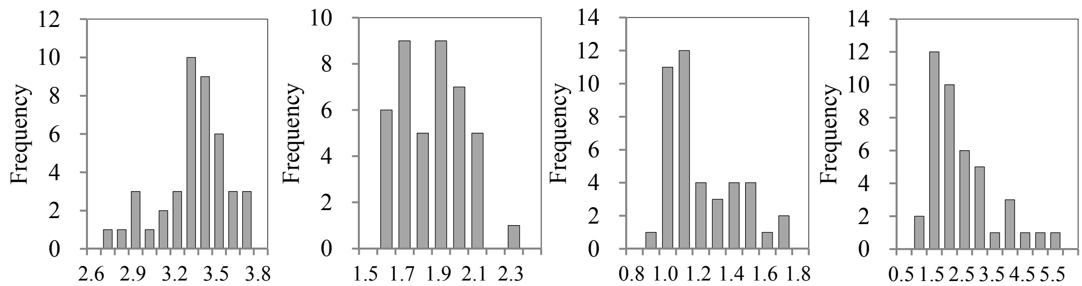
Figure 2 Yearly variations of four moments and relationship between skewness and kurtosis at Tokyo from 1961 to 2002

Table 1 Mean and standard deviation value of correlation coefficients between four moments

	$\mu-\sigma$	$\mu-\beta_3$	$\mu-\beta_4$	$\sigma-\beta_3$	$\sigma-\beta_4$	$\beta_3-\beta_4$
mean	0.59	-0.09	-0.06	0.17	0.09	0.91
standard deviation	0.33	0.30	0.26	0.32	0.27	0.05

※μ: mean, σ: standard deviation, β_3: skewness, β_4: kurtosis

Histograms of four moments at Tokyo from 1961 to 2002 can also be drawn as Figure 3. It is observed that the histograms of skewness and kurtosis show a tail which makes the distribution similar to a lognormal distribution rather than a normal distribution. Among 155 sites, the common features can also be observed in most sites. For the simulation of annual maxima, a proper assumption of the probability distribution model for four moments may contribute to a better prediction.



(a) μ (in m/sec) (b) σ (in m/sec) (c) β_3 (4) β_4

Figure 3 Histograms of four moments at Tokyo from 1961 to 2002

2.2 *Regional variation of four moments*

After examining statistical characteristics of yearly variations of four moments in 155 sites, differences from site to site are further examined as the effect of regional variation of four moments. Mean and standard deviation of yearly four moments are calculated and denominated as moment parameters, such as $E(\mu)$, $E(\sigma)$, $E(\beta_3)$, $E(\beta_4)$, $\sigma(\mu)$, $\sigma(\sigma)$, $\sigma(\beta_3)$, and $\sigma(\beta_4)$. From the histograms of these 155 sets of moment parameters, it indicates that some sites show relatively large scattering in yearly skewness and kurtosis. Once the estimation of skewness and kurtosis is large, the simulation of annual maxima will results in longer tail in the probability distribution model so that the simulation will be overestimated (Lo and Kanda, 2010).

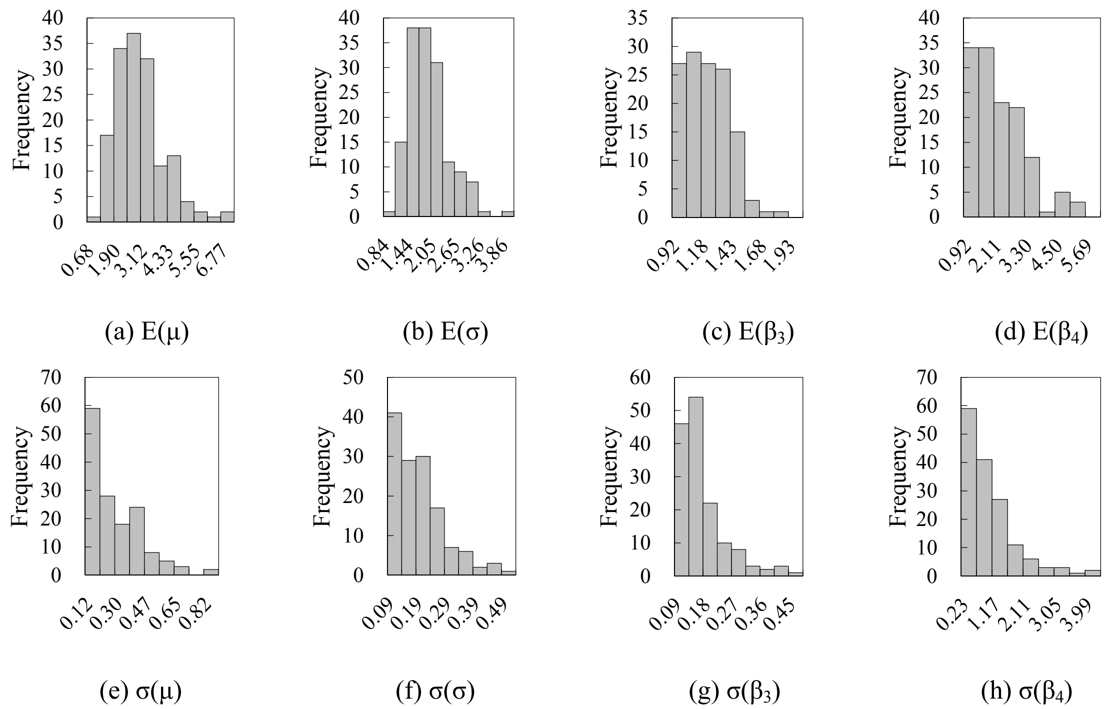


Figure 4 Histograms of moment parameters calculated from four moments at 155 sites in Japan

3 APPLICATION OF POLYNOMIAL TRANSLATION METHOD

3.1 *Brief introduction*

From the aforementioned statistical nature of parent distributions in all sites, it is essential to simulate annual maximum wind speeds with the proper estimation of four moments, especially those with extremely large skewness or kurtosis. A set of non-Gaussian random variables, Y, whose four moments are given, is written in a polynomial form with respect to a set of Gaussian random variables, X, as

$$Y = a + bX + cX^2 + dX^3 \tag{1}$$

where the coefficients of the polynomial form can be approximated by the following equations,

$$E(Y) = a + c = 0 \tag{2}$$

$$Var(Y) = b^2 + 6bd + 2c^2 + 15d^2 = 1 \tag{3}$$

$$\beta_3(Y) = 2c\left(b^2 + 24bd + 105d^2 + 2\right) \tag{4}$$

$$\beta_4(Y) = 24\left\{bd + c^2\left(1 + b^2 + 28bd\right) + d^2\left(12 + 48bd + 141c^2 + 225d^2\right)\right\} \tag{5}$$

An available approximation based on least square method for nonlinear parameters was introduced by Edgeworth (1898) to solve Equation (2) ~ (5) simultaneously. Table 2 shows the application sheet provided by Edgeworth to estimate the coefficients of the polynomial form. By substituting the given skewness and kurtosis, coefficients, a, b, c, and d, can be obtained by Equation (6) and Table 2. A set of non-Gaussian random variables can then be obtained by assuming X a set of Gaussian random variables in Equation (1). After adding the given mean and multiplying the given standard deviation, a set of non-Gaussian random variables, which possess the statistical nature of given four moments, is translated from a set of Gaussian random variables.

$$b = \sum_{j=1}^{16} T_j b_j \quad c = \sum_{j=1}^{16} T_j c_j \quad d = \sum_{j=1}^{16} T_j d_j \quad a = -c \tag{6}$$

Table 2 Application sheet for approximation of coefficients of the polynomial form

j	T_j	b_j		c_j		d_j	
		$\beta_4<1.5$	$\beta_4>=1.5$	$\beta_4<1.5$	$\beta_4>=1.5$	$\beta_4<1.5$	$\beta_4>=1.5$
1	1	1.0000	0.9698	0.0000	0.0012	0.0000	0.0112
2	β_3	-0.0014	-0.0305	0.1668	0.1566	0.0007	0.0129
3	β_4	-0.1238	-0.0765	0.0000	-0.0009	0.0412	0.0236
4	β_3^2	0.1224	0.0558	0.0019	-0.0024	-0.0469	-0.0177
5	β_4^2	0.0353	0.0054	0.0000	0.0002	-0.0131	-0.0018
6	β_3^3	-0.0491	-0.0348	0.0653	0.0466	0.0258	0.0216
7	β_4^3	-0.0085	-0.0002	0.0001	0.0000	0.0033	0.0001
8	$\beta_3\beta_4$	0.0027	0.0181	-0.0397	-0.0155	-0.0009	-0.0061
9	$\beta_3^2\beta_4$	-0.0768	-0.0130	0.0178	0.0236	0.0314	0.0087
10	$\beta_3\beta_4^2$	-0.0075	-0.0041	0.0183	0.0026	0.0021	0.0016
11	$\beta_3^3\beta_4$	0.0134	0.0029	-0.0068	0.0023	-0.0108	-0.0009
12	$\beta_3\beta_4^3$	0.0007	0.0003	-0.0018	-0.0002	0.0010	-0.0001
13	$\beta_3^2\beta_4^2$	-0.0101	-0.0002	-0.0071	-0.0029	0.0018	-0.0005
14	$\beta_3^2\beta_4^3$	0.0103	0.0001	0.0136	0.0001	0.0002	0.0000
15	$\beta_3^3\beta_4^2$	-0.0322	-0.0002	-0.0167	-0.0011	0.0165	-0.0001
16	$\beta_3^3\beta_4^3$	0.0127	0.0000	0.0207	0.0000	-0.0033	0.0000

3.2 *Testing of application sheet for polynomial translation method*

To verify the applicability of the application sheet (Table 2) and the polynomial translation method in simulating non-Gaussian random variables, a testing flow is attempted as Figure 5. A specific set of four moments is first given to generate a new set of non-Gaussian random variables. Comparing the given set and the resultant set of four moments, the applicability can be verified.

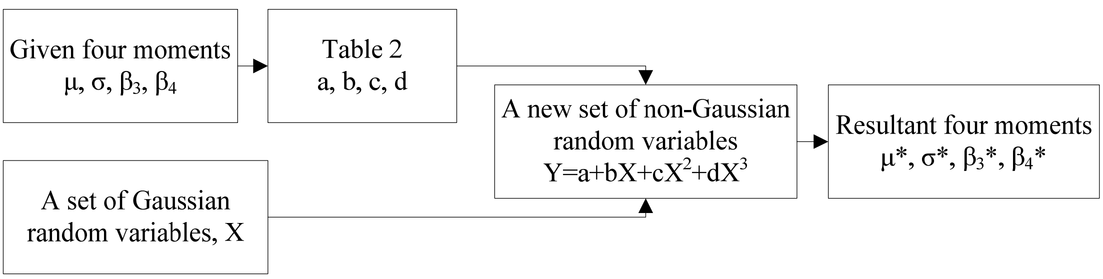


Figure 5 Testing flow for verification of polynomial translation method

All the four moments calculated from 155 sites are then utilized to verify the application range. The maximum and minimum values of all four moments are defined as the testing ranges. To simplify the testing procedure, whenever one given moment is tested, other three given moments are fixed at their mean values. As shown in Figure 6, the testing flow is carried out for the verification of each given moment. In the topmost four figures, the first moment value, mean, is varying from 1.003 to 9.887 and other three moment values are fixed at 2.041, 1.175, and 2.087 respectively.

For the varying given mean and given standard deviation, the simulation of non-Gaussian random variables can fit the original statistical nature well. However, when the given skewness is larger than 1.4, the polynomial translation method based on Table 2 will provide a biased resultant statistical nature. The same tendency occurs in the given kurtosis when the given value is larger than about 12.

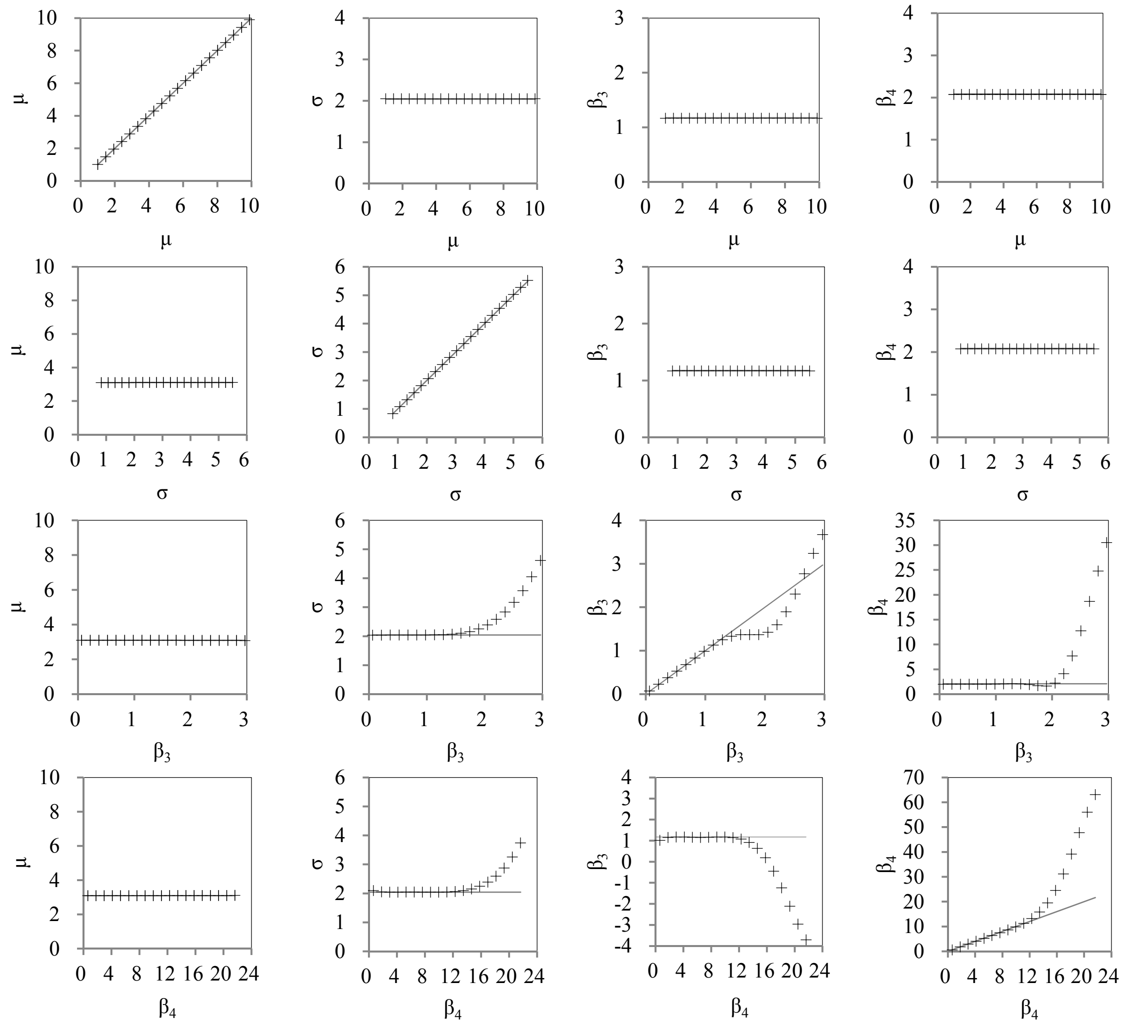


Figure 6 Verification results of each varying given moment
(−: given moment values; +: resultant moment values)

3.3 *Modification of application sheet based on observed statistical nature*

From the testing results in 3.2, it is indicated that extremely large given skewness or kurtosis is not suitable for the approximation sheet proposed by Edgeworth. To avoid the complexity of solving the nonlinear equations (2) ~ (5) and to improve the approximation method proposed by Edgeworth, the modification is conducted by utilizing the observed four moments from 155 sites. As shown in Figure 7, all sets of four moments from 155 sites are substituted into the testing flow in Figure 5 again. Given and resultant moment values are drawn respectively to show the biases. It can be presumed that the bias in standard deviation figure is caused by large given skewness or kurtosis. In the figures of skewness and kurtosis, there seems a bilinear relationship between given and resultant values.

To eliminate the biases caused by extremely large skewness or kurtosis, an inverse presumption is attempted. As illustrated in Figure 8, errors between given and resultant values can be first calculated and categorized as two groups, those with errors less than 20% and those with error larger than 20%. The former group follows the slope, L1, which represents the applicable range of given values; the latter group follows the slope, L2, which needs to be modified. In order to make all the resultant values follow L1 slope, a smaller given value is inversely estimated as shown in Figure 9. The inverse estimation is attempted as Equation (7) and (8). Figure 10 shows the modified resultant values versus given values for four moments.

$$\beta_3' = (\beta_3 - 1.751)\cdot\left(\frac{0.307}{1.249}\right) + 1.751 \quad (7)$$

$$\beta_4' = (\beta_4 - 14.468)\cdot\left(\frac{4.035}{25.532}\right) + 14.468 \quad (8)$$

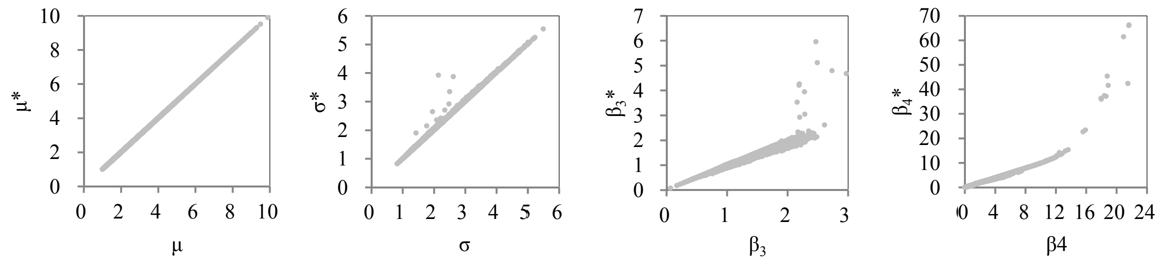


Figure 7 Verification results of 155 sets of given four moments

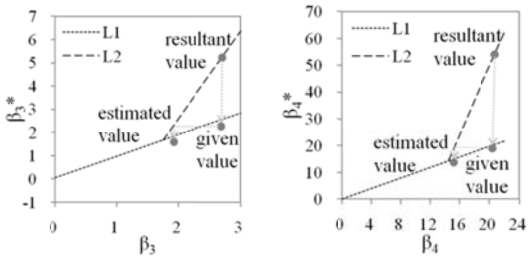


Figure 8 Bilinear relationships in β_3 and β_4

Figure 9 Inverse estimation diagram for β_3 and β_4

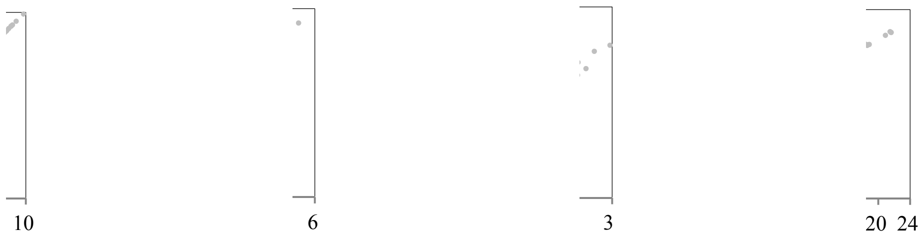


Figure 10 Verification results of 155 sets of given four moments after inverse calculation

4 SIMULATION OF ANNUAL MAXIMUM WIND SPEEDS

4.1 *Simulation procedure based on polynomial translation method*

Based on the statistical nature of four moments and the application of polynomial translation method aforementioned, a simulation procedure is introduced as Figure 11 (Choi and

Kanda, 2003, 2005). To estimate the annual maximum wind speed distribution for a specific site, 8 moment parameters, $E(\mu)$, $E(\sigma)$, $E(\beta_3)$, $E(\beta_4)$, $\sigma(\mu)$, $\sigma(\sigma)$, $\sigma(\beta_3)$, and $\sigma(\beta_4)$, are calculated in advance. By utilizing these moment parameters and the observation of probability distribution models, the generation of skewness and kurtosis can be conducted as normally distributed or log-normally distributed. According to the linear relationship between yearly skewness and kurtosis, a fully correlated assumption is also included. Then the polynomial translation method is applied to generate non-Gaussian samples. From these samples, the maximum value is picked as the annual maximum wind speed. In this research, 100 sets of four moments are generated as samples of 100 years. Every year has its annual maximum value and can be drawn as an extreme value distribution on the Gumbel model paper. To keep the randomness of simulation procedure, the extreme value distribution drawing is repeated for 11 times and taken the median values as the best estimation of annual maximum wind speed distribution for the specific site.

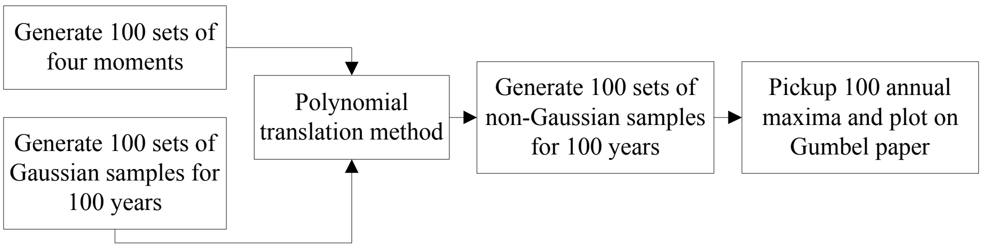


Figure 11 Simulation procedure for 100 annual maxima based on polynomial translation method

4.2 *Simulation results of several sites in Japan*

The aforementioned simulation procedure and the general statistical nature of 155 meteorological sites are then applied to estimate the annual maximum wind speeds. Four sites, Aikawa, Sapporo, Ishigakijima, and Kumijima, are chosen to demonstrate the agreement of simulation results and observed records.

Table 3 lists moment parameters of these four sites. Since the standard deviation of skewness and kurtosis is quite large in Ishigakijima and Kumijima, it is predictable to generate a longer tail in the probability distribution of skewness and kurtosis and further results in large annual maximum wind speeds. For Aikawa and Sapporo relatively, the values of moment parameters are considered moderate among 155 sites.

Table 3 Moment parameters of four meteorological sites

	$E(\mu)$	$E(\sigma)$	$E(\beta_3)$	$E(\beta_4)$	$\sigma(\mu)$	$\sigma(\sigma)$	$\sigma(\beta_3)$	$\sigma(\beta_4)$
Aikawa	4.32	3.33	1.31	1.40	0.41	0.30	0.12	0.48
Sapporo	2.51	1.71	0.94	0.88	0.28	0.22	0.14	0.49
Ishigakijima	4.51	2.15	1.28	6.32	0.38	0.23	0.65	6.47
Kumijima	3.70	1.96	0.92	5.05	0.18	0.15	0.60	5.30

Figure 12 shows the simulation results based on two probability distribution models for generated skewness and kurtosis. For Aikawa and Sapporo, skewness and kurtosis are generated based on moderate moment parameters so that the assumption of probability distribution can be normal or lognormal. However, for Ishigakijima and Kumijima, a lognormal distribution may be more appropriate to generate skewness and kurtosis. The errors between the observed annual maxima and the simulation results are 2.97%, 9.67%, 16.93%, and 16.21% by Equation (9).

$$Er(\%) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{E_i - E_i^*}{E_i^*}\right)^2} \times 100 \tag{9}$$

where E_i represents i-th estimated annual maxima; E_i^* represents i-th observed annual maxima; and n represents the number of simulation years. In this research, n is assumed to be 100 and observed annual maxima are linearly interpolated to calculate the error. Among 155 sites, the worst two fitting cases have been shown as Ishigakijima and Kumijima.

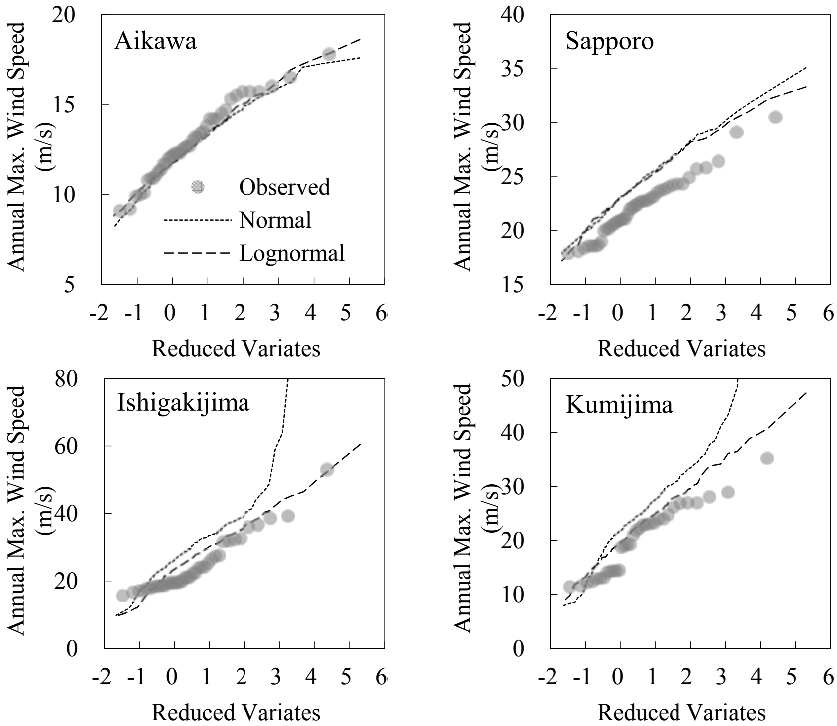


Figure 12 Simulation results for Aikawa, Sapporo, Ishigakijima, and Kumijima

5 CONCLUSIONS

A simulation procedure based on the polynomial translation method for estimation of annual maximum wind speeds was introduced. The modification to the approximation method proposed by Edgeworth was further conducted to provide a wider application range of skewness or kurtosis for non-Gaussian random variables. With the observation of moment parameters, statistical nature of parent distributions could be reflected well to the simulation results. The simulation results of 155 sites indicate that once the statistical nature, four moments, are properly estimated, the simulation procedure based on polynomial translation method can provide a fairly good prediction.

6 REFERENCES

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