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The retailer's optimal ordering policy with trade credit in different financial environments

Mei-Chuan Cheng [a], Chun-Tao Chang [b,*], Liang-Yuh Ouyang [c]

[a] Department of International Business, Hsin Sheng College of Medical Care and Management, Longtan, Taoyuan 325, Taiwan
[b] Department of Statistics, Tamkang University, Tamsui, Taipei 251, Taiwan
[c] Department of Management Sciences, Tamkang University, Tamsui, Taipei 251, Taiwan

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ABSTRACT

In business transactions, it is quite common for the supplier to offer the retailer a permissible delay in payments in order to stimulate the demand of the retailer. The retailer can either pay off all accounts at the end of the credit period or delay incurring interest charges on the unpaid and overdue balance due to the difference between interest earned and interest charged. In this study, we consider different financial environments when the supplier provides a permissible delay in payments. The proper mathematical models are developed to find the optimal order quantity and payoff time for maximizing the retailer's total profit for each financial environment. Furthermore, two theorems are established to determine the optimal solutions. Finally, numerical examples are presented to illustrate the proposed model. A sensitivity analysis is performed and economic interpretations are proposed.
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1. Introduction

In today's competitive business environment, offering retailers a permissible delay in payments is, for suppliers, an effective method of attracting new customers and increasing sales as it represents a form of price reduction. Additionally, the retailer is usually not obliged to pay interest during this period of permissible payment delay. Therefore, a retailer can earn interest on the accumulated revenue received, while delaying payments up until the end of the permissible period. However, if payment is delayed beyond that period, interest will be charged on the outstanding amount. In Taiwan, for example, most publishers allow delayed payments from bookstores. Interest does not begin accruing for the outstanding amount if it is paid within the permissible delay period. Therefore, the bookseller can earn interest on the accumulated revenue received by deferring payment until the end of this permissible period.

The existing literature has thoroughly discussed the inventory problem with delayed payments. Goyal [10] first developed an economic order quantity (EOQ) model under the condition of permissible delay in payments, in which he calculated interest income based on the purchasing cost of goods sold within the permissible delay period. Teng [24] amended Goyal's [10] model by calculating interest earned based on the selling price of goods. Chang [2] established an EOQ model with deteriorating items under inflation, where a supplier provides a permissible delay in payments for a large order that is greater than or equal to the predetermined quantity. Ouyang et al. [21] developed a general EOQ model with trade credit and partial backlogging for a retailer to determine its optimal shortage interval and replenishment cycle. Goyal et al. [11] established an appropriate EOQ model for a retailer where the supplier offers a progressive interest charge and found that the retailer would order a greater quantity and pay less total costs per year if the supplier provided a short-term teaser interest rate. Jaggi et al.

* Corresponding author.
E-mail address: chuntao@stat.tku.edu.tw (C.-T. Chang).

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[13] studied the retailer's optimal replenishment decisions with credit-linked demand under a permissible delay in payments. Liao [16] proposed an EOQ model with exponentially deteriorating items under two-level trade credit. Chang et al. [7] incorporated the concepts of vendor–buyer integration and order-size-dependent trade credit. They presented a stylized model to determine the optimal strategy for an integrated vendor–buyer inventory system under the condition of trade credit being linked to order quantity. Teng [25] established an EOQ model for a retailer who receives full trade credit from its supplier, and offers either partial or full trade credit to his or her customers. Chang et al. [5] addressed the shortcomings in Liao [16], and proposed a generalized model with both up-stream and down-stream trade credits. Other related articles on this topic can be found in works by Dave [9], Mandal and Phaujdar [18], Sarker et al. [23], Jamal et al. [14], Chang and Wu [6], Chang and Teng [4], Chung et al. [8], Ouyang et al. [20–22], Chang et al. [3], Ho [12], Liu [17], Min [19], Krichen et al. [15], Teng and Chang [26], Teng et al. [27,28] and Taso [29,30].

During the permissible period, a retailer is permitted to pay back the amount owing without accruing any interest to the supplier. In order to take the advantage of this opportunity, the retailer can sell the items and earn interest on the accumulated revenue received instead of paying off the amount, even after the permissible credit period expires, as interest will be charged if the payment is delayed beyond the credit period. Therefore, there is a trade-off between the total interest earned and the total interest charged to the supplier during the overdue period. In today's financial markets, the retailer may invest the money into stock markets or into developing new products, thus gaining a return on investment that may be higher than the interest charged. If the interest earned is higher than the interest charged, a reasonable retailer may not return money to the supplier until the end of the replenishment cycle. On the other hand, if the interest earned is less than the interest charged, a reasonable retailer will pay off the total purchase cost to the supplier as soon as possible, following the end of the credit period. Therefore, a more practical option for the retailer is either to pay off the entire amount owed to the supplier at the end of the credit period or to delay incurring interest charges on the unpaid and overdue balance. Hence, the determination of a retailer's payoff time is affected by the amount of interest income and interest payments. In previous papers, although much research has been devoted to studying inventory problems relating to trade credit, few of the research papers have considered the financial environment and their effects on a retailer's optimal ordering policy and payoff time. Therefore, in this paper, our goal is to discuss the EOQ inventory model under the conditions that the interest earned per dollar per unit time is higher than the interest charged per dollar per unit time, and the interest earned per dollar per unit time is lower than the interest charged per dollar per unit time. Furthermore, two theorems are proposed to determine the optimal replenishment cycle time and payoff time such that the total profit per unit time is maximized. Finally, we provide some numerical examples to illustrate the solution procedure and present the effects of the parameters on the optimal replenishment cycle time, order quantity, payoff time and total profit per unit time.

2. Notation and assumptions

The mathematical model in this paper is developed on the basis of the following notation and assumptions.
Notation:

D	the demand per unit time
p	unit selling price
c	unit purchase cost, with $c < p$
h	unit holding cost per unit time excluding interest charges
s	ordering cost per order
Q	the order quantity
Q^*	the optimal order quantity
I_c	interest charges per \$ in stock per unit time by the supplier
I_e	interest earned per \$ per unit time
M	permissible delay in settling account
T	the replenishment cycle time
T_i^*	the optimal replenishment cycle time for case i, $i = 1,2$
$TAP_i(T)$	the total profit per unit time for case i, $i = 1,2$
TAP_i^*	the optimal total profit per unit time for case i, i.e., $TAP_i^* = TAP_i(T_i^*), i = 1, 2$

Assumptions:

1. The inventory system involves only one item and the planning horizon is infinite.
2. Shortages are not allowed and the demand for the item is constant with time.
3. We adopt a methodology similar to Aggarwal and Jaggi [1] to calculate the interest earned. That is, when $T \geqslant M$, the retailer utilizes the sales revenue to earn interest throughout the inventory cycle T and when $T \leqslant M$, the retailer earns interest on sales revenue up to the permissible period M.

4. The retailer has two choices to pay off the total amount owed to the supplier: the retailer can decide to pay off all accounts either at the end of the credit period M or at any time point during $(M,T]$. If the retailer chooses the second option, the retailer must pay the supplier interest accrued.

3. Mathematical formulation

In this section, some appropriate inventory models are developed for each possible case. Our purpose is to maximize the total profit per unit time. First, we consider the total profit per replenishment cycle which consists of the following elements:

(a) Sales revenue = $p\ DT$,
(b) Cost of purchasing = $c\ DT$,
(c) Cost of placing order = s,
(d) Cost of carrying inventory (excluding interest payable) = $h\ DT^2/2$,
(e) Interest payable per cycle to the supplier, and
(f) Interest earned per cycle.

In regard to the interest charges and interest earned (i.e., (e) and (f)), two possible cases exist, based on the values of I_c and I_e, namely: (i) $I_e \geqslant I_c$ and (ii) $I_e < I_c$.

Case 1. $I_e \geqslant I_c$

This case situation indicates that the interest earned per dollar per unit time, I_e, is greater than or equal to the interest charges per dollar per unit time, I_c. Based on the values of T and M, the following two possible sub-cases exist: (i) $T \geqslant M$ and (ii) $T \leqslant M$.

Case 1.1. $T \geqslant M$

This situation indicates that the replenishment cycle time T is greater than or equal to the permissible delay in payments M. Since $I_e \geqslant I_c$, the retailer may never return money to the supplier until the end of the replenishment cycle time T. So the retailer's interest payable per cycle is cI_c times the area of rectangle BATM (i.e., $c\ I_cD\ T(T-M)$) and the interest earned per cycle is pI_e times the area of triangle AOT (i.e., $p\ I_eD\ T^2/2$) as shown in Fig. 1(a). Thus, the total profit per replenishment cycle is:

$$\begin{aligned} Z_{11}(T) &= \text{sales revenue} - \text{purchasing cost} - \text{ordering cost} - \text{carrying cost} - \text{interest payable} + \text{interest earned} \\ &= (p-c)DT - s - hDT^2/2 - cI_cDT(T-M) + pI_eDT^2/2. \end{aligned} \tag{1}$$

Case 1.2. $T \leqslant M$

In this situation, the replenishment cycle time T is less than or equal to the permissible delay in settling account M. The retailer will pay off the total amount owed to the supplier at the end of the trade credit period M. Thus, no interest charges are paid for the items. At the same time, the retailer uses the sales revenue to earn interest at the rate of I_e during the period $[0,M]$. In the time interval $[0,T]$, the retailer's interest earned is pI_e times the area of triangle GOT (i.e., $p\ I_eD\ T^2/2$) as shown in Fig. 1(b). In addition, the retailer has $p\ D\ T + p\ I_eD\ T^2/2$ at the end of the replenishment cycle time T. By using this amount, they are able to obtain the interest earned $I_e(p\ D\ T + p\ I_eD\ T^2/2)(M-T) = pI_e(D\ T + I_eD\ T^2/2)(M-T)$ in the time interval $[T,M]$, that is, pI_e times the area of rectangle HIMT as shown in Fig. 1(b). Hence, the interest earned during the period $[0,M]$ is $p\ I_eD\ T^2/2 + pI_e(D\ T + I_eD\ T^2/2)(M-T) = p\ I_eD\ T[T/2 + (1 + I_eT/2)(M-T)]$.

Therefore, the total profit per replenishment cycle is:

$$\begin{aligned} Z_{12}(T) &= \text{sales revenue} - \text{purchasing cost} - \text{ordering cost} - \text{carrying cost} + \text{interest earned} \\ &= (p-c)DT - s - hDT^2/2 + pI_eDT[T/2 + (1 + I_eT/2)(M-T)]. \end{aligned} \tag{2}$$

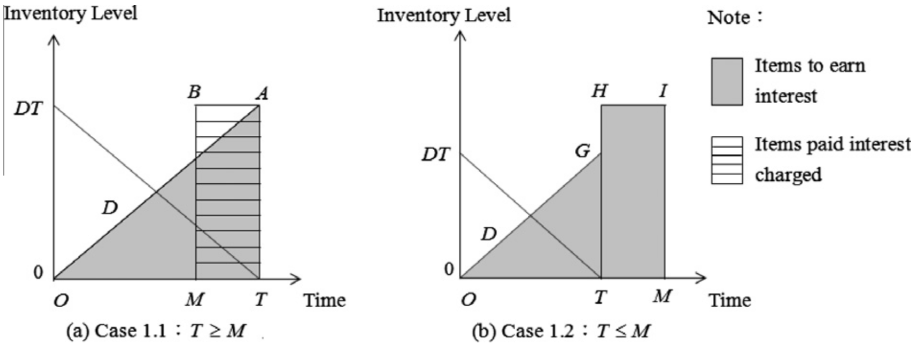


Fig. 1. Graphical representation of interest earned and interest charged for $I_e \geqslant I_c$.

Therefore, the total profit per unit time for Case 1 (i.e., $I_e \geqslant I_c$) is as follows:

$$TAP_1(T) = \begin{cases} TAP_{11}(T), & \text{if} \quad T \geqslant M, \\ TAP_{12}(T), & \text{if} \quad T \leqslant M, \end{cases}$$

where

$$TAP_{11}(T) = Z_{11}(T)/T = (p - c)D - s/T - hDT/2 - cI_cD(T - M) + pI_eDT/2, \tag{3}$$

and

$$TAP_{12}(T) = Z_{12}(T)/T = (p - c)D - s/T - hDT/2 + pI_eD[T/2 + (1 + I_eT/2)(M - T)]. \tag{4}$$

It is noted that $TAP_{11}(M) = TAP_{12}(M)$. Hence, $TAP_1(T)$ is a continuous function on $T > 0$.

Case 2. $I_e < I_c$

In this case, the interest earned per dollar per unit time, I_e, is less than the interest charges per dollar per unit time, I_c. Similarly, based on the values of T and M, we have the following two possible sub-cases: (i) $T \geqslant M$ and (ii) $T \leqslant M$.

Case 2.1. $T \geqslant M$

When $I_e < I_c$, the retailer will pay off the total purchase cost to the supplier as soon as possible. During $[0,M]$ period, the retailer sells products and uses the revenue to earn interest. The interest earned during the period $[0,M]$ is pI_e times the area of triangle AOM (i.e., $p\ I_eD\ M^2/2$) as shown in Fig. 2(a). Therefore, the retailer has $p\ D\ M + p\ I_eD\ M^2/2$ at time M. Since the retailer buys $D\ T$ units at time 0, the retailer owes the supplier $c\ D\ T$ at time M. Based on the difference between the money $p\ D\ M + p\ I_eD\ M^2/2$ the retailer has and the purchase cost $c\ D\ T$, there are two possible situations: (a) $p\ D\ M + p\ I_eD\ M^2/2 < c\ D\ T$, which means the retailer cannot pay off the unpaid balance at time M due to insufficient cash, (b) $p\ D\ M + p\ I_eD\ M^2/2 \geqslant c\ D\ T$, which means the retailer can pay off the total purchase cost to the supplier at time M. We discuss these two situations below.

Case 2.1–1. $p\ D\ M + p\ I_eD\ M^2/2 < c\ D\ T$ (i.e., $T > pM(1 + I_eM/2)/c$)

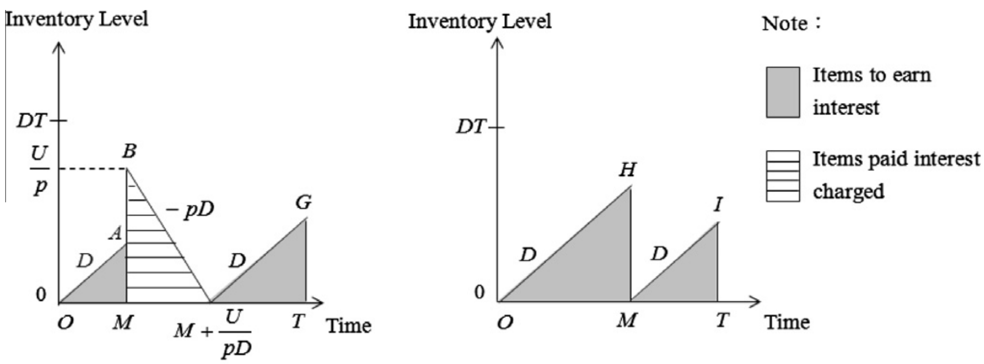
In this situation, the money in the retailer's account is less than the purchase cost at time M. The retailer pays $p\ D\ M + p\ I_eD\ M^2/2$ to the supplier at time M and finances the difference $U \equiv c\ D\ T - (p\ D\ M + p\ I_eD\ M^2/2)$. Thereafter, the retailer gradually reduces the financed loan from constant sales and revenue received. Hence, the interest payable per replenishment cycle is pI_c times the area of triangle $BM\ (M + U/pD)$ as shown in Fig. 2(a) (i.e., $pI_c(U/p)[U/(pD)]/2 = I_cU^2/(2pD)$), where $U/(pD)$ is the time period that the retailer pays off the unpaid balance to the supplier. Furthermore, after the time point $M + U/(pD)$, the retailer continuously sells products and uses the revenue to earn interest, which is equal to pI_e times the area of triangle $GT\ (M + U/pD)$ as shown in Fig. 2(a) (i.e., $p\ I_eD(T - M - U/pD)^2/2$). Therefore, the total profit per replenishment cycle is:

$$\begin{aligned} Z_{21}(T) &= \text{sales revenue} - \text{purchasing cost} - \text{ordering cost} - \text{carrying cost} - \text{interest payable} + \text{interest earned} \\ &= (p - c)DT - s - hDT^2/2 - I_cU^2/(2pD) + pI_eDM^2/2 + pI_eD(T - M - U/pD)^2/2, \end{aligned} \tag{5}$$

where $U = c\ D\ T - (p\ D\ M + p\ I_eD\ M^2/2)$.

Case 2.1–2. $p\ DM + p\ I_eDM^2/2 \geqslant c\ DT$ (i.e., $T \leqslant p\ M(1 + I_eM/2)/c$)

In this situation, the money in the retailer's account is greater than or equal to the purchase cost at time M. The retailer pays off the total amount owed to the supplier $c\ DT$ at time M. Hence, no interest charges are paid for the items. In addition, at time pointM, the retailer has amount $p\ D\ M + p\ I_eD\ M^2/2 - c\ DT$ on hand. By using the amount, the retailer is able to obtain



(a) Case 2.1-1：$T > pM(1+I_eM/2)/c$ (b) Case 2.1-2：$T \leq pM(1+I_eM/2)/c$

Fig. 2. Graphical representation of interest earned and interest charged for $I_c > I_e$.

the interest earned $I_e(p\ D\ M + p\ I_eD\ M^2/2 - c\ DT)(T - M)$ during the time period $[M,T]$. Furthermore, after the time point M, the retailer continuously sells products and uses the revenue to earn interest, which is equal to pI_e times the area of triangle MIT as shown in Fig. 2(b) (i.e., $p\ I_eD(T - M)^2/2$). Hence, the interest earned during the period $[0,T]$ is.

$$pI_eDM^2/2 + I_e\Big(pDM + pI_eDM^2/2 - cDT\Big)(T-M) + pI_eD(T-M)^2/2.$$

Therefore, the total profit per replenishment cycle is:

$$\begin{aligned} Z_{22}(T) &= \text{sales revenue} - \text{purchasing cost} - \text{ordering cost} - \text{carrying cost} + \text{interest earned} \\ &= (p-c)DT - s - hDT^2/2 + pI_eDM^2/2 + I_e(pDM + pI_eDM^2/2 - cDT)(T-M) + pI_eD(T-M)^2/2 \\ &= (pDM + pI_eDM^2/2 - cDT)[1 + I_e(T-M)] + pD(T-M) + pI_eD(T-M)^2/2 - s - hDT^2/2. \end{aligned} \tag{6}$$

Case 2.2. $T \leqslant M$

This case is the same as Case 1.2. Thus, the total profit per replenishment cycle is:

$$Z_{23}(T) = Z_{12}(T) = (p-c)DT - s - hDT^2/2 + pI_eDT[T/2 + (1 + I_eT/2)(M-T)]. \tag{7}$$

Therefore, the total profit per unit time for Case 2 (i.e., $I_e < I_c$) is as follows:

$$TAP_2(T) = \begin{cases} TAP_{21}(T), & \text{if} \quad T > pM(1 + I_eM/2)/c, \\ TAP_{22}(T), & \text{if} \quad M \leqslant T \leqslant pM(1 + I_eM/2)/c, \\ TAP_{23}(T), & \text{if} \quad T \leqslant M, \end{cases}$$

where

$$TAP_{21}(T) = Z_{21}(T)/T = (p-c)D - s/T - hDT/2 - I_cU^2/(2pDT) + pI_eDM^2/(2T) + pI_eD(T - M - U/pD)^2/(2T), \tag{8}$$

where $U = c\ D\ T - (p\ D\ M + p\ I_eD\ M^2/2)$:

$$\begin{aligned} TAP_{22}(T) &= Z_{22}(T)/T \\ &= \Big\{(pDM + pI_eDM^2/2 - cDT)[1 + I_e(T-M)] + pD(T-M) + pI_eD(T-M)^2/2\Big\}/T - s/T - hDT/2, \end{aligned} \tag{9}$$

and

$$TAP_{23}(T) = Z_{23}(T)/T = (p-c)D - s/T - hDT/2 + pI_eD[T/2 + (1 + I_eT/2)(M-T)], \tag{10}$$

It is noted that $TAP_{22}(M) = TAP_{23}(M)$. Hence, $TAP_2(T)$ is continuous at $T = M$.

4. Theoretical results

In this section, we present the procedure to obtain the solution and also establish two theorems to determine the optimal solution to the aforementioned cases.

Case 1. $I_e \geqslant I_c$

Case 1.1. $T \geqslant M$

To maximize the total profit per unit time, taking the first-order derivative of $TAP_{11}(T)$ in (3) with respect to T, we obtain:

$$dTAP_{11}(T)/dT = s/T^2 - hD/2 - cI_cD + pI_eD/2 = s/T^2 - D(h + 2cI_c - pI_e)/2. \tag{11}$$

If $h + 2c\ I_c - p\ I_e \leqslant 0$, then $d\ TAP_{11}(T)/d\ T > 0$, which implies $TAP_{11}(T)$ is a strictly increasing function of T. In this situation, it will be profitable for the retailer to prolong the replenishment cycle time as long as possible. Hence, the optimal replenishment cycle time is $T \to \infty$. Nevertheless, this is impossible in the real market. Thus, for the situation $T \geqslant M$ in Case 1 (i.e., $I_e \geqslant I_c$), the condition $h + 2c\ I_c - pI_e > 0$ (i.e., $I_e < (h + 2c\ I_c)/p$) must be satisfied. From this point onwards, we assume the condition $I_c \leqslant I_e < (h + 2c\ I_c)/p$ holds in Case 1.1. Next, motivated by (11), we define a new function $G_{11}(T)$ as follows:

$$G_{11}(T) = s/T^2 - D(h + 2cI_c - pI_e)/2, \tag{12}$$

for $T \in [M,\infty)$. Due to the derivative of $G_{11}(T)$ with respect to T is:

$$dG_{11}(T)/dT = -2s/T^3 < 0, \tag{13}$$

$G_{11}(T)$ is a strictly decreasing function in $T \in [M,\infty)$. Furthermore, we have:

$$\lim_{T\to\infty} G_{11}(T) = -D(h + 2cI_c - pI_e)\Big/2 < 0,$$

and

$$G_{11}(M) = \left[2s - M^2D(h + 2cI_c - pI_e)\right]/(2M^2).$$

For convenience, we let $\Delta_1 \equiv M^2D(h + 2c\, I_c - pI_e)$; then, we have the following results.

Lemma 1. *For $I_c \leqslant I_e < (h + 2c\, I_c)/p$,*

(a) *If $2s \geqslant \Delta_1$, then the value of $T \in [M,\infty)$ (say T_{11}) which satisfies $d\, TAP_{11}(T)/d\, T = 0$ exists uniquely.*
(b) *If $2s < \Delta_1$, then the value of $T \in [M,\infty)$ which satisfies $d\, TAP_{11}(T)/d\, T = 0$ does not exist.*

Proof. See Appendix A □

Lemma 2. *For $I_c \leqslant I_e < (h + 2c\, I_c)/p$,*

(a) *If $2s \geqslant \Delta_1$, then $TAP_{11}(T)$ has a maximum value at the point $T = T_{11}$, where $T_{11} \in [M,\infty)$ and satisfies $d\, TAP_{11}(T)/d\, T = 0$.*
(b) *If $2s < \Delta_1$, then $TAP_{11}(T)$ has a maximum value at the boundary point $T = M$.*

Proof. See Appendix B □

Case 1.2. $T \leqslant M$

Taking the first-order derivative of $TAP_{12}(T)$ in (4) with respect to T, we have:

$$dTAP_{12}(T)/dT = s/T^2 - hD/2 + pI_e^2DM/2 - pI_eD/2 - pI_e^2DT. \tag{14}$$

Next, we let function $G_{12}(T)$ denotes the right hand side of (14), i.e.,

$$G_{12}(T) = s/T^2 - hD/2 + pI_e^2DM/2 - pI_eD/2 - pI_e^2DT, \tag{15}$$

for $T \in (0,M]$. Due to the derivative of $G_{12}(T)$ with respect to T is:

$$dG_{12}(T)/dT = -2s/T^3 - pI_e^2D < 0, \tag{16}$$

$G_{12}(T)$ is a strictly decreasing function in $T \in (0,M]$. Furthermore, we have

$$\lim_{T\to 0^+} G_{12}(T) = \infty \quad \text{and} \quad G_{12}(M) = \left\{2s - M^2D[h + pI_e(I_eM + 1)]\right\}\Big/(2M^2).$$

For convenience, letting $\Delta_2 \equiv M^2D[h + p\, I_e(I_e\, M + 1)]$, we have the following results.

Lemma 3. *For $I_e \geqslant I_c$,*

(a) *If $2s \leqslant \Delta_2$, then the value of $T \in (0,M]$ (say T_{12}) which satisfies $d\, TAP_{12}(T)/d\, T = 0$ exists uniquely.*
(b) *If $2s > \Delta_2$, then the value of $T \in (0,M]$ which satisfies $d\, TAP_{12}(T)/d\, T = 0$ does not exist.*

Proof. The proof is similar to that of Lemma 1, hence we omit it here. □

Lemma 4. *For $I_e \geqslant I_c$,*

(a) *If $2s \leqslant \Delta_2$, then $TAP_{12}(T)$ has a maximum value at the point $T = T_{12}$, where $T_{12} \in (0,M]$ and satisfies $d\, TAP_{12}(T)/d\, T = 0$.*
(b) *If $2s > \Delta_2$, then $TAP_{12}(T)$ has a maximum value at the boundary point $T = M$.*

Proof. The proof is similar to that of Lemma 2, hence we omit it here.

Note that $\Delta_1 < \Delta_2$ for $I_e \geqslant I_c$. Thus, from Lemmas 2 and 4, we can develop the following theorem to obtain the optimal replenishment cycle time T_1^* for Case 1. □

Theorem 1. *For $I_e \geqslant I_c$:*

Situation	Condition	$TAP_1(T_1^*)$	T_1^*
$I_e < (h + 2c\ I_c)/p$	$2s < \Delta_1$	$TAP_{12}(T_{12})$	T_{12}
	$\Delta_1 \leqslant 2s \leqslant \Delta_2$	$Max\{TAP_{11}(T_{11}), TAP_{12}(T_{12})\}$	T_{11} or T_{12}
	$2s > \Delta_2$	$TAP_{11}(T_{11})$	T_{11}
$I_e \geqslant (h + 2c\ I_c)/p$	$2s \leqslant \Delta_2$	$TAP_{12}(T_{12})$	T_{12}
	$2s > \Delta_2$	$TAP_{12}(M)$	M

Proof. It immediately follows from Lemmas 2 and 4 and the fact that $TAP_{11}(M) = TAP_{12}(M)$.

Once the optimal replenishment cycle time T_1^* is obtained, the optimal order quantity per cycle $Q^* = DT_1^*$ follows.

Case 2. $I_e < I_c$

Case 2.1–1. $T > p\ M(1 + I_eM/2)/c$

Taking the first-order derivative of $TAP_{21}(T)$ in (8) with respect to T, we have:

$$dTAP_{21}(T)/dT = s/T^2 + pDI_e^2M^4(I_c - I_e)/(8T^2) + pDM^2(I_c - I_e)/(2T^2) + pDI_cI_eM^3/(2T^2) + D\left[c^2(I_e - I_c)/p - (h + 2cI_e - pI_e)\right]/2. \tag{17}$$

If $h + 2c\ I_e - p\ I_e \leqslant c^2(I_e - I_c)/p$, then $d\ TAP_{21}(T)/d\ T > 0$, which implies $TAP_{21}(T)$ is a strictly increasing function of T. In this situation, it will be profitable for the retailer to prolong the replenishment cycle time as long as possible. Hence, the optimal replenishment cycle time is $T \to \infty$. As mentioned previously, this is impossible in the real market and thus for the situation $T > p\ M(1 + I_eM/2)/c$ in Case 2 (i.e., $I_e < I_c$), the condition $h + 2c\ I_e - p\ I_e > c^2(I_e - I_c)/p$ (i.e., $I_c > I_e - p(h + 2c\ I_e - pI_e)/c^2$) must be satisfied. From now on, we assume the condition $I_c > max\ \{I_e, I_e - p(h + 2c\ I_e - pI_e)/c^2\}$ holds true in Case 2.1–1. Next, motivated by (17), we let function $G_{21}(T)$ denote the right hand side of (17), i.e.,

$$G_{21}(T) = s/T^2 + pDI_e^2M^4(I_c - I_e)/(8T^2) + pDM^2(I_c - I_e)/(2T^2) + pDI_cI_eM^3/(2T^2) + D\left[c^2(I_e - I_c)/p - (h + 2cI_e - pI_e)\right]/2, \tag{18}$$

for $T \in (p\ M(1 + I_eM/2)/c, \infty)$. Due to the derivative of $G_{21}(T)$ with respect to T is:

$$dG_{21}(T)/dT = -2s/T^3 - pDM^2(I_c - I_e)/T^3 - pI_e^2DM^4(I_c - I_e)/\left(4T^3\right) - pDI_cI_eM^3/T^3 < 0. \tag{19}$$

$G_{21}(T)$ is a strictly decreasing function in $T \in (p\ M(1 + I_e\ M/2)/c, \infty)$. Furthermore, we have

$$\lim_{T\to\infty} G_{21}(T) = D\left[c^2(I_e - I_c) - p(h + 2cI_e - pI_e)\right]/(2p) < 0,$$

and

$$\lim_{T\to(pM(1+I_eM/2)/c)^+} G_{21}(T) = \{2s - \left[p^2M^2D(1 + I_eM/2)^2(h + 2cI_e - pI_e)\right]\Big/c^2 + pI_e^2DM^3\}\Big/\left[2p^2M^2(1 + I_eM/2)^2/c^2\right].$$

For convenience, letting $\Delta_3 \equiv p^2M^2D(1 + I_eM/2)^2(h + 2cI_e - pI_e)/c^2 - pI_e^2DM^3$, we have the following result. □

Lemma 5. *For $I_c > max\ \{I_e, I_e - p(h + 2c\ I_e - pI_e)/c^2\}$:*

(a) *If $2s \geqslant \Delta_3$, then the value of $T \in (p\ M\ (1 + I_eM/2)/c, \infty)$ (say T_{21}) which satisfies $d\ TAP_{21}\ (T)/d\ T = 0$ exists uniquely and $TAP_{21}(T)$ has a maximum value at the point $T = T_{21}$.*
(b) *If $2s < \Delta_3$, then the value of $T \in (p\ M(1 + I_e\ M/2)/c, \infty)$ which maximizes $TAP_{21}(T)$ does not exist.*

Proof The proof is similar to that of Lemmas 1 and 2, hence we omit it here.

Case 2.1–2. $M \leqslant T \leqslant p\ M(1 + I_eM/2)/c$

Taking the first-order derivative of $TAP_{22}(T)$ in (9) with respect to T, we obtain:

$$dTAP_{22}(T)/dT = pI_e^2DM^3/(2T^2) + s/T^2 - D(h + 2cI_e - pI_e)/2. \tag{20}$$

If $h + 2c\,I_e - p\,I_e \leqslant 0$, then $d\,TAP_{22}(T)/d\,T > 0$, which implies $TAP_{22}(T)$ is a strictly increasing function of $T \in [M, p\,M(1 + I_eM/2)/c]$. In this situation, the optimal replenishment cycle time is the boundary point $T = p\,M(1 + I_eM/2)/c$. On the other hand, if $h + 2c\,I_e - p\,I_e > 0$, we let function $G_{22}(T)$ denote the right hand side of (20), i.e.,

$$G_{22}(T) = pI_e^2DM^3/(2T^2) + s/T^2 - D(h + 2cI_e - pI_e)/2, \tag{21}$$

for $T \in [M, p\,M(1 + I_eM/2)/c]$. Due to the derivative of $G_{22}\,(T)$ with respect to T is:

$$dG_{22}(T)/dT = -\left(2s + pI_e^2DM^3\right)/T^3 < 0. \tag{22}$$

$G_{22}(T)$ is a strictly decreasing function in $T \in [M, p\,M\,(1 + I_eM/2)/c]$. Furthermore, we have

$$G_{22}(pM(1 + I_eM/2)/c) = (2s - \Delta_3)/\left[2p^2M^2(1 + I_eM/2)^2/c^2\right],$$

and

$$G_{22}(M) = (2s - \Delta_4)/(2M^2),$$

where Δ_3 is defined as above and $\Delta_4 \equiv M^2D(h + 2cI_e - pI_e) - pI_e^2DM^3$. It is obvious that where $\Delta_4 < \Delta_3$, then we have the following results. □

Lemma 6. *For $I_c > I_e$ and $h + 2c\,I_e - p\,I_e > 0$:*

(a) *If $\Delta_4 \leqslant 2s \leqslant \Delta_3$, then the value of $T \in [M, p\,M(1 + I_eM/2)/c]$ (say T_{22}) which satisfies $d\,TAP_{22}(T)/d\,T = 0$ exists uniquely.*
(b) *If $2s < \Delta_4$ or $2s > \Delta_3$, then the value of $T \in [M, p\,M(1 + I_eM/2)/c]$ which satisfies $d\,TAP_{22}(T)/d\,T = 0$ does not exist.*

Proof. The proof is similar to that of Lemma 1, hence we omit it here. □

Lemma 7. *For $I_c > I_e$:*

(a) *If $h + 2c\,I_e - p\,I_e \leqslant 0$, then $TAP_{22}(T)$ has a maximum value at the boundary point $T = p\,M(1 + I_eM/2)/c$.*
(b) *If $h + 2c\,I_e - p\,I_e > 0$ and*
(i) *If $2s < \Delta_4$, then $TAP_{22}(T)$ has a maximum value at the boundary point $T = M$.*
(ii) *If $\Delta_4 \leqslant 2s \leqslant \Delta_3$, then $TAP_{22}\,(T)$ has a maximum value at the point $T = T_{22}$, where $T_{22} \in [M, p\,M(1 + I_eM/2)/c]$ and satisfies $d\,TAP_{22}(T)/d\,T = 0$.*
(iii) *If $2s > \Delta_3$, then $TAP_{22}(T)$ has a maximum value at the boundary point $T = p\,M(1 + I_eM/2)/c$.*

Proof.

(a) It is obvious from (20).
(b) The proof is similar to that of Lemma 2, hence we omit it here. □

Case 2.2. $T \leqslant M$

For this case, we know that the total profit per unit time $TAP_{23}\,(T)$ is the same as that of Case 1.2 (i.e., $TAP_{23}(T) = TAP_{12}\,(T)$), and hence the solution procedure is the same as that of Case 1.2. That is, we have the following result:

Lemma 8. *For $I_c > I_e$:*

(a) *If $2s \leqslant \Delta_2$, then $TAP_{23}(T)$ has a maximum value at the point $T = T_{23}$, where $T_{23} \in (0, M]$ and satisfies $d\,TAP_{23}(T)/d\,T = 0$.*
(b) *If $2s > \Delta_2$, then $TAP_{23}(T)$ has a maximum value at the boundary point $T = M$.*

Note that $\Delta_2 > \Delta_4$ and $\Delta_3 > \Delta_4$ for $I_c > I_e$. For simplicity, we let $W \equiv p\,M(1 + I_eM/2)/c$. Thus, from the above arguments, we can develop the following theorem to obtain the optimal replenishment cycle time T_2^* for Case 2.

Theorem 2. *For $I_e < I_c$,*

Situation	Conditions		$TAP_2(T_2^*)$	T_2^*
$h + 2c\,I_e - p\,I_e > 0$	$\Delta_2 \geqslant \Delta_3$	$2\,s < \Delta_4$	$TAP_{23}(T_{23})$	T_{23}
		$\Delta_4 \leqslant 2s \leqslant \Delta_3$	$Max\{TAP_{22}(T_{22}), TAP_{23}(T_{23})\}$	T_{22} or T_{23}
		$\Delta_3 < 2s \leqslant \Delta_2$	$Max\{TAP_{21}(T_{21})\ TAP_{22}(W), TAP_{23}(T_{23})\}$	T_{21} or W or T_{23}
		$2s > \Delta_2$	$Max\{TAP_{21}(T_{21}), TAP_{22}(W), TAP_{23}(M)\}$	T_{21} or W or M
	$\Delta_2 < \Delta_3$	$2s < \Delta_4$	$TAP_{23}(T_{23})$	T_{23}
		$\Delta_4 \leqslant 2s \leqslant \Delta_2$	$Max\{TAP_{22}(T_{22}), TAP_{23}(T_{23})\}$	T_{22} or T_{23}
		$\Delta_2 < 2s \leqslant \Delta_3$	$TAP_{22}(T_{22})$	T_{22}
		$2s > \Delta_3$	$Max\{TAP_{21}(T_{21}), TAP_{22}(W), TAP_{23}(M)\}$	T_{21} or W or M
$c^2(I_e - I_c)/p < h + 2c\,I_e - p\,I_e \leqslant 0$	$\Delta_2 \geqslant \Delta_3$	$2s \leqslant \Delta_3$	$Max\{TAP_{22}(W), TAP_{23}(T_{23})\}$	W or T_{23}
		$\Delta_3 < 2s \leqslant \Delta_2$	$Max\{TAP_{21}(T_{21}), TAP_{22}(W), TAP_{23}(T_{23})\}$	T_{21} or W or T_{23}
		$2s > \Delta_2$	$Max\{TAP_{21}(T_{21}), TAP_{22}(W), TAP_{23}(M)\}$	T_{21} or W or M
	$\Delta_2 < \Delta_3$	$2s \leqslant \Delta_2$	$Max\{TAP_{22}(W), TAP_{23}(T_{23})\}$	W or T_{23}
		$\Delta_2 < 2s \leqslant \Delta_3$	$Max\{TAP_{22}(W), TAP_{23}(M)\}$	W or M
		$2s > \Delta_3$	$Max\{TAP_{21}(T_{21}), TAP_{22}(W), TAP_{23}(M)\}$	T_{21} or W or M
$h + 2c\,I_e - p\,I_e \leqslant c^2(I_e - I_c)/p < 0$		$2s \leqslant \Delta_2$	$Max\{TAP_{22}(W), TAP_{23}(T_{23})\}$	W or T_{23}
		$2s > \Delta_2$	$Max\{TAP_{22}(W), TAP_{23}(M)\}$	W or M

Proof. It immediately follows from Lemmas 5, 7 and 8 and the fact that $TAP_{22}(M) = TAP_{23}(M)$.

Once the optimal replenishment cycle time T_2^* is obtained, the optimal order quantity per cycle $Q^* = DT_2^*$ follows. □

5. Numerical examples

In order to illustrate the solution procedure and investigate the effect of changes in some main parameter values on the optimal solution, some numerical examples are given below.

The supplier offers a permissible delay if the payment is made within 30 days (i.e., $M = 1/12 = 0.083333$ years). However, if the payment is not made in full by the end of 30 days, then 15% interest (i.e., $I_c = 0.15$) is charged per year on the outstanding amount. Suppose $D = 2000$ units/year, $h = \$3$/unit/year, $p = \$40$, $c = \$20$ and $s = \$200$ per order.

Example 1. If the retailer invests its revenue in the stock market and achieves an $I_e = 20\%$ return on investment per year, given that $I_e = 0.2 > I_c = 0.15$, from Theorem 1, $2s = 400 > \Delta_2 = 154.628$, the optimal total profit per unit time for Case 1 is $TAP_1(T^*) = TAP_{11}(T_{11}) = \$39605.5708$ and the optimal replenishment cycle time $T^* = T_{11} = 0.447214$ years. Thus, the optimal payoff time is $T^* = T_{11} = 0.447214$ years and the optimal order quantity is$Q^* = D\,T^* = D\,T_{11} \approx 894$ units. In this situation, since the optimal payoff time is equal to the optimal replenishment cycle time, and is larger than the credit period M, the retailer should not return the total purchase cost to the supplier until the end of the replenishment cycle.

Table 1
The optimal solution for different I_e and $p(I_c = 0.15)$.

Interest rate I_e (%)	Selling price p	Replenishment cycle T^*	Order quantity Q^*	Total profit per year TAP^*	Payoff time
5	25	$T_{21} = 0.201615$	403	8343.8500	$M + U/pD = 0.161118$
10		$T_{21} = 0.198531$	397	8398.0000	$M + U/pD = 0.158478$
15		$T_{11} = 0.195180$	390	8450.6078	$T_{11} = 0.195180$
20		$T_{11} = 0.223607$	447	8711.1436	$T_{11} = 0.223607$
5	30	$T_{21} = 0.213814$	428	18434.3000	$M + U/pD = 0.142369$
10		$T_{21} = 0.212474$	425	18519.7000	$M + U/pD = 0.141302$
15		$T_{11} = 0.210819$	422	18602.6314	$T_{11} = 0.210819$
20		$T_{11} = 0.258199$	516	18950.8047	$T_{11} = 0.258199$
5	35	$T_{21} = 0.226044$	452	28515.3000	$M + U/pD = 0.128995$
10		$T_{21} = 0.228435$	457	28643.0000	$M + U/pD = 0.130187$
15		$T_{11} = 0.230940$	462	28767.9472	$T_{11} = 0.230940$
20		$T_{11} = 0.316228$	632	29235.0869	$T_{11} = 0.316228$
5	40	$T_{21} = 0.238721$	477	38591.6000	$M + U/pD = 0.119187$
10		$T_{21} = 0.247407$	495	38771.6000	$M + U/pD = 0.123356$
15		$T_{11} = 0.258199$	516	38950.8047	$T_{11} = 0.258199$
20		$T_{11} = 0.447214$	894	39605.5708	$T_{11} = 0.447214$

Example 2. In this example, we assume $I_e = 5\%$ per year. Since $I_e = 0.05 < I_c = 0.15$, from Theorem 2, $h + 2c\,I_e - p\,I_e = 3 > 0$, $\Delta_2 = 69.5596 < \Delta_3 = 167.245$ and $2\ s = 400 > \Delta_3 = 167.245$, we get the optimal total profit per unit time for Case 2 $TAP_2(T^*) = Max\{TAP_{21}(T_{21}),\ TAP_{22}\ (W), TAP_{23}(M)\} = Max\{38591.6, 38468.5, 37516.7\} = \$38591.6$, and the optimal replenishment cycle time is $T^* = T_{21} = 0.238721$ years. Thus, the optimal payoff time is $M + U/pD = 0.119187$ years and the optimal order quantity is $Q^* = D\,T^* = D\,T_{21} \approx 477$ units. In this situation, since the optimal payoff time is less than the optimal replenishment cycle time, but is larger than the credit period, the retailer should pay off the entire amount owed to the supplier after the credit period, but before the end of replenishment cycle.

Example 3. Due to the uncertainties in any decision-making situation, sensitivity analysis will provide significant assistance in the decision-making process. In this case, sensitivity analysis is performed by changing I_e or p. Using the proposed theorems, the computational results are shown in Table 1.

Table 1 shows that for a fixed p, a higher interest rate value I_e results in a higher value for the optimal total profit per year TAP^*. That is, the change in I_e will lead to a positive change in TAP^*. For a fixed I_e, a higher selling price p results in higher values for the optimal replenishment cycle time T^*, the optimal economic order quantity Q^* and the optimal total profit per year TAP^*. That is, the change in p will cause positive changes in T^*, Q^* and TAP^*. A simple economic interpretation is that a higher value of I_e or p implies a higher value of benefit from the permissible delay. In addition, the change in p results in a positive change in the optimal payoff time if $I_c \leqslant I_e$, but a negative change in the optimal payoff time if $I_c > I_e$. This means that if the selling price increases, then the retailer should prolong the payoff time when the interest charged per dollar per year is less than or equal to the interest earned per dollar per year. Conversely, the retailer should shorten the payoff time when the interest charged per dollar per year is larger than the interest earned per dollar per year.

Example 4. Supposing $I_e = 5\%$ per year, the sensitivity analysis is conducted by changing s or p in this example. From Theorem 2, we obtain the computational results as shown in Table 2.

The computational results show that the change in s causes positive changes in the optimal replenishment cycle time T^* and the optimal economic order quantity Q^*, but a negative change in the optimal total profit per year TAP^*; i.e., for fixed p, as s increases, T^* and Q^* increase, but TAP^* decreases. Furthermore, as s is increasing, the optimal payoff time is non-decreasing. The economic interpretation is that the retailer should order a lower quantity to increase the number of orders and receive the benefit from the permissible delay if the ordering cost, s, is low. Conversely, the retailer needs to order a greater quantity to reduce the number of orders if the ordering cost, s, is more expensive. Additionally, for fixed ordering cost s, the change in selling price p, results in a positive change in the optimal total profit per year (TAP^*). Furthermore, the optimal payoff time is non-decreasing when the ordering cost is increasing. Therefore, the retailer may prolong or leave unchanged the payoff time if the order cost is increasing.

6. Conclusions

In this paper, we assumed that the retailer is allowed a specified credit period to pay back amounts owed without penalty. Under different financial situations, the retailer can decide to pay off the total amount owed to the supplier at the end of the permissible delay period, or at the end of the replenishment cycle; alternatively, the retailer requires additional time to pay

Table 2
The optimal solution for different s and p ($I_e = 0.05$ and $I_c = 0.15$).

Ordering cost s	Selling price p	Replenishment cycle T^*	Order quantity Q^*	Total profit per year TAP^*	Payoff time
25	25	$T_{23} = 0.076657$	153	9556.4442	$M = 0.083333$
65		$T_{21} = 0.124157$	248	9172.6500	$M + U/pD = 0.099152$
100		$T_{21} = 0.148179$	296	8915.6100	$M + U/pD = 0.118369$
200		$T_{21} = 0.201615$	403	8343.8500	$M + U/pD = 0.161118$
25	30	$T_{23} = 0.074495$	149	19579.2279	$M = 0.083333$
65		$T_{22} = 0.136323$	273	19212.9292	$M = 0.083333$
100		$T_{22} = 0.169068$	338	18983.7141	$M = 0.083333$
200		$T_{21} = 0.213814$	428	18434.3000	$M + U/pD = 0.142369$
25	35	$T_{23} = 0.072506$	145	29602.5316	$M = 0.083333$
65		$T_{22} = 0.141476$	283	29247.6768	$M = 0.083333$
100		$T_{22} = 0.175456$	351	29026.8096	$M = 0.083333$
200		$T_{21} = 0.226044$	452	28515.3000	$M + U/pD = 0.128995$
25	40	$T_{23} = 0.070670$	141	39626.3146	$M = 0.083333$
65		$T_{22} = 0.147262$	295	39283.7913	$M = 0.083333$
100		$T_{22} = 0.182627$	365	39071.5984	$M = 0.083333$
200		$T_{21} = 0.238721$	477	38591.6000	$M + U/pD = 0.119187$

off the account after the end of the credit period. In this study, we have developed a retailer's inventory model and proposed two theorems that assist the retailer in quickly and precisely determining the optimal replenishment cycle time, order quantity and payoff time for maximizing total profit per unit time. Finally, we provided some numerical examples to illustrate the solution procedure and investigated the impact of sensitivity analysis on the optimal solution, using various parameters. In addition, we proposed some economic interpretations.

The proposed model can be extended in several ways. For instance, potential future research includes investigating the effect of extending the constant demand rate to stochastic fluctuating demand patterns, and taking account of time values. It should also be possible to generalize the model to allow for shortages, quantity discounts, deteriorating items and other items. In addition, it may address other policies for trade credit offered by suppliers, such as the cash discount policy. In this situation, it is suitable to explore the effect of a cash discount on the inventory model.

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Appendix A. Proof of lemma 1

A.1. Proof of part (a)

If $2s \geqslant \Delta_1$, then $G_{11}(M) = (2s - \Delta_1)/\ (2M^2) \geqslant 0$. Due to $G_{11}(T)$ is a strictly decreasing function in $T \in [M, \infty)$ and $\lim_{T\to\infty} G_{11}(T) < 0$, from the intermediate value theorem, we can find a unique value $T_{11} \in [M, \infty)$ such that $G_{11}(T_{11}) = 0$. That is, $T_{11} \in [M, \infty)$ is the unique value which satisfies $d\ TAP_{11}(T)/d\ T = 0$.

A.2. Proof of part (b)

If $2s < \Delta_1$, then $G_{11}(M) = (2s - \Delta_1)/(2M^2) < 0$, which implies $G_{11}(T) < 0$ for all $T \in [M, \infty)$. Thus, we cannot find a value $T \in [M, \infty)$ such that $G_{11}(T) = 0$. That is, the value of $T \in [M, \infty)$ which satisfies $d\ TAP_{11}(T)/d\ T = 0$ does not exist.

The proof is completed. □

Appendix B. Proof of lemma 2

B.1. Proof of part (a)

If $2s \geqslant \Delta_1$, then from the proof of Lemma 1(a), we can find a unique value $T_{11} \in [M, \infty)$ which satisfies $d\ TAP_{11}(T)/d\ T = 0$. Furthermore, we have:

$$d^2 TAP_{11}(T)/dT^2|_{T=T_{11}} = -2s/T_{11}^3 < 0.$$

Hence, T_{11} is the unique value which maximizes the value $TAP_{11}(T)$.

B.2. Proof of part (b)

If $2s < \Delta_1$, then from the proof of Lemma 1(b), we know that $G_{11}(T) < 0$ for all $T \in [M, \infty)$. Thus, $d\ TAP_{11}(T)/d\ T = G_{11}(M) < 0$, which implies $TAP_{11}(T)$ is a strictly decreasing function in $T \in [M, \infty)$, Therefore, $TAP_{11}(T)$ has a maximum value at the boundary point $T = M$.

The proof is completed. □

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