A fuzzy real option approach for investment project valuation

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ARTICLE INFO

ABSTRACT

The main purpose of this paper is to propose a fuzzy approach for investment project valuation in uncertain environments from the aspect of real options. The traditional approaches to project valuation are based on discounted cash flows (DCF) analysis which provides measures like net present value (NPV) and internal rate of return (IRR). However, DCF-based approaches exhibit two major pitfalls. One is that DCF parameters such as cash flows cannot be estimated precisely in the uncertain decision making environments. The other one is that the values of managerial flexibilities in investment projects cannot be exactly revealed through DCF analysis. Both of them would entail improper results on strategic investment projects valuation. Therefore, this paper proposes a fuzzy binomial approach that can be used in project valuation under uncertainty. The proposed approach also reveals the value of flexibilities embedded in the project. Furthermore, this paper provides a method to compute the mean value of a project's fuzzy expanded NPV that represents the entire value of project. Finally, we use the approach to practically evaluate a project.

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1. Introduction

DCF-based approaches to project valuation implicitly assume that a project will be undertaken immediately and operated continuously until the end of its expected useful life, even though the future is uncertain. By treating projects as independent investment opportunities, decisions are made to accept projects with positive computed NPVs. Traditional NPV techniques only focus on current predictable cash flows and ignore future managerial flexibilities, therefore, may undervalue the projects and mislead the decision makers. Furthermore, for high-risk investment projects, the traditional NPV method may adopt higher discount rates to discount project cash flows for trade-off or compensation. However, higher discount rates may result in the underestimation of project value and the rejection of a potential project. For instance, investments such as new drug development or crude oil exploitation may carry high risk, but may also bring higher returns.

Since DCF-based approaches ignore the upside potentials of added value that could be brought to projects through managerial flexibilities and innovations, they usually underestimate the upside value of projects (Bowman & Moskowitz, 2001; Dixit & Pindyck, 1995; Luehrman, 1998; Trigeorgis, 1993; Yeo & Qiu, 2003). In particular, as market conditions change in the future, investment project may include flexibilities by which project value can be raised. Such flexibilities are called real options or strategic options. The real options approach to projects valuation seeks to correct the

deficiencies of the traditional valuation methods through recognizing that managerial flexibilities can bring significant values to projects. According to real options theory, an investment is of higher value in a more uncertain or volatile market because of investment decision flexibilities.

Real options approach, as a strategic decision making tool, borrows ideas from financial options because it explicitly accounts for future flexibility value. Real options analysis is based on the assumption that there is an underlying source of uncertainty, such as the price of a commodity or the outcome of a research project. Over time, the outcome of the underlying uncertainty is revealed, and managers can adjust their strategy accordingly.

The objectives of this paper are to develop a fuzzy binomial approach to evaluate a project embedded with real options, to propose a method suitable for computing the mean value of fuzzy NPV, and to explore the value of multiple options existing in projects. The paper is organized as follows. Section 2 provides a survey of real options analysis. We especially focus on pricing, applications and recent developments of real options analysis. Section 3 presents a fuzzy binomial approach to evaluate a project under vague situations. This section also proposes a method to compute the mean value of fuzzy NPV. Section 4 illustrates a project valuation based on our approach. In the example, the values of the real options are also assessed. Section 5 discusses the results and findings in the example. Finally, conclusions are drawn in Section 6.

2. Related works

Based on real options theory, Chen, Zhang, and Lai (2009) presented an approach to evaluate IT investments subject to multiple

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risks. By modeling public risks and private risks into a unified framework, they utilized the binomial model to evaluate an ERP development project. Wu, Ong, and Hsu (2008) argued that ERP may be best represented by a non-analytical, compound option model. However, most IT studies that employ the options theory only consider a single option, use an analytical model such as the Black–Scholes model (1973), and cannot deal with multi-option situations. Therefore, Wu et al. employed the binomial tree approach to implement an active ERP management which involves uncertainties over time. Hahn and Dyer (2008) proposed a recombining binomial lattice approach for modeling real options and valuing managerial flexibility to address a common issue in many practical applications—underlying stochastic processes that are mean-reverting. The models were tested by implementing the lattice in binomial decision tree format and applying to a real application by solving for the value of an oil and gas switching option. Reyck, Degraeve, and Vandenborre (2008) proposed an alternative approach for valuing real options based on the certainty-equivalent version of the NPV formula, which eliminates the need to identify market-priced twin securities. Moreover, Bowe and Lee (2004) also utilized the log-transformed binomial lattice approach to evaluate the Taiwan High-Speed Rail (THSR) project.

In DCF, parameters such as cash flows and discount rates are difficult to estimate (Carlsson & Fuller, 2003). In particular, innovative investment projects may count on the subjective judgments of decision makers due to lack of past data for reference. These parameters are essentially estimated under uncertainty. With respect to uncertainty, probability is one way to depict whereas possibility is another. Fuzzy set theory provides a basis for the theory of possibility (Zadeh, 1999). Fuzzy logic may be viewed as an attempt at formalization of two remarkable human capabilities. One is the capability to converse, reason and make rational decisions in an environment of imprecision, uncertainty and incompleteness of information and the other one is to perform a wide variety of physical and mental tasks without any measurements and computations (Zadeh, 2008). The outstanding feature of fuzzy logic is that in fuzzy logic everything is—or is allowed to be—a matter of degree. In the generalized theory of uncertainty, uncertainty is linked to information through the concept of granular structure—a concept that plays a key role in human interaction with the real world (Zadeh, 2005). Thus, these parameters can be characterized with possibilistic distributions instead of probabilistic distributions, and can be estimated by making use of fuzzy numbers.

By modeling the stock price in each state as a fuzzy number, Muzzioli and Torricelli (2004) obtained a possibility distribution of the risk-neutral probability in a multi-period binomial model, then computed the option price with a weighted expected value interval, and thus determined a "most likely" option value within the interval. Muzzioli and Reynaerts (2008) also addressed that the key input of the multi-period binomial model is the volatility of the underlying asset, but it is an unobservable parameter. The volatility parameter can be estimated either from historical data (historical volatility) or implied from the price of European options (implied volatility). Providing a precise volatility estimate is difficult; therefore, they used a possibility distribution to model volatility uncertainty and to price an American option in a multi-period binomial model. Carlsson and Fuller (2003) mentioned that the imprecision in judging or estimating future cash flows is not stochastic in nature, and that the use of the probability theory leads to a misleading level of precision. Their study introduced a real option rule in a fuzzy setting in which the present values of expected cash flows and expected costs are estimated by trapezoidal fuzzy numbers. They determined the optimal exercise time with the help of possibilistic mean value and variance of fuzzy numbers. The proposed model that incorporates subjective judgments and statistical uncertainties may give investors a better understanding of the problem when making investment decisions. Carlsson, Fuller, Heikkila, and Majlender (2007) also developed a methodology for valuing options on R&D projects, in which future cash flows were estimated by trapezoidal fuzzy numbers. In particular, they presented a fuzzy mixed integer programming model for the R&D optimal portfolio selection problem.

In addition to the binomial model, the Black–Scholes model is another way to evaluate the option's value. Owing to fluctuations in the financial market from time to time, some input parameters in the Black–Scholes formula cannot be expected to always be precise. Wu (2004) applied the fuzzy set theory to the Black–Scholes formula. Under the assumptions of fuzzy interest rate, fuzzy volatility and fuzzy stock price, the European option price turns into a fuzzy number. This allows the financial analyst to pick a European option price with an acceptable degree of belief. Lee, Tzeng, and Wang (2005) adopted the fuzzy decision theory and Bayes' rule as a basis for measuring fuzziness in the practice of option analysis. Their study also employed "Fuzzy Decision Space" that consisted of four dimensions—fuzzy state, fuzzy sample information, fuzzy action and evaluation function—to describe the decisions of investors. These dimensions were used to derive a fuzzy Black–Scholes option pricing model under fuzzy environments. Thiagarajah, Appadoo, and Thavaneswaran (2007) also addressed that most stochastic models involve uncertainty arising mainly from lack of knowledge or from inherent vagueness. Traditionally, these stochastic models are solved using probability theory and fuzzy set theory. In their study, using adaptive fuzzy numbers, they modeled the uncertainty of characteristics such as interest rate, volatility, and stock price. They also replaced fuzzy interest rate, fuzzy stock price and fuzzy volatility with possibilistic mean values in the fuzzy Black–Scholes formula.

Making a R&D portfolio decision is difficult, because the long lead times of R&D and the market and technology dynamics lead to unavailable or unreliable collected data for portfolio management. Wang and Hwang (2007) developed a fuzzy R&D portfolio selection model to hedge against the R&D uncertainty. Since traditional project valuation methods often underestimated the risky project, a fuzzy compound-options model was used to evaluate the value of each R&D project. The R&D portfolio selection problem was formulated as a fuzzy zero-one integer programming model that could handle both uncertain and flexible parameters to determine the optimal project portfolio.

From the viewpoint of fuzzy random variables, Yoshida, Yasuda, Nakagami, and Kurano (2006) discussed, under uncertainty in financial engineering, an American put option model that was based on the Black–Scholes stochastic model. In their study, probability is applied as the uncertainty such that something occurs or not with probability, and fuzziness is applied as the uncertainty such that the exact values cannot be specified because of a lack of knowledge regarding the present stock market. By introducing fuzzy logic to the log-normal stochastic processes for the financial market, they presented a model with uncertainty of both randomness and fuzziness in output.

The Garman–Kohlhagen (G–K) model is a closed-form solution of the European currency options pricing model based on the Black–Scholes model, but the input variables of the G–K model are usually regarded as real numbers. However, it is more suitable and realistic to price currency options with fuzzy numbers because these variables are only available with imprecise data or data related in a vague way. Therefore, Liu (2009) started from the fuzzy environments of currency options markets, introduced fuzzy techniques, and created a fuzzy currency options pricing model. By turning exchange rate, interest rates and volatility into triangular fuzzy numbers, the currency option price turns into a fuzzy number. This allows financial investors to pick any currency option price with an acceptable degree of belief.

From a modeling perspective, real options valuation methods have tended to follow financial option pricing techniques. The Black–Scholes models are used to evaluate simple real option scenarios such as delay decisions, research and development, licenses, patents, growth opportunities, and abandonment scenarios (Miller & Bertus, 2005). Despite its theoretical appeal, however, the practical use of real option valuation techniques in industry has been limited by the complexity of these techniques, the resulting lack of intuition associated with the solution process, or the restrictive assumptions required for obtaining analytical solutions. On the other hand, Cox, Ross, and Rubinstein (1979) developed a binomial discrete-time option valuation technique that has gained similar popularity to evaluate real options due to its intuitive nature, ease of implementation, and wide applicability to variety of option attributes. In addition, analytical models such as the Black–Scholes formula focus on a single option and cannot deal with multi-option situations. Therefore, the binomial model is adopted as a basis to develop the fuzzy valuation approach in this study.

3. The valuation approach

3.1. Expanded net present value

The NPV approach assumes a fixed scenario, in which a company starts and completes a project that then generates cash flows during some expected lifetime without any contingencies. The approach anticipates no contingency for delaying or abandoning the project if market conditions turn sour. However, the assumption about NPV does not fit the actual situation. In reality, if the market is unfavorable, the project could be postponed to undertake until market conditions turn better; or, the project may be abandoned during the operation to reduce losses; or, the project may be expanded or extended as market conditions turn around. The flexibilities of these investment decisions indicate that decision makers are capable of restricting loss risks and retaining the potential to raise profits infinitely. As a result, the valuation should include these flexibilities which are embedded as real options in investment projects.

In considering option value, the traditional NPV can be expanded as: expanded NPV = static NPV + value of option from active management (Trigeorgis, 1993). The expanded NPV is also called strategic NPV. Static NPV is the NPV obtained using the traditional discount method; it is also called passive NPV.

3.2. The fuzzy binomial valuation approach

In this study, a fuzzy binomial valuation approach is proposed to evaluate investment projects that are embedded with real options. The value of the project is represented by its expanded NPV, which can be evaluated by the valuation approach. However, the parameters are estimated by fuzzy numbers when the expanded NPV is estimated; thus, the expanded NPV is called fuzzy expanded NPV (*FENPV*) in this study.

The proposed valuation approach is based on Cox et al. (1979). Assuming there is a call option with the present value of underlying asset S_0 and exercising price K, the value of the underlying asset has P_u probability to rise to uS_0 or P_d probability to drop to dS_0 in the next period. The factors u and d represent the jumping up and down factors of the underlying asset's present value, respectively. A single period binomial tree of the underlying asset value is shown in Fig. 1.

The option will be exercised at period $t = 1$ if the underlying value is higher than K, and forgone if the underlying value is lower than K. The dynamics of the option value is shown in Fig. 2.

If the option is sold at price C_0, then the pricing approach is generally based on the assumption of replicating portfolio and can thus be determined by the following expression
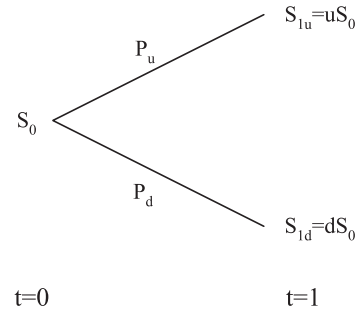

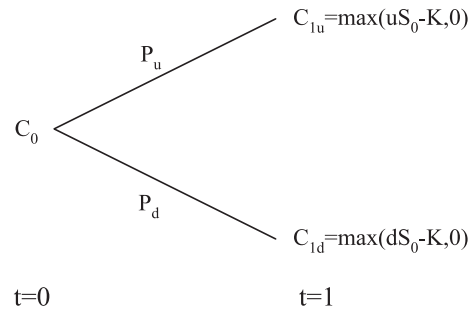**Fig. 1.** The single period binomial tree of underlying asset value.

Fig. 2. The dynamics of option value.

$$C_0 = \frac{1}{(1+r)}[P_u C_{1u} + P_d C_{1d}] \qquad (1)$$

in which r is risk-free interest rate, and P_u and P_d are risk-neutral probabilities, which are determined by the following formulas.

$$P_u = \frac{(1+r)-d}{(u-d)} \qquad (2)$$

$$P_d = \frac{u-(1+r)}{(u-d)} = 1 - P_u \qquad (3)$$

Therefore, the price or present value of the call option is the discounted result of the option values C_{1u} and C_{1d} with risk-neutral probabilities. Also, under the assumption of no arbitrage opportunities, the condition $0 < d < 1 < (1+r) < u$ must be satisfied. Furthermore, the expected return of the underlying asset should be zero based on the no-arbitrage assumption:

$$P_u\left(\frac{uS_0}{1+r} - S_0\right) + P_d\left(\frac{dS_0}{1+r} - S_0\right) = 0 \qquad (4)$$

That is

$$\frac{uP_u}{1+r} + \frac{dP_d}{1+r} = 1 \qquad (5)$$

Thus, we have the following risk-neutral probabilities equations:

$$\begin{cases} P_u + P_d = 1 \\ \frac{uP_u}{1+r} + \frac{dP_d}{1+r} = 1 \end{cases} \qquad (6)$$

From (1)–(3), we know that the main factors affecting the call option value are jumping factors u and d; it is not easy, however, to estimate their values in a precise manner due to the uncertainty of the underlying volatility.

The cash flow models applied to many financial decision making problems often involve some degree of uncertainty. In the case of deficient data, most decision makers tend to rely on experts'

knowledge of financial information when carrying out their financial modeling activities. The nature of this knowledge often tends to be vague rather than random. Fuzzy theory, which is aimed at rationalizing the uncertainty caused by vagueness or imprecision, has provided a promising basis for manipulating such vague knowledge (Sheen, 2005). In the relevant application of financial decision making, there is an example of employing triangular fuzzy numbers to examine the profitability indexes (Chiu & Park, 1994).

In strategic or innovative investment projects, information often tends to be vague rather than random. Therefore, this study considers possibilistic uncertainty rather than probabilistic uncertainty and employs fuzzy numbers instead of statistics to estimate the parameters. For lightening computation efforts, we utilize the triangular fuzzy numbers $\tilde{u} = [u_1, u_2, u_3]$ and $\tilde{d} = [d_1, d_2, d_3]$ to represent the jumping factors of the underlying asset. Therefore, the risk-neutral probabilities equations can be rewritten as

$$\begin{cases} \tilde{P}_u \oplus \tilde{P}_d = \tilde{1} \\ \frac{\tilde{u} \otimes \tilde{P}_u}{1+r} \oplus \frac{\tilde{d} \otimes \tilde{P}_d}{1+r} = \tilde{1} \end{cases} \tag{7}$$

where $\widetilde{P_u} = [P_{u_1}, P_{u_2}, P_{u_3}]$ and $\tilde{P}_d = [P_{d_1}, P_{d_2}, P_{d_3}]$. Thus, we have

$$\begin{cases} [P_{u_1}, P_{u_2}, P_{u_3}] \oplus [P_{d_1}, P_{d_2}, P_{d_3}] = [1, 1, 1] \\ \frac{[u_1, u_2, u_3] \otimes [P_{u_1}, P_{u_2}, P_{u_3}]u}{1+r} \oplus \frac{[d_1, d_2, d_3] \otimes [P_{d_1}, P_{d_2}, P_{d_3}]}{1+r} = [1, 1, 1] \end{cases} \tag{8}$$

which are

$$\begin{cases} P_{u_i} + P_{d_i} = 1 \\ \frac{u_i \times P_{u_i}}{1+r} + \frac{d_i \times P_{d_i}}{1+r} = 1 \end{cases} \quad \text{for } i = 1, 2, 3 \tag{9}$$

It can be solved by considering the following relationship

$$P_{u_i} = \frac{(1+r) - d_i}{u_i - d_i} \tag{10}$$

$$P_{d_i} = \frac{u_i - (1+r)}{u_i - d_i} \tag{11}$$

Since the risk-free interest rate r and the exercising price K are usually known, they are crisp values, whereas, the option values C_{1u} and C_{1d} become fuzzy numbers as a result of the jumping factors being fuzzified. That is, $\tilde{C}_{1u} = \max(\tilde{u}S_0 - K, 0)$ and $\tilde{C}_{1d} = \max(\tilde{d}S_0 - K, 0)$. The ranking of two triangular fuzzy numbers $\tilde{A} = [a_1, a_2, a_3]$ and $\tilde{B} = [b_1, b_2, b_3]$ can be derived from $\max(\tilde{A}, \tilde{B}) = [\max(a_1, b_1), \max(a_2, b_2), \max(a_3, b_3)]$. Thus, the pricing formula for the fuzzy call option is

$$\tilde{C}_0 = \frac{1}{1+r} \left[\tilde{P}_d \otimes \tilde{C}_{1d} \oplus \tilde{P}_u \otimes \tilde{C}_{1u} \right] \tag{12}$$

In practical application, the present value of the underlying asset is determined by the NPV of the investment project; the exercising price is the additional outlay to exercise the embedded option.

Managerial flexibility to adopt future actions introduces an asymmetry or skewness in the probability distribution of the project NPV (Yeo & Qiu, 2003). In the absence of such managerial flexibility, the probability distribution of project NPV would be considerably symmetric. However, in the existence of managerial flexibility such as the exercising of options, enhanced upside potential is introduced and the resulting actual distribution is skewed to the right.

In essence, identical results are obtained in the case of possibilistic distribution which is adopted by this study to characterize the NPV of an investment project. In other words, the characteristic of right-skewed distribution also appears in the FENPV of an investment project when the parameters (such as cash flows) are characterized with fuzzy numbers. Although many studies have proposed a variety of methods to compute the mean value (Carlsson & Fuller,
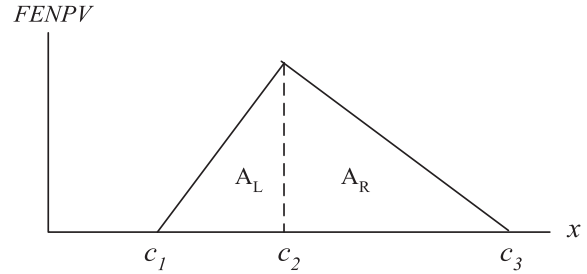

Fig. 3. A FENPV with right-skewed distribution.

2001; Fuller & Majlender, 2003) and median value (Bodjanova, 2005) of fuzzy numbers, these works did not consider the right-skewed characteristic present in the FENPV. Therefore, this study proposes a new method to compute the mean value of the FENPV based on its right-skewed characteristic. This mean value can be used to represent the FENPV with a crisp value. Moreover, different FENPVs can be compared according to their mean values.

Let $\tilde{C} = [c_1(\alpha), c_3(\alpha)]$ be a fuzzy number and $\lambda \in [0, 1]$. Then, the mean value of $\tilde{C}$ is defined as

$$E(\tilde{C}) = \int_0^1 [(1 - \lambda)c_1(\alpha) + \lambda c_3(\alpha)] d\alpha \tag{13}$$

The weighted index λ is called the pessimistic-optimistic index in Yoshida et al. (2006), but the index is determined by a subjective decision in Yoshida et al. (2006). However, this study considers that the index can be determined objectively. Fig. 3 illustrates a case in which the FENPV is represented by a right-skewed triangular fuzzy number. The right-skewed characteristic of FENPV—meaning that the more skew to the right, the more optimistic the payoff of the project—provides a clue to determining λ with $\lambda = \frac{A_R}{A_L + A_R}$, where A_L and A_R are the left-part area and right-part area of the FENPV, respectively. Thus, when λ is determined objectively and substituted into Eq. (13), the mean value of the FENPV can be computed as follows

$$E(FENPV) = \frac{(1 - \lambda)c_1 + c_2 + \lambda c_3}{2} \tag{14}$$

4. An illustrative example

An enterprise must continually develop new products and introduce them into the market to create profit. Therefore, evaluating projects of new product development is a crucial task that should be an ongoing effort of an enterprise. In this case, a local biotechnology company in Taiwan proposes a new product development project that needs evaluation. The project must go through two stages before the new product can be introduced into the market. Stage one of the project will require two years and an investment of $I_1 = 40$ (million NT\$) toward product development. When this is done, the project will proceed to the second stage, which will require one year and an outlay of $I_2 = 80$ (million NT\$) to acquire the equipment and raw material for mass production. Experts estimate that the project will create cash inflows with a present value of 100 (million NT\$). If we use the biannual risk-free interest rate $r = 3\%$ as the discounting rate and frame six months as one period, the NPV of the project can be calculated as follows:

$$NPV = 100 - 40 - \frac{80}{(1 + 0.03)^4} = -11.08 \text{(million)}$$

This negative NPV suggests that the project should be rejected.

The above results are obtained under the assumption that cash inflows can be generated with certainty. However, this assumption is unrealistic. In fact, the cash inflows will vary with fluctuations in market conditions, such as the market demand of the new product.
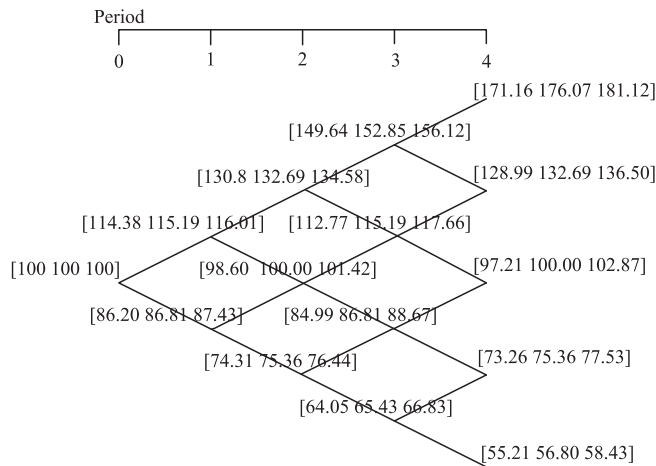
Fig. 4. Binomial tree of the project's cash inflows.

Fig. 5. The decision tree of the project with the option to defer.

Fig. 6. The decision tree of the project with the option to abandon.

According to experts' estimation, the new product may have a rate of $20\% \times (1 \pm 5\%)$ fluctuation per year with regard to its market demand. Since the volatility is estimated under uncertainty, a triangular fuzzy number is employed to characterize the possibilistic uncertainty of the volatility. Based on the estimation, the triangular fuzzy number $\tilde{\rho} = [(1 - 0.05) \times 0.2, 0.2, (1 + 0.05) \times 0.2] = [0.19, 0.25em0.2, 0.21]$ is used to express the fuzzy volatility. From the fuzzy volatility $\tilde{\rho}$, the fuzzy jumping factors $\tilde{u}$ and $\tilde{d}$ can be determined as $\tilde{u} = \exp(\tilde{\rho} \otimes \sqrt{\tau})$ and $\tilde{d} = 1/\tilde{u}$, where τ is the chosen time interval expressed in the same unit as $\tilde{\rho}$ and exp denotes the exponential function. In this case, the value of τ is 0.5 because there are six months (0.5 y) in each period. As a result, we have $\tilde{u} = [1.1438, 1.1519, 1.1601]$ and $\tilde{d} = [0.8620, 0.8681, 0.8743]$. The fuzzy risk-neutral probabilities are $\tilde{P}_u = [0.5448, 0.5704, 0.5962]$ and $\tilde{P}_d = [0.4038, 0.4296, 0.4552]$, respectively. With the above conditions, a binomial tree of the project's cash inflows can be established, as shown in Fig. 4. (For simplicity, the numbers in the binomial tree are represented to two digits after the decimal point.)

Nevertheless, the project may have some decision flexibilities when the project is undertaken. For instance, when the market conditions are unfavorable, the project can be deferred one period to undertake or the project can abandon its second stage investment to prevent losses from mass production. Therefore, the project with deferring option and abandoning option will be evaluated in the following subsections, respectively. Moreover, the project with a sequential multiple options which is combined with deferring option and abandoning option will also be evaluated.

4.1. Option to defer

First of all, considering the situation that decision maker defers one period to undertake the first stage investment and commits to undertake the second stage investment. In this case, the project's total outlay that discounted to period one is calculated as follows:

$$I\_defer = 40 \times (1 + 0.03) + \frac{80}{(1 + 0.03)^3} = 114.41$$

The decision tree is shown in Fig. 5, where $V = 100$, $\tilde{I}\_defer = [114.41, 114.41, 114.41]$ and $\tilde{0} = [0, 0, 0]$. The root value in Fig. 5 is the FENPV of the project with deferring option and can be calculated as follows:

$$FENPV = \left[\tilde{P}_u \otimes [\tilde{C}_{1u} \oplus \tilde{P}_d \otimes \tilde{C}_{1d}\right]/(1 + 0.03) = [0, 0.43, 0.92]$$

The mean value of the FENPV is 0.46 (million), and the value of the option to defer the first stage investment is 0.46 − (−11.08) = 11.54 (million NT$).
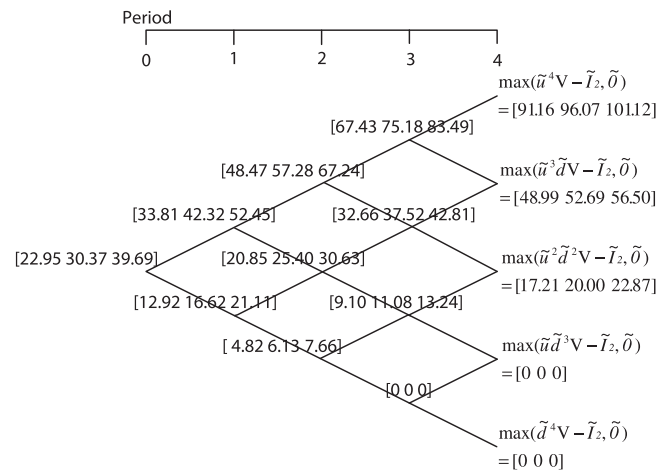
4.2. Option to abandon

Furthermore, when the decision maker only possesses the option to abandon the second stage investment, this implies that the decision maker has already completed the first stage investment without deferring. The decision tree is shown in Fig. 6, in which $\tilde{I}_2 = [80, 80, 80]$. From the root value in Fig. 6, we can conclude that the FENPV of the project with option to abandon the second stage investment is FENPV = [22.95, 30.37, 39.69] − $\tilde{I}_1$ = [−17.05, −9.64, −0.31], where $\tilde{I}_1 = [40, 40, 40]$. In this case, the mean value of the FENPV is −8.68 (million), and thus, the value of the option to abandon the second stage investment is −8.68 − (−11.08) = 2.4 (million).

4.3. Sequential multiple options

Finally, when the project involves these two options but with different expiration days, these two options form a sequential multiple options. In the sequential multiple options, decision makers have the options not only to abandon the second stage investment but also to defer the first stage investment. Therefore, the decision in period one is $\max(\tilde{C}_{1u} - \tilde{I}_1, \tilde{0})$ and $\max(\tilde{C}_{1d} - \tilde{I}_1, \tilde{0})$, where $\tilde{C}_{1u}$ and $\tilde{C}_{1d}$ are the project values in the up and down cases, respectively, during period one. Based on the values at period two, we can find that $\tilde{C}_{1u} = [33.81, 42.32, 52.45]$ and $\tilde{C}_{1d} = [12.92, 16.62, 21.11]$. The FENPV of the project with sequential multiple options is FENPV = [0, 1.28, 7.21], its mean value is 3.60 (million), and the value of the sequential multiple options is 3.60 − (−11.08) = 14.68 (million) (See Fig. 7).

5. Discussion

In Table 1, we summarize the evaluation results of the new product development project that embedded with three different real options, respectively.

From the evaluation results, we can observe that if the project does not have any decision flexibility, the project's NPV is −11.08
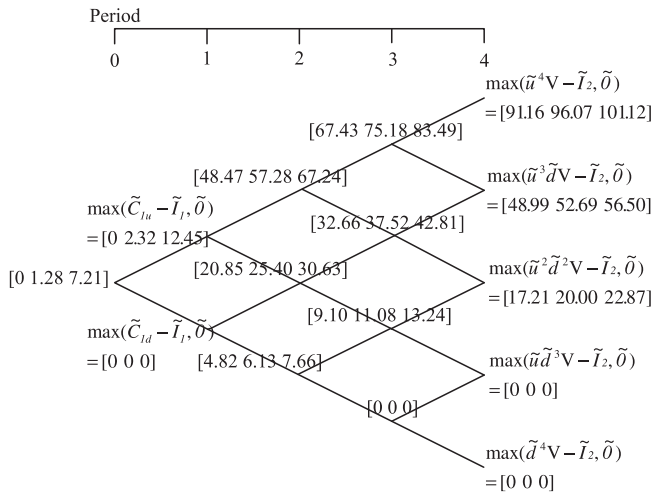
Fig. 7. The decision tree of the project with sequential multiple options.

Table 1
A summary of the results (in million NT$).

Type of option	FENPV of the project	Mean value of the FENPV	Option value
Option to defer	[0, 0.43, 0.92]	0.46	11.54
Option to abandon	[−17.05, −9.635, −0.31]	−8.68	2.4
Multiple options	[0, 1.28, 7.21]	3.60	14.68

(million NT$) and the project should therefore be rejected. However, when the project is embedded with some decision flexibilities, the decisions will be different. Confronting uncertain market conditions, the decision flexibilities, such as deferring investment in the first stage or abandoning investment in the second stage, have specific values. In this paper, we have verified the values of these flexibilities from the aspect of fuzzy real options.

When the project involves the option to defer investment in the first stage, the mean value of the project's *FENPV* is 0.46 (million NT$). The overall value of the project is positive, thus, the project become acceptable. Moreover, the value of the option to defer is 11.54 (million NT$). The option value stems from the flexibility that decision maker can defer investment in the first stage to avoid the downward losses at project initiation.

Moreover, when the project includes the option to abandon the second stage investment, the mean value of the project's *FENPV* is −8.68 (million NT$). Although this mean value is negative, it is still greater than the original NPV = −11.08 (million NT$). This reveals that the second stage option can still prevent losses when the market conditions are downward and can retain the upside potential of profit when the market conditions are upward. Therefore, this option to abandon the second stage investment has a value of 2.4 (million NT$)-lower than the value of option to defer. The reason is that the first stage investment has been completed without deferring, whatever the market conditions are. Thus, even though the market conditions are downward at the initiation of the project, the decision maker will only be able to prevent losses at the second stage. Due to the smaller extent of hedging, the second stage option has a lower option value than the first stage option.

Lastly, when both options form a sequential multiple options, the mean value of the project's *FENPV* is 3.60 (million NT$), which represents the total value of the project. Since this value is positive, the project is acceptable. The value of the sequential multiple options is 14.68 (million NT$). This option value is higher than the

value of a single option. This result shows that the multiple options provide greater value than a single option because multiple options provide more flexibility. However, the value of multiple options does not equate directly to the addition of the values of both options. The value cannot be raised linearly because of the nonlinear operations in the valuation model and the trade-off between both options in the hedging process.

6. Conclusions

Since the options-related commodities could not be priced objectively, they were not widely accepted in the market. Until 1973, Black–Scholes proposed a valuation model that allowed investors to price the options; investors no longer needed to rely on subjective judgments. Since then, transactions and innovations in options have continually developed. Options have become the most popular financial commodities and have satisfied the market's needs for hedge and arbitrage.

There is a similar problem in traditional capital budgeting. Because estimating the value of flexibilities is difficult, the traditional capital budgeting methods cannot discover the value of managerial flexibilities or other alternatives within investment projects. As a result, a potential investment project will be undervalued and rejected. The rejection of a potential project may incur substantial losses. Nevertheless, once flexibilities are regarded as real options, they can be evaluated using option valuation models. The values of these flexibilities become measurable and the entire value of an investment project can be revealed.

The binomial valuation technique has gained popularity in the valuation of real options due to its intuitive nature, ease of implementation, and capability of dealing with multiple options. Furthermore, in an uncertain economic decision making environment, information such as cash flows, interest rate, cost of capital, and so forth possess some vagueness but not randomness (Kahraman, Ruan, & Tolga, 2002). Consequently, this study has proposed the fuzzy binomial valuation approach to evaluate investment projects with embedded real options in uncertain decision making environments.

Finally, combining real options theory with other theories has been a significant development in recent years. Smit and Trigeorgis (2006), (2007) combined real options theory with game theory to serve as an analytical instrument for competitive strategies in an uncertain environment. They unify two major strands of economic theory—real options and games—into a single, coherent framework and demonstrate how these ideas can be applied to formulating corporate strategy. The integrated options-and-games perspective is particularly relevant for oligopolistic and innovative industries such as consumer electronics, telecommunications or pharmaceuticals.

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