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Annals of Operations Research

ISSN 0254-5330
Volume 186
Number 1

Ann Oper Res (2011)
186:213-229
DOI 10.1007/
s10479-010-0801-3

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Published online: 4 November 2010

Abstract In this paper, we propose an integrated model to incorporate inventory control decisions—such as economic order quantity, safety stock and inventory replenishment decisions—into typical facility location models, which are used to solve the distribution network design problem. A simultaneous model is developed considering a stochastic demand, modeling also the risk poling phenomenon. Multi-objective decision analysis is adopted to allow use of a performance measurement system that includes cost, customer service levels (fill rates), and flexibility (responsive level). This measurement system provides more comprehensive measurement of supply chain system performance than do traditional, single measure approaches. A multi-objective location-inventory model which permits a comprehensive trade-off evaluation for multi-objective optimization is initially presented. More specifically, a multiobjective evolutionary algorithm is developed to determine the optimal facility location portfolio and inventory control parameters in order to reach best compromise of these conflicting criteria. An experimental study using practical data was then illustrated for the possibility of the proposed approach. Computational results have presented promising solutions in solving a practical-size problem with 50 buyers and 15 potential DCs and proved to be an innovative and efficient approach for so called difficult-to-solve problems.

Keywords Supply chain integration · Vendor-managed inventory · Multi-objective supply chain model · Multi-objective evolutionary algorithm

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1 Introduction

Enterprises are facing competitive environments by implementing new strategies and technologies in response to the challenges and customer demands. Recently, two generic strategies for supply chain design emerged: *efficiency* and *responsiveness*. Efficiency aims to reduce operational costs; responsiveness, on the other hand, is designed to react quickly to satisfy customer demands. Gunasekaran et al. (2008) has concurred the need to develop cost effective solutions to organizations that are highly flexible and responsive to changing market/buyer demands. However, there are practical challenges for firms when they try to simultaneously reduce operating costs and customer service.

Most previous work on integrated supplier-buyer inventory systems does not consider ordering cost reduction. It has been a trend for firms to invest in logistics technology and methodology development in order to gain competitive advantage from lowered logistic costs and customer loyalty. For example, it is possible to reduce ordering cost and lead time by using *vendor-managed inventory* (VMI). The potential benefits from VMI are very compelling and can be summarized as reduced inventory costs for the supplier and buyer and improved customer service levels, such as reduced order cycle times and higher fill rates (Waller et al. 1999; Achabal et al. 2000). These benefits have been realized by successful retailers and suppliers, most notably Wal-Mart and key suppliers like Proctor and Gamble (Cetinkaya and Lee 2000).

Recently, Daskin et al. (2002) introduced an integrated location-inventory model with risk pooling (LMRP) that incorporates inventory costs at distribution centres (DCs) into location problems. LMRP assumes direct shipments from DCs to buyers which extended the uncapacitated fixed-charge problems to incorporate inventory decisions at the DCs. The uncapacitated assumption at DCs is usually not the case in practice. Capacity limitation may affect the number and locations of the facilities, the inventory that can be stored at the facilities and consequently the order frequency as well as the assignment of buyers to the facilities. Our research builds upon the initial LMRP model by first using an analytical mathematical model in VMI to determine the tradeoffs between dyadic supplier—buyer relationships in VMI. First, a capacitated version of the similar model is established. Second, two performance metrics corresponding to customer service are incorporated to make our contribution. We present a capacitated Multi-Objective Location-Inventory Problem (MOLIP) which results in a Mixed-Integer Non-Linear Programming (MINLP) formulation.

Evolutionary optimization algorithms are efficient-solving and easy-adaptive, especially those where traditional methods failed to provide good solutions (e.g. MINLP). Recently, multiobjective evolutionary algorithms (MOEAs) have become prevailing for they drive a population of solutions toward an approximation of the efficient frontier. There are many efficient MOEAs which are possible to find Pareto optimal solutions as well as widely distributed set of solutions; NSGAII (Deb et al. 2002) is one of the most successful approaches. In our study, a hybrid evolutionary approach based on NSGAII is incorporated for MOLIP.

This paper is organized as follows. Section 2 discusses relevant literature review. Section 3 details the MOLIP model formulation. Section 4 proposes a hybrid evolutionary approach with a heuristic procedure for MOLIP. Section 5 illustrates the experimental results and evaluates the proposed algorithm for MOLIP. Finally, conclusions and suggestions with future research directions are provided in Sect. 6.

2 Literature review

The implementation of VMI requires the coordination and integration of processes between buyers and suppliers. In general, buyers share demand and inventory status information with their suppliers so that suppliers can take over the inventory control and purchasing function from the buyers. As a result, we examine literature on both VMI supply chain and integrated supply chain.

2.1 VMI supply chains

Waller et al. (1999) provided a simple diagram of a VMI relationship where the supplier is in control of the buyer's inventory to ensure that predetermined service levels are maintained. So in this respect, the customer effectively takes a passive role in the supply chain. Disney (2001) provided a fuller description of the development of VMI and a number of benefits that VMI brings to the supply chain. Cetinkaya and Lee (2000) revealed an analytical model that the order release policy in use with VMI influences the level of inventory required at the buyer, thus directly affecting a supplier's inventory costs. Dong and Xu (2002) evaluated how VMI affected a supply channel and concluded that VMI can, in the long run, increase the profitability of both the supplier and buyer in the supply chain. Yao et al. (2007) developed an analytical model that explores how important supply chain parameters affect the cost savings to be realized in VMI.

2.2 Integrated supply chain

Research on integrated supply chain design is a relatively new study area. Jayaraman (1998) developed an integrated model which jointly examined the effects of facility location, transportation modes, and inventory-related issues. However, Jayaraman's study did not contain any demand and capacity restrictions. Erlebacher and Meller (2000) proposed a model to minimize the sum of fixed operating cost and inventory costs incurred by the DCs, together with the transportation costs between manufacturers and DCs, and between DCs and retailers. Nozick and Turnquist (2001) proposed an integrated location-inventory model to consider both cost and service responsiveness trade-offs based on an uncapacitated facility location problem. Miranda and Garrido (2004) studied a MINLP model to incorporate inventory decisions into typical facility location models. They solved the distribution network problem by incorporating a stochastic demand and risk pooling phenomenon. Sabri and Beamon (2000) presented an integrated multi-objective multi-product multi-echelon model that simultaneously addresses strategic and operational planning decisions by developing an integrated model which includes cost, customer service levels and flexibility. Gaur and Ravindran (2006) studied a bi-criteria optimization model to represent the inventory aggregation problem under risk pooling, finding out the tradeoffs in costs and responsiveness.

Recently, Daskin et al. (2002) and Shen et al. (2003) presented a location inventory model with risk pooling (LMRP) that incorporates safety stock placement into a location problem for a two-stage network. There are several variations of the LMRP model. Ozsen (2004) presents a capacitated version of LMRP which determines the ordering policy at the DCs so that the inventory aggregation does not exceed DC capacities. Shen and Daskin (2005) extended the LMRP model including cost and service objectives. They developed practical methods for quick and meaningful evaluation of cost/service trade-offs. In contrast to LMRP and its variants that consider inventory cost only at the DC level, Teo and Shu (2004) and Romeijn et al. (2007) proposed a warehouse-retailer network design problem in which both DCs and retailers carried inventory. These are actually the two major streams of integrated distribution network design problems.

It is found that studies of the facility location and distribution decisions within both VMI supply chain and integrated supply chain have tended to be limited. Also, the inclusion of these decisions would make the problem undoubtedly complex and difficult to solve. From the survey, some noteworthy innovative research aspects that are incorporated in our research include: (i) *Multi-objective location inventory problem*. Very few studies have addressed this problem; (ii) *Multi-objective evolutionary algorithms* (MOEAs). Most previous works have focused on traditional optimization techniques, but few have performed these techniques successfully and efficiently. In contrast, MOEAs have been successfully developed for various optimization problems, creating potential for the proposed MOLIP.

3 Mathematical formulation

3.1 Problem description and model assumptions

3.1.1 Overview of our research problem

Multi-echelon supply chains are commonly used to support manufacturing and after-market sales organizations. Such a supply chain network must satisfy buyers' demands at specified service levels and at the lowest possible cost. We study the design of a multi-echelon supply chain distribution network in which various products are shipped from a supplier (e.g., an outside vendor) to a set of DCs and from there distributed to a set of buyers. We assume that the DCs and the buyers will replenish using a single-sourcing strategy, i.e., each DC will be replenished from a single supplier, and each buyer will be replenished from a single DC. The design of such a supply chain therefore involves the issue of location of DCs as well as assigning located DCs to buyers. Figure 1 illustrates our multi-echelon supply chain system.

However, our problem considers both *strategic* and *tactical* decisions in the supply chain system. The strategic decision involves the *location* problem, which determines the number and the locations of DCs and assigns buyers to DCs, whereas the tactical decision deals with the *inventory* problem which determines the levels of safety stock inventory at DCs to provide certain service levels to buyers. The centralized inventory policy is considered under VMI which refers to the holding safety stocks aggregated at DCs. The integrated problem is called the *location-inventory* distribution network problem. Figure 2 shows the overall schematic diagram of the hierarchy of the model considered in our study.

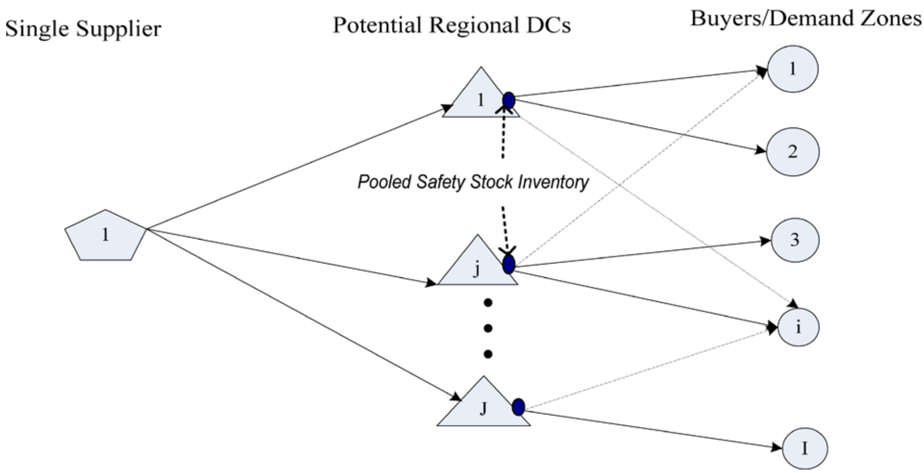

Fig. 1 Multi-echelon supply chain distribution network problem

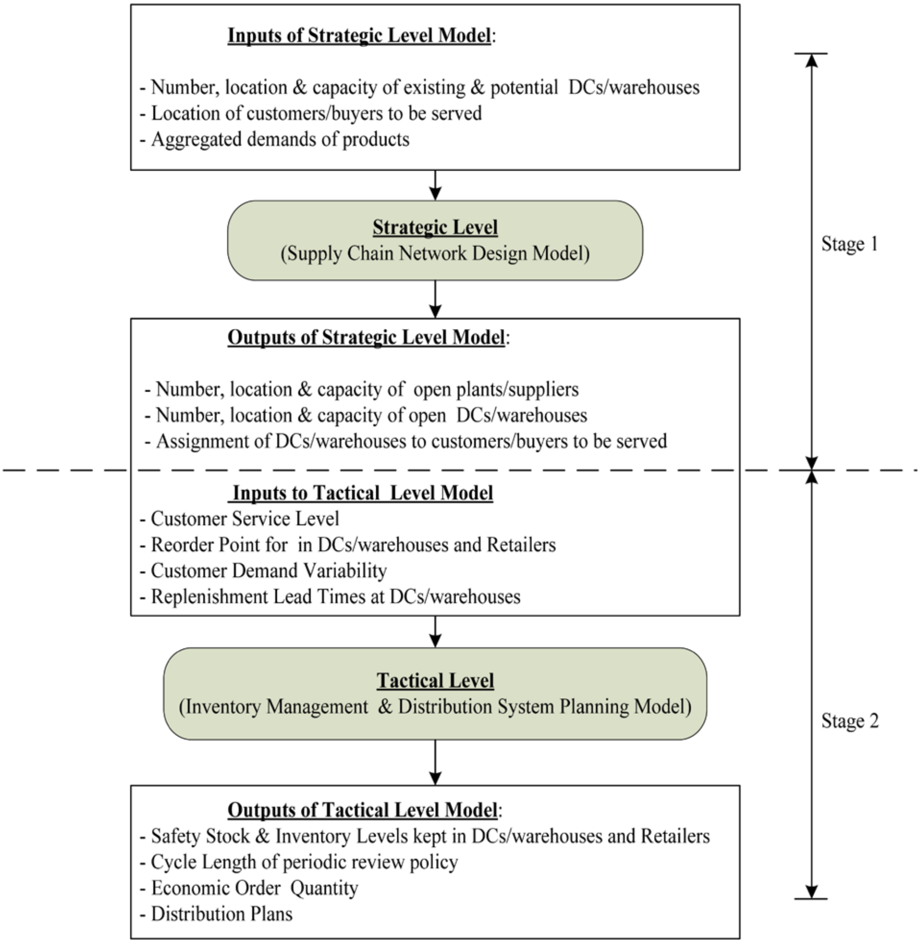

Fig. 2 Overview of the integrated strategic and tactical planning model

3.1.2 VMI coordination mechanism

VMI is one of the widely discussed coordination mechanisms for improving supply chain efficiency. The main feature of VMI indicates the centralized system within, with which the supplier as a sole decision maker decides the order quantity based on information available from both buyers and suppliers to minimize the total cost of the whole supply chain system. The supplier has full authority over inventory management at the buyer's DC to pay all costs associated with the supplier's production cost, both the buyer's and the supplier's ordering cost, the inventory holding cost and distribution cost. The supplier monitors, manages and replenishes the inventory of the buyer. Thus, the decisions on order replenishment quantity and order shipping are given to the supplier in the VMI system, rather than to the buyer as in tradition systems. Figure 3 presents the operational cost structure between the partners in the VMI system. The proposed model is mainly based on this cost structure.

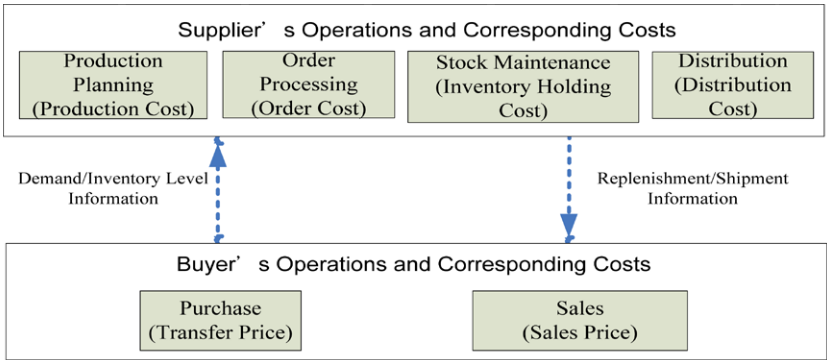

Fig. 3 Cost structure of the VMI system

3.2 Mathematical assumptions and notations

Basic assumptions are used when modeling our problem. It is assumed that all the products are produced by a single supplier and one specific product for a buyer should be shipped from a single DC. Reverse flows, in-transit inventory, and pipeline inventory are not considered. All the buyers' demands are uncertain and the storage capacities of the supplier are unlimited but are capacitated at the open DCs. More assumptions will be stated when we illustrate the mathematical model. Here, the mathematical notations used in the model are described as follows:

Indices:

K Set of product classifications
J Set of candidate DCs
I Set of buyers

Parameters:

f_j Fixed annual facility operating cost of opening a DC at site j
h_j^k Unit inventory holding cost per time (annually) at DC j for product k
o_j^k Ordering cost at DC j for product k
u_j Total capacity volume of the corresponding DC j
tc_{ji}^k Unit cost of shipping product k from DC j to buyer i
rc_j^k Unit cost of producing and shipping product k from the supplier to DC j
$D_{\max}$ Maximal coverage distance
τ_j The set of buyers that could attend DC j within the covering distance $D_{\max}$
ζ_j^k Average lead time in days for product k to be shipped to DC j from the supplier
d_i^k Average *daily* demands for product k at buyer i
σ_i^k Standard deviation of *daily* demands for product k at buyer i
d_{wj}^k Average aggregated *daily* demand of product k at DC j;
$z_{1-\alpha}$ Standard normal value where $P(z \le z_{1-\alpha}) = 1 - \alpha$
ψ Number of days per year

Decision variables:

Y_j Binary integer (=1, if DC j is chosen to be operated, and 0 otherwise)
X_{ji}^k Binary integer (=1, if DC j serves buyer i for shipping product k and 0 otherwise)
Q_{wj}^k Aggregate economic order quantity for product k at DC j

3.3 Mathematical models

To begin modeling this problem, we assume that the assignment of buyers to a DC is known a priori and that all the products are produced by a single supplier. The daily demand for product k at each buyer i is independent and normally distributed, i.e. $N(d_i^k, (\sigma_i^k)^2)$. Furthermore, at any site of DC j, we assume a continuous review inventory policy (Q_j, r_j) to meet a stochastic demand pattern. Also, we consider that the supplier takes an average lead time ζ_j^k (in days) for shipping product k from the supplier to DC j so as to fulfill an order.

From basic inventory theory (Eppen 1979), we know that if the demands at each buyer are uncorrelated, then the aggregate demand for product k during lead time ζ_j^k at the DC j is normally distributed with a mean of $\zeta_j^k d_{wj}^k$, where $d_{wj}^k = \sum_{i\in I} d_i^k X_{ji}^k$, and a variance of $\zeta_j^k \sum_{i\in I}(\sigma_i^k)^2 X_{ji}^k$. Let us consider the centralized supply chain system under VMI, which refers to aggregating the safety stock pooled at different DCs. Then, the total amount of safety stock for product k at DC j with risk pooling is $z_{1-\alpha}\sqrt{\zeta_j^k \sum_{i\in I}(\sigma_i^k)^2 X_{ji}^k}$, where $1-\alpha$ refers to the *level of service* for the system and $z_{1-\alpha}$ is the standard normal value with $P(z \le z_{1-\alpha}) = 1-\alpha$.

In the proposed model, the total cost is based on the cost structure of the VMI system in Fig. 3 and is decomposed into the following items: (i) *facility cost*, which is the annual fixed cost of operating facility incurred when DCs are opened at different locations for all products, (ii) *transportation cost*, which captures the *inbound* shipping cost from the supplier to specific DCs as well as the *outbound* shipment costs from DCs to the buyers, (iii) *operating cost*, which is the cost incurred when an order is placed and is calculated by multiplying the ordering cost per replenishment with the order frequency placed in one year, (iv) *cycle stock cost*, which is the cost for holding the cycle stock inventory during the year where the average cycle stock for any product k per unit time aggregated at DC j is measured by $Q_{wj}^k/2$, and (v) *safety stock cost*, which is the cost of holding sufficient inventory at DCs in order to provide a specific service level to their buyers. The total cost Z_1 is represented as follows:

$$Z_1 = \sum_{j\in J} f_j \cdot Y_j + \psi \sum_{k\in K}\sum_{j\in J}\sum_{i\in I}\left(\mathrm{rc}_j^k + \mathrm{tc}_{ji}^k\right)\cdot d_i^k \cdot X_{ji}^k + \sum_{k\in K}\cdot\sum_{j\in J} o_{\mathrm{j}}^k \cdot \frac{\psi\cdot d_{wj}^k}{Q_{wj}^k}$$
$$+\sum_{k\in K}\sum_{j\in J} h_j^k \cdot \frac{Q_{wj}^k}{2}\cdot Y_j + \sum_{k\in K}\sum_{j\in J} h_j^k \cdot z_{1-\alpha}\sqrt{\zeta_j^k \sum_{i\in I}(\sigma_i^k)^2 \cdot X_{ji}^k} \qquad (1)$$

Based on Z_1, the optimal order quantity $(Q_{wj}^k)^*$ for product k at each DC j can be obtained by differentiating (1) in terms of Q_{wj}^k, each DC j and each product k equal zero to minimize the total supply chain cost. We can obtain $(Q_{wj}^k)^* = \sqrt{2\cdot o_j^k \cdot (\psi \cdot d_{wj}^k)/h_j^k}$ for $\forall$ open DC $j, \forall k$. In this case, there is no capacity constraint for the order quantities Q_{wj}^k since we assume the storage capacity at the supplier is unlimited. Thus, replacing $(Q_{wj}^k)^*$ in the third and fourth terms of Z_1 in (1), we can obtain a non-linear cost function of Z_1. As follows, we propose an innovative mathematical model for the Multi-Objective Location-Inventory Problem (MOLIP).

$$\begin{aligned}\text{Min}\quad Z_1 = &\sum_{j\in J} f_j \cdot Y_j + \sum_{k\in K}\sum_{j\in J}\sum_{i\in I} \Psi_{ji}^k \cdot X_{ji}^k \\ &+ \sum_{k\in K}\sum_{j\in J}\left(\Gamma_j^k \cdot \sqrt{\sum_{i\in I} \mathrm{D}_i^k \cdot X_{ji}^k}\right) \\ &+ \sum_{k\in K}\sum_{j\in J}\left(h_j^k \cdot \sqrt{\sum_{i\in I} \Lambda_{ji}^k \cdot X_{ji}^k}\right)\end{aligned} \tag{2}$$

$$\text{Max}\quad Z_2 = \left(\sum_{k\in K}\sum_{j\in J}\sum_{i\in I} d_i^k \cdot X_{ji}^k\right) \Big/ \sum_{k\in K}\sum_{i\in I} d_i^k \tag{3}$$

$$\text{Max}\quad Z_3 = \left(\sum_{k\in K}\sum_{j\in J}\sum_{i\in \tau_j} d_i^k \cdot X_{ji}^k\right) \Big/ \sum_{k\in K}\sum_{j\in J}\sum_{i\in I} d_i^k \cdot X_{ji}^k \tag{4}$$

$$\text{s.t.}\quad \sum_{j\in J} X_{ji}^k \le 1 \quad \forall i\in I;\ \forall k\in K \tag{5}$$

$$X_{ji}^k \le Y_j \quad \forall i\in I; \forall j\in J; \forall k\in K \tag{6}$$

$$\sum_{k\in K}\sum_{i\in \mathrm{I}} d_i^k \cdot X_{ji}^k + \sum_{k\in K}\sqrt{\sum_{i\in I}\Lambda_{ji}^k \cdot X_{ji}^k} \le u_j \cdot \mathrm{Y}_j \quad \forall j\in J \tag{7}$$

$$X_{ji}^k \in \{0,1\}, Y_j \in \{0,1\}, \quad \forall i\in I;\ \forall j\in J;\ \forall k\in K \tag{8}$$

where $\Psi_{ji}^k = \psi \cdot (\mathrm{rc}_j^k + tc_{ji}^k)\cdot d_i^k$; $\Gamma_j^k = \sqrt{2\cdot o_j^k \cdot h_j^k}$ $\mathrm{D}_i^k = \psi \cdot d_i^k$; $\Lambda_{ji}^k = (z_{1-\alpha})^2 \cdot \zeta_j^k \cdot (\sigma_i^k)^2$ $\forall i\in I; \forall j\in J; \forall k\in K$.

Equations (2)–(4) give the objectives. While (2) of Z_1 is to minimize the total cost, (3) of Z_2 and (4) of Z_3 give the objectives by referring to the maximization customer service level by specifying two performance measurements: (i) *volume fill rate* (VFR), defined as the fraction of total demand that can be satisfied from inventory without shortage; (ii) *responsive level* (RL), the percentage of fulfilled demand volume that can satisfied within an exogenously specified coverage distance $D_{\max}$ for DCs. Constraint (5) restricts a buyer to be served by a single DC if possible. Constraint (6) stipulates that buyers can only be assigned to candidate sites that are selected as open DCs. Constraints (7) indicates the maximal capacity restrictions on the opened DCs, These constraints ensure the fact that for every product that flows through the DC, a part of the DC is held in safety stock and the rest of it is used to satisfy demands by the assigned buyers. Constraints (8) determines binary constraints.

The proposed MOLIP model does not only determine the DC locations, the assignment of buyers to DCs, but also finds out endogenously both the optimal order quantities and safety-stock levels at DCs. Since two of the three objective functions (Z_1 and Z_3) are nonlinear, the formulation results in an intractable multi-objective MINLP model.

Table 1 The NSGAII-based evolutionary algorithm

Randomly generate $P(1)$
Evaluate $P(1)$
Nondominated sort $P(1)$
Generate $C(1)$ form $P(1)$, apply binary tournament selection, crossover, and mutation.
Evaluate $C(1)$
while $t \leq T$ **do**
$R(t) = P(t) \cup C(t)$
Nondominated sort $R(t)$
Sort $R(t)$ using $\geq n$ {see Definition 1}
Select $P(t+1)$ from the first L chromosome of $R(t)$
Generate C($t+1$) from $P(t+1)$, apply binary tournament selection, crossover, and mutation
Mutate $C(t+1)$
Evaluate $C(t+1)$
$t \leftarrow t+1$
end while

4 Problem solving methodology

4.1 NSGAII-based evolutionary algorithm

Multiobjective optimization problems give rise to a set of Pareto-optimal solutions, none of which can be said to be better than other in all objectives. Unlike most traditional optimization approaches, evolutionary algorithms (EAs) work with a population of solutions and thus are likely candidates for finding multiple Pareto-optimal solutions simultaneously (Coello Coello 2006; Michalewicz 1996). Recently, since the pioneering work by Schaffer (1985), multiobjective evolutionary algorithms (MOEAs) have prevailed. NSGA-II (Deb et al. 2002) is one of the best techniques for generating Pareto frontiers in MOEAs. NSGA-II is a computationally efficient algorithm implementing the idea of a selection method based on classes of *dominance* of all the solutions. For each solution in the population, one has to determine how many solutions dominate it and the set of solutions to which it dominates. Then, it ranks all solutions to form non-dominated fronts according to a *non-dominated sorting* process, hence, classifying the chromosomes into several fronts of nondominated solutions. To allow for diversification, NSGA-II also estimates the solution density surrounding a particular solution in the population by computing a *crowding distance* operator.

A NSGAII-based evolutionary algorithm is proposed, as shown in Table 1. This algorithm starts by generating a random population $P(1)$ of size L. For each chromosome in $P(1)$ the algorithm evaluates its cost and coverage using the encoded solution expressions. Then, the algorithm applies nondominated sorting of $P(1)$ and assigns each chromosome to the front to which it belongs. Next, the algorithm applies binary tournament selection (to form the crossover pool), crossover, and mutation operators to generate the children population $C(1)$ of size L. Once initialized, the main algorithm repeats T generations. The algorithm applies *non-dominated sorting* to $R(t)$, resulting in a population from the union of parents $P(t)$ and children $C(t)$. The algorithm obtains the next generation population $P(t+1)$ after selecting L chromosomes from the first fronts of $R(t)$. Next, it applies selection, crossover, and mutation operators to generate the children $C(t+1)$.

During the selection process, chromosome fitness depends on the evaluation of the decoded solution in the objective functions and its comparison with other chromosomes. The

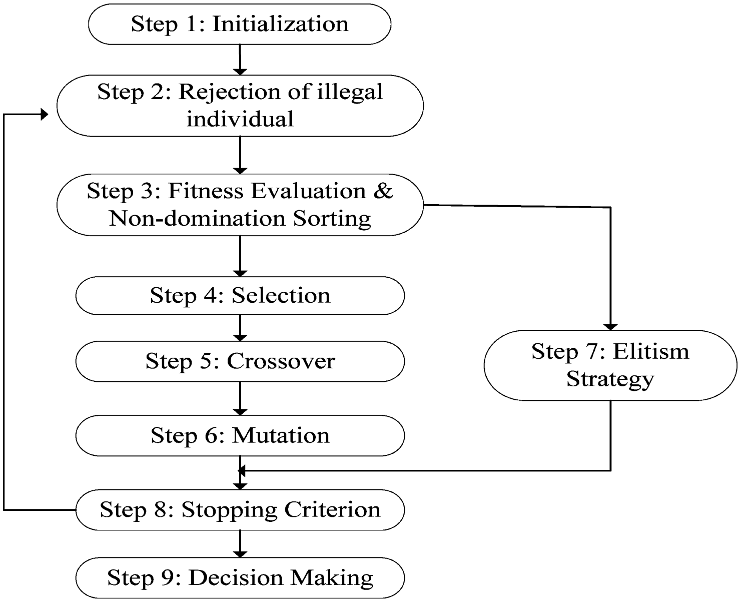

Fig. 4 A hybrid evolutionary algorithm for MOLIP

non-domination sorting updates a tentative set of Pareto optimal solutions by ranking a population according to non-domination. After that, each individual p in the population is given two attributes: (i) *non-domination rank* in the optimization objectives (p.rank); (ii) *local crowding distance* in the objectives space directions (p.distance). If both chromosomes are at the same rank, the one with fewer chromosomes around in the front is preferred. A partial order ($\geq n$) defined in Definition 1 is used to decide which of the two chromosomes is fitter. If chromosomes are at the same level, the one with fewer chromosomes around the front is preferred.

Definition 1 Let $p, q \in R(t)$ be two chromosomes in population $R(t)$. We say that p is better fitted than $q (p \geq nq)$, either if (p.rank $<$ q.rank) or ((p.rank $=$ q.rank) and (p.distance $>$ q.distance)).

4.2 A hybrid evolutionary algorithm for MOLIP

Here, a hybrid evolutionary algorithm is proposed for MOLIP and its basic block diagram is shown in Fig. 4. Cycles of fitness evaluation, selection, crossover, and mutation repeat until some stopping criteria are met. However, our algorithm first focuses on fitness evaluation according to a partial order ($\geq n$) which is used to decide which chromosomes are fitter. Suppose that $Z_k(p)$ and $Z_k(q)$ be the k-th objective function evaluated at two decoded chromosomes p and q, respectively. Here in MOLIP, $Z_1(\cdot)$ indicates *cost*, $Z_2(\cdot)$ indicates *volume fill rate* and $Z_3(\cdot)$ indicates *responsiveness level*. According to Definition 1, it is said that $p \geq q$ if $Z_1(p) \leq Z_1(q)$, $Z_2(p) \geq Z_2(q)$ and $Z_3(p) \geq Z_3(q)$; and either $Z_1(p) < Z_1(q)$ or $Z_2(p) > Z_2(q)$ or $Z_3(p) > Z_3(q)$.

The chromosome representation of the MOLIP is represented as well in Fig. 5. The solution is encoded in a binary string of length $m = |I|$, where the j-th position indicates if DC

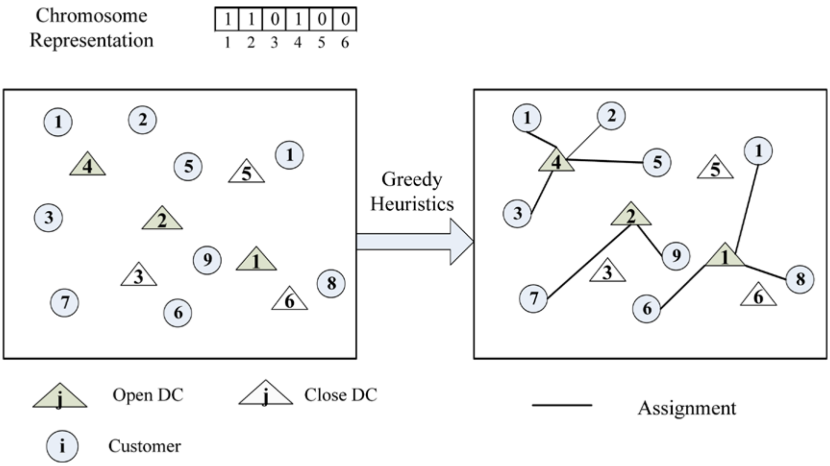

Fig. 5 Chromosome representation for the MOLIP problem

j is open (value of 1) or closed (value of 0). This binary encoding only considers if a given DC j is open or closed (variables Y_j).

A MOLIP solution also involves the assignment of buyers to open DCs (variables X^k_{ji}). This assignment is performed by a procedure that minimizes cost without deteriorating capacity limitation which limits the amount of demands to be assigned to each candidate DC. A greedy heuristics is used to obtain the buyer-DC assignments where the buyers are sorted in the descending order of their demand flows and assign them in the sorted order to the DC according to the following rules:

Rule 1 If the buyer i is covered (*i.e.*, there are DCs within a distance of $D_{\max}$), it is assigned to the DC with sufficient capacity (if one exists) which can serve it with the minimal difference between the remaining capacity of an open DC j and the demand flow of the buyer i through DC j. That is, the DC assignment attempts to be as full as possible.

Rule 2 If the buyer i cannot be covered or there is no successful assignment from the coverage set, it is then assigned to the candidate DC (with sufficient capacity) that increases the total cost by the least amount, regardless of its distance to the DC.

5 Model applications and computational results

5.1 A base-line problem

There are no MOLIP instances in the public domain, nor are any available previous studies to serve for benchmarking. For this reason, a base-line problem was developed by taking the size of a Gamma company's supply chain network with 15 DCs and 50 buyers as reference. The potential DC locations are randomly generated within a square of 100 distance units of width. Other model parameters are given in Table 2. For the sake of simplicity, Euclidean distance is used for measuring distribution distances. The company intended to determine the number of open DCs needed for order assignments. However, the capacity limitation of DCs affects the assignments of buyers. The managers also need to evaluate tradeoffs among

Table 2 Model parameters for the base-line problem

Parameters	Value
Annual cost of operating a DC j	U(900,1000)
Annual holding cost at DC j for product k	U(.2,.4)
Unit ordering cost at DC j for product k	U(8,10)
Capacity of DC j	U(500,700)
Unit variable transportation cost	$1
Unit production and shipping cost for product k from the supplier to DC j	U(1,3)
Maximal covering distance	25 Km
Lead time (daily)	U(2,4)
Working days per year	260
Average daily demand for product k at buyer i	U(60,80)
Standard deviation of daily demands	U(2,4)
Standard normal value (service level = 0.95)	1.96

three criteria: total cost (TC), volume fill rate (VFR) and responsive level (RL). To obtain the approximate Pareto front, we attempted to solve the specified problem using the proposed hybrid evolutionary approach. Through the GA approach, the base-line model (# of DCs = 15, # of buyers = 50) with product number ($k = 2$) resulted in 765 binary variables and 815 constraints.

In addition, defining a *reference* point is the first step in allowing the MOLIP to obtain tradeoff solutions. The reference point is a vector formed by the single-objective optimal solutions and is the best possible solution that may be obtained for a multi-objective problem. With a given reference point, the MOLIP problem can then be solved by locating the alternative(s) or decision(s) which have the minimum distance to the reference point. Thus, the problem becomes how to measure the distance to the reference point. For the MOLIP problem, the decision maker is asked to determine weights by prior knowledge of objectives once all the alternatives in the Pareto front are generated. Moreover, the reference point can be found simply by optimizing one of the original objectives at a time subjective to all constraints. Due to the incommensurability among objectives, we measure this distance by using normalized Euclidean distance between two points in k-dimensional vector space, $score = \{\sum_{t=1}^{k} w_t[(f_t^* - f_t)/f_t^*]^2\}^{1/2}$, where f is an alternative solution in the Pareto front, f^* is the reference point and w_t is the relative weight for the t-th objective. Then, all alternatives are ranked based on the value of *score* in descending order. The highest ranked alternative (with the minimal value of *score*) is then considered as the "optimal" solution among alternatives for the given MOLIP problem.

There are still some parameters: population size = 100; maximum number of generations = 200; cloning = 20%; crossover rate = 80%; mutation rate varies from 5% to 10% as the number of generations increases. The evolutionary procedure was coded in MATLAB environment and the experiments were executed on a Pentium IV processor at 3.2 GHz under Window XP with 1 GB of RAM. The base-line solution required 128.72 secs of CPU time to be obtained.

5.2 Computational results

For demonstrate the utility of the proposed hybrid evolutionary algorithm, the base-line problem was solved. We consider four cases by varying weights for all three objectives. In

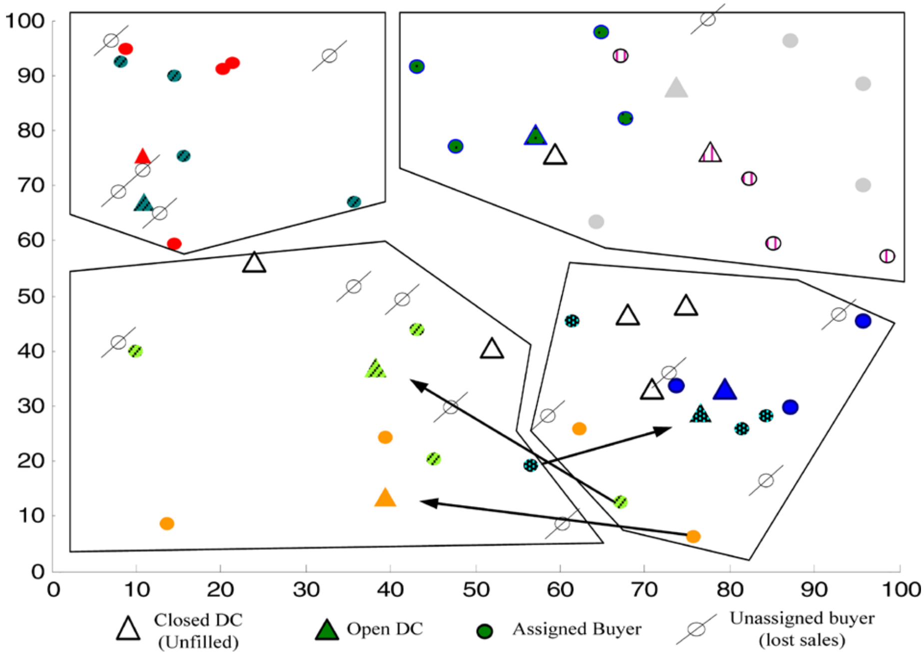

Fig. 6 Graphical display of the base-line solution of alternative 33

case 1, the first objective *total cost* is given a weight double the ones for the volume fill rate and the responsive level ($w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$); in case 2, the second objective *volume fill rate* is given a weight double the ones for the total cost and the responsive level ($w_1 = 0.25$, $w_2 = 0.5$, $w_3 = 0.25$); in case 3, the third objective responsive level is given a weight double the ones for the total cost and responsive level ($w_1 = 0.25$, $w_2 = 0.25$, $w_3 = 0.5$); finally, in case 4, all the objectives are given the same weight ($w_1 = 0.33$, $w_2 = 0.33$, $w_3 = 0.33$). Table 3 reported the computational solutions with their *original objective functions*, *normalized objective functions* and their respective *scores* in different cases.

It is illustrated that different sets of DCs are opened for all four cases of alternatives as shown in Table 3. In case 1, when supply chain cost is the first priority to be considered, the alternative 20 is chosen with the minimal total cost of \$182,131,489, the maximal volume fill rate of 56.06% and the maximal responsive level of 47.73%, respectively, where 7 out of 15 candidate DCs are required to open. In cases 2 and 3 with customer-driven consideration, we have the same alternative 38 as the optimal solution with the total cost of \$268,545.33, the volume fill rate of 77.52%, and the responsive level of 71.31%, respectively, where 10 out of 15 candidate DCs are aggregated. Finally, in case 4, when all weights of three objectives are considered of the same importance, the chosen compromise solution will be the alternative 33 with the total cost of \$237,469,731, the volume fill rate of 69.82% and the responsive level of 63.61%. There will be 9 DCs required to be opened. Figure 6 illustrates the graphical representation of the chosen "optimal" solution of the alternative 33 under case 4 for the base-line model. It is worth mentioning that most of these aggregated DCs should be assigned to buyers as close to them as possible within the maximal coverage (within 25 km). Unfortunately, with the DC capacity restrictions, there are possible buyers not to be assigned or to be assigned to DCs further than the maximal coverage. For example in

Table 3 Computational results of the base-line problem

Alter-native	# of DCs	Original Objectives			Normalized Objectives			Scores			
		Z_1 (thousand)	Z_2	Z_3	Z_1	Z_2	Z_3	Case 1	Case 2	Case 3	Case 4
1	1	$59,285.31	6.55%	0.00%	1.00	0.00	0.00	0.707	0.866	0.866	0.816
2	1	$67,365.72	8.71%	4.35%	0.97	0.02	0.05	0.683	0.839	0.832	0.788
3	3	$79,573.74	23.61%	17.13%	0.93	0.18	0.19	0.579	0.708	0.707	0.667
4	3	$86,048.25	23.61%	17.13%	0.91	0.18	0.19	0.580	0.708	0.708	0.668
5	3	$96,553.66	25.38%	18.89%	0.88	0.20	0.20	0.570	0.693	0.692	0.654
6	3	$98,558.65	25.53%	16.95%	0.87	0.20	0.18	0.578	0.699	0.704	0.663
7	3	$101,695.05	25.56%	19.08%	0.86	0.20	0.21	0.571	0.692	0.691	0.654
8	3	$103,167.29	25.70%	17.23%	0.85	0.20	0.19	0.578	0.698	0.703	0.662
9	3	$105,372.22	25.70%	19.22%	0.85	0.20	0.21	0.571	0.692	0.691	0.654
10	4	$106,511.61	29.83%	23.34%	0.84	0.25	0.25	0.541	0.654	0.653	0.618
11	4	$112,395.29	29.83%	23.34%	0.82	0.25	0.25	0.544	0.655	0.654	0.620
12	4	$117,149.21	31.87%	25.39%	0.81	0.27	0.28	0.532	0.638	0.637	0.604
13	4	$120,944.94	31.87%	25.39%	0.79	0.27	0.28	0.533	0.638	0.637	0.605
14	5	$122,740.49	38.02%	33.65%	0.79	0.34	0.36	0.482	0.576	0.568	0.544
15	5	$148,054.34	40.05%	33.69%	0.70	0.36	0.36	0.496	0.572	0.571	0.547
16	6	$152,292.45	42.00%	33.59%	0.69	0.38	0.36	0.494	0.563	0.567	0.542
17	6	$156,963.04	44.10%	33.69%	0.67	0.40	0.36	0.491	0.552	0.563	0.536
18	6	$159,561.65	48.11%	39.74%	0.66	0.44	0.43	0.460	0.512	0.516	0.497
19	6	$161,714.96	48.11%	41.75%	0.66	0.44	0.45	0.456	0.507	0.505	0.490
20	**7**	$182,131.49	56.06%	47.73%	0.59	0.53	0.52	**0.440***	0.457	0.461	0.453
21	7	$187,349.43	54.08%	49.84%	0.57	0.51	0.54	0.448	0.466	0.458	0.457
22	7	$191,651.44	56.06%	47.73%	0.56	0.53	0.52	0.455	0.464	0.468	0.462
23	7	$193,037.16	56.06%	49.84%	0.55	0.53	0.54	0.451	0.459	0.457	0.456
24	7	$204,913.12	58.03%	49.84%	0.51	0.55	0.54	0.465	0.459	0.461	0.462
25	8	$207,105.51	58.03%	49.84%	0.51	0.55	0.54	0.469	0.460	0.463	0.464
26	**8**	$210,591.91	60.01%	53.65%	0.49	0.57	0.58	0.460	0.443	0.441	0.448
27	8	$221,662.50	60.01%	53.80%	0.46	0.57	0.58	0.479	0.453	0.451	0.461
28	8	$222,260.02	60.01%	53.80%	0.46	0.57	0.58	0.480	0.454	0.451	0.462
29	8	$228,458.50	61.98%	49.84%	0.43	0.59	0.54	0.497	0.460	0.473	0.477
30	8	$230,417.85	63.95%	51.81%	0.43	0.61	0.56	0.491	0.448	0.460	0.467
31	8	$234,501.32	61.98%	55.77%	0.41	0.59	0.60	0.494	0.451	0.449	0.465
32	9	$235,584.83	67.87%	61.66%	0.41	0.66	0.67	0.472	0.412	0.409	0.432
33	**9**	$237,469.73	69.82%	63.61%	0.40	0.68	0.69	0.468	0.401	0.399	**0.424***
34	9	$251,929.95	71.75%	65.54%	0.36	0.70	0.71	0.492	0.407	0.405	0.436
35	10	$263,376.20	73.68%	63.58%	0.32	0.72	0.69	0.516	0.418	0.423	0.455
36	10	$266,566.37	75.60%	65.48%	0.31	0.74	0.71	0.517	0.412	0.417	0.451
37	10	$267,109.92	75.60%	69.39%	0.31	0.74	0.75	0.512	0.406	0.404	0.444
38	**10**	$268,545.33	77.52%	71.31%	0.30	0.76	0.77	0.511	**0.398***	**0.396***	0.438
39	11	$280,849.15	81.36%	73.23%	0.26	0.80	0.79	0.531	0.402	0.403	0.449
40	11	$290,302.16	83.27%	75.15%	0.23	0.82	0.81	0.549	0.408	0.409	0.460
41	12	$297,291.87	87.06%	78.95%	0.20	0.86	0.86	0.558	0.407	0.408	0.463

Table 3 (*Continued*)

Alter-native	# of DCs	Original Objectives Z_1 (thousand)	Z_2	Z_3	Normalized Objectives Z_1	Z_2	Z_3	Scores Case 1	Case 2	Case 3	Case 4
42	12	$299,336.16	88.94%	78.96%	0.20	0.88	0.86	0.562	0.407	0.409	0.465
43	12	$299,987.23	88.94%	80.81%	0.20	0.88	0.88	0.562	0.406	0.407	0.464
44	12	$308,170.27	88.94%	80.82%	0.17	0.88	0.88	0.581	0.419	0.420	0.479
45	12	$333,874.21	90.82%	82.73%	0.08	0.90	0.90	0.638	0.456	0.457	0.524
46	13	$341,991.71	96.36%	86.47%	0.06	0.96	0.94	0.654	0.463	0.464	0.535
47	13	$346,735.92	96.36%	86.81%	0.04	0.96	0.94	0.664	0.471	0.471	0.543
48	14	$358,601.18	100.00%	92.31%	0.00	1.00	1.00	0.691	0.489	0.489	0.564

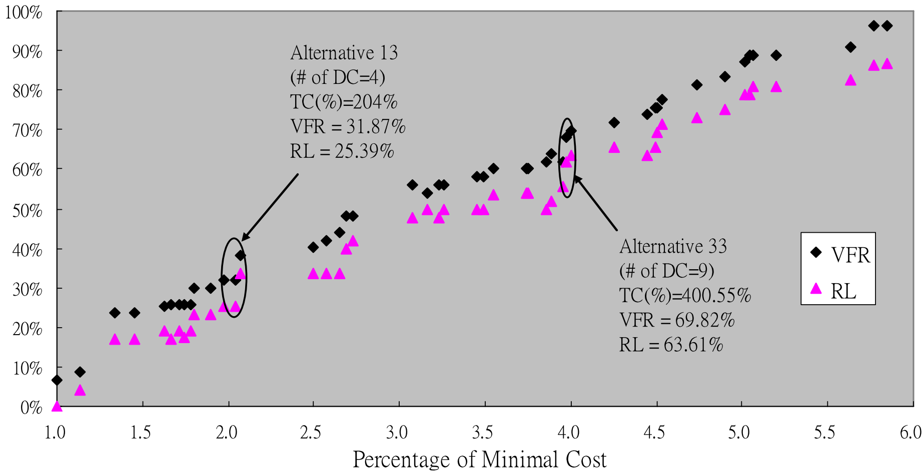

Fig. 7 Approximate Pareto solutions of the base-line problem

alternative 33, there are about 30.18% unassigned demands (∅) and there are also 36.39% aggregated buyers (→) will be assigned to DCs further than the maximal coverage.

The approximate Pareto solutions obtained by the hybrid evolutionary algorithm are illustrated in Fig. 7. For ease of understanding existing tradeoffs between cost and volume fill rate and responsive level respectively, we present it as a percentage of the minimal cost instead of using it in an absolute term. As shown in Fig. 7, we can see that the decision maker could easily adopt a minimal cost policy (i.e., minimal cost solution) that ignores customer's responsiveness. However, there should be a clear requirement to evaluate tradeoffs among total costs and customer responsiveness. It is possible to increase volume fill rate (VFR) from 31.87% to 69.82% and responsive level (RL) from 25.39% to 63.61%, if the percentage over the minimal cost increases from 204% to 400.55% (about 2 times) when the number of open DCs is increased from 4 to 9. Based on the approximate Pareto solutions, we can easily make decisions according to the decision maker's preference. For example, if the decision maker's goal is to maintain volume fill rate (VFR) at about the level of 70% from the current status of 31.87%, it is necessary to spend extra costs to open additional

DCs up to 9 to enhance customer's volume fill rate and also to increase responsive level at the same time.

6 Concluding remarks

In this study, we presented a mixed-integer non-linear programming model for multi-objective optimization of supply chain network and a multi-objective evolutionary algorithm approach. The model includes elements of total cost, customer service level (fill rate), and flexibility (responsive level) as its objectives and also integrates the effects of facility location, distribution, and inventory issues under a vendor managed inventory (VMI) coordination mechanism. To deal with multi-objective and enable the decision maker to evaluate a greater number of alternative solutions, a well-known NSGAII-based evolutionary algorithm was implemented.

The proposed model provides a means by which inventory, distribution and facility location strategies can be evaluated. Such joint examination of those strategies could lead to further thorough investigations of competitive strategies. For example, the model could be used to vary the number of suppliers and warehouses and evaluate its effect on the transportation modes and the amount of inventory (cycle stock and safety stock) that needs to be carried by these suppliers and warehouses based on their locations in the distribution network. For many purchasing managers, transportation costs are erroneously taken as fixed, and thereby not a relevant cost for the contract negotiation. This is probably the case in a regulated transportation environment where similar carriers in a given mode were required to charge the same price for the same service. Further, many buyers look at transportation cost as only a small part of the unit price of an item. However, the results of the integrated model indicate that firms have to reconsider their transportation, inventory, and location strategies in the light of changing market conditions.

In future works, we intend to adapt the proposed hybrid evolutionary algorithm to other integrated location, inventory and distribution systems that have different characteristics or network structures. For instance, a network system may have stockpiles or inventories within the suppliers and the customer sites, and the shortage penalty needs to be considered in the overall supply chain operating cost. In addition, the inclusion of other inventory decisions would be a direction worth pursuing. Such inventory decisions could include frequency and size of the shipments from suppliers to the DCs and from DCs to the retailers based on different replenishment policies, and lead time in addition to safety-stock inventory in the model. Finally, the dynamic or multi-period planning problems, in which the decisions are conducted for a specific number of planning periods, should be considered. For examples, not only the demands on the customer zones are non-deterministic but also the product lead-times can be stochastic.

Other possible research directions are to explore more competitive MOEAs or other existing optimization technologies, such as Lagrangian relaxation, particle swarm optimization, ant colony optimization, or other soft intelligent computing techniques. Comparative studies of these techniques are worth investigating in the future. In addition, some possible methods of hybridizations include the adaption of new genetic operators for integrated systems and the incorporation of other heuristic search techniques into the evolutionary algorithms, such as hill-climbing or local repair procedure.

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