Binding Transition in Quantum Hall Edge States

Hsien-chung Kao,[1] Chia-Hung Chang,[2] and Xiao-Gang Wen[3,2]

[1]*Department of Physics, Tamkang University, Tamsui, Taiwan 25137, Republic of China*
[2]*Physics Division, National Center for Theoretical Sciences, P.O. Box 2-131, Hsinchu, Taiwan 300, Republic of China*
[3]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

We study a class of Abelian quantum Hall (QH) states which are topologically unstable (T unstable). We find that the T-unstable QH states can have a phase transition on the edge which causes a binding between electrons and reduces the number of gapless edge branches. After the binding transition, the single-electron tunneling into the edge gains a finite energy gap, and only certain multielectron cotunneling (such as three-electron cotunneling for $\nu = 9/5$ edges) can be gapless. We point out that the binding transition can also be viewed as an edge reconstruction transition.

PACS numbers: 73.40.Hm, 73.20.Dx

A large class of quantum Hall (QH) liquids [1] (almost all those observed in experiments) is called Abelian QH state. The topological orders [2,3] in the Abelian QH liquids are labeled by a rank κ integer symmetric matrix K (called the K matrix) and a κ dimensional integer vector t (called the charge vector) [4]. (K, t) describe the internal correlation in QH liquids and provide much more information than filling fraction. In this paper we will show that a QH liquid may have special properties when (K, t) take a special form.

The effective theory for the (K, t) QH liquid is given by the U(1) Chern-Simons (CS) theory [4]:

$$L_{\text{bulk}} = -\frac{1}{4\pi} K_{ij} a_{i\mu} \partial_\nu a_{j\lambda} \epsilon^{\mu\nu\lambda} - eA_\mu t_i \partial_\nu a_{i\lambda} \epsilon^{\mu\nu\lambda}. \tag{1}$$

The above pure CS effective theory is dual to the Ginzburg-Landau-CS effective theory obtained earlier [5]. (K, t) determine all the universal properties of the Abelian QH states. For example, all the allowed quasiparticles are labeled by κ dimensional nonzero vector l with integer elements. The electric charge and statistics of a quasiparticle are given by

$$\theta = \pi l^T K^{-1} l, \qquad Q_q = -e t^T K^{-1} l, \tag{2}$$

while the filling fraction is given by $\nu = t^T K^{-1} t$. (K, t) also determines the structure of edge excitations [3] (at least for the sharp edges [6]).

According to Haldane [7], an Abelian quantum Hall theory is T unstable if there exist quasiparticles (labeled by m) that are both bosonic and charge neutral, i.e., $m^T K^{-1} m = 0$ and $t^T K^{-1} m = 0$. Such a vector m will be called neutral null vector. Since these quasiparticles carry trivial quantum numbers, their creation and annihilation operators can appear in the Hamiltonian without breaking any symmetries. By including them in Eq. (1), a more generic effective theory can be obtained. In this paper we will study the physical consequences of the these operators on the edge states. Near the edge, these neutral quasiparticle operators simply describe all kinds of charge

transfers between different edge branches. We find that, under the right conditions, they can cause a phase transition in the edge states. Such a transition will be called binding transition in this paper.

Many QH states have neutral null vectors, such as the $\nu = 9/5$ state with

$$K = \text{Diag}(1, 1, -5), \qquad t^T = (1, 1, 1), \tag{3}$$

where $\text{Diag}(a_1, a_2, \ldots)$ is a diagonal matrix with diagonal elements $a_1, a_2, \ldots$. Before the binding transition, the edge state of the above QH state has three branches. The single-electron tunneling is gapless. After the binding transition, as we will see, the edge state has only one gapless branch, and the single-electron tunneling opens up a finite energy gap. Only three-electron cotunneling is gapless.

The binding transition can also appear on the boundary between two different QH liquids. The edge state between two different QH liquids (K_1, t_1) and (K_2, t_2) is described by $K = K_1 \oplus (-K_2)$, $t = t_1 \oplus t_2$. The edge state will be called T unstable if the (K, t) has nonzero neutral null vector m. For a sequence of hierarchical QH liquids $\nu = 1/3, 2/5, 3/7, \ldots$ the edge state between any two QH states in the sequence is T unstable since they are based on the same $\nu = 1/3$ state. The edge excitations from the base $1/3$ state can annihilate each other. More nontrivial cases of T unstable boundary can appear between hierarchical states based on different QH liquids. We find T unstable edge states between $(2/5, 2/9)$, $(2/5, 3/13)$, $(2/9, 2/7)$, $(2/5, 1/7)$, $(2/5, 2/13)$, $(1/9, 2/9)$, ... QH states. The simplest T unstable edge state is the one between $\nu = 2$ and $\nu = 1/5$ states. Such a edge state is equivalent to that of the $\nu = 9/5$ state and has the same binding transition.

To show the above results, let us concentrate on the simplest T-unstable systems, the $\nu = 9/5$ state described by Eq. (3). The quasiparticle operators are labeled by integer vectors l. The electron operators are those of the quasiparticle operators which carry charge e and have statistics $\theta = (2n + 1)\pi$ (i.e., the Fermi statistics).

There are infinite many electron operators, which can be labeled by two integers (k_1, k_2):

$$\boldsymbol{l}_e^T = [k_1, k_2, -5(1 - k_1 - k_2)]. \qquad (4)$$

The $\nu = 9/5$ state has two neutral null vectors:

$$\boldsymbol{m}_1^T = (-1, 2, 5), \qquad \boldsymbol{m}_2^T = (2, -1, 5). \qquad (5)$$

The edges of quantum Hall systems are described by a chiral Luttinger liquid (χLL) theory. In imaginary time, the corresponding χLL action contains N bosonic fields ϕ_i and has the form [3]

$$S_{\text{edge}} = \frac{1}{4\pi} \int dx\, d\tau [iK_{ij}\partial_x\phi_i\partial_\tau\phi_j + V_{ij}\partial_x\phi_i\partial_x\phi_j]. \qquad (6)$$

On the edge, quasiparticles are created by the vertex operators, $V_l = \exp(il_j\phi_j)$. The correlation function of V_l has a form $\prod_{k=1}^{N^+}(x + iv_k^+\tau)^{-\alpha_k}\prod_{k=1}^{N^-}(x - iv_k^-\tau)^{-\beta_k}$. Here N^+ and N^- are the numbers of positive and negative eigenvalues of K; $v_k^\pm, \alpha_k, \beta_k$ are nonnegative real numbers which depend on V and K. The sum of the exponents $\Delta(\boldsymbol{l}) \equiv (\sum_{k=1}^{N^+}\alpha_k + \sum_{k=1}^{N^-}\beta_k)/2$ is the scaling dimension of the operator V_l. Since the scaling dimension $\Delta(\boldsymbol{l})$ is a function of V, it is useful to write V in such a way that isolates the parts of V affecting $\Delta(\boldsymbol{l})$. We will follow the approach used in Ref. [8].

To calculate $\Delta(\boldsymbol{l})$, we can simultaneously diagonalize K and V through a basis change $\phi_i = M_{ij}\tilde{\phi}_j$. This can be done in two steps. First, we find an M_1 that brings K to the pseudoidentity I_{N^-,N^+}, i.e.,

$$M_1^T K M_1 = I_{N^-,N^+} = \begin{pmatrix} -I_{N^-} & 0 \\ 0 & I_{N^+} \end{pmatrix}. \qquad (7)$$

For the state $\nu = 9/5$, we find

$$M_1 = \begin{pmatrix} \frac{-5\lambda^2}{2} & \frac{-2+5\lambda^2}{2} & \frac{-5\lambda}{\sqrt{5}} \\ 2 + 5\lambda + 5\lambda^2 & -5\lambda - 5\lambda^2 & \frac{5+10\lambda}{\sqrt{5}} \\ \frac{-2-4\lambda-5\lambda^2}{2} & \frac{4\lambda+5\lambda^2}{2} & \frac{-2-5\lambda}{\sqrt{5}} \end{pmatrix}, \qquad (8)$$

and $(N^-, N^+) = (1, 2)$. Then we find the second basis change M_2 in the proper pseudo-orthogonal group $SO(N^-, N^+)$ that diagonalizes V while leaving the pseudoidentity invariant. In the new basis $\tilde{\boldsymbol{\phi}} = (M_1 M_2)^{-1}\boldsymbol{\phi}$,

$$\tilde{\boldsymbol{l}} = M_2^T M_1^T \boldsymbol{l},$$
$$\tilde{K} = I_{N^-,N^+} = M_2^T M_1^T K M_1 M_2, \qquad (9)$$
$$\tilde{V} = M_2^T M_1^T V M_1 M_2.$$

The functions $K(\boldsymbol{l})$ and $\Delta(\boldsymbol{l})$ are basis independent. Now that both $\tilde{V}$ and $\tilde{K}^{-1}$ become diagonal, the correlation functions are trivial: $\langle e^{i\tilde{\phi}_j(x,\tau)}e^{-i\tilde{\phi}_j(0,0)}\rangle \propto \frac{1}{x \mp iv_j\tau}$, where the sign depends on whether $\tilde{\phi}_j$ appears with -1 or $+1$ in I_{N^-,N^+}. Consequently, the scaling dimension of the operator $\exp(il_j\tilde{\phi}_j)$ is found to be $\Delta(\boldsymbol{l}) = \tilde{l}_j\tilde{l}_j = l_i\Delta_{ij}l_j$, with $2\Delta = M_1 M_2 M_2^T M_1^T$. Drawing analogy from special relativity [8], we can factor M_2 into a product of a

symmetric positive matrix B analogous to the Lorentz boost and an orthogonal matrix R: $M_2 = BR$. $2\Delta(\boldsymbol{l})$ becomes

$$2\Delta = M_1 M_2 M_2^T M_1^T = M_1 B^2 M_1^T. \qquad (10)$$

For the $\nu = 9/5$ state, the matrices B and R can be parametrized as

$$B = \begin{pmatrix} \gamma & \gamma\beta_1 & \gamma\beta_2 \\ \gamma\beta_1 & 1 + \frac{\gamma^2\beta_1^2}{\gamma+1} & \frac{\gamma^2\beta_1\beta_2}{\gamma+1} \\ \gamma\beta_1 & \frac{\gamma^2\beta_1\beta_2}{\gamma+1} & 1 + \frac{\gamma^2\beta_2^2}{\gamma+1} \end{pmatrix},$$

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}, \qquad (11)$$

where $\gamma = 1/\sqrt{1 - \beta^2}$. Note that $V = (M_1^T)^{-1} \times (M_2^T)^{-1}\tilde{V}(M_1)^{-1}(M_1)^{-1}$; thus $(\lambda, \beta_1, \beta_2, \varphi)$ and the three diagonal elements in $\tilde{V}$, (v_1, v_2, v_3), can be viewed as a parametrization of V. Since V contains only six independent parameters, we may set one of the above seven parameters to zero.

For neutral null vectors $\boldsymbol{m}_{1,2}$, the quasiparticle operators

$$V_{\boldsymbol{m}_a} \equiv e^{ip_a x}\exp\{i(\boldsymbol{m}_a)_j\phi_j\}, \qquad a = 1, 2 \qquad (12)$$

carry trivial quantum number [9] and the generic edge Hamiltonian/action contains a term $\Gamma_1 V_{\boldsymbol{m}_1} + \Gamma_2 V_{\boldsymbol{m}_2} +$ H.c. The problem here is how this quasiparticle term affects the dynamics of low lying edge excitations. First let us consider when the quasiparticle term becomes a relevant perturbation.

The scaling dimensions of $V_{\boldsymbol{m}_{1,2}}$ are found to be $\Delta(\boldsymbol{m}_1) = \gamma^2\{-1 + \beta_1\}^2$ and $\Delta(\boldsymbol{m}_2) = \frac{1}{4}\gamma^2\{-(14 + 30\lambda + 45\lambda^2) + (-4 + 30\lambda + 45\lambda^2)\beta_1 - 6\sqrt{5}(1 + 3\lambda)\beta_2\}^2$. Note that $\Delta(\boldsymbol{m}_{1,2})$ depend only on $(\lambda, \beta_1, \beta_2)$, and in the rest of the paper we will set one of the redundant parameter $\beta_2 = 0$.

In one area of the β_1-λ plan, we find that both $\Delta(\boldsymbol{m}_{1,2}) > 2$ (see Fig. 1). In this case, both the neutral quasiparticle operators are irrelevant and can be dropped at low energies. Therefore in this area of the β_1-λ plan, the neutral quasiparticle operators do not cause any instability, and the edge theory is still described by Eq. (6),
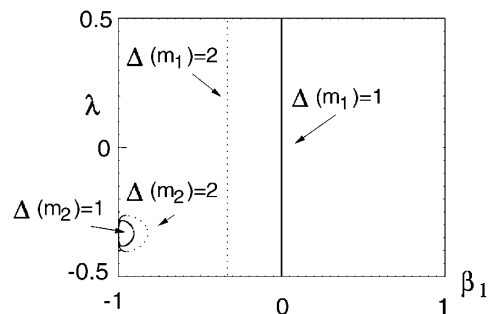

FIG. 1. Plot of scaling dimension of the two neutral null operators for $\nu = 9/5$ edge as functions of (β_1, λ). Dashed lines indicate when operators become marginal $[\Delta(\boldsymbol{m}) = 2]$ and solid lines indicate the soluble case where $[\Delta(\boldsymbol{m}) = 1]$.

and has three gapless branches. We also see in Fig. 1 that in another area, $\Delta(\boldsymbol{m}_1) < 2$ and $\Delta(\boldsymbol{m}_2) > 2$. In this case only the $V_{\boldsymbol{m}_1}$ is relevant. In the following, we will drop the $V_{\boldsymbol{m}_2}$ term and consider the effect of the $V_{\boldsymbol{m}_1}$ term.

In general, it is very difficult to solve our model with a relevant quasiparticle operator. However, in a special case when $(\tilde{\boldsymbol{m}}_1)_3 = 0$ [where $\tilde{\boldsymbol{m}}_1 = (M_1 M_2)^T \boldsymbol{m}_1$], the problem can be simplified. This is because when $(\tilde{\boldsymbol{m}}_1)_3 = 0$ the mode described by $\tilde{\phi}_3$ is decoupled from the modes described by $\tilde{\phi}_{1,2}$ even in the presence of the $V_{\boldsymbol{m}_1}$ quasiparticle operator. In this case, at least the sector described by $\tilde{\phi}_3$ can be solved, which describes a gapless edge mode. The condition for $(\tilde{\boldsymbol{m}}_1)_3 = 0$ can be satisfied if and only if $\varphi = 0, \pi$.

Now let us concentrate on the dynamics of the decoupled sector described by $\tilde{\phi}_{1,2}$. First we will show that if in addition to $\varphi = 0, \pi$, we also have $\Delta(\boldsymbol{m}_1) = 1$, then the $\tilde{\phi}_{1,2}$ sector can be solved exactly. The condition for the operator $V_{\boldsymbol{m}_1}$ to have a scaling dimension 1 is $\beta_1 = 0$. Under this condition (note we have already set $\beta_2 = 0$ and $\varphi = 0, \pi$), we have $\tilde{m}_1 = (-1, 1, 0)$. We can fermionize $\exp(i\tilde{\phi}_1) \propto e^{\alpha_1 x}\psi_1$, $\exp(i\tilde{\phi}_2) \propto e^{i\alpha_2 x}\psi_2$ with properly chosen $\alpha_{1,2}$ to obtain $V_{\boldsymbol{m}_1} \propto \psi_1^\dagger \psi_2$ and

$$\frac{1}{4\pi}\left[\sum_{a=1,2}\psi_a^\dagger[\partial_t + (-)^a v_a \partial_x + \mu]\psi_a \right.$$
$$\left. + (\Gamma \psi_1^\dagger \psi_2 + \text{H.c.}) + \partial_x \tilde{\phi}_3 \partial_\tau \tilde{\phi}_3 + v_3(\partial_x \tilde{\phi}_3)^2\right],$$
$$(13)$$

with $\mu = p_1(v_1 + v_2)$. Now we see that the system described by Eq. (13) is a free fermion theory and is exactly soluble. We note that when $\mu \neq 0$, the two fermions $\psi_{1,2}$ have different Fermi momenta, and the $\Gamma \psi_1^\dagger \psi_2$ term cannot mix the fermions at the Fermi surface. Even though $\psi_1^\dagger \psi_2$ has a scaling dimension less than 2, when $\Gamma < \mu$ the $\psi_{1,2}$ sector remains to be gapless. The low lying excitations are described by gapless free fermions $\psi_{1,2}$ and bosons $\tilde{\phi}_3$, and form three gapless branches. We will call such a phase "three-branch phase." Only when $\Gamma > \mu$, the $\psi_{1,2}$ sector can have a finite energy gap, and there is only one gapless branch described by $\tilde{\phi}_3$. Such a phase will be called "one-branch phase."

We note that the one-branch phase is very stable. In respect to the one-branch fixed point, any change in V (or in $\lambda, \beta_1, \varphi, v_1, v_2, v_3$) corresponds to an irrelevant or exactly marginal perturbation. (Only a change in v_3 corresponds to an exactly marginal perturbation.) In particular, if we change φ away from $0, \pi$, it will flow back to $0, \pi$ at low energies. Therefore, the edge can be in the one-branch phase for a finite volume in the parameter space of V if Γ is large enough.

Now consider the correlation of a generic quasiparticle operator $V_l \equiv e^{il\cdot\phi} = e^{i\tilde{l}\cdot\tilde{\phi}}$, where $\tilde{l} = M_2^T M_1^T l$. (Note that, when $l = l_e$, V_l will describe an electron operator.) In the three-branch phase, V_l has an algebraic correlation

which can be calculated through the bosonization. In the one-branch phase, φ flows to $0, \pi$, $(\tilde{\boldsymbol{m}}_1)_3 = 0$ and the $\tilde{\phi}_3$ sector decouples with the $\tilde{\phi}_{1,2}$ sector. We may write V_l as $V_l = V_l' e^{i(\tilde{l})_3 \tilde{\phi}_3}$ and $V_l' = e^{i(\tilde{l})_1 \tilde{\phi}_1 + i(\tilde{l})_1 \tilde{\phi}_2}$. It is easy to find $\langle e^{i(\tilde{l})_3 \tilde{\phi}_3} e^{-i(\tilde{l})_3 \tilde{\phi}_3}\rangle = (x - vt)^{-(\tilde{l})_3^2}$. Thus we can concentrate on the $\tilde{\phi}_{1,2}$ sector and the correlation of V_l'.

First we note that if we write the edge partition function in the form of imaginary-time path integral and expand it in power of Γ, the edge partition function will have the same form as the partition function of a 2D Coulomb gas. The "particles" in the Coulomb gas correspond to $V_{\boldsymbol{m}_1}$ and $V_{\boldsymbol{m}_1}^\dagger$. The interaction potential is given by $-\ln\langle V_{\boldsymbol{m}_1}(z)V_{\boldsymbol{m}_1}^\dagger(0)\rangle = \Delta(\boldsymbol{m}_1)\ln|z|^2$ if we assume, for the time being, $p_1 = 0$. When $\Delta(\boldsymbol{m}_1) < 2$, the Coulomb gas is in the plasma phase and the $\tilde{\phi}_{1,2}$ sector has a finite energy gap. In the Coulomb gas picture, calculating the correlation of V_l' corresponds to calculating the change ΔE in the energy of the Coulomb gas when we insert two test charges corresponding to V_l' and $V_l'^\dagger$. (The correlation function is given by $e^{-\Delta E}$.) From the correlation $-\ln\langle V_l'(z)V_{\boldsymbol{m}_1}^\dagger(0)\ldots\rangle = (\tilde{l})_1(\tilde{\boldsymbol{m}}_1)_1 \ln z^* + (\tilde{l})_2(\tilde{\boldsymbol{m}}_1)_2 \ln z + \ldots$, we see that if $-(\tilde{l})_1(\tilde{\boldsymbol{m}}_1)_1 + (\tilde{l})_2(\tilde{\boldsymbol{m}}_1)_2 = \tilde{l}\tilde{K}^{-1}\tilde{\boldsymbol{m}}_1 = lK^{-1}\boldsymbol{m}_1 = 0$, then V_l' will indeed correspond to a charged particle in the Coulomb gas, since the interaction potential is real. Thus in the plasma phase, V_l' will have a finite and constant correlation at long distance due to the complete screening of the plasma phase which gives $\Delta E = 0$. Now V_l' can be replaced by a pure number and the correlation of V_l is just $(x - vt)^{-(\tilde{l})_3^2}$. Note that $(\tilde{\boldsymbol{m}}_1)_1^2 = (\tilde{\boldsymbol{m}}_1)_2^2$ and $\tilde{l}\tilde{K}^{-1}\tilde{\boldsymbol{m}}_1 = 0$ requires $(\tilde{l})_1^2 = (\tilde{l})_2^2$; thus $(\tilde{l})_3^2 = -(\tilde{l})_1^2 + (\tilde{l})_2^2 + (\tilde{l})_3^2 = \tilde{l}\tilde{K}^{-1}\tilde{l} = lK^{-1}l$. Therefore, $\langle V_l V_l^\dagger\rangle = (x - vt)^{-lK^{-1}l}$ in the plasma phase when $lK^{-1}\boldsymbol{m}_1 = 0$.

When $lK^{-1}\boldsymbol{m}_1 \neq 0$, the interaction potential is a complex function [4]. As the particles in the Coulomb gas move around the test charge, the partition function can have arbitrary phases which average out to zero, unless the two test charges sit at the same space-time point. Therefore, we expect V_l' to have a short ranged correlation in the plasma phase. As a consequence V_l also has a short ranged correlation in the plasma phase when $lK^{-1}\boldsymbol{m}_1 \neq 0$.

In the above we have assumed that $p_1 = 0$. If $p_1 \neq 0$ then we need $\Gamma > \mu$ to open an energy gap in the $\tilde{\phi}_{1,2}$ sector and to be in the one-branch phase. All the above results remain to be valid if we regard the plasma phase mentioned above as the one-branch phase.

Now let us apply the above results to the correlation of the electron operator given by V_{l_e}, where l_e is given in Eq. (4). We find that $l_e K^{-1}\boldsymbol{m}_1 = 5 - 3(2k_1 + k_2)$, which can never vanish for integer $k_{1,2}$. This means that the electron correlation is short ranged in space-time in the one-branch phase. *It costs a finite energy to add (remove) an electron to (from) the edge in the one-branch phase.*

We next consider a more general n-electron operator described by $l_{ne}^T = [k_1, k_2, -5(n - k_1 - k_2)]$. We find that $l_{ne}K^{-1}m_1 = 5n - 3(2k_1 + k_2)$. Thus, in the one-branch phase, n-electron operator is gapless if and only if n is in multiples of 3. The correlation of the $3m$-electron operator $V_{l_{(3m)e}}$ has an algebraic decay if $k_2 = 5m - 2k_1$. The exponent is $5m^2$.

For a generic quasiparticle operator V_l, we find that $lK^{-1}m_1 = 0$ requires $l_3 = 2l_2 - l_1$. Only those quasiparticles are gapless. The exponent of the quasiparticle correlation is found to be $(2l_1 + l_2)^2/5$ and the charge of the quasiparticle is $3(2l_1 + l_2)/5$.

We see that the edge excitations in the one-branch phase are exactly those of the 1/5 Laughlin state. But the particles that form the Laughlin state carry charge $3e$. Such a state is described by $K = (5)$ and $t = (3)$. Thus the transition from the three-branch phase to the one-branch phase on the edge of the $\nu = 9/5$ state can be viewed as a binding transition in which electrons form triplet bound states.

There is another way to view the above results. We know that after edge reconstruction [6], the K matrix that describes the bulk state and the K matrix that describes the edge state may be different: $K_{\text{edge}} = K_{\text{bulk}} \oplus K' \oplus (-K')$. Through a $SL(3, Z)$ transformation, the (K, t) of the $\nu = 9/5$ state Eq. (3) is equivalent to $K = \text{Diag}(5, 1, -1)$, $t^T = (3, 1, 1)$. This (K, t) can be viewed as the edge K matrix K_{edge}, describing the reconstructed edge of the charge-$3e$ Laughlin state $(K, t) = (5, 3)$. Therefore, the $\nu = 9/5$ state Eq. (3) is really the charge-$3e$ Laughlin state $(K, t) = (5, 3)$. The apparent difference on edge is due to a edge reconstruction.

To summarize, we know that $K = \text{Diag}(1, 1, -5)$ QH state usually has three edge branches. However, if the charge transfer described by V_{m_1} (see Fig. 2) is strong enough, it will cause a binding transition on the edge. A strong V_{m_1} can be caused by a strong interaction between the three edge branches. If it is relevant [i.e., $\Delta(m_1) < 2$] and carries zero momentum [i.e., $p_1 = 0$ in Eq. (12), even a weak V_{m_1} will cause a binding transition.

To see the physical effect of the binding transition, let us consider tunneling between the $\nu = 9/5$ and the $\nu = 1$ (metallic) states [10]. Before the binding transition, all three branches of the $\nu = 9/5$ state contribute to the exponent of the tunneling conductance at finite temperature. After binding transition, the first two branches become gapped, and there is only one gapless mode. Furthermore, the single electron tunneling also opens up a finite gap. Only three-electron cotunneling is gapless, which gives
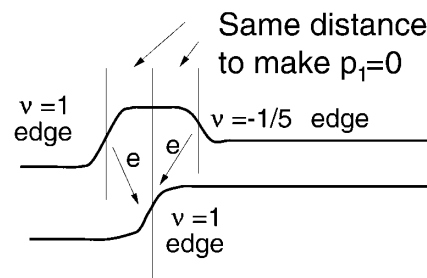

FIG. 2. The edge of the $\nu = 9/5$ bilayer state and the charge transfer caused by the V_{m_1} operator.

$I \propto V^{13}$ at zero temperature and $\frac{dI}{dV}|_{V=0} \propto T^{12}$ at finite temperatures.

Experimentally, the $\nu = 9/5$ state is not spin polarized. The tunneling process that causes the binding transition also flip spins for single-layer systems. Thus, strong spin-orbit coupling is necessary to see the binding transition in single layer systems. It should be easier to observe the binding transition in the bilayer $\nu = 9/5$ state.

[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982); R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2] X.-G. Wen, Phys. Rev. B **40**, 7387 (1989); Int. J. Mod. Phys. B **4**, 239 (1990); X.-G. Wen and Q. Niu, Phys. Rev. B **41**, 9377 (1990).

[3] For a review, X.-G. Wen, Adv. Phys. **44**, 405 (1995).

[4] B. Blok and X.-G. Wen, Phys. Rev. B **42**, 8133 (1990); Phys. Rev. B **42**, 8145 (1990); N. Read, Phys. Rev. Lett. **65**, 1502 (1990); J. Fröhlich and T. Kerler, Nucl. Phys. **B354**, 369 (1991); X.-G. Wen and A. Zee, Phys. Rev. B **46**, 2290 (1992).

[5] S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58**, 1252 (1987); S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989); N. Read, Phys. Rev. Lett. **62**, 86 (1989).

[6] A. H. MacDonald, S. R. Eric Yang, and M. D. Johnson, Aust. J. Phys. **46**, 345 (1993); C. de C. Chamon and X. G. Wen, Phys. Rev. B **49**, 8227 (1994).

[7] F. D. M. Haldane, Phys. Rev. Lett. **74**, 2090 (1995).

[8] J. E. Moore and X.-G. Wen, Phys. Rev. B **57**, 10 138 (1998).

[9] $\exp\{i(m_{1,2})_j \phi_j\}$ may carry a finite momentum, and the factor $e^{ip_{1,2}x}$ is included to shift the momentum of $V_{m_{1,2}}$ to zero.

[10] The tunneling between the $\nu = 1/3$ and the $\nu = 1$ (metal) states has been studied by A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996).