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Modeling and Vibration Control of a Drum-Type Washing Machine via MR Fluid Dampers

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Abstract—In this paper, a multibody dynamic model is developed for a front loading type washing machine in details. This model is constructed for verifying the bearing model between the tub and drum, and for analyzing the suspension system composed by two springs and magnetorheological dampers between the case and basket. The obtained results help engineers to design washing machines efficiently.

Keywords – drum type washing machine, MR fluid damper

I. INTRODUCTION

Washing machines had been developed to be automatic machines with several milestones. Automatic washing machines can be characterized into two types according to loading types, namely, top loading model with vertical axis and front loading type with horizontal axis. Front loading type had become standard in Europe, and top loading models were popular in America and Asia. Recently, front loading type has been increased in the United States and Asia.

To reduce vibration and oscillatory walk, one type of dynamic balancer that can be used of washing machines is a hydraulic balancer, which contains salt water and is attached at the upper rim of the basket. The liquid in the balancer moves to the opposite side of unbalance automatically due to inherent nature of fluids when the rotational speed is higher than the critical speed of the spinning drum [1]. Another active balancing that counteracts vibrations is to use two balancing masses. In this method, two balancing masses move along the rim of the basket. The rotation plane of the balancing masses can be easily chosen to be wherever judged suitable, always targeting at the reduction of the induced moments [2].

For the vibration caused by the imbalance of clothes during the varying spin-drying stage in a drum-type washing machine requiring the varying damping force to be reduced to get the results of vibration reduction that we need. Recently, a new way of using low-cost, controllable magnetorheological (MR) sponge dampers was developed [3], which is appropriate for the next-generation high-performance horizontal-axis washing machines when a moderate-force is desired. The control system is implemented via a semi-active magnetorheological (MR) damper located on the suspension that links the basket to the cabinet. The suspension is to damp the basket movements and to reduce the vibrations transmitted

to the chassis, which are strictly related to the perceived acoustic noise. The idea is to use electronically-controlled MR dampers to adapt the on-line damping characteristics to reduce vibration level of the washing machine panels was proposed in [4].

This paper is organized as follows. Section 2 describes...

II. WASHING MACHINE MODEL

A model of a horizontal-axis washing machine is shown in Fig. 1. Three coordinate systems are adopted in this system:

XYZ : inertial coordinate system, attached to the ground at the initial position of center of mass of washing machine, G (see Fig. 1).

$x_b y_b z_b$: body coordinate system, attached to the basket at its center of mass, G_b (Fig. 2).

$x_d y_d z_d$: body coordinate system, attached to the drum at its center of mass, G_d (see Fig. 3).

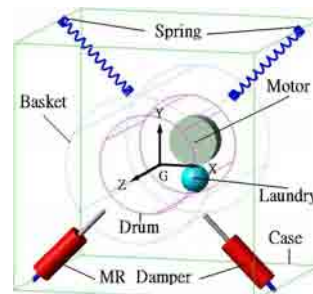


Fig. 1. Schematic View of a Horizontal-axis Washing Machine

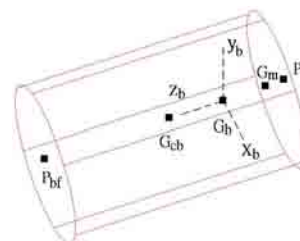


Fig. 2. Body Coordinate System for Basket

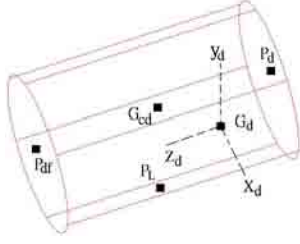


Fig. 3. Body Coordinate System for Drum

In this washing machine system model, we assume that the basket is a thin cylinder with the front end opened, the rear side sealed with a thin disk, and the motor is attached to the rear side. We also assume that the drum is a thin cylinder with a similar structure as the basket.

A. Bearing between Basket and Drum

The bearing system installed between the tub and the drum was modeled with three degrees of freedom (see Fig. 4) [5], that is,

- 1) the first one is for the vertical motion in planar joint,
- 2) the second one is for horizontal motion in planar joint, and
- 3) the third one is for rotation of the drum.

Although this bearing model does not correctly predict the acceleration in the vertical direction as was pointed out in [5] when the drum is loaded with unbalanced laundry, this model is adopted in this work for simplicity.

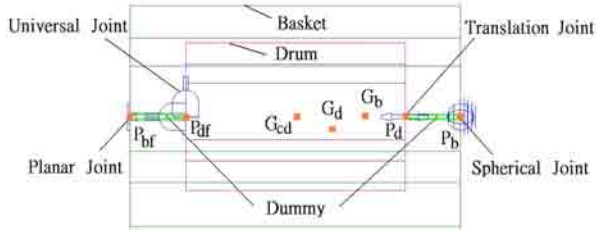


Fig. 4. Schematic View of Bearing Model between Basket and Drum

B. Contact Force between Tub and Drum

To present real motions between the drum and bearings, contact force was used. The algorithm of contact was selected as shown in equation (2.1). This makes normal force F_n and tangential force F_t when the drum impacts the front bearing as shown in Fig. 5.

Normal force F_n was calculated from equation (2.1), and tangential force F_t was applied with assumption of Coulomb friction [5].

$$F_n = k \times g^e + \text{Step}(g, 0, 0, d_{\max}, c_{\max}) \frac{dg}{dt}, \quad (2.1)$$

where

- k is the stiffness of boundary surface,
- g is the gap size between two boundaries,

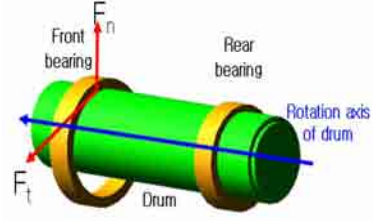


Fig. 5. Contact Force between Tub and Drum [5]

d_{\max} means the penetration depth, and c_{\max} is the damping coefficient.

III. THREE WALK MODES FOR A WASHING MACHINE

It has been shown in [2] that the minimum critical spin speed of the impending rotational slip

$$\omega_{rot} = \sqrt{\frac{fg(m_c \frac{x_1}{x_2} + m_L)}{m_L(r_d - r_L)\sqrt{1 + f^2}}}, \quad (3.1)$$

where

r_d and r_L are the radius of drum and laundry, respectively,

f is a constant Coulomb friction coefficient,

and the other variables are defined in Fig. 6. In the mean time, as $\frac{x_1}{x_2} = 1$, the washer will not rotate, the critical spin speed becomes for the impending translational slip

$$\omega_{tra} = \sqrt{\frac{fg(m_c + m_L)}{m_L(r_d - r_L)\sqrt{1 + f^2}}}, \quad (3.2)$$

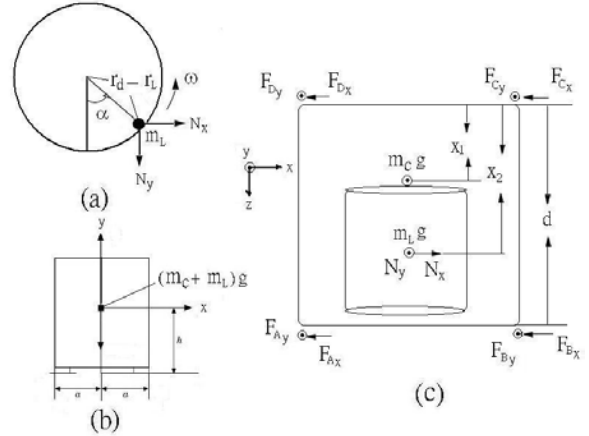


Fig. 6. (a) Laundry Forces on The Drum (b) The Front View of a Horizontal Axis Washing Machine (c) The Free Body Diagram of The Horizontal Axis Washing Machine in The Bird's Eye View [2]

From [6] the critical spin speed of the impending tipping condition is

$$\omega_{tip} = \sqrt{\frac{a}{h} \frac{g(m_c + m_L)}{m_L(r_d - r_L)\sqrt{1 + (\frac{a}{h})^2}}}, \quad (3.3)$$

where the variables are defined in Fig. 6 (b). A washing machine in three dimensions can in principle have three walk

modes : tip, rotational slip and translational slip. With the comparisons among equations (3.1), (3.2) and (3.3), we can find the relationship among ω_{rot} , ω_{tra} and ω_{tip} as $\omega_{rot} < \omega_{tra} < \omega_{tip}$ under two conditions as followings

$$\frac{x_1}{x_2} \leq 1, \quad f \leq \frac{a}{h}.$$

When the laundry is rotated across the angle α , three walk modes for a washing machine will be induced under these conditions as followings

When $\omega_{tra} < \omega < \omega_{tip}$, the washing machine will be moved along the positive x-direction axis,

When $\omega_{tip} < \omega$, the washing machine will be moved up and down heavily and turned over finally.

Where ω is the spin speed of the drum. ω_{rot} , ω_{tra} , ω_{tip} and ω are all constant values. On the contrary, a washing machine has no movements under this condition when the laundry is rotated across the angle α

$$\omega < \omega_{rot} < \omega_{tra} < \omega_{tip}.$$

In the practical analysis of walk, tipping is considered a safety hazard in the industry and is not allowed to occur. So, from equations (3.2) and (3.3), the design criterion for a washing machine becomes [6]

$$f < \frac{a}{h}.$$

IV. CONTROL SCHEME FOR THE WASHING MACHINE

A. Modified Bouc-Wen Model for MR Damper

A modified Bouc-Wen model (see Fig. 7) for better predicting the response of the MR damper in the region of the yield point was proposed by [7] and is adopted in this work.

The equations governing the force F_{rh} exerted by the MRF damper are

$$\begin{aligned} \dot{x}_1 - \dot{y} &= \frac{1}{c_o + c_1} [-k_o(x_1 - y) - \alpha z + c_1 \dot{x}_{p1}], \\ \dot{z} &= (\dot{x}_1 - \dot{y}) \{ \delta - |z|^n [\beta + \gamma \text{sgn}(\dot{x}_1 - \dot{y}) \text{sgn}(z)] \}. \end{aligned}$$

$$\begin{aligned} F_{rh} &= -c_1(\dot{x}_1 - \dot{y}) + c_1 \dot{x}_{p1} + k_1(x_{p1} - \bar{x}_o), \\ &= \frac{c_1}{c_o + c_1} [k_o(x_1 - y) + \alpha z] \\ &\quad + \frac{c_o c_1}{c_o + c_1} \dot{x}_{p1} + k_1(x_{p1} - \bar{x}_o). \end{aligned}$$

The voltage dependent parameters are modeled by

$$\alpha = \alpha_a + \alpha_b u, \quad c_o = c_{o_a} + c_{o_b} u, \quad c_1 = c_{1_a} + c_{1_b} u.$$

Furthermore, the command voltage is accounted for through the first-order filter

$$\dot{u} = -\eta(u - v), \quad u(0) = 0.$$

The command input v is confined to be finite positive to reflect the real situation of the saturation of the magnetic field in the MR fluid damper [7].

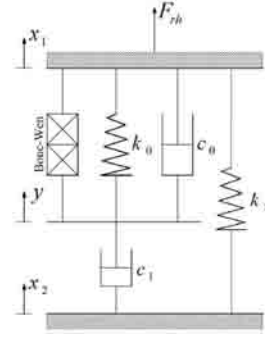


Fig. 7. Modified Bouc-Wen Model

B. Controller Synthesis

In this work the schematic of a semiactive control system based on an MR fluid damper is illustrated in Fig. 8. The MR damper based semiactive control system consists of a system controller and a voltage algorithm. The system controller generates the desired voltage according to the dynamic responses of the plant while the voltage algorithm adjusts the desired voltage to meet the range of the value for the voltage between 0v ~ 2.25v being under the saturation condition.

The two measurements a_r and a_L in Fig. 8 are as followings

$$\begin{aligned} a_r &= a_{top} \cdot \vec{j}_r + a_{side} \cdot \vec{j}_r, \\ a_L &= a_{top} \cdot \vec{j}_L + a_{side} \cdot \vec{j}_L. \end{aligned}$$

We present a PI control strategy to control the vibration of the suspension system. The values of the desired voltage for V_{rd} and V_{Ld} in Fig. 8 can be defined as followings

$$\begin{aligned} V_{rd} &= K_P \cdot a_r + K_I \cdot \int a_r dt, \\ V_{Ld} &= K_P \cdot a_L + K_I \cdot \int a_L dt. \end{aligned}$$

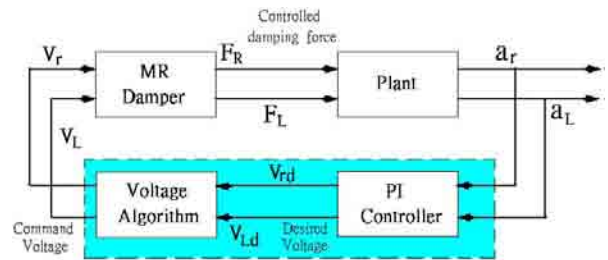


Fig. 8. Semi-active Control Systems for a Plant Integrated with MR Fluid Damper

The voltage algorithm for varying the applied voltage V_r and V_L can be obtained in Fig. 8 as followings

If $V_{rd} > 2.25v$, $V_r = 2.25v$. The same result is also for V_{Ld} .

If $0v < V_{rd} < 2.25v$, $V_r = V_{rd}$. The same result is also for V_{Ld} .

If $V_{rd} < 0v$, we substitute V_{rd} with $|V_{rd}|$ and then follows the above principles as followings.

If $|V_{rd}| > 2.25v$, $V_r = 2.25v$. The same result is also for $|V_{Ld}|$.

If $0v < |V_{rd}| < 2.25v$, $V_r = |V_{rd}|$. The same result is also for $|V_{Ld}|$.

V. NUMERICAL EXAMPLES

In this example, the PI controller synthesis is implemented to provide the desired voltages V_{rd} and V_{Ld} , and the voltage algorithm is adopted to get the applied voltage V_r and V_L for two MR dampers in the range between $0v \sim 2.25v$. The system parameters with Recurdyn are given in Table I, Table II and Table III for the washing machine, MR damper and contact force, respectively. The parameters LE , W , H , MA mean the length, width, height and mass, respectively. The parameters C , V , U mean the coefficient, value and unit, respectively.

TABLE I
PARAMETERS FOR WASHING MACHINE

| Body | LE | W, H | MA |
|---------|------------|------------|-------|
| Case | 500mm | 500mm | 100kg |
| Basket | 450mm | 150mm | 8kg |
| Drum | 300mm | 100mm | 4kg |
| Motor | 37.5mm | 50mm | 7kg |
| Laundry | 28.28727mm | 28.28727mm | 13kg |

TABLE II
PARAMETERS FOR MR DAMPER (LORD, RD-1005-3) IN MODIFIED
BOUC-WEN MODEL

| C | V | U | C | V | U |
|------------|-----------|--------------------|-------------|------|------------------|
| c_{oa} | 2.1 | $N \text{ s/mm}$ | α_b | 69.5 | $N/V \text{ mm}$ |
| c_{ob} | 0.35 | $N \text{ s/V mm}$ | γ | 3.63 | mm^{-2} |
| c_{1a} | 28.3 | $N \text{ s/mm}$ | β | 3.63 | mm^{-2} |
| c_{1b} | 0.295 | $N \text{ s/V mm}$ | δ | 301 | |
| k_o | 4.69 | N/mm | n | 2 | |
| k_1 | 30.425176 | N/mm | η | 190 | s^{-1} |
| α_a | 14 | N/mm | \bar{x}_o | 0 | mm |

TABLE III
PARAMETERS FOR CONTACT FORCE

| C | V | U | C | V | U |
|-----------|-------|---------------|-----------|-----|------------------|
| k | 2.0E5 | N/mm | c_{max} | 1 | $N \text{ s/mm}$ |
| g | 20 | mm | β | 2.5 | |
| e | 2.2 | | μ_s | 0.3 | |
| d_{max} | 10 | mm | μ_k | 0.1 | |

The parameters used in PI controller are chosen to be

$$K_P = 2.9, \quad K_I = 12.35.$$

It is clear that the vibrations of the basket and case are suppressed after two MR dampers work. The open-loop (left) and closed-loop (right) simulations are shown in Fig. 9 - 16 for comparison purposes.

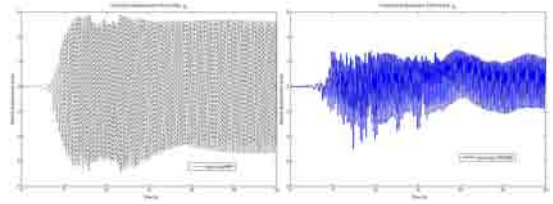


Fig. 9. Horizontal Displacement of The Basket

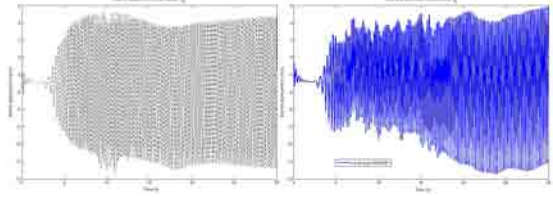


Fig. 10. Vertical Displacement of The Basket

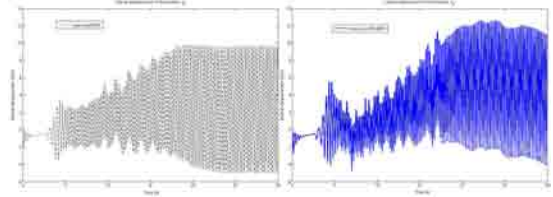


Fig. 11. Lateral Displacement of The Basket

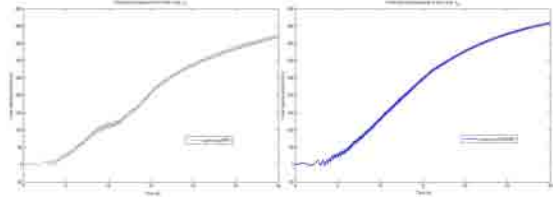


Fig. 12. Horizontal Displacement of The Case

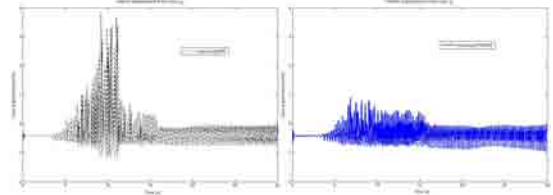


Fig. 13. Vertical Displacement of The Case

VI. CONCLUSIONS

In this paper, a multibody dynamic model of a drum-type washing machine with MR fluid dampers was generated in a commercial package "RecurDyn". The resultant displacements from Recurdyn-Simulink simulation were conducted by utilizing this virtual system. The following conclusions were obtained.

- 1) Owing to this virtual system of a washing machine, we able change the parameters of the washing machine to

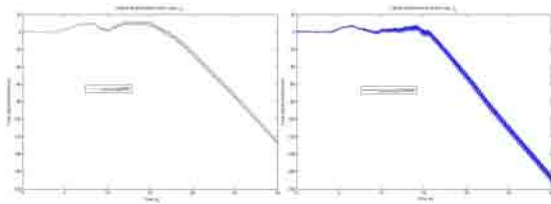


Fig. 14. Lateral Displacement of The Case

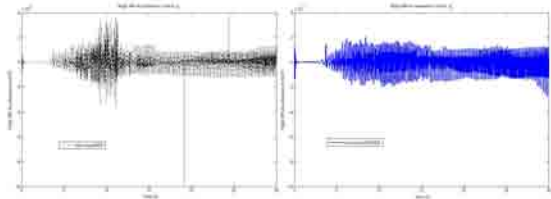


Fig. 15. Case Acceleration in Right MR Damper Direction

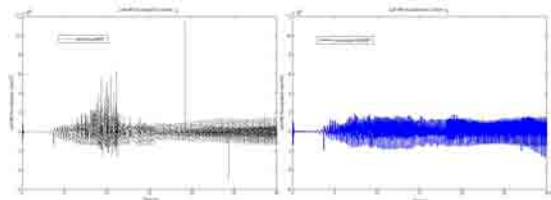


Fig. 16. Case Acceleration in Left MR Damper Direction

what are required, and relocate the springs and MR dampers at the ours will efficiently.

- 2) From the comparison of the above figures, we can see that the PI control strategy is the best for reducing the vibrations of the bakset and the case at the same time.

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APPENDIX: NOMENCLATURE

| | |
|---------------------|---|
| P_L | center of mass of the laundry. |
| m | mass. |
| ϕ, θ | Euler angles. |
| r, h | radius and length, respectively. |
| a_{top}, a_{side} | acceleration in the center of top and left location of the case, respectively. |
| j_r, j_L | direction of movements for the right and left MR damper, respectively. |
| k | stiffness of the boundary surface interaction. |
| c, f | damping and friction coefficient. |
| g | gap size between two boundaries. |
| σ | gap size variation w.r.t. time. |
| e | spring force of restitution for non-linear characteristics, $e \geq 2.1$ the results run better using this exponent value in the impact function. |
| β | damping force of restitution for non-linear characteristics. |
| x_1-y, x_{p1} , | internal and external relative displacement and hysteretic component, respectively. |
| z | initial displacement. |
| \bar{x}_o | initial displacement. |
| η | the delay time of the MR damper. |
| v | command voltage sent to the current driver. |
| Subscripts, | |
| b, d, c | basket, drum, case, respectively. |
| L, m | laundry and motor, respectively. |
| cb, cd | thin cylinder belonging to the basket and drum, respectively. |
| bp, dp | thin disk belonging to the basket and drum, respectively. |
| bf, df | center point of the basket and drum in the front side, respectively. |