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Observer synthesis for the T–S fuzzy system with uncertainty and output disturbance

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Abstract. The paper proposes a novel fuzzy observer synthesis for the Takagi–Sugeno (T–S) fuzzy system with uncertainty and output disturbance. First, an augmented fuzzy model is built by integrating the system state and the output disturbance into a new variable. Then, based on Lyapunov theory and LMIs tools, two main theorems are derived for particular and general cases of fuzzy systems, respectively. In each main theorem, three key conditions are proposed, under which the fuzzy observer is synthesized to estimate the system state and the output disturbance simultaneously. According to the main theorems, a methodical procedure for the fuzzy observer synthesis is also provided. Finally, the effectiveness of the observer is demonstrated by a numerical example.

Keywords: Augmented model, fuzzy system models, linear matrix inequalities (LMIs), state observer, uncertainty and disturbance

1. Introduction

In practical control application, some state variables may not be available, but prerequisite in the control design. Therefore, the state estimation is necessary to achieve the control objective [12]. Over the past decades, the study of state observers design for linear systems has been one of the most active research areas. Various approaches for the observer design have been proposed, such as transfer function methods, geometric-observer, differential algebraic model, and singular value decomposition [15, 16, 28]. Moreover, the applications of state observer can be also found in a wide range of different fields [15].

Since the time-varying uncertainties (structured or unstructured) are inevitable in many practical linear systems, the control of the linear systems with time-varying uncertainties has become an important research topic in the engineering areas [5, 17, 24]. The conventional observer, Luenberger's observer [12], fails to consider handling the uncertainties which appear in the estimated systems. However, the paper [3] proposed a proportional integral observer design for single-output uncertain linear systems in which either measurement noise or modeling error is attenuated. The proportional multiple-integral observer design was provided by [6] for descriptor systems with measurement noise. There is a pioneer paper investigating the observer-controller design for T–S fuzzy system [9] in which the stability criterion and robust area with respect to a single-grid-point (SGP) in the parameter space are derived. Its alternative to settle an uncertain nonlinear system with inaccessible system states subjected to large parameter

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uncertainties is to design a number of local observer-controllers dedicated to different operating conditions of the plant (grid-point).

The fuzzy control techniques have been widely applied to the systems with unclear model. The fuzzy control with IF–THEN rules can combine other control methods to reach the control objective due to its capability of modeling complex nonlinear systems [4, 10, 11, 22, 25, 26]. Among various kinds of fuzzy models, Takagi–Sugeno (T–S) fuzzy model [18] is a typical and important class of fuzzy model. It has recently gained much popularity because of its special rule consequent structure. Furthermore, T–S fuzzy system has many successful applications in the functional approximation and controller synthesis. Therefore, T–S fuzzy model has been a popular model for the system analysis and control synthesis in nonlinear control systems [18, 20, 23].

Many papers have been studying the observer design for fuzzy T–S systems by using different techniques, such as the linear matrix-inequality approach, the sliding mode technique, and the adaptive method to design the fuzzy observer [1, 13, 14, 21, 27]. However, most of the existing fuzzy-observer cannot deal with the estimation problem for the fuzzy systems with output disturbance [8], since the output disturbance maybe amplified by the conventional observer gain matrices. Although there were remarkable successes in the observer design for linear crisp system with measurement noise [7], the observer technique for the crisp system cannot be directly utilized in the fuzzy observer design because of the existence time-varying weights in the fuzzy inference engine. Therefore, the way to improve the fuzzy observer design for T–S fuzzy systems with output disturbances is worth being studied. The paper [8] studied the observer design technique for the T–S fuzzy system with output disturbance. However [8], did not deal with the T–S fuzzy system with uncertain parameters.

In this paper, the problem to be handled is the state/disturbance estimation subjected to the influence of unknown output disturbance and uncertainty in the T–S fuzzy system. The work proposes a novel fuzzy observer to accurately estimate the system state and the output disturbance at the same time where the uncertainty and output disturbance exist in the T–S fuzzy simultaneously. First, an augmented model is generated for the uncertain T–S fuzzy system. Then, based on Lyapunov theory and LMIs tools [2, 19], two main theorems are derived for particular and general cases of fuzzy systems, respectively. In each main theorem, three key conditions are proposed under which the fuzzy observer is synthesized to estimate the system state and the output disturbance simultaneously. According to the main theorems, a methodical procedure of the fuzzy observer synthesis is also provided. Finally, the effectiveness of the observer is demonstrated by a numerical example.

The paper is organized as follows. After the introduction, in Section 2, several notations and problem statement are presented. In Section 3, the fuzzy state/disturbance observer is synthesized and the conditions ensuring global and asymptotic convergence of estimation error are derived as a set of LMI terms. A numerical example is given in Section 4 to demonstrate the observer synthesis procedure and the effectiveness of the proposed observer. The last section gives a conclusion.

2. Problem description

A T–S fuzzy model described by IF–THEN rules is considered, which represents the local linear input-output relation from a nonlinear uncertain system. The *ith* rule of the fuzzy model is shown as the following form

Plant Rule i

IF θ_i is $\mu_{i1}$and ... and θ_s is μ_{is}, THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i(t))x(t) + B_i u(t), \quad i = 1, \ldots, r, \\ y(t) = C_i x(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the unavailable state, $u(t) \in R^m$ is the control, $y(t) \in R^p$ is the output, θ_j $(j = 1, 2, \ldots, s)$ are the premise variables, and μ_{ij} $(i = 1, 2, \ldots, r,\ j = 1, 2, \ldots, s)$ are the fuzzy sets that are characterized by membership functions, r is the number of IF–THEN rules and s is the number of the premise variables. The matrices A_i, B_i and C_i are with appropriate dimensions and the time-varying uncertainty $\Delta A_i(t)$ is bounded as $\|\Delta A_i\| \le \varepsilon_i$. The overall fuzzy model achieved from plant rules in (1) is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \beta_i(\theta)\{(A_i + \Delta A_i(t))x(t) + B_i u(t)\} \\ y(t) = \sum_{i=1}^{r} \beta_i(\theta) C_i x(t), \end{cases} \tag{2}$$

where $\theta = [\theta_1, \ldots, \theta_s]$, $\beta_i(\theta) = (\psi_i(\theta))/(\sum_{i=1}^{r}\psi_i(\theta))$, $\psi_i(\theta) = \prod_{j=1}^{s}\mu_{ji}(\theta)$. Hence, $\beta_i(\theta)$ is regarded as the normalized weight of each IF–THEN rule and $\beta_i(\theta) \geq 0$, $\sum_{i=1}^{r} \beta_i(\theta) = 1$. The aforementioned model does not include the output disturbance. In practical cases, it is possible that the output contains an unknown disturbance as (3)

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \beta_i(\theta)\{(A_i + \Delta A_i(t))x(t) + B_i u(t)\} \\ y(t) = \sum_{i=1}^{r} \beta_i(\theta) C_i x(t) + \omega(t), \end{cases} \tag{3}$$

where $\omega(t) \in R^p$ stands for the unknown output disturbance. Since the state x is unavailable and the disturbance ω is unknown, the objective of this study is to synthesize a fuzzy observer to estimate the state x and the output disturbance ω. The conventional fuzzy observer is constructed as

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} \beta_i(\theta) \left\{ A_i \hat{x}(t) + B_i u(t) - L_i \left(y(t) - \sum_{j=1}^{r} C_j \hat{x}(t) \right) \right\}, \tag{4}$$

where $\hat{x}(t) \in R^n$ is the estimate of the state *x(t)* and $L_i \in R^{n\times p}$ is a gain matrix to be found. By using $e(t) = x(t) - \hat{x}(t)$, the following error dynamics is obtained from (4)

$$\dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \beta_i(\theta) \beta_j(\theta) \times \{(A_i + L_i C_j)e(t) + L_i \omega(t) + \Delta A_i(t) x(t)\}. \tag{5}$$

It is seen that the output disturbance $\omega(t)$ is amplified by L_i, *i = 1,..., r*, times by using the fuzzy observer (4) [12], namely, increasing estimation rate may cause the disturbance to be amplified. Furthermore, $\Delta A_i(t)x(t)$ cannot be handled by (4) either. Therefore, based on the above analysis, the conventional fuzzy observer should be modified to handle such system which contains uncertainty and unknown disturbance simultaneously.

3. Fuzzy observer synthesis

First, let us consider the particular case which has equal output matrices in (3) i.e., $C_i = C$ for all i. Subsequently, this technique will be applied to the general case in which the system contains different output matrices.

3.1. The particular case

If the output matrices of all subsystems are equal.

$$C_1 = C_2 = \ldots = C_r = C \tag{6}$$

Then, (3) becomes

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \beta_i(\theta)\{(A_i + \Delta A_i)x(t) + B_i u(t)\} \\ y(t) = Cx(t) + \omega(t). \end{cases} \tag{7}$$

Rewrite (7) as the following augmented uncertain fuzzy system

$$\begin{cases} \overline{E}\dot{\overline{x}}(t) = \sum_{i=1}^{r} \beta_i(\theta)\{(\overline{A}_i + \Delta\overline{A}_i)\overline{x}(t) + \overline{B}_i u(t) + \overline{N}\omega(t)\} \\ y(t) = \overline{C}\overline{x}(t) = C_0 \overline{x}(t) + \omega(t), \end{cases} \tag{8}$$

where

$$\begin{gathered} \overline{x}(t) = \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix}, \ \overline{E} = \begin{bmatrix} I_n & O_p \\ O_p & O_p \end{bmatrix}, \ \overline{A}_i = \begin{bmatrix} A_i & O_p \\ O_p & -I_p \end{bmatrix}, \ \Delta\overline{A}_i = \begin{bmatrix} \Delta A_i & O_p \\ O_p & O_p \end{bmatrix}, \\ \overline{B}_i = \begin{bmatrix} B_i \\ O_{p\times m} \end{bmatrix}, \ \overline{N} = \begin{bmatrix} O_{n\times p} \\ I_p \end{bmatrix}, \ \overline{C} = \begin{bmatrix} C & I_p \end{bmatrix}, \text{ and } C_0 = \begin{bmatrix} C & O_p \end{bmatrix}. \end{gathered} \tag{9}$$

Moreover, O_p denotes a $p \times p$ zero square matrix and $O_{n\times p}$ is a $n \times p$ zero matrix. Since, $\|\Delta A_i\| \leq \varepsilon_i$, it yields $\|\Delta\overline{A}_i\| \leq \varepsilon_i$. Suppose $\Delta\overline{A}_i$ can be represented as the form below

$$\Delta\overline{A}_i = M U_i^T, \tag{10}$$

where M is a constant matrix with dimensions, but U_i is an uncertainty matrix and satisfies $\|U_i\| \leq \gamma_i$. M and U_i will be determined later. Add $\overline{L}\dot{y}(t)$ to both sides of (8)

$$\begin{cases} (\overline{E} + \overline{LC})\dot{\overline{x}}(t) = \sum_{i=1}^{r} \beta_i(\theta) \left\{ (\overline{A}_i + \Delta\overline{A}_i)\overline{x}(t) + \overline{B}_i u(t) + \overline{N}\omega(t) + \overline{L}\dot{y}(t) \right\} \\ y(t) = \overline{C}\overline{x}(t) = C_0 \overline{x}(t) + \omega(t), \end{cases} \tag{11}$$

where $\overline{L} \in R^{(n+p)\times p}$ will be determined later as well. Let $\omega(t) = y(t) - C_0\overline{x}(t)$, then (11) becomes

$$(\overline{E}+\overline{LC})\dot{\overline{x}}(t)=\sum_{i=1}^{r}\beta_i(\theta)\{\overline{A}_{0i}\overline{x}(t)+\Delta\overline{A}_i\overline{x}(t)+\overline{B}_i u(t) +\overline{N}y(t)+\overline{L}\dot{y}(t)\}, \tag{12}$$

where $\overline{A}_{0i} = \overline{A}_i - \overline{N}C_0$. Suppose that, the pairs $(\overline{A}_{0i}, C)$ is observable, the fuzzy observer is proposed.

$$\begin{cases} (\overline{E}+\overline{LC})\dot{\hat{\overline{x}}}(t) = \sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{A}_{0i}\hat{\overline{x}}(t)+\overline{B}_i u(t)+\overline{H}_i(y(t)-\hat{y}(t))+\Lambda_i(t)+\overline{N}y(t)+\overline{L}\dot{y}(t)\right\} \\ \hat{y}(t) = \overline{C}\hat{\overline{x}}(t), \end{cases} \tag{13}$$

where $\hat{\overline{x}}(t) \in R^{n+p}$ is the estimate of $\overline{x}(t)$, and $\hat{y}(t) \in R^p$ is the observer output. Here, the gain $\overline{H}_i \in R^{(n+p)\times p}$ and the vector $\Lambda_i(t) \in R^{n+p}$ will be determined to force the estimation error converging to zero.

$$\text{Let} \quad \overline{e}(t) = \overline{x}(t) - \hat{\overline{x}}(t), \tag{14}$$

$$e_y(t) = y(t) - \hat{y}(t) = \overline{C}\overline{e}(t). \tag{15}$$

From (12) – (15), the error dynamics is shown as follows

$$\begin{aligned}(\overline{E}+\overline{LC})\dot{\overline{e}}(t) &= \sum_{i=1}^{r}\beta_i(\theta)\{(\overline{A}_{0i}-\overline{H}_i\overline{C})\overline{e}(t) +\Delta\overline{A}_i\overline{x}(t)-\Lambda_i(t)\} \\ &= \sum_{i=1}^{r}\beta_i(\theta)\left\{(\overline{A}_{0i}-\overline{H}_i\overline{C})\overline{e}(t)+\Delta\overline{A}_i\overline{x}(t) +\Delta\overline{A}_i\hat{\overline{x}}(t)-\Delta\overline{A}_i\hat{\overline{x}}(t)-\Lambda_i(t)\right\} \\ &= \sum_{i=1}^{r}\beta_i(\theta)\left\{(\overline{A}_{0i}-\overline{H}_i\overline{C}+\Delta\overline{A}_i)\overline{e}(t) +\Delta\overline{A}_i\hat{\overline{x}}(t)-\Lambda_i(t)\right\}.\end{aligned} \tag{16}$$

Because

$$rank\begin{bmatrix}\overline{E}\\ \overline{C}\end{bmatrix} = rank\begin{bmatrix} I_n & 0\\ 0 & 0\\ \hline C & I_p\end{bmatrix} = n+p \tag{17}$$

Then, we can find a $\overline{L} \in R^{(n+p)\times p}$ such that $(\overline{E}+\overline{LC})^{-1}$ exists. Rewrite (16) as the form below

$$\begin{aligned}\dot{\overline{e}}(t) &= \sum_{i=1}^{r}\beta_i(\theta)\Big\{(\overline{E}+\overline{LC})^{-1}\big[(\overline{A}_{0i}-\overline{H}_i\overline{C}+\Delta\overline{A}_i)\overline{e}(t) +\Delta\overline{A}_i\hat{\overline{x}}(t)-\Lambda_i(t)\big]\Big\} \\ &= \sum_{i=1}^{r}\beta_i(\theta)\{(\overline{A}_{1i}-\overline{H}_{1i}\overline{C}+\Delta\overline{A}_{1i})\overline{e}(t) +\Delta\overline{A}_{1i}\hat{\overline{x}}(t)-\Lambda_{1i}(t)\} \\ &= \sum_{i=1}^{r}\beta_i(\theta)\left\{(\overline{A}_{2i}+\Delta\overline{A}_{1i})\overline{e}(t)+\Delta\overline{A}_{1i}\hat{\overline{x}}(t)-\Lambda_{1i}(t)\right\},\end{aligned} \tag{18}$$

where

$$\overline{A}_{1i}=(\overline{E}+\overline{LC})^{-1}\overline{A}_{0i},\ \overline{H}_{1i}=(\overline{E}+\overline{LC})^{-1}\overline{H}_i,$$
$$\overline{A}_{2i}=\overline{A}_{1i}-\overline{H}_{1i}\overline{C},\ \Delta\overline{A}_{1i}=(\overline{E}+\overline{LC})^{-1}\Delta\overline{A}_i, \tag{19}$$

and $\Lambda_{1i}(t)=(\overline{E}+\overline{LC})^{-1}\Lambda_i(t)$. Furthermore, $\|\Delta\overline{A}_i\| \le \varepsilon_i$ implies $\|\Delta\overline{A}_{1i}\| \le \varepsilon_{1i}$, where $\varepsilon_{1i} = \|(\overline{E}+\overline{LC})^{-1}\|\varepsilon_i$. Thus, we have the following theorem.

Theorem 3.1.1. Consider the fuzzy system (3) and fuzzy observer (13). Suppose $\Delta\overline{A}_i$ can be represented as the form (10) and $\|\Delta\overline{A}_{1i}\| \le \varepsilon_{1i}$, then, the estimation errors (18) of state and the output disturbance converge to zero asymptotically if the following three conditions hold simultaneously.

Condition 1: There exist a symmetric positive definite matrix $P = P^T$ and matrix $\overline{H}_i$ such that

$$\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P < 0, \tag{20}$$

where the relationship between $\overline{H}_i$ and $\overline{A}_{2i}$ is shown in (19).

Condition 2: The matrix M satisfies

$$M = \left(\overline{E}+\overline{LC}\right)P^{-1}\overline{C}^T, \quad \forall\, i \in r, \tag{21}$$

Condition 3: The term $\Lambda_{1i}(t)$ is given as the form,

$$\Lambda_{1i}(t)=\begin{cases}\dfrac{\gamma_i\left\|\hat{\overline{x}}(t)\right\|\left\|e_y(t)\right\|}{e_y^T(t)\,e_y(t)}P^{-1}\overline{C}^T e_y(t), & \text{if } e_y \neq 0,\\ 0, & \text{if } e_y = 0\end{cases} \tag{22}$$

where γ_i is the known bound of U_i.

Proof. Let

$$V(t) = \overline{e}^T(t)P\overline{e}(t). \tag{23}$$

The time derivative of *V(t)* along function trajectories of system (18) is as follows.

$$\begin{aligned}
\dot{V}(t) &= \dot{\overline{e}}^T(t)P\overline{e}(t) + \overline{e}^T(t)P\dot{\overline{e}}(t) \\
&= \sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{e}^T(t)\left(\overline{A}_{2i}^T P + P\overline{A}_{2i}\right)\overline{e}(t) + 2\overline{e}^T(t)\,\Delta\overline{A}_{1i}^T P\overline{e}(t) + 2\hat{\overline{x}}^T(t)\,\Delta\overline{A}_{1i}^T P\overline{e}(t) - 2\Lambda_{1i}^T(t)\,P\overline{e}(t)\right\} \\
&\le \sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{e}^T(t)\left(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P\right)\overline{e}(t) + 2\hat{\overline{x}}^T(t)\,\Delta\overline{A}_{1i}^T P\overline{e}(t) - 2\Lambda_{1i}^T(t)\,P\overline{e}(t)\right\}
\end{aligned} \tag{24}$$

Case 1: When $e_y(t) \neq 0$, from (22), we have

$$\begin{aligned}
\Lambda_{1i}^T(t)\,P\overline{e}(t) &= \left[\frac{\gamma_i\left\|\hat{\overline{x}}\right\|\left\|e_y\right\|}{e_y^T e_y}e_y^T(t)\,\overline{C}P^{-1}\right]P\overline{e}(t) \\
&= \frac{\gamma_i\left\|\hat{\overline{x}}\right\|\left\|e_y\right\|e_y^T e_y}{e_y^T e_y} = \gamma_i\left\|\hat{\overline{x}}\right\|\left\|e_y\right\|,
\end{aligned} \tag{25}$$

and with the aids of (10), (19) and (21), it yields

$$\begin{aligned}
&\hat{\overline{x}}^T(t)\,\Delta\overline{A}_{1i}^T P\overline{e}(t) = \hat{\overline{x}}^T(t)\left(U_i\overline{C}P^{-1}\right)P\overline{e}(t) \\
&= \hat{\overline{x}}^T(t)\,U_i e_y(t) \le \left\|\hat{\overline{x}}^T(t)\,U_i e_y(t)\right\| \le \gamma_i\left\|\hat{\overline{x}}(t)\right\|\left\|e_y(t)\right\|.
\end{aligned} \tag{26}$$

Therefore, based on Condition 1,

$$\begin{aligned}
\dot{V}(t) \le &\sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{e}^T(t)\left(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P\right)\overline{e}(t)\right. \\
&\left. -2\gamma_i\left\|\overline{x}(t)\right\|\left\|e_y(t)\right\| + 2\gamma_i\left\|\overline{x}(t)\right\|\left\|e_y(t)\right\|\right\} \\
= &\sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{e}^T(t)\left(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P\right)\overline{e}(t)\right\} < 0.
\end{aligned}$$

Case 2: When $e_y(t) = 0$, let $\Lambda_{1i}(t) = 0$, then (24) becomes

$$\begin{aligned}
\dot{V}(t) \le &\sum_{i=1}^{r}\beta_i(\theta)\Big\{\overline{e}^T(t)(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P)\overline{e}(t) \\
&+2\hat{\overline{x}}^T(t)\Delta\overline{A}_{1i}^T P\overline{e}(t)\Big\} \\
= &\sum_{i=1}^{r}\beta_i(\theta)\Big\{\overline{e}^T(t)(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P)\overline{e}(t) \\
&+2\hat{\overline{x}}^T(t)U_i e_y(t)\Big\} \\
= &\sum_{i=1}^{r}\beta_i(\theta)\Big\{\overline{e}^T(t)(\overline{A}_{2i}^T P + P\overline{A}_{2i} + 2\varepsilon_{1i}P)\overline{e}(t)\Big\} < 0.
\end{aligned} \tag{27}$$

Finally, the proof is successfully completed.

3.2. The general case

The output matrices of all subsystems may not be equal, i.e., $C_i \neq C_j,\ i \neq j$. Then, the T-S fuzzy system can be rewritten as a new form

$$\begin{cases}
\dot{x}(t) = \sum_{i=1}^{r}\beta_i(\theta)\left\{(A_i + \Delta A_i)x(t) + B_i u(t)\right\} \\
y(t) = Cx(t) + \sum_{i=1}^{r}\beta_i(\theta)(C_i - C)x(t) + \omega(t),
\end{cases} \tag{28}$$

where C is the matrix chosen from the set $\{C_1,\ C_2, \ldots,\ C_r\}$. Let $\omega_0(t) = \sum_{i=1}^{r}\beta_i(\theta)(C_i - C)x(t) + \omega(t)$, (28) becomes

$$\begin{cases}
\dot{x}(t) = \sum_{i=1}^{r}\beta_i(\theta)\left\{(A_i + \Delta A_i)\,x(t) + B_i u(t)\right\} \\
y(t) = Cx(t) + \omega_0(t).
\end{cases} \tag{29}$$

Similar to (8), (29) can be rewritten as

$$\begin{cases}
\overline{E}\dot{\overline{x}}_0(t) = \sum_{i=1}^{r}\beta_i\theta)\left\{(\overline{A}_i + \Delta\overline{A}_i)\overline{x}_0(t) + \overline{B}_i u(t) + \overline{N}\omega_0(t)\right\} \\
y(t) = \overline{C}\overline{x}_0(t) = C_0\overline{x}_0(t) + \omega_0(t),
\end{cases} \tag{30}$$

where $\overline{x}_0^T = [\, x(t) \quad \omega_0(t)\,]$ and the state-space system coefficients $\overline{A}_i$, $\overline{B}_i$, $\overline{N}$, $\overline{C}$, C_0, $\overline{E}$, $\Delta\overline{A}_i$ are defined as the same as those in (9). Add $\overline{L}\dot{y}(t)$ to both sides of (30), it yields

$$\begin{cases} (\overline{E}+\overline{LC})\dot{\overline{x}}_0(t) = \sum_{i=1}^{r}\beta_i(\theta)\left\{(\overline{A}_i+\Delta\overline{A}_i)\overline{x}_0(t)+\overline{B}_i u(t)+\overline{N}\omega_0(t)+\overline{L}\dot{y}(t)\right\} \\ y(t) = C_0\overline{x}_0(t)+\omega_0(t), \end{cases} \tag{31}$$

Substitute $\omega_0(t) = y(t) - C_0\overline{x}_0(t)$ into (31)

$$(\overline{E}+\overline{LC})\dot{\overline{x}}_0(t) = \sum_{i=1}^{r}\beta_i(\theta)\Big\{\overline{A}_{0i}\overline{x}_0(t)+\Delta\overline{A}_i\overline{x}_0(t) + \overline{B}_i u(t)+\overline{N}y(t)+\overline{L}\dot{y}(t)\Big\} \tag{32}$$

Let the T-S fuzzy observer be as the following form.

$$\begin{cases} (\overline{E}+\overline{LC})\dot{\hat{\overline{x}}}_0(t) = \sum_{i=1}^{r}\beta_i(\theta)\left\{\overline{A}_{0i}\hat{\overline{x}}_0(t)+\overline{B}_i u(t)+\overline{H}_i(y(t)-\hat{y}(t))+\Lambda_i(t)+\overline{N}y(t)+\overline{L}\dot{y}(t)\right\} \\ \hat{y}(t) = \overline{C}\hat{\overline{x}}_0(t) \\ \begin{bmatrix}\hat{x}(t)\\ \hat{\omega}(t)\end{bmatrix} = \begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r}\beta_i(\theta)(C_i-C) & I_p\end{bmatrix}^{-1}\hat{\overline{x}}_0(t) \end{cases} \tag{33}$$

where $\hat{\overline{x}}_0(t)$ is the estimate of $\overline{x}_0(t)$. Let $\overline{e}_1(t) = \overline{x}_0(t) - \hat{\overline{x}}_0(t)$, and $e_{1y}(t) = y(t) - \hat{y}(t) = \overline{C}\overline{e}_1(t)$.

Therefore, it yields

$$\overline{e}_1(t) = \sum_{i=1}^{r}\beta_i(\theta)\Big\{(\overline{A}_{2i}+\Delta\overline{A}_{1i})\overline{e}_1(t) + \Delta\overline{A}_{1i}\hat{\overline{x}}_0(t)-\Lambda_{2i}(t)\Big\}, \tag{34}$$

where $\overline{A}_{2i}$ and $\Delta\overline{A}_{1i}$ are the same as those in (19) and $\Lambda_{2i}(t) = \left(\overline{E}+\overline{LC}\right)^{-1}\Lambda_i(t)$.

Theorem 3.2.1. Consider the fuzzy system (3) and fuzzy observer (33). Suppose $\Delta\overline{A}_i$ can be represented as the form (10) and $\|\Delta\overline{A}_{1i}\| \le \varepsilon_{1i}$, then the estimation errors (34) of state and the output disturbance converge to zero asymptotically, if the Conditions 1 and 2 in Theorem 3.1.1 and the following Condition 4 hold simultaneously.

Condition 4: The term $\Lambda_{2i}(t)$ is given as the form,

$$\Lambda_{2i}(t) = \begin{cases} \dfrac{\gamma_i\left\|\hat{\overline{x}}_0(t)\right\|\left\|e_{1y}(t)\right\|}{e_{1y}{}^T(t)\,e_{1y}(t)}P^{-1}\overline{C}^T e_{1y}(t), & \text{if } e_{1y}\neq 0 \\ 0, & \text{if } e_{1y}=0 \end{cases}. \tag{35}$$

Proof.

Because the forms of fuzzy system (32) and fuzzy observer (33) are similar to the forms of fuzzy system (12) and fuzzy observer (13), respectively, based on Theorem 3.1.1, if all conditions in Theorem 3.2.1 hold simultaneously,

$$\lim_{t\to\infty}\left\{\begin{bmatrix}x(t)\\ \omega_0(t)\end{bmatrix}-\begin{bmatrix}\hat{x}(t)\\ \hat{\omega}_0(t)\end{bmatrix}\right\} = 0, \tag{36}$$

where $\hat{x}(t)$ is the estimate of $x(t)$. Since $\beta_i(t)$ is bounded for any $i \in r$, it obtains

$$\lim_{t\to\infty}\begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r}\beta_i(\theta)(C_i-C) & I_p\end{bmatrix}^{-1}\left\{\begin{bmatrix}x(t)\\ \omega_0(t)\end{bmatrix}-\begin{bmatrix}\hat{x}(t)\\ \hat{\omega}_0(t)\end{bmatrix}\right\} = 0. \tag{37}$$

Since $\omega_0(t) = \sum_{i=1}^{r}\beta_i(\theta)(C_i-C)x(t)+\omega(t)$ and the third equation of (33), we have

$$\begin{aligned}&\begin{bmatrix}x(t)\\ \omega(t)\end{bmatrix}-\begin{bmatrix}\hat{x}(t)\\ \hat{\omega}(t)\end{bmatrix} \\ &= \begin{bmatrix}x(t)\\ \omega(t)\end{bmatrix}-\begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r}\beta_i(\theta)(C_i-C) & I_p\end{bmatrix}^{-1}\begin{bmatrix}\hat{x}(t)\\ \hat{\omega}_0(t)\end{bmatrix} \\ &= \begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r}\beta_i(\theta)(C_i-C) & I_p\end{bmatrix}^{-1} \\ &\times\left\{\begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r}\beta_i(\theta)(C_i-C) & I_p\end{bmatrix}\begin{bmatrix}x(t)\\ \omega(t)\end{bmatrix}-\begin{bmatrix}\hat{x}(t)\\ \hat{\omega}_0(t)\end{bmatrix}\right\}\end{aligned}$$

$$= \begin{bmatrix} I_n & 0 \\ \sum_{i=1}^{r} \beta_i(\theta)(C_i - C) & I_p \end{bmatrix}^{-1} \times \left\{ \begin{bmatrix} x(t) \\ \omega_0(t) \end{bmatrix} - \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}_0(t) \end{bmatrix} \right\}. \tag{38}$$

Based on (37) and (38), it yields

$$\lim_{t\to\infty} \left\{ \begin{bmatrix} x(t) \\ \omega(t) \end{bmatrix} - \begin{bmatrix} \hat{x}(t) \\ \hat{\omega}(t) \end{bmatrix} \right\} = 0. \tag{39}$$

Therefore, the proof is completed.

Based on Theorem 3.1.1 and Theorem 3.2.1, we construct a procedure as follows to synthesize the fuzzy observer. Before the observer synthesis, some known parameters are stated firstly. For the system (3), ΔA_i is the uncertainty satisfying (10), where $\|\Delta A_i\| \le \varepsilon_i$ and $\|U_i\| \le \gamma_i$. Their bounds ε_i and γ_i are known in advance.

Step 1. Find the matrix $\overline{L}$ such that $\left(\overline{E} + \overline{LC}\right)^{-1}$ exists.

Step 2. With the aids of (26), we have $\|\Delta \overline{A}_{1i}\| \le \|P^{-1}\overline{C}^T\|\|U_i\| \le\le \|P^{-1}\|\|\overline{C}\|\gamma_i \le \varepsilon_{1i}$. Therefore,

$$\|P^{-1}\| \le \frac{\varepsilon_{1i}}{\|\overline{C}\|\,\gamma_i} \tag{40}$$

Give a positive value p to satisfy $p \le (\varepsilon_{1i})/(\|\overline{C}\|\gamma)_i$ and assume $\|P^{-1}\| \le p$.

Step 3. Define $Q_i = P\overline{H}_{1i}$ and $\varepsilon_{1i} = \|P^{-1}\|\|\overline{C}\|\gamma_i$ to rewrite (20) as

$$\overline{A}_{1i}^T P + P\overline{A}_{1i} - \overline{C}^T Q_i^T - Q_i\overline{C} + 2\varepsilon_{1i}P < 0, \quad \forall i \in r. \tag{41}$$

By means of LMI method, P and $\overline{H}_{1i}$ in (41) are obtained. Let us substitute P into (40) to check whether $\|P^{-1}\| \le p$ holds? If not, go back to *Step 2* to give another p and re-run LMI again to solve (41) until $\|P^{-1}\| \le p$ holds, then go to next step.

Step 4. Get M from (21).

Step 5. Base on the above values γ_i, $\Lambda_{2i}(t)$ is obtained from (35).

Finally, the fuzzy observer is completely synthesized.

Remark 3.2.2. In *Step* 3, using LMI method to solve (41) and check $\|P^{-1}\| \le p$ are two iterative processes. We can give the initial p to be equal to $(\varepsilon_{1i})/(\|\overline{C}\|\gamma_i)$, and decrease p to $p - \Delta p$ until the suitable P is found, where Δp is a very small value.

4. A numerical example

Consider an uncertain nonlinear system described by the following T-S fuzzy system.

Rule 1. If $y_1^2(t)$ is μ_1 (small), then

$$\begin{cases} \dot{x}(t) = (A_1 + \Delta A_1)x(t) + B_1u(t) \\ y(t) = C_1x(t). \end{cases}$$

Rule 2. If $y_1^2(t)$ is μ_2 (large), then

$$\begin{cases} \dot{x}(t) = (A_2 + \Delta A_2)x(t) + B_2u(t) \\ y(t) = C_2x(t). \end{cases}$$

where $y_1 \in \begin{bmatrix} 0 & 1 \end{bmatrix}$, and A_i, B_i and C_i are the same as those in the example in [8], that is,

$$x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^T,$$

$$y(t) = \begin{bmatrix} y_1(t) & y_2(t) & y_3(t) \end{bmatrix}^T,$$

$$A_1 = \begin{bmatrix} -1 & -2 & 0 \\ 2 & -1 & 0 \\ 1 & 0 & -3 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -0.5 & -1 \\ 1 & 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Identical to the definition in the paper [19], let $\mu_1 = 1 - y_1^2(t)$ and $\mu_2 = y_1^2(t)$, then

$$\begin{cases} \beta_1(\theta) = \dfrac{\psi_i(\theta)}{\sum_{i=1}^{r}\psi_i(\theta)} = 1 - y_1^2(t) \\ \beta_2(\theta) = \dfrac{\psi_i(\theta)}{\sum_{i=1}^{r}\psi_i(\theta)} = y_1^2(t). \end{cases} \tag{42}$$

In practical, the fuzzy system may be disturbed by some output disturbance as follows.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \beta_i(\theta)(A_i + \Delta A_i)x(t) + B_iu(t) \\ y(t) = \sum_{i=1}^{2} \beta_i(\theta)C_ix(t) + \omega(t), \end{cases} \tag{43}$$

where ω(t) represents the unknown disturbance output. Suppose the time varying uncertainties ΔA_i, i=1, 2 are bounded by ε_1 = 0.1199 and ε_2 = 0.2512, respectively, and satisfy (10) and U_i is bounded by γ_1 = 0,01 and γ_2 = 0,02. Assume ΔA_i is formulated as below

$$\Delta A_1(t) = \begin{bmatrix} 0.1199 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.009 \end{bmatrix} \eta(t),$$

$$\Delta A_1(t) = \begin{bmatrix} 0.2512 & 0 & 0 \\ 0 & 0.135 & 0 \\ 0 & 0 & 0.03 \end{bmatrix} \eta(t).$$

where $\eta(t)$ is a random function whose magnitude is uniformly distributed on the interval [0,1] as shown in Fig. 1.

Now, according to the procedure in Section 3, let us start the fuzzy observer synthesis.

Step 1. Choose $L^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

and $C = C_1$, such that $\left(\overline{E} + \overline{LC}\right)^{-1}$ exists in which

$$\left(\overline{E} + \overline{LC}\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Step 2. Based on Remark 1, give an initial p to be equal to $(\varepsilon_{1i}))/(\|\overline{C}\|\gamma_i) = 6.6040$.

Step 3. Utilize LMI-Toolbox of MATLAB and obtain the feasible solution P and $\overline{H}_{1i}$ of (41) based on Remark 1, where the matrix P satisfies $\|P^{-1}\| \leq 6.6039 = p$, and

$$Q_1 = \begin{bmatrix} 38.3451 & 38.0742 & 56.3557 \\ -37.8561 & -0.7170 & -58.5018 \\ 1.5680 & 57.9994 & 1.2280 \\ -0.3128 & 39.0103 & -1.2798 \\ -39.4255 & -0.2609 & -58.1619 \\ 1.4195 & 58.3273 & -0.0041 \end{bmatrix};$$

$$Q_2 = \begin{bmatrix} 102.3509 & 103.0127 & -53.3371 \\ -102.6177 & 0.2073 & 14.0316 \\ 37.8068 & -14.4852 & 0.8765 \\ -0.1320 & 102.1026 & -37.4577 \\ -102.7044 & -0.0911 & 14.9580 \\ 37.5749 & -14.4843 & -0.1129 \end{bmatrix};$$

and

$$\overline{H}_1 = \begin{bmatrix} 27.3448 & 12.4525 & 40.2877 \\ -28.1548 & -22.2236 & -43.4245 \\ 2.9060 & 93.3812 & 5.6746 \\ 27.9251 & 56.5990 & 40.4392 \\ -84.8985 & -28.6739 & -127.0387 \\ 2.4487 & 106.5783 & 2.2373 \end{bmatrix};$$

$$\overline{H}_2 = \begin{bmatrix} 65.1253 & 79.7908 & -40.2541 \\ -89.4097 & 5.6022 & 9.8636 \\ 61.5275 & -23.9548 & 4.9397 \\ 75.0537 & 150.6050 & -66.6657 \\ -225.9075 & -75.5513 & 60.4480 \\ 69.0866 & -26.8033 & 1.9691 \end{bmatrix}.$$

Step 4. Obtain M from (21)

$$M = \begin{bmatrix} 0.7342 & -0.7342 & -0.2599 \\ 0.0000 & 0.7342 & -0.3724 \\ 0.0000 & 0.0000 & 1.6081 \\ 1.4685 & -0.7342 & 0.0000 \\ -0.7342 & 2.2027 & 0.0000 \\ 0.0000 & 0.0000 & 1.8280 \end{bmatrix}.$$

$$P = \begin{bmatrix} 1.0985 & 0.0050 & 0.1473 & 0.2634 & -0.2684 & 0.0552 \\ 0.0050 & 1.2579 & 0.2537 & -0.0050 & 0.1090 & 0.2311 \\ 0.1473 & 0.2537 & 0.6561 & -0.1473 & -0.1064 & 0.5819 \\ 0.2634 & -0.0050 & -0.1473 & 1.0985 & 0.2684 & -0.0552 \\ -0.2684 & 0.1090 & -0.1064 & 0.2684 & 0.9845 & -0.1758 \\ 0.0552 & 0.2311 & 0.5819 & -0.0552 & -0.1758 & 0.9040 \end{bmatrix};$$

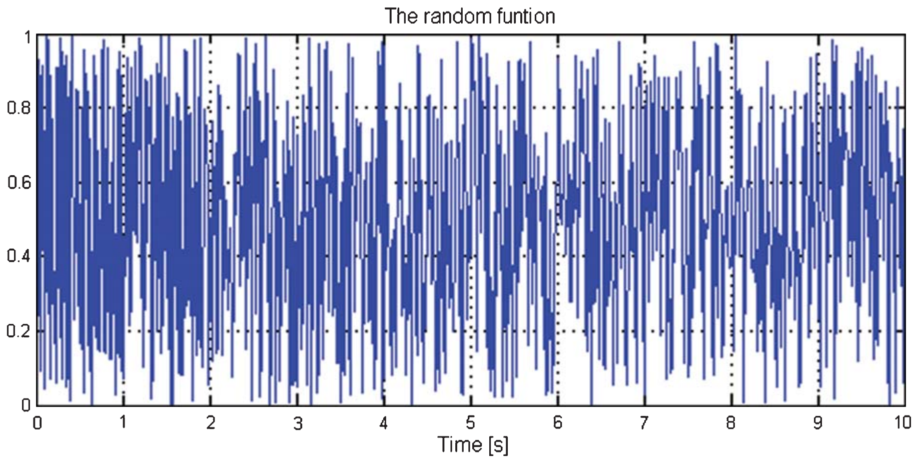

Fig. 1. The random function $\eta(t)$.

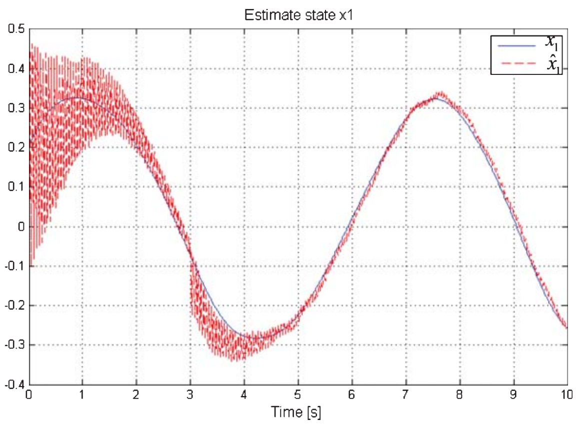

Fig. 2. State $x_1(t)$ and the estimated state $\hat{x}_1(t)$.

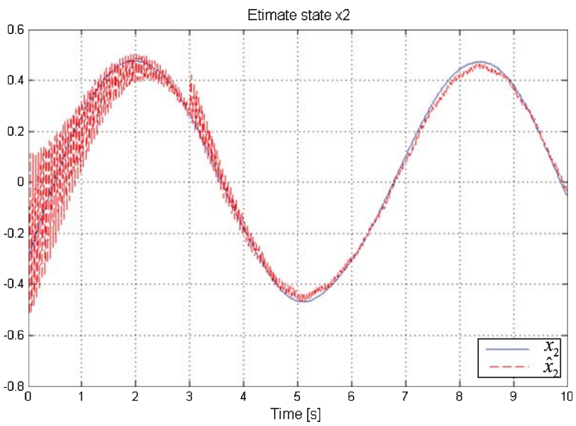

Fig. 3. State $x_2(t)$ and the estimated state $\hat{x}_2$).

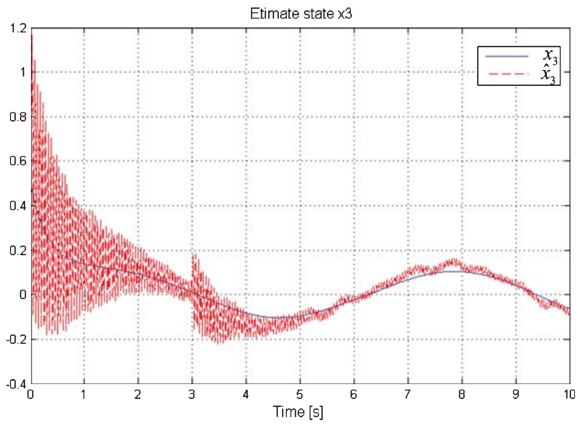

Fig. 4. State $x_3(t)$ and the estimated state $\hat{x}_3(t)$.

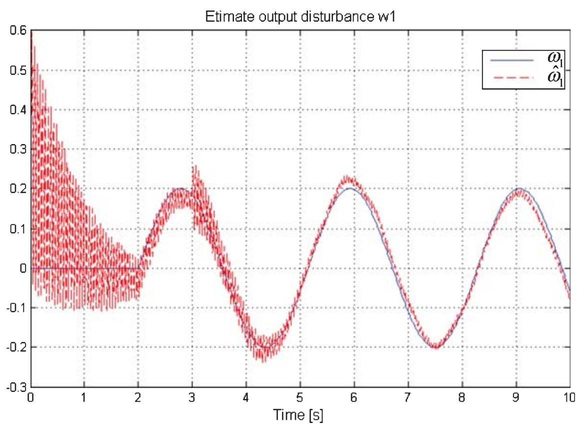

Fig. 5. Output disturbance $\omega_1(t)$ and the estimated output disturbance $\hat{\omega}_1(t)$.

Step 5. With the above value of γ_i and the obtained P and $\overline{C}$, the forms of $\Lambda_{2i}(t)$ are

$$\Lambda_{21}(t)=\begin{cases}\dfrac{0.01\left\|\hat{\bar{x}}_0(t)\right\|\left\|e_{1y}(t)\right\|}{e_{1y}^T(t)\,e_{1y}(t)}P^{-1}\overline{C}^T e_{1y}(t), & \text{if } e_{1y}(t)\neq 0\\ 0, & \text{if } e_{1y}(t)=0.\end{cases}$$

$$\Lambda_{22}(t)=\begin{cases}\dfrac{0.02\left\|\hat{\bar{x}}_0 t\,()\right\|\left\|e_{1y}(t)\right\|}{e_{1y}^T(t)\,e_{1y}(t)}P^{-1}\overline{C}^T e_{1y}(t), & \text{if } e_{1y}(t)\neq 0\\ 0, & \text{if } e_{1y}(t)=0.\end{cases}$$

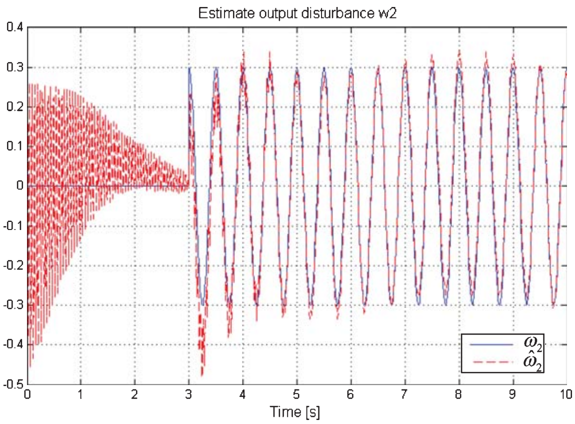

Fig. 6. Output disturbance $\omega_2(t)$ and the estimated output disturbance $\hat{\omega}_2(t)$.

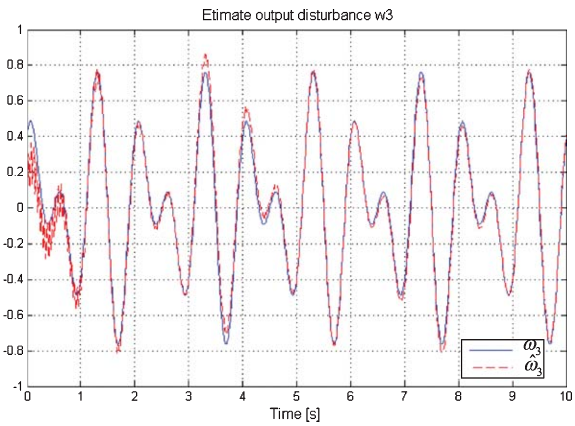

Fig. 7. Output disturbance $\omega_2(t)$ and the estimated output disturbance $\hat{\omega}_2(t)$.

Suppose *u(t)* = *sin(t)* and $\omega(t)=\begin{bmatrix}\omega_1(t) & \omega_2(t) & \omega_3(t)\end{bmatrix}^T$,

where $\omega_1(t)=\begin{cases}0.2\sin(3(t-2)\pi), & t\geq 2\\ 0, & \text{else}\end{cases}$,

$\omega_2(t)=\begin{cases}0.3\cos(4(t-3)\pi), & t\geq 3\\ 0, & \text{else}\end{cases}$, and

$\omega_3(t)=0.4(\sin 2\pi t+\cos 3\pi t), \quad t\geq 0$

Let the initial conditions be selected as: $x(0)=(0.2,-0.3,0.5)^T$ and $\hat{x}(0)=(0,0,0,0,0,0)^T$. The simulation results are shown in Figs 2–7. Figures. 2–4 show the curves of state estimation. Figs 5–7 express the estimated output disturbance $\hat{\omega}_i(t)$ with the different form of output disturbance $\omega_i(t)$, respectively.

5. Conclusions

In this paper, we have studied the problem of fuzzy observer synthesis for T-S fuzzy systems with unknown output disturbance and time-varying norm-bounded parameter uncertainty. Either with the same output matrices or with the different output matrices in the fuzzy system, the fuzzy observer is successfully synthesized. The conditions for guaranteeing the asymptotic stability of error dynamics in term of an LMI have been given. A numerical example has shown the effectiveness of the proposed fuzzy observer.

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