

*Review of Development Economics*, 10(2), 243–261, 2012

DOI:10.1111/j.1467-9361.2012.00659.x

# Intersectoral Spillovers, Relative Prices and Development Traps

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## Abstract

Why is the economic growth rate so low in poor countries? This paper offers an explanation by using a simple two-sector AK growth model with intersectoral linkages and high relative prices of intermediate goods. Intersectoral linkages lead to two balanced growth paths (BGPs). The high-growth BGP is a source. The low-growth BGP is a sink because it has a small final goods sector, small intersectoral spillovers from the final goods sector to the intermediate goods sector, and small marginal products in the intermediate goods sector, yielding high relative prices of intermediate goods. The low-growth BGP is an attractor and thus development trap. To produce a big push effect, this paper analyzes the first-best policy and finds that a subsidy to own consumption and a provision of public goods to the final goods sector can internalize the external effect and render the low-growth BGP infeasible. As a result, there is only the high-growth BGP.

## 1. Introduction

Existing studies documented a negative relationship between the relative price of investment and output as well as economic growth. Restuccia and Urrutia (2001), for example, reported that the relative price of investment to consumption, a measure of policy distortions and barriers to investment, is two to three times higher in poor countries than in rich countries. As a result, the real investment rate of wealthy countries such as Norway and the USA is two to three times higher than that of poor countries such as Mali and Kenya. Hsieh and Klenow (2007) showed that poor countries appeared to be plagued by low efficiencies in producing investment goods which resulted in a high relative price of investment goods. Indeed, of the 67 explanatory variables studied by Sala-i-Martin et al. (2004), the evidence for relative price of investment is among the three strongest to explain cross-country economic growth differentials.

This paper presents an endogenous growth model to study why countries are poor as a result of a high relative price of investment goods. The present model is based on a simple AK growth framework with the final goods sector and the intermediate goods sector developed by Rebelo (1991). A central feature in the model is the existence of positive intersectoral externalities. Evidence indicates that when an economy develops, there is an increase in intersectoral interdependence between the downstream industry such as food, textile and plastics and the upstream industry such as machinery, manmade fiber and chemicals. For example, in studying the rapid-growth experience in China between 1987 and 1997, Andreosso-O'Callaghan and Yue (2004) found that backward (upstream) and forward (downstream) linkages have generally increased in

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China, indicating an increase in intersectoral interdependence, with some sectors (e.g. agriculture, food, textile and chemicals) being dominant in this process. As another example, Kelegama and Foley (1999) studied the garment industry in Sri Lanka and found that backward linkages are useful to reduce the lead time and remain competitive in the international market and as a result, the garment industry in Sri Lanka has contributed significantly to foreign exchange earnings and employment creation in the country. Finally, Utama and Peridy (2010) analyzed the productivity spillover effects of foreign direct investment (FDI) inflows to ASEAN (Association of Southeast Asian Nations) countries and found strong evidence that FDI causes productivity growth via backward and forward spillovers.<sup>1</sup> Thus, there is prevailing evidence of backward and forward spillovers.

The existence of intersectoral spillovers has important theoretical consequences. It is found that if there are moderate backward spillovers, namely spillovers from the final goods sector to the intermediate goods sector, there is a non-monotonic relationship between the changes in the relative price of investment goods and the resource allocation between the two sectors. The non-monotonic relationship yields two balanced growth paths (BGPs). One of the two BGPs has a high economic growth rate and the other BGP has a low economic growth rate. The high-growth BGP is a source, and the low-growth BGP is a sink. The low-growth BGP is a sink because of a high relative price of intermediate goods (in terms of final goods). The low-growth BGP is associated with a small final goods sector and a large intermediate goods sector. Since the final goods sector is small, there are small intersectoral spillovers from the final goods sector to the intermediate goods sector. Moreover, the intermediate goods sector is large. The marginal products in the intermediate goods sector are thus small, thereby leading to a high relative price of intermediate goods. As a result, the agent allocates more resources toward the intermediate goods sector and fewer resources toward the final goods sector. Thus, the final goods sector is small and the intermediate sector is large. The low-growth BGP is thus an attractor and thus development trap (Steger, 2000; Easterly, 2006; Mosley and Suleiman, 2007; Aizenman and Spiegel, 2010).

The theoretical findings have two contributions to existing literature that studies AK growth models. First, this paper demonstrates that even in a simple AK growth framework, it is possible to produce multiple BGPs. In existing literature, AK growth models may yield multiple BGPs only when an additional complexity such as endogenous labor–leisure choices (Palivos, 2001; Park, 2009) or cash-in-advance constraints (Suen and Yip, 2005; Chen and Guo, 2008) is introduced. In this model the necessary condition for the existence of multiple BGPs is that there are moderate intersectoral effects from the final goods sector to the intermediate goods sector. To best of the authors' knowledge, this paper presents the simplest AK growth model with multiple BGPs. Second, most of the AK growth models with two BGPs show that the BGP with a high growth rate is a sink and the other is a source (Park and Philippopoulos, 2004; Suen and Yip, 2005; Park, 2009). Therefore, it is difficult to use these existing models to discuss development trap. In the presence of dual BGPs in this paper, the low-growth BGP is a sink. Thus, the model can analyze the development trap.

To escape from the development trap, the paper studies the first-best policy. The first-best policy levies a lump-sum tax and subsidizes the final goods sector. Because the subsidy internalizes the spillover originated from the final goods sector, there is only the high-growth BGP. A version of this model is also studied with productive public goods in the final goods sector. It is found that if the government levies the lump-sum tax in proportion to average consumption and uses the tax revenue to provide productive public goods to the final goods sector, the public good provision also internalizes the

spillover originated from the final goods sector. Both policies render the low-growth BGP infeasible. As a result, there is only the high-growth BGP and the economy can escape from the development trap.

As further developed below, section 2 sets up the basic model and studies the BGP. Section 3 analyzes the development trap and provides policies for the big push. Finally, section 4 offers preliminary conclusions in anticipation of further research.

## 2. Environment

The present model is built on the model of Rebelo (1991, Section II) wherein this model abstracts from human capital so the focus is on an analysis of the growth rate of capital. This paper departs from the Rebelo model by examining intersectoral externalities in production.

The economy has two sectors of production. Both sectors use capital as the only inputs and are of social constant returns to scale. One sector (sector  $x$ ) produces goods used for consumption only and the other sector (sector  $y$ ) produces goods used for investment only. Sector  $x$  will also be referred to as the final goods sector and sector  $y$  as the intermediate goods sector.

The two technologies are assumed as follows.

$$x(t) = [k_x(t)]^{1-\alpha_1} e_x^{\alpha_1}, \quad 0 \leq \alpha_1 < 1, \quad (1a)$$

$$y(t) = A[k_y(t)]^{1-\alpha_2} e_y^{\alpha_2}, \quad 0 \leq \alpha_2 < 1, \quad (1b)$$

where  $A > 0$  is a productivity coefficient in sector  $y$  with the productivity coefficient in sector  $x$  being normalized at unity. The representative firm's own capital input in sector  $x$  (resp.  $y$ ) is  $k_x$  (resp.  $k_y$ ) and  $e_x$  (resp.  $e_y$ ) is the externality in sector  $x$  (resp.  $y$ ) with  $\alpha_1$  (resp.  $\alpha_2$ ) representing the degree to which the externality affects the productivity.

Let two remarks be made about the production functions. First, the production technology uses only capital. As in Rebelo (1991), there is only one kind of capital that is broadly thought of as a composite of various types of physical and human capital. With this reinterpretation, sector  $y$  produces a composite of capital that includes human as well as physical capital. While both technologies display decreasing returns at the private level, they exhibit constant returns at the social level because of an externality that is ignored by infinitesimal firms. An implication of decreasing private returns is positive profits. Unless the number of firms is fixed, a fixed entry cost must be assumed in order to determine the number of firms along the equilibrium path. As will be clear in the analysis below, the required external effect is small for the main results. Under the setup of social constant returns, this indicates a small degree of decreasing returns which results in a small amount of profits. In an environment with a small amount of profits, a small fixed cost is sufficient to deter new entrants. It is assumed that profits are distributed in a lump-sum fashion to the households who own these firms. This setup is in line with two-sector endogenous growth models with sector-specific externalities (Benhabib et al., 2000; Mino, 2001).

Second, this paper follows the neoclassical school of thought and assumes perfect capital mobility. This school views capital as putty; it can be continuously reshaped to accommodate other factors. Under this school of thought, capital can be reallocated without cost between sectors and therefore, the fraction of physical capital allocated to the two sectors is a jump variable. The neoclassical benchmark is set for simplicity and is not too far from the reality if one is thinking of the aggregate economy as a whole.

This neoclassical assumption is frequently used in a two-sector model; see, e.g. Rebelo (1991), Mulligan and Sala-i-Martin (1993), Bond et al. (1996), and Benhabib et al. (2000). This paper follows the neoclassical view.

Externalities in production may be in the form of aggregate externalities (Benhabib and Farmer, 1994; Farmer and Guo, 1994), sector-specific externalities (Benhabib et al., 2000; Mino, 2001) and intersectoral externalities (Drugeon et al., 2003; Chen and Lee, 2009). Thus, the externalities are assumed to take the following forms

$$e_x = [\bar{k}_x(t)]^{1-\beta_1-\gamma_1} [\bar{k}_y(t)]^{\beta_1} [\bar{k}(t)]^{\gamma_1}, \quad (2a)$$

$$e_y = [\bar{k}_y(t)]^{1-\beta_2-\gamma_2} [\bar{k}_x(t)]^{\beta_2} [\bar{k}(t)]^{\gamma_2}, \quad (2b)$$

where  $\bar{k}_x(t)$  is average capital in sector  $x$ ,  $\bar{k}_y(t)$  is average capital in sector  $y$  and  $\bar{k}(t)$  is average capital in the economy at time  $t$ . Parameter  $\beta_i \in [0, 1]$  is the proportion from intersectoral externalities,  $\gamma_i \in [0, 1]$  is the proportion from aggregate externalities, and hence  $1 - \beta_i - \gamma_i$ , the proportion from sector-specific externalities in sector  $i = 1, 2$ . There are only intersectoral externalities if  $\beta_1 = \beta_2 = 1$  and  $\gamma_1 = \gamma_2 = 0$ , only sector-specific externalities if  $\beta_1 = \beta_2 = \gamma_1 = \gamma_2 = 0$ , and only aggregate externalities if  $\beta_1 = \beta_2 = 0$  and  $\gamma_1 = \gamma_2 = 1$ .

Finally, to close the model, the representative household's preference is described by a discounted lifetime utility, with a felicity exhibiting a constant, intertemporal elasticity of substitution as follows.

$$\int_0^{\infty} e^{-\rho t} \frac{c^{1-\sigma} - 1}{1-\sigma} dt, \quad \sigma \geq 0, \rho > 0,$$

where  $\sigma$  is the reciprocal of the intertemporal elasticity of substitution and  $\rho$  is the discount rate.

### Optimization

Given the rental rate of capital  $r$ , the representative firm in sector  $x$  chooses the amount of capital in order to maximize  $\pi_1 = x - rk_x$  and the optimal condition is  $r = MP_x = (1 - \alpha_1)x/k_x$ . This condition equates the rental rate to the marginal product of capital. Similarly, the representative firm in sector  $y$  maximizes  $\pi_2 = py - rk_y$ , where  $p$  is the relative price of goods  $y$  in terms of goods  $x$ . The optimal condition also equates the rental rate to the value of the marginal product of capital in terms of good  $x$ :  $r = MP_y = p(1 - \alpha_2)y/k_y$ . Note that  $\pi_1 = (1 - \alpha_1)x > 0$  and  $\pi_2 = p(1 - \alpha_2)y > 0$  as the technologies  $x$  and  $y$  are decreasing returns from private perspectives.

As an owner of capital and firms, the representative household receives income flows of rentals and profits:  $rk + \pi$ , where  $k = k_x + k_y$  and  $\pi = \pi_1 + \pi_2$ . The household decides the allocation of income flows between consumption and investment under the following budget constraint:  $rk + \pi = c + pI$  where  $I$  is the investment. The amount of investment net of depreciation accumulates capital:  $\dot{k} = I - \delta k$  where  $\delta$  is the depreciation rate. The representative household's problem is to maximize the discounted lifetime utility, subject to the budget constraint and the capital accumulation. Let  $\mu$  denote the Lagrange multiplier and  $\lambda$  the shadow price of capital. The Hamiltonian is

$$H = e^{-\rho t} [(c^{1-\sigma} - 1)/(1-\sigma) + \mu(rk + \pi - c - pI) + \lambda(I - \delta k)].$$

The household optimally chooses consumption, investment and capital accumulation. These three conditions may be simplified to the following two conditions.

$$p = \lambda c^\sigma, \tag{3a}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - \frac{1}{p}. \tag{3b}$$

Since  $x$  is a final good and  $y$  is an intermediate good, the following two market clearance conditions need to satisfy.

$$c(t) = x(t), \tag{4a}$$

$$\dot{k} = y(t) - \delta k(t), \text{ given } k(0). \tag{4b}$$

Denote  $v(t) = k_x(t)/k(t)$  as the fraction of capital allocated in the final goods sector in  $t$ . Then, the fraction of capital allocated in the intermediate goods sector is  $k_y(t)/k(t) = 1 - v(t)$ .

**DEFINITION.** *The perfect-foresight equilibrium (PFE) is  $\left\{ r, p, \frac{\dot{\lambda}}{\lambda}, \frac{\dot{k}}{k}, \frac{c}{k}, \frac{x}{k}, \frac{y}{k}, v \right\}$ , that satisfies*

- (i) *the technologies (1a) and (1b);*
- (ii) *the optimization of the representative firm,  $r = (1 - \alpha_1)x/k_x$  and  $r = p(1 - \alpha_2)y/k_y$ ;*
- (iii) *the optimization of the representative household, (3a) and (3b);*
- (iv) *the clearance of two goods markets, (4a) and (4b).*

To analyze the PFE, the conditions are transformed with variables that grow perpetually into stationary variables. The eight-equation conditions are simplified into one equation as follows.

First, by using firms' optimal conditions and (1a)–(2b), one obtains

$$p = \frac{MP_x}{MP_y} = \frac{1 - \alpha_1}{(1 - \alpha_2)A} \left( \frac{1 - v}{v} \right)^{(\alpha_1\beta_1 + \alpha_2\beta_2)} \frac{(1 - v)^{\alpha_2\gamma_2}}{v^{\alpha_1\gamma_1}}. \tag{5}$$

The above relationship indicates at optimum, the relative price of intermediate goods is equal to the relative marginal product between the final goods sector and the intermediate goods sectors.

Second, by using (5), (3b) is rewritten as

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - (1 - \alpha)A \left( \frac{v}{1 - v} \right)^{\alpha_2\beta_2} \left( \frac{1}{1 - v} \right)^{\alpha_2\gamma_2}. \tag{6}$$

Moreover, by using (1b) and (2b), (4b) is rewritten as

$$\frac{\dot{k}}{k} = A(1 - v) \left( \frac{v}{1 - v} \right)^{\alpha_2\beta_2} \left( \frac{1}{1 - v} \right)^{\alpha_2\gamma_2} - \delta. \tag{7}$$

Finally, differentiating (5) and using (1a), (3a), (4a), (6) and (7) yields

$$\dot{v} = M(v)\Lambda(v) \equiv \Omega(v), \tag{8}$$

where  $M(v) \equiv v(1-v)/\{\sigma(1-v) - (\sigma-1)\alpha_1[\beta_1 + \gamma_1(1-v)] + \alpha_2(\beta_2 + \gamma_2v)\}$ ,

$$\Lambda(v) \equiv -\frac{\dot{\lambda}}{\lambda} - \sigma \frac{\dot{k}}{k} = (\sigma-1)\delta - \rho - A \left( \frac{v}{1-v} \right)^{\alpha_2\beta_2} \left( \frac{1}{1-v} \right)^{\alpha_2\gamma_2} [\sigma(1-v) - (1-\alpha_2)].$$

Equation (8) is a first-order nonlinear differential equation in the fraction of capital in the final goods sector and summarizes the equilibrium dynamics. Note that  $M(0) = M(1) = 0$ . Once  $v$  is determined from (8), other variables can be determined.

It is worth noting that the shadow price of capital in (6) is monotonic in the fraction of capital allocated to sector  $y$ ,  $v$ , as is the relative price of investment goods in (5). However, the growth rate of capital in (7) is not monotonic in  $v$ . As will be shown later,  $\Lambda(v)$  in (8) is about equal to a change in the relative price of intermediate goods with a negative sign. A change in the relative price of intermediate goods is determined by the growth rate of the shadow price of capital and the growth rate of capital, so the change is also non-monotonic, as is  $\Lambda(v)$ . A change in the relative price of intermediate goods proves to be crucial for capital allocation between the two sectors.

The non-monotonic  $\Lambda(v)$  is crucial for the existence of multiple BGPs, which will be proved in the next section. It is straightforward to see that  $\Lambda(v)$  is monotonic in  $v$  if there are aggregate externalities and/or sector-specific externalities (i.e.  $\beta_1 = \beta_2 = 0$ ), but  $\Lambda(v)$  is non-monotonic in  $v$  if there are intersectoral externalities (i.e.  $\beta_1 > 0$ ,  $\beta_2 > 0$ ). Indeed,  $\Lambda(v)$  is non-monotonic in  $v$  as long as  $\beta_2 > 0$  and thus there are moderate intersectoral externalities from the final goods sector to the intermediate goods sector. To ease the theoretical analysis below, this paper will focus on the simplest case wherein the final goods sector has no externalities ( $\alpha_1 = 0$ ) and the intermediate goods sector has only intersectoral externalities ( $\beta_2 = 1$  and  $\gamma_2 = 0$ ). Thus,  $x(t) = k_x(t)$  and  $y(t) = A[k_y(t)]^{1-\alpha_2}[\bar{k}_x(t)]^{\alpha_2}$  with  $0 < \alpha_2 < 1$ . It should be emphasized that the theoretical findings hold true when one allows for externalities in the final goods sector and other types of externalities in the intermediate goods sector.

### *Balanced Growth Path*

Now, the BGP is determined. Along a BGP,  $\frac{\dot{c}}{c} = \frac{\dot{k}}{k}$  are constant, denoted as  $g_c$  as are  $\frac{c}{k}$ ,  $\frac{x}{k}$ ,  $\frac{y}{k}$  and  $v$ . Under  $\beta_2 = 1$ ,  $\gamma_2 = 0$  and  $\alpha_1 = 0$ ,  $\dot{v} = 0$  is

$$(\sigma-1)\delta - \rho = A \left( \frac{v}{1-v} \right)^{\alpha_2} [\sigma(1-v) - (1-\alpha_2)], \quad (9)$$

which determines  $v$  in the BGP.

To determine  $v$  in the BGP in (9), notice that the left-hand side of (9), denoted as *LHS*, is a constant. The right-hand side of (9), denoted as *RHS*( $v$ ), changes in  $v$ . Note that if  $\alpha_2 = 0$ , then *RHS*( $v$ ) is monotonic in  $v$ . When  $\alpha_2 > 0$ , *RHS*( $v$ ) is non-monotonic in  $v$ . Then, there is a possibility of multiple BGPs.

The possibility of multiple BGPs is analyzed in two cases as follows. First, case 1,  $(\sigma-1)\delta - \rho > 0$ , is analyzed. This case applies when the intertemporal elasticity of substitution ( $1/\sigma$ ) is as small as  $\delta/(\delta + \rho) < 1$ . Under this case, consistency in (9) implies  $v < \bar{v} \equiv 1 - (1-\alpha_2)/\sigma$ . Then, *RHS*( $v$ ) is zero at both  $v = 0$  and  $\bar{v}$  in Figure 1(a). Condition for  $\bar{v} > 0$  is  $1/\sigma < 1/(1-\alpha_2) > 1$ , which is automatically met in this case. Under  $\alpha_2 > 0$ , it is obvious that *RHS*( $v$ ) is a locus with an inverted U shape, with a positive (negative) value for all  $v < (>)\bar{v}$ . It follows that there exist two interior BGPs if the value of *LHS* is smaller than the maximal value of *RHS*( $v$ ), which is obtained at  $v = \tilde{v}$ . Result

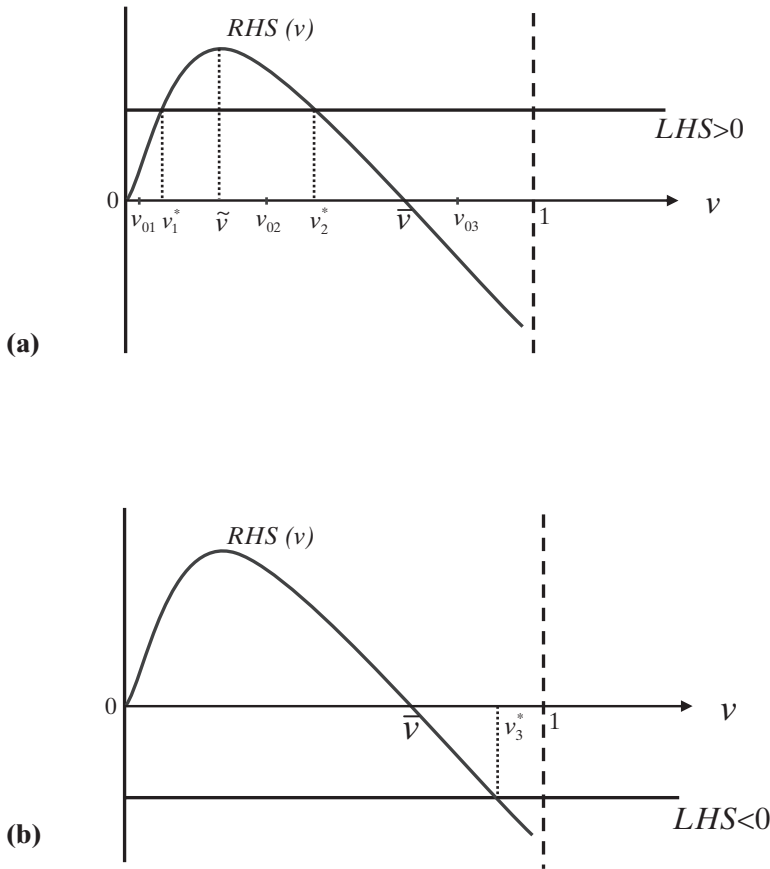


Figure 1. (a) Existence of Two BGPs; (b) Existence of One BGP

$LHS < RHS(\tilde{v})$  is ensured if the productivity in the intermediate goods sector is sufficiently large such that:

$$\text{CONDITION B. } A(\sigma - 1)\alpha_2^{\alpha_2}(1 - \alpha_2)^{1 - \alpha_2} \geq (\sigma - 1)\delta - \rho.$$

Then, there are two interior BGPs.<sup>2</sup> See  $v_1^*$  and  $v_2^*$  in Figure 1(a). The BGP at  $v_1^*$  has a small final goods sector and thus a large intermediate goods sector, while the BGP at  $v_2^*$  is associated with a large final goods sector and a small intermediate goods sector. As will be seen later, economic growth is increasing in  $v$ . The BGP at  $v_2^*$  will be called the low-growth BGP and at  $v_1^*$  the high-growth BGP.

Second, case 2,  $(\sigma - 1)\delta - \rho < 0$ , is analyzed. This case applies when the intertemporal elasticity of substitution is as large as  $\delta/(\delta + \rho) < 1$ . Under this condition, (9) implies  $v > \bar{v} \equiv 1 - (1 - \alpha_2)/\sigma$ . As  $RHS(0) = 0$  and  $RHS(v)$  approaches monotonically to negative infinity as  $v$  is close to 1, there is a unique BGP (see  $v_3^*$  in Figure 1(b)).

To summarize the number of interior BGPs, let the parameter space be  $\Theta: \theta \equiv \{\alpha_2, \rho, \delta, \sigma, A\}$ , and  $\theta \in \Theta$ , where  $\Theta = (0, 1)^3 \times \Re \times \Re_+$ . Define the following sets.

$$\Theta_1 \equiv \{\theta \in \Theta \mid (\sigma - 1)[\delta - A\alpha_2^{\alpha_2}(1 - \alpha_2)^{1 - \alpha_2}] < \rho < (\sigma - 1)\delta\},$$

$$\Theta_2 \equiv \{\theta \in \Theta \mid \rho > (\sigma - 1)\delta\}.$$



Thus, under  $\alpha_2 > 0$ , there are two interior BGPs if  $\theta \in \Theta_1$  and one interior BGP if  $\theta \in \Theta_2$ .

If  $v^*$  is substituted in a BGP into (6) and (7), the balanced growth rates for  $\lambda$  and  $k$  in a BGP are obtained; similarly, substituting  $v^*$  into (1a) and (1b) yields the ratios  $\left\{ \frac{x}{k}, \frac{y}{k} \right\}$  and into (5) yields the path of  $p$ . Thus, all variables are determined in a BGP. In particular, by using (3a) and (5), the economic growth rate in a BGP,  $g_c = \frac{1}{\sigma} \left\{ -\frac{\dot{\lambda}}{\lambda} \right\}$ , is obtained, which using (6) becomes

$$g_c = \frac{1}{\sigma} \left\{ (1 - \alpha_2) A \left( \frac{v}{1-v} \right)^{\alpha_2} - \delta - \rho \right\}. \quad (10)$$

The balanced growth rate of consumption, and thus the economic growth rate, is increasing in  $v$ . Thus, the BGP  $v_1^*$  has low growth, and the BGP  $v_2^*$  has high growth. This effect is originated from the diminishing  $MP_y$ . In the high-growth BGP, more resources are allocated from the intermediate goods sector to the final goods sector. The marginal product of capital increases in the intermediate goods sector and so does the economic growth rate.

To summarize the result:

**PROPOSITION 1.** *In a two-sector economy with the final goods sector and the intermediate goods sector and with positive intersectoral externalities, there exist two interior BGPs if and only if  $\theta \in \Theta_1$  and an interior BGP if and only if  $\theta \in \Theta_2$ .*

In the existing literature, AK growth models may yield multiple BGPs only when an additional complexity such as endogenous labor-leisure choices (Palivos, 2001; Park, 2009) or cash-in-advance constraints (Suen and Yip, 2005; Chen and Guo, 2008) are introduced. In the model the necessary condition for the existence of multiple BGPs is that there are moderate intersectoral effects from the final goods sector to the intermediate goods sector,  $\alpha_2 > 0$ . To the authors' knowledge, this paper presents the simplest AK growth model with multiple BGPs.

### 3. The Development Trap and Relative Prices

The stability property is now analyzed. The dynamic system is characterized by equation (8). The stability of a BGP is determined by the eigenvalue of the Jacobean matrix of a dynamic system. The dynamic system includes only a control variable  $v$  whose value may be adjusted instantaneously and does not include any state variable. As a result, the steady state is a source and thus the dynamic path toward the BGP is determinate if the eigenvalue is positive; the steady state is a sink and thus the dynamic path toward the BGP is indeterminate if the eigenvalue is negative.

Under case 1 ( $\theta \in \Theta_1$ ), the eigenvalue is negative at the BGP  $v_1^*$  and is positive at the BGP  $v_2^*$ . As a result, the BGP  $v_1^*$  is a sink and  $v_2^*$  is a source. Figure 2(a) illustrates this case. It is clear that locus  $\dot{v} = \Omega(v)$  is decreasing in  $v$  in the neighborhood of  $v_1^*$  and increasing in  $v$  in the neighborhood of  $v_2^*$ . The equilibrium path converges to BGP  $v_2^*$  only when the economy starts at  $v_2^*$ . Otherwise, starting from any  $v < v_2^*$ , the equilibrium paths will converge toward BGP  $v_1^*$ . Initial  $v$  will not lie above  $v_2^*$ , as the equilibrium paths will converge to the degenerate equilibrium with  $v = 1$  which violates the transversality condition.



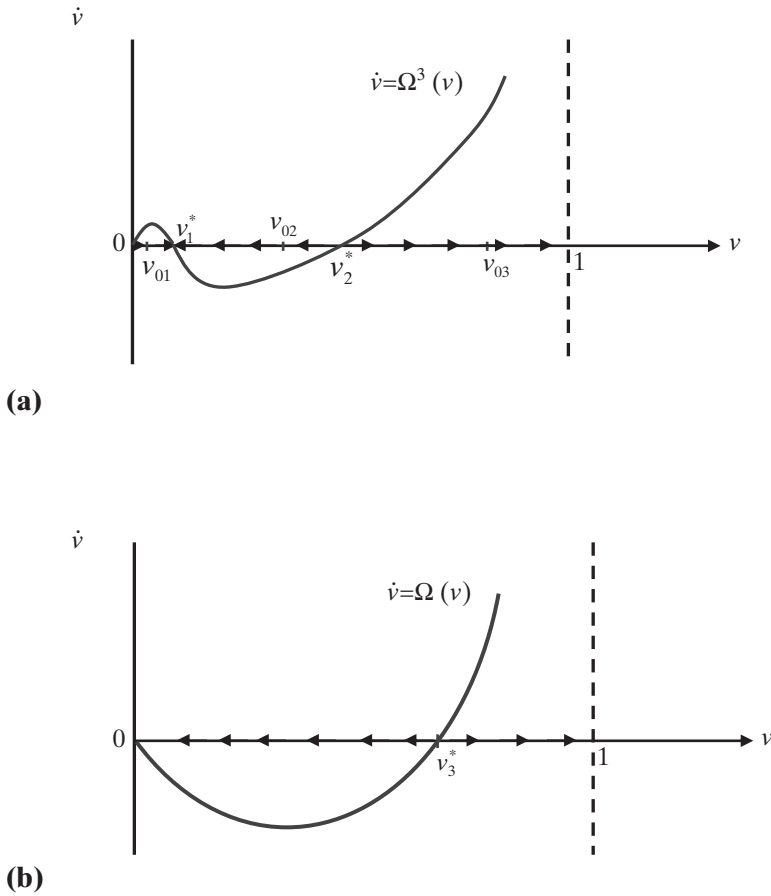


Figure 2. (a) Dynamic Paths for  $\theta \in \Theta_1$ :  $v_1^*$  is a Sink and  $v_2^*$  is a Source; (b) Dynamic Path for  $\theta \in \Theta_2$ :  $v_3^*$  is a Source

Under case 2 ( $\theta \in \Theta_2$ ), the eigenvalue is positive at the BGP  $v_3^*$ . Thus, the equilibrium path is unique and moves to BGP  $v_3^*$ . See Figure 2(b). All other equilibrium paths are the degenerate equilibrium as they move to either  $v = 0$  or  $v = 1$  that violates the transversality condition.

To see why the BGP  $v_1$  in Figure 2(a) is a sink and thus an attractor, the relative price of intermediate goods is  $p = \lambda c^\sigma$  and thus,

$$\frac{\dot{p}}{p} = \frac{\dot{\lambda}}{\lambda} + \sigma \frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda} + \sigma \frac{\dot{k}}{k} + \sigma \frac{\dot{v}}{v} = RHS(v) - LHS + \sigma \frac{\dot{v}}{v}, \tag{11}$$

where the second equality uses (1a) and (4a) and the final equality uses (8).

While  $\dot{\lambda}/\lambda$  is negative and always decreasing in  $v$ ,  $\dot{k}/k$  is positive and is first increasing and then decreasing in  $v$ . Suppose that the agent chooses  $v = v_{01}$  that is below the smaller BGP  $v_1^*$  in Figures 1(a) and 2(a). At  $v_{01}$ ,  $v$  is so small that the positive growth rate of capital is small and dominated by the negative growth rate of the shadow price of capital. Thus,  $RHS(v_{01}) - LHS < 0$  and the growth rate of the relative price of intermediate goods is negative. As a result of a decreasing relative price of intermediate goods, the representative agent reallocates the resources out of the intermediate goods

sector toward the final goods sector and thus  $\dot{v} > 0$  until  $v = v_1^*$  when the relative price no longer changes, so  $\dot{v} = 0$ .

Alternatively, suppose that the agent chooses  $v > v_1^*$ . Consider  $v_{02} < v_2^*$  in Figures 1(a) and 2(a). Now a larger size of the final goods sector generates a larger intersectoral spillover. Thus, the positive growth rate of capital is larger and dominates the negative growth rate of the shadow price of capital. Then,  $RHS(v_{02}) - LHS > 0$  and the growth rate of the relative price of intermediate goods is positive. As a result of an increasing relative price of intermediate goods, the representative agent reallocates the resources out of the final goods sector into the intermediate goods sector and  $\dot{v} < 0$  until  $v = v_1^*$  when the relative price no longer changes, so  $\dot{v} = 0$ .<sup>3</sup>

Summarizing the above analysis, the following is obtained:

**PROPOSITION 2.** *In a two-sector economy with the final goods sector and the intermediate goods sector, if there are positive intersectoral externalities, then*

- (i) *when  $\theta \in \Theta_1$ , unless the initial equilibrium is at the high-growth BGP, the equilibrium path always moves toward the low-growth BGP;*
- (ii) *when  $\theta \in \Theta_2$ , the equilibrium path is unique at the high-growth BGP.*

Thus, under the case with two interior BGPs, for any initial state  $k(0)$ , if the agent chooses an initial fraction of capital allocated to the consumption good sector  $v(0)$  exactly at  $v_2^*$ , there is a unique equilibrium path  $\{p(t), \lambda(t), k(t), c(t), x(t), y(t)\}$  that is uniquely determined by  $v_2^*$ . However, if the agent chooses initial  $v(0) \neq v_2^*$ , then for every  $v(t) < v_2^*$ , there is an equilibrium path  $\{p(v(t)), \lambda(v(t)), k(v(t)), c(v(t)), x(v(t)), y(v(t))\}$ . As there is a continuum of  $v(t)$  for  $v(t) < v_2^*$ , there is a continuum of equilibrium paths that will eventually move toward the low-growth BGP  $\{p(v_1^*), \lambda(v_1^*), k(v_1^*), c(v_1^*), x(v_1^*), y(v_1^*)\}$ . The low-growth BGP,  $v_1^*$ , represents development trap because it is an attractor and the economic growth rate is low.

Most of the AK growth models with two BGPs in the existing literature show that the BGP with a high growth rate is a sink and the other is a source (Park and Philippopoulos, 2004; Suen and Yip, 2005; Park, 2009). Therefore, it is difficult to use these existing models to discuss development trap. In the presence of dual BGPs in this paper, the low-growth BGP is a sink. Thus, this model can analyze the development trap.

### *Relative Price of Investment Goods*

The low-growth BGP is an attractor and thus development trap because of the high relative price of intermediate goods owing to inefficiencies of the intermediate goods sector. Near the low-growth BGP, the size of the intermediate goods sector is large but the size of the final goods sector is small. The intersectoral spillover toward the intermediate goods sector is thus small and this leads to a small marginal product of capital in the intermediate goods sector. As a result, the relative price of intermediate goods is high. A high price of intermediate goods gives the agent incentives to allocate more resources to the intermediate goods sector and fewer resources to the final goods sector. Therefore, the low-growth BGP is an attractor.

According to (5), with other things being equal, the relative price of intermediate goods is increasing in the fraction of aggregate capital allocated in the intermediate goods sector and thus decreasing in the fraction of aggregate capital allocated in the final goods sector,  $v$ . At the low-growth BGP  $v_1^*$ , the fraction of aggregate capital allocated in the intermediate goods sector is large. Because of diminishing returns, the

marginal product of capital in the intermediate goods sector is low relative to the final goods sector, leading to a high price of intermediate goods relative to final goods. In contrast, at the high-growth BGP  $v_2^*$ ,  $1 - v$  is small. The relative price of intermediate goods is low.

At the low-growth BGP  $v_1^*$ , high relative prices of investment goods lead to a low growth rate of investment and yields a low economic growth rate. At the high-growth BGP  $v_2^*$ , the relative price of investment goods is low which brings forth a high growth rate of investment and thereby a high economic growth rate.

The high relative price of investment goods in the low-growth BGP  $v_1^*$  is due to low marginal products in the intermediate goods sector. This result corroborates with the findings in Hsieh and Klenow (2007) that poor countries appear to be plagued by a low efficiency in producing investment goods which results in a high relative price of investment goods.

The major reasons underlying these results are not due to institutional factors such as high-dependency ratios, high discount rates, and subsistence consumption needs. The key mechanism here is the market failure underlying the existence of positive intersectoral spillovers. There is prevailing evidence of positive intersectoral spillovers as mentioned in section 1. The results rely on the existence of two interior BGPs. To quantitatively assess whether the conditions for two BGPs are in a plausible range, the model is calibrated and quantitatively envisaged in the next subsection.

*Quantitative Simulation*

The parameter values representative of the economy in the USA are chosen and the model is calibrated consistent with a 2% long-run, real economic growth rate. The time preference rate is chosen at  $r = 0.025$  in accordance with Benhabib and Perli (1994). Moreover, the intertemporal elasticity of substitution is chosen at  $1/s = 1/2$  and the depreciation rate at  $\delta = 0.1$ , as in Mulligan and Sala-i-Martin (1993). Finally, following Nishimura and Venditti (2002), a small degree of the externalities is chosen at  $\alpha_2 = 0.083$ . Based upon these parameter values, the productivity coefficient is calibrated in the intermediate goods sector and obtain  $A = 0.1279$ . With these parameter values, the two BGPs are obtained at  $v_1^* = 1\%$  and  $v_2^* = 44\%$ . The long-run economic growth rates at the two BGPs are  $g_c = 0.3\%$  and  $2\%$ , respectively. As an inference, the economic growth rate of wealthy countries is six times as high as that of poor countries.

First, the model is simulated to assess whether the conditions for  $\theta \in \Theta_1$  are met. The conditions in  $\theta \in \Theta_1$  include

$$\{(i) \rho > (\sigma - 1)[\delta - A\alpha_2^{\alpha_2}(1 - \alpha_2)^{1-\alpha_2}], \quad (ii) (\sigma - 1)\delta > \rho\}. \tag{12}$$

As can be seen, the condition (i) in (12) is met based on the benchmark parameter values and the calibrated value of  $A$ , as long as  $\alpha_2 > 0$ . The condition (ii) in (12) can be met if the values for  $\sigma$  and  $\delta$  are larger and the value for  $\rho$  is smaller.

With regard to the time preference rate, the value is in  $\rho \in [0.02, 0.05]$ , ranging from  $\rho = 0.05$  in Benhabib and Farmer (1996),  $\rho = 0.04$  in Kydland and Prescott (1982) and  $\rho = 0.02$  in Jones et al. (1993). The value of the depreciation rate is in  $\delta \in [0.05, 0.19]$ , ranging from 0.059 in Nadiri and Prucha (1996), 0.06 in Judd (1987), 0.1 in Kydland and Prescott (1982), 0.12 in Judd (1987), and between 0.05 and 0.19 in Rosen (1976). These numbers indicate the value of  $\rho/\delta$  is in the range of  $[0.1053, 1]$ .

As for the intertemporal elasticity of substitution, Ogaki and Reinhart (1998) estimated the value using the US data and obtained  $\sigma \in [2.22, 3.125]$ . Yogo (2004)

estimated data using cross-country data and uncovered  $\sigma > 2$  across eleven developed countries which was smaller than  $\sigma > 5$  he estimated for the US economy. Given the above data, the condition (ii),  $(\sigma - 1)\delta > \rho$ , is consistent with the empirical studies. As the results stand, the conditions for  $\theta \in \Theta_1$  are easily met quantitatively. It follows there are two interior BGPs in the model.

For the smaller BGP to be a sink, the model requires a degree of externalities at no more than 0.083. In comparison to existing two-sector endogenous growth models establishing local indeterminacy, this required degree of externalities is mild. Benhabib and Perli (1994), for example, required the degree of aggregate externalities (via average human capital) at 0.225 in their model with inelastic labor supply and the degree of sector-specific externalities (via labor) at 0.11 in their model with elastic labor supply. In Ben-Gad (2003), under zero tax rates and the share of physical capital in the human capital production sector at 0.25, the required degree of sector-specific externality was even larger, at 0.3.

A robustness check was conducted of the parameter values that the conditions for  $\theta \in \Theta_1$  are met quantitatively. The check was performed under  $\{\alpha_2 \in [0.01, 0.40], \rho \in [0.02, 0.05], \delta \in [0.05, 0.19], \sigma \in [1.5, 7]\}$  whose values are sufficiently wide to cover those of values estimated and used. Three of these four above parameter values given above are then controlled for and the model is quantified with  $A$  calibrated in consistence with the 2% real economic growth rate. The range of the remaining fourth parameter value is found so the conditions for  $\theta \in \Theta_1$  are met. The resulting ranges are:  $\{\alpha_2 \in (0, 0.4], \sigma \in [1.49, \infty), \rho \in [-0.26, 0.39], \delta \in [0.025, 0.21]\}$ . Obviously, the wide ranges for any one of the four parameter values indicate that they cover the values obtained in empirical studies. In particular, for parameter values for  $\rho$ ,  $\delta$  and  $\sigma$  obtained in the above, only a negligible positive value of intersectoral externalities in investment goods sector is required ( $\alpha_2 > 0$ ). Finally, if aggregate externalities ( $\gamma_1 > 0$ ,  $\gamma_2 > 0$ ), sector specific externalities ( $\beta_1 + \gamma_1 < 1$ ,  $\beta_2 + \gamma_2 < 1$ ) and intersectoral externalities in the final goods sector ( $\beta_1 > 0$ ) are positive, it remains that a negligible positive value of intersectoral externalities in investment goods sector is required.

### *First-best Policy*

The first-best policy is now analyzed in order to escape from development trap. As will be seen, a lump-sum tax and a consumption subsidy can attain the Pareto optimum. The reasons are explained in the following under the simplified case of  $\alpha_1 = 0$ ,  $\beta_2 = 1$  and  $\gamma_2 = 0$ .

First, the social planner's problem is to maximize household's lifetime utility, subject to two resource constraints in (4a) and (4b). The first-order conditions are:

$$c^{-\sigma}k = \psi y \frac{v - \alpha_2}{v(1-v)}, \quad (13a)$$

$$c^{-\sigma}v + \psi \left[ \frac{y}{k} - \delta \right] = \rho\psi - \dot{\psi}, \quad (13b)$$

where  $\psi$  is the social shadow price of capital. Thus, (13a), (13b), (4a), and (4b) can be used to determine efficient allocation.

Next, the first-best policy is analyzed so the allocation in the market equilibrium can duplicate the efficient allocation. While the optimization conditions of the firms are

identical to those in section 2, in the household's problem the flow budget constraint is now

$$rk + \pi + T = (1 + \tau_c)c + pI,$$

where  $\tau_c$  is the consumption tax rate and  $T$  is the lump-sum subsidy in the final goods sector.

The household maximizes the lifetime utility subject to the above flow budget constraint and  $\dot{k} = I - \delta k$ . The household's first-order conditions in (3a) and (3b) are modified to

$$p = \lambda c^\sigma (1 + \tau_c), \tag{14a}$$

$$\left(\frac{r}{p} - \delta\right)\lambda = \rho\lambda - \dot{\lambda}, \tag{14b}$$

where  $\lambda$  is the private shadow price of capital. By using the firm's first-order conditions, (14a) and (14b) become, respectively,

$$c^{-\sigma}k = \lambda y(1 - \alpha_2) \frac{1 + \tau_c}{1 - \nu}, \tag{15a}$$

$$c^{-\sigma\nu} + \lambda \left[\frac{y}{k} - \delta\right] - c^{-\sigma} \frac{\tau_c \nu}{1 + \tau_c} - \lambda \alpha_2 \frac{y}{k} = \rho\lambda - \dot{\lambda}. \tag{15b}$$

Thus, (15a), (15b), (4a) and (4b) can be used to determine the allocation in the market equilibrium.

In order for the allocation in the market equilibrium to duplicate efficient allocation, it suffices if (15a) is the same as (13a) and (15b) is the same as (13b). This is achieved if the following conditions are met.<sup>4</sup>

$$-\tau_c c = T, \tag{16a}$$

$$\frac{\nu - \alpha_2}{\nu(1 - \nu)} = (1 - \alpha_2) \frac{1 + \tau_c}{1 - \nu}, \tag{16b}$$

$$c^{-\sigma} \frac{\tau_c \nu}{1 + \tau_c} + \lambda \alpha_2 \frac{y}{k} = 0. \tag{16c}$$

From (16b), a subsidy to consumption is obtained,

$$\tau_c^* = -\frac{\alpha_2}{\nu} \frac{1 - \nu}{1 - \alpha_2} < 0. \tag{17a}$$

Intuitively, a subsidy to consumption corrects the spillovers from the final goods sector to the intermediate goods sector. By examining (1b) and (2b), the external effect is  $\alpha_2 y / \nu$ . The external effect benefits the marginal product of own capital which is  $[(1 - \alpha_2)y] / (1 - \nu)$ . Thus, the subsidy rate to the final goods sector is  $\left[\frac{\alpha_2 y}{\nu}\right] / \left[\frac{(1 - \alpha_2)y}{1 - \nu}\right] = \frac{\alpha_2}{\nu} \frac{1 - \nu}{1 - \alpha_2}$  as shown in (17a).

Notice that (16c) is implied if one substitutes (17a) into (16a). Finally, (16a) and (17a) imply

$$T^* = -\tau_c^* c > 0, \quad (17b)$$

in which  $c$  is average consumption in the society.

Thus, the first-best policy is to levy a lump-sum tax in proportion to average consumption in the society and subsidize own consumption. By using non-distortionary lump-sum taxes to subsidize the final goods sector, the government can internalize the external effect from the final goods sector to the intermediate sector.

Now, the BGP is investigated under Pareto optimum. As in Section 2, the equilibrium system is simplified to

$$\dot{v} = M_1(v)\Lambda(v)(v - \alpha_2), \quad (18)$$

where  $M_1(v) \equiv \frac{v(1-v)}{[\sigma - (1-\alpha_2)](1-v) + \alpha_2 v}(v - \alpha_2) + v(1-v)$ ,

$$\Lambda(v) \equiv -\left(\frac{\dot{\mu}}{\mu} + \sigma \frac{\dot{k}}{k}\right) = (\sigma - 1)\delta - \rho - A\left(\frac{v}{1-v}\right)^{\alpha_2} [\sigma(1-v) - (1-\alpha_2)].$$

While both  $v = \alpha_2$  and  $\Lambda(v) = 0$  lead to  $\dot{v} = 0$ , the consistency in (13a) requires  $v > \alpha_2$  in order to have positive capital. Thus,  $v = \alpha_2$  is not a feasible BGP. Notice that when  $\Lambda(v) = 0$ , the expression is identical to (9). Under Condition B,  $\Lambda(v) = 0$  gives two BGPs as illustrated in Figure 1(a). Condition B requires  $LHS < RHS(a_2) = A(\sigma - 1)(\alpha_2)^{\alpha_2}(1 - \alpha_2)^{1-\alpha_2}$ , as  $RHS(\alpha_2) < RHS(\bar{v})$ . This implies that  $v_1^* < \alpha_2$ ; however,  $v_1^* < \alpha_2$  violates (14a) and hence  $v_1^*$  is also not one of BGPs. As a result, there is a unique BGP for all parameter space. The BGP is  $v_2^*$  in Figure 1(a) if  $\theta \in \Theta_1$  and  $v_3^*$  in Figure 1(b) if  $\theta \in \Theta_2$ , which is the same as the high-growth BGP in section 2. No indeterminacy occurs in this economy. To summarize the result:

**PROPOSITION 3.** *In a two-sector economy with the final goods sector and the intermediate goods sector and with intersectoral externalities, when the government levies a lump-sum tax in proportion to average consumption in the society and subsidizes own consumption, the economy can escape from development trap.*

To see the economic growth rate in a BGP in the first-best policy, (13a) is differentiated. An equation is obtained identical to (10) in the competitive equilibrium in section 2. The level of  $v^*$  in the BGP in the social planning economy is the same as the level of the high-growth BGP at the competitive equilibrium in section 2. As a result, the long-run economic growth rate in the social planning economy is the same as the economic growth rate at the high-growth BGP in the competitive equilibrium. Thus, as the low-growth BGP is eliminated, the social welfare on the BGP is maximized at the high-growth BGP.

### *Public Goods*

It is shown that if the government uses the lump-sum taxes to provide a public good for the final goods sector, the economy can attain the first-best allocation and escape from development trap. The reason is that the public good provision in the final goods sector internalizes the spillover effect originated from the final goods sector. As a result, there is a unique BGP that is associated with the high-growth BGP.

Specifically, in the simplest case of  $\alpha_1 = 0$ ,  $\beta_2 = 1$  and  $\gamma_2 = 0$ , the two production functions are

$$x(t) = [k_x(t)]^{1-\omega} [G(t)]^\omega \quad \text{and} \quad y(t) = [k_y(t)]^{1-\alpha_2} [\bar{k}_x(t)]^{\alpha_2},$$

where  $G$  is public goods *a la* Barro (1990). The government uses a lump-sum tax on final goods  $T = \tau x$  to finance the public expenditure, so  $T = G$ .

By using the same method in deriving the first-best policy as in last subsection, it has been shown that the allocation in the market equilibrium under the public goods policy can replicate the first-best allocation if the tax revenue is

$$T = \left( \frac{1-\alpha_2}{1-\omega} \frac{v^*}{v^*-\alpha_2} \right)^{\frac{1}{\omega}} (vk), \tag{19}$$

where  $v^*$  is evaluated at BGP. The tax in (19) is equivalent to the lump-sum tax rate of  $\tau = \left( \frac{1-\alpha_2}{1-\omega} \frac{v^*}{v^*-\alpha_2} \right)^{\frac{1-\omega}{\omega}}$ .

In equilibrium, the two market equilibrium conditions are:  $c = x - G$  and  $\dot{k} = y - \delta k$ . Moreover, by using the firm's optimization conditions in the two sectors and the balanced government budget, the household optimization condition for consumption becomes

$$c^\sigma \varphi = \frac{1}{A} \left( \frac{1-v}{v} \right)^{\alpha_2} \frac{v}{v-\alpha_2}, \tag{20}$$

where  $\varphi$  is the shadow price of capital. Note that (20) requires  $v > \alpha_2$  in order to be consistency with positive consumption.

With the use of the growth rate of the shadow price of capital, the growth rate of capital, and the consumption market equilibrium condition, (20) is differentiated to obtain

$$\dot{v} = M_2(v) \Lambda_2(v), \tag{21}$$

where  $M_2(v) \equiv \left( \frac{(1-\alpha_2)\alpha_2(\Delta^{1/\omega-1}/\omega-1)}{(1-\omega)(\Delta-\Delta^{1/\omega})(v-\alpha_2)^2} + \frac{(\sigma+\alpha_2-1)}{v} + \frac{\alpha_2}{1-v} + \frac{1}{v-\alpha_2} \right)^{-1}$ ,

$$\Lambda_2(v) \equiv - \left( \frac{\dot{\varphi}}{\varphi} + \sigma \frac{\dot{k}}{k} \right) = (\sigma-1)\delta - \rho - A \left( \frac{v}{1-v} \right)^{\alpha_2} [\sigma(1-v) - (1-\alpha_2)],$$

$$\Delta \equiv \frac{1-\alpha_2}{1-\omega} \frac{v}{v-\alpha_2}.$$

First, the requirement  $v > \alpha_2$  in (20) implies  $M_2(v) > 0$ . Next, in BGP when  $\dot{v} = 0$   $\Lambda_2(v) = 0$  leads to an expression identical to (9). Under Condition B,  $\Lambda_2(v) = 0$  gives two BGPs as illustrated in Figure 1(a). Condition B implies that  $v_1^* < \alpha_2$ ; however,  $v_1^* < \alpha_2$  violates consistency in (20) and hence is not a feasible BGP. As a result, there is a unique feasible BGP. The BGP is  $v_2^*$  in Figure 1(a) if  $\theta \in \Theta_1$  which is the same as the high-growth BGP in last subsection.

Thus, with a lump-sum tax on the final goods sector at the tax rate of (19), a public good provision to the final goods sector can internalize the spillover effect



from the final goods sector to the intermediate goods sector. As a result, the social welfare on the BGP is maximized at the high-growth BGP. To summarize the results,

**PROPOSITION 4.** *In a two-sector economy with the final goods sector and the intermediate goods sector and with intersectoral externalities, when the government levies a lump-sum tax in proportion to average consumption in the society at the tax rate as specified in (19) and provides public goods to the final goods production, the economy can escape from development trap and attain the first-best allocation.*

#### 4. Concluding Remarks

This paper studies a two-sector AK growth model of final goods and intermediate goods based on the model developed by Rebelo (1991). A central feature in the model is the intersectoral externality. It is found that, no matter whether or not there are other types of externalities, the model creates two BGPs if there is a moderate intersectoral externality from the final goods sector to the intermediate sector. The high-growth BGP is a source, and the low-growth BGP is a sink and development trap. Unlike existing AK growth models that yield multiple BGPs by adding an additional complexity such as endogenous labor–leisure choices or cash-in-advance constraints, this model presents the simplest AK growth model with multiple BGPs. Moreover, unlike most existing AK growth models with two BGPs wherein the high-growth BGP is a sink, the low-growth BGP is a sink in the present model. As a result, the model can analyze the development trap.

Policies are analyzed in order to yield a big-push effect. It is found that a subsidy to the final goods sector financed by lump-sum taxes in proportion to average consumption can achieve the first best as the subsidy to the final goods internalizes the spillovers originated from this sector. Moreover, it is also found that a public good provision to the final goods sector financed by lump-sum taxes can internalize the spillover effect from the final goods sector to the intermediate goods sector. In both policies, the only feasible BGP is the high-growth BGP.

Finally, there are limitations to this model. In particular, to simplify the model, only capital has been considered. As a result, the dynamics is one-dimensional and the transitional dynamics cannot be analyzed. To analyze transitional dynamics, if human capital *a la* Lucas (1988) is introduced, the dynamic equilibrium system is three-dimensional, which determines the fraction of capital allocation between sectors, the consumption to capital ratio and the human to physical capital ratio. While it is possible to carry out a mathematical solution of the dynamic property, it is difficult to illustrate transitional dynamics in a three-dimensional diagram. Alternatively, if a public capital stock *a la* Futagami et al. (1993) is introduced, the dynamic equilibrium system determines the fraction of capital allocation between sectors, the consumption to capital ratio and the public capital to physical capital ratio, but it is three-dimensional. Finally, if consumption habit stock *a la* Chen (2007) is introduced, the dynamic equilibrium system determines the fraction of capital allocation between sectors, the consumption to habit ratio and the capital to habit ratio, but it is still three-dimensional. Nevertheless, there may be other state variables that can be added into this model and produce a two-dimensional dynamic equilibrium system. This is a direction for further research.

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## Notes

1. For more evidence concerning the importance of backward and forward linkages, see Liu (2008) and Debaere et al. (2010) who used data in China, Wang (2010) who used data in Canada, and Reinhardt (2000) who used data in Malaysia and Thailand. It should also be noted that the importance of intersectoral spillovers in economic development can be traced to Hirschman (1958) with the existence of the spillover being first confirmed by Kaldor (1967) and followed by many others.
2. The corner point  $v = 0$  is not a BGP as  $\dot{v} < 0$  at  $v = 0$  and the economy will not stay there.
3. In the case of two BGPs, it is impossible for the agent to choose initial  $v > v_2^*$  in Figures 1(a) and 2(a), for, should the agent make an arbitrary choice, the economy would have moved eventually to point  $v = 1$ . However, point  $v = 1$  is not a BGP as  $\dot{v} > 0$  at point  $v = 1$ . Moreover, at  $v = 1$  the economy does not accumulate capital and the economy is in stagnation. The welfare is

lower. Therefore, it is not optimal for the representative agent to choose an equilibrium path with  $v$  in the range of  $(v_2^*, 1]$ .

4. While the condition (16a) makes the individual's flow budget constraint the same in the economy with and without taxes, the condition (16b) makes (15a) to equalize (13a) and the condition (16c) makes (15b) to equalize (13b).