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The Capture Region of a General 3D TPN Guidance Law for Missile and Target with Limited Maneuverability

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Abstract

In this paper, a novel method is used to analyze the capture area of general 3D TPN guidance laws. With the aid of the orthogonal by not orthonormal modified polar coordinate (MPC), and three modified polar variables (MPVs), the expression of relative dynamics between target and missile becomes simple, and no trigonometric functions are involved, which makes possible the analysis of capture region for both target and missile with limited maneuverability. It can be shown that the determination of the desired capture area requires only the first two of the MPVs no matter whether the maneuverability of missile and target are limited or not. The boundary of the capture region on the two MPVs phase plane can be shown is composed of stable, unstable manifolds and a particular trajectory which will be indicated in the context. For the case of unlimited missile acceleration and measurable target acceleration, the capture region can be found analytically, while for the other cases, the capture region can be obtained graphically.

1 Introduction

Although three-dimensional missile guidance have been widely used and studied in the guidance literature as was pointed out in [1] and the reference therein. Deriving the related capture condition is still one of the main topic being discussed [2–9]. However, most of the literature either required a small heading error assumption [2, 3], or can be applied only to time-invariant navigation constant and two-dimensional case [2–6], most of all, their derivation were developed in polar or spherical coordinate and involved lots of trigonometric functions, and hence, inevitably, were complicated [6–8]. Only a few papers discussed capturability of guidance law against 3D maneuvering target [8, 9]. However, their results are conservative in general, besides, to the author's knowledge none of them considered the limited maneuverability of missile. In this paper, a novel method is used to analyze the capture area of general three-dimensional true proportional navigation (TPN) guidance laws. With the aid of the orthogonal but not orthonormal modified polar coordinate (MPC) [10], and three modified polar variables (MPVs), at the cost of two redundant state variables and two constraints of course, the expression of relative dynamics between target and missile becomes very simple, and no trigonometric functions are involved, which is quite different from the traditional way. In addition, it was shown in this paper, the three MPVs are the all variables we need to characterize the target missile relative dynamics. As will be seen, only first two of the MPVs, u and v (which will be defined later in the context), are required to analyze the capture condition. Hence, for the maneuvering target with measurable acceleration, we are able to use the classic phase plane method to express the capture condition of the corresponding guidance law analytically. The result is valid for both time-varying and time-invariant navigation constant. For the maneuvering target, we assume two different escape model, namely, worst case target (a target uses maximum available power) or intelligent target (the target applies TPN guidance law). The capture region is able to be shown on the (u, v) plane graphically in general. It is found that when target acceleration can be obtained, those two escape models provide almost the same capture region, however, if the target acceleration can not be provided, then the intelligent target model tends to give larger capture region.

2 Dynamic Equations in the MPC

Let the relative position vector (line of sight),r, of target and missile be defined as

$$r = r_T - r_M = \rho e_r, \tag{2.1}$$

where r_T and r_M are the position vectors of target and missile in an inertial coordinate $OXYZ$ respectively, ρ is the length of line of sight vector, and e_r is the unit vector along line of sight vector. The components of the state vector of the modified polar coordinate system are defined as follows [10]:

$$x_P = \begin{bmatrix} x_{P1} & x_{P2} & x_{P3} & x_{P4} \end{bmatrix}^T \tag{2.2}$$

$$\triangleq \begin{bmatrix} e_r & \dot{e}_r & \frac{1}{\rho} & \frac{1}{\rho}\frac{d\rho}{dt} \end{bmatrix}^T. \tag{2.3}$$

The state dynamics, are given by differentiating each

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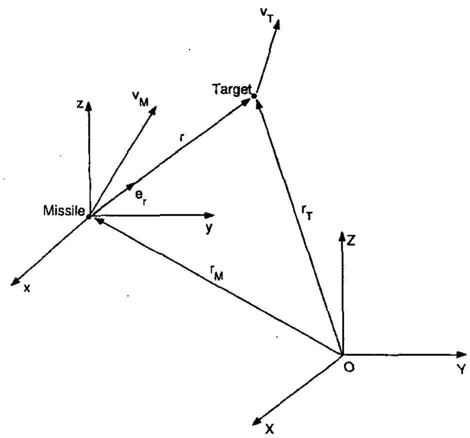

Figure 1: Engagement geometry

component of state vector x_P:

$$\frac{dx_P}{dt} = f(x_P) + g(x_P)(a_T - a_M), \tag{2.4}$$

where a_T and a_M are accelerations of target and missile respectively, and

$$f(x_P) = \begin{bmatrix} x_{P2} \\ -2x_{P4}x_{P2} - (x_{P2}^T x_{P2})x_{P1} \\ -x_{P3}x_{P4} \\ (x_{P2}^T x_{P2}) - x_{P4}^2 \end{bmatrix}, \tag{2.5a}$$

$$g(x_P) = \begin{bmatrix} 0_{3\times3} \\ x_{P3}(I_3 - x_{P1}x_{P1}^T) \\ 0_{1\times3} \\ x_{P3}x_{P1}^T \end{bmatrix}. \tag{2.5b}$$

To describe the relative dynamics between missile and target in a 3-Dimensional space, six states are required. However, we utilized eight states instead, hence, there are two constraints on the above given states, namely,

$$x_{P1}^T x_{P1} = 1, x_{P1}^T x_{P2} = 0. \tag{2.6}$$

Note that x_{P2} is not a unit vector in general, indeed its magnitude is equal to the magnitude of the angular velocity of line of sight. Since the angular velocity of line of sight, Ω, can be expressed as

$$\Omega = \frac{r \times \dot{r}}{\rho^2} = x_{P1} \times x_{P2}, \tag{2.7}$$

it follows that

$$(x_{P1} \times x_{P2})^T (x_{P1} \times x_{P2}) = x_{P2}^T x_{P2} = \Omega^T \Omega. \tag{2.8}$$

By taking this orthogonal but not orthonormal coordinate system (x_{P1}, x_{P2}, Ω), the analysis in of guidance law in this paper becomes easy.

If we express a_T and a_M in the (x_{P1}, x_{P2}, Ω) coordinate system as

$$a_T \triangleq a_{T1}x_{P1} + a_{T2}\frac{x_{P2}}{\sqrt{\Omega^T\Omega}} + a_{T\Omega}\frac{\Omega}{\sqrt{\Omega^T\Omega}}, \tag{2.9}$$

$$a_M \triangleq a_{M1}x_{P1} + a_{M2}\frac{x_{P2}}{\sqrt{\Omega^T\Omega}} + a_{M\Omega}\frac{\Omega}{\sqrt{\Omega^T\Omega}}, \tag{2.10}$$

and after applying the constraints (2.6), we then have the following three coupled scalar differential equations:

$$\frac{d}{dt}x_{P4} = \Omega^T\Omega - x_{P4}^2 + x_{P3}(a_{T1} - a_{M1}), \tag{2.11}$$

$$\frac{d}{dt}\sqrt{\Omega^T\Omega} = -2x_{P4}\sqrt{\Omega^T\Omega} + x_{P3}(a_{T2} - a_{M2}), \tag{2.12}$$

$$\frac{d}{dt}x_{P3} = -x_{P3}x_{P4}. \tag{2.13}$$

Remark 2.1. From (2), it is obvious that the components in the Ω direction of both a_T and a_M do not influence the range between target and missile. However, it can be used to increase the system observability [12].

For the reason that will be explained later, we introduce the following modified polar variables (MPVs),

$$u \triangleq \frac{x_{P4}}{\sqrt{x_{P3}}}, \quad v \triangleq \sqrt{\frac{\Omega^T\Omega}{x_{P3}}}, \quad w \triangleq \sqrt{x_{P3}}. \tag{2.14}$$

Then, it can be shown that

$$\frac{du}{dt} = \left\{-\frac{1}{2}u^2 + v^2 + a_{T1} - a_{M1}\right\}w, \tag{2.15a}$$

$$\frac{dv}{dt} = \left\{-\frac{3}{2}uv + a_{T2} - a_{M2}\right\}w, \tag{2.15b}$$

$$\frac{dw}{dt} = -\frac{1}{2}uw^2. \tag{2.15c}$$

Next, we change the independent variable from t to τ as defined by

$$d\tau \triangleq w dt, \tag{2.16}$$

which in turn yields

$$\frac{du}{d\tau} = -\frac{1}{2}u^2 + v^2 + a_{T1} - a_{M1}, \tag{2.17a}$$

$$\frac{dv}{d\tau} = -\frac{3}{2}uv + a_{T2} - a_{M2}, \tag{2.17b}$$

$$\frac{dw}{d\tau} = -\frac{1}{2}uw. \tag{2.17c}$$

In this paper, we adopt the definition of "capture" of target given in [1], and define the capture region as the follows:

Definition 2.1. The capture of target by missile is characterized by a finite final time t_f at which the range $\rho(t_f)$ is equal to zero. This can be formulated as

$$\exists t_f < \infty \text{ such that } \rho(t_f) = 0. \tag{2.18}$$

To avoid the turn rate of the line of sight, i.e. $\|\dot{e}_r\|$, from infinite, we also require that

$$\rho(t_f)\|\dot{e}_r(t_f)\| = 0 \text{ or } \frac{\Omega^T(t_f)\Omega(t_f)}{x_{P3}(t_f)} = 0. \qquad (2.19)$$

Definition 2.2. The capture region is the region on the $(x_{P4}, \sqrt{\Omega^T\Omega})$-plane or (u, v)-plane such that whenever the initial states $(x_{P4}(t_0), \sqrt{\Omega^T(t_0)\Omega(t_0)})$ or $(u(0), v(0))$ are started inside this region, the state trajectories will lead to $(x_{P4}(t_f), \sqrt{\Omega^T(t_f)\Omega(t_f)}) = (-\infty, \Omega_f)$ or $(u(t_f), v(t_f)) = (-\infty, 0)$, where Ω_f is a nonnegative finite number.

3 Unlimited a_M and a_T with Measurable a_T

We at first consider the case when the target acceleration a_T can be obtained and the magnitude of missile's acceleration a_M is unlimited. The following result is similar to the one in [1], however by virtue of the MPVs, the sufficient condition (equation (83) in [1]) can be relaxed.

Theorem 3.1. Let the guidance law be

$$a_M = a_T - \alpha(x_P)\frac{\Omega^T\Omega}{x_{P3}}x_{P1} - \beta(x_P)\frac{x_{P4}}{x_{P3}}x_{P2} - \gamma(x_P)\Omega, \qquad (3.1)$$

then we have the following two cases:

Case 1: If the following scalar functions satisfy

$$\alpha(x_P) < -1, 2 < \beta(x_P), |\gamma(x_P)| < \infty,$$

and the missile starts its trajectory with any initial condition except

$$x_{P4}(t_0) \geq 0 \text{ and } \Omega^T(t_0)\Omega(t_0) = 0,$$

then the capture of the target always occurs in a finite time t_f,

$$t_f < t_0 - \frac{1}{x_{P4}(t_0)},$$

with $\|a_M(t)\| < \infty, \forall\, t \geq t_0$.

Case 2: If the constant α_{max} and scalar functions $\alpha(x_P), \beta(x_P), \gamma(x_P)$ satisfy

$$-1 \leq \alpha(x_P) \leq \alpha_{max}, 2 < \beta(x_P), |\gamma(x_P)| < \infty,$$

and the missile starts its trajectory with initial condition such that

$$x_{P4}(t_0) < 0, (\alpha_{max} + 1)\Omega^T(t_0)\Omega(t_0) < x_{P4}^2(t_0),$$

then the capture of the target always occurs in a finite time t_f,

$$-\frac{1}{x_{P4}(t_0)} \leq t_f - t_0 \leq \frac{x_{P4}(t_0)}{(\alpha_{max} + 1)\Omega^T(t_0)\Omega(t_0) - x_{P4}^2(t_0)},$$

with $\|a_M(t)\| < \infty, \forall\, t \geq t_0$. Furthermore, if $\alpha(x_P) = \alpha, \beta(x_P) = \beta$ for all $x_P \in \mathbf{R}^8$, where α, β are constants and $\alpha > -1, \beta > 2$. Then the capture of target always occurs in a finite time t_f if and only if

$$x_{P4}(t_0) < 0, (\alpha+1)\Omega^T(t_0)\Omega(t_0) < (\beta-1)x_{P4}^2(t_0). \quad (3.2)$$

Proof: For details and an alternative way of the proof please refer to [11, 12].

4 Limited a_M and a_T with Measurable a_T

Now we consider the case when target acceleration, a_T, is measurable, and both the magnitude of target and missile's acceleration (manerverability) are limited, that is,

$$a_T = \mathrm{sat}_T\left[(x_{P1}^Ta_T)x_{P1} + (x_{P2}^Ta_T)\frac{x_{P2}}{\Omega^T\Omega} + (\Omega^Ta_T)\frac{\Omega}{\Omega^T\Omega}\right],$$

$$a_M = \mathrm{sat}_M\left[a_T - \alpha(x_P)\frac{\Omega^T\Omega}{x_{P3}}x_{P1} - \beta(x_P)\frac{x_{P4}}{x_{P3}}x_{P2} - \gamma(x_P)\Omega\right]$$

where sat_T and sat_M represent the saturation function for target and missile respectively. Since (x_{P1}, x_{P2}, Ω) is an orthogonal coordinate system, for simplicity, it is reasonable to assume that $\mathrm{sat}_T[\cdot]$ and $\mathrm{sat}_M[\cdot]$ both are independent saturation functions, hence,

$$\begin{aligned} a_T = {} & \mathrm{sat}_{T1}\left[x_{P1}^Ta_T\right]x_{P1} + \mathrm{sat}_{T2}\left[\frac{x_{P2}^Ta_T}{\sqrt{\Omega^T\Omega}}\right]\frac{x_{P2}}{\sqrt{\Omega^T\Omega}} \\ & +\mathrm{sat}_{T\Omega}\left[\frac{\Omega^Ta_T}{\sqrt{\Omega^T\Omega}}\right]\frac{\Omega}{\sqrt{\Omega^T\Omega}}, \end{aligned} \qquad (4.1)$$

and

$$\begin{aligned} a_M = {} & \mathrm{sat}_{M1}\left[a_{T1} - \alpha(x_P)\frac{\Omega^T\Omega}{x_{P3}}\right]x_{P1} \\ & +\mathrm{sat}_{M2}\left[a_{T2} - \beta(x_P)\frac{x_{P4}}{x_{P3}}\sqrt{\Omega^T\Omega}\right]\frac{x_{P2}}{\sqrt{\Omega^T\Omega}} \\ & + \mathrm{sat}_{M\Omega}\left[a_{T\Omega} - \gamma(x_P)\sqrt{\Omega^T\Omega}\right]\frac{\Omega}{\sqrt{\Omega^T\Omega}}, \end{aligned} \qquad (4.2)$$

where for $i = 1, 2, \Omega$,

$$\mathrm{sat}_{Ti}[x] = \begin{cases} -a_{Timax} & \text{if } x < -a_{Timax}, \\ x & \text{if } -a_{Timax} \leq x \leq a_{Timax}, \\ a_{Timax} & \text{if } a_{Timax} < x, \end{cases}$$

$$\mathrm{sat}_{Mi}[x] = \begin{cases} -a_{Mimax} & \text{if } x < -a_{Mimax}, \\ x & \text{if } -a_{Mimax} \leq x \leq a_{Mimax}, \\ a_{Mimax} & \text{if } a_{Mimax} < x. \end{cases}$$

It is easy to see that equations (2.17) can be written as

$$\frac{du}{d\tau} = -\frac{1}{2}u^2 + v^2 + a_{T1} + \mathrm{sat}_{M1}\left[-a_{T1} + \alpha(x_P)v^2\right], (4.3a)$$

$$\frac{dv}{d\tau} = -\frac{3}{2}uv + a_{T2} + \mathrm{sat}_{M2}\left[-a_{T2} + \beta(x_P)uv\right], \quad (4.3b)$$

$$\frac{dw}{d\tau} = -\frac{1}{2}uw, \qquad (4.3c)$$

respectively. For small u and v, the behavior of the state trajectory around $(0,0)$ is similar to the one discussed in Theorem 3.1, hence, $\beta(x_P) > 2$, is enough to guarantee the existence of capture area.

Equations (4.3) are highly nonlinear, the exact capture region can only be determined graphically at this moment. In this paper we consider the following two cases regarding the target escape model.

4.1 Target uses maximum available power

From target's aspect, it is reasonable to assume that target utilizes maximum available power to maximize $\frac{du}{d\tau}$ and $\frac{dv}{d\tau}$ to escape, therefore,

$$a_{T1} = a_{T1max}, a_{T2} = a_{T2max}, a_{T\Omega} = 0. \tag{4.4}$$

Figure 2 shows the mushroom type phase portraits on the (u, v) plane of the differential equations (4.3) when $\alpha = 1, \beta(x_P) = 3, a_{T1max} = a_{T2max} = 8g_0, a_{M1max} = a_{M2max} = 20g_0$. Apparently, the boundary of the capture is defined by one manifold connecting the unstable equilibrium point $(-12.522, 6.261)$ and the saddle equilibrium point $(0,0)$, and one manifold defined by $u \leq 0, v = 0$, and one state trajectory connecting the equilibrium point $(-12.522, 6.261)$ and $(-\infty, 0)$.

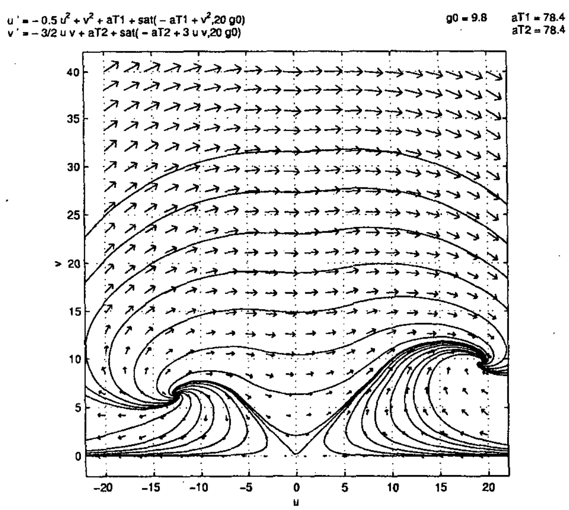

Figure 2: The phase portrait for $\alpha = 1, \beta = 3, a_{T1max} = a_{T2max} = 8g_0$, $a_{M1max} = a_{M2max} = 20g_0$, and $a_{T1} = a_{T1max}, a_{T2} = a_{T2max}$, a_T is measurable.

4.2 Intelligent target

For an intelligent target using the three dimensional IPN [8], we have

$$a_T = \text{sat}_T \left[\lambda_T \frac{\Omega^T \Omega}{x_{P3}} x_{P1} - \lambda_T \frac{x_{P4}}{x_{P3}} x_{P2} \right], \tag{4.5}$$

and similarly, by assuming independent saturation, we have

$$\begin{aligned} a_{T1} &= \text{sat}_{T1} \left[\lambda_T \frac{\Omega^T \Omega}{x_{P3}} \right] = \text{sat}_{T1} \left[\lambda_T v^2 \right], \\ a_{T2} &= \text{sat}_{T2} \left[-\lambda_T \frac{x_{P4}}{x_{P3}} \sqrt{\Omega^T \Omega} \right] = \text{sat}_{T2} \left[-\lambda_T uv \right], \\ a_{T\Omega} &= 0. \end{aligned}$$

Again, Figure 3 shows the mushroom type phase portraits on the (u, v) plane of the differential equations (4.3) when $\alpha = 1, \beta(x_P) = 3, a_{M1max} = a_{M2max} = 20g_0$, and the intelligent target is performing an escape with $\lambda_T = 5, a_{T1max} = a_{T2max} = 8g_0$. Similarly, the boundary of the capture is defined by one manifold connecting the unstable equilibrium point $(-12.522, 6.261)$ and the saddle equilibrium point $(0,0)$, and one manifold defined by $u \leq 0, v = 0$, and one state trajectory connecting the equilibrium point $(-12.522, 6.261)$ and $(-\infty, 0)$. It can also be seen that finite saturation level brings the unstable equilibrium point from infinite distance $(-\infty, \infty)$ to finite distance. Comparing the above two cases reveals that both maintain almost the same capture region.

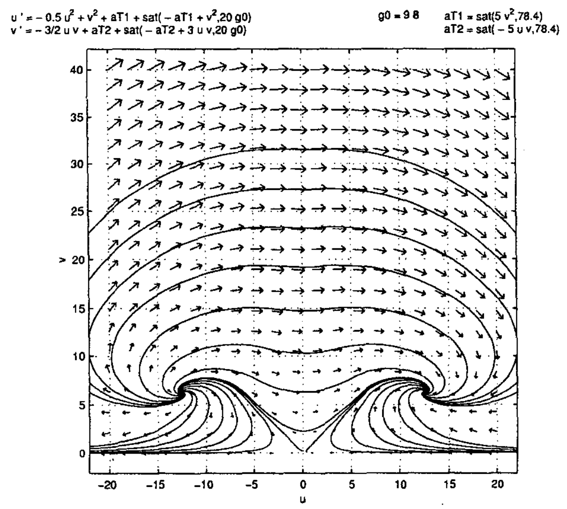

Figure 3: The phase portrait for $\alpha = 1, \beta = 3, \lambda_T = 5, a_{T1max} = a_{T2max} = 8g_0$, $a_{M1max} = a_{M2max} = 20g_0$, and an intelligent target with a_T is measurable.

5 Limited a_M and a_T with Unmeasurable a_T

Next, let's consider the case when the target acceleration, a_T, can not be obtained, and the target maneuverability is limited. By assuming independent saturation of the missile's acceleration, the guidance law is the following,

$$a_M = \text{sat}_{M1}\left[-\alpha(x_P)\frac{\Omega^T\Omega}{x_{P3}}\right]x_{P1}$$
$$+\text{sat}_{M2}\left[-\beta(x_P)\frac{x_{P4}}{x_{P3}}\sqrt{\Omega^T\Omega}\right]\frac{x_{P2}}{\sqrt{\Omega^T\Omega}}$$
$$+\text{sat}_{M\Omega}\left[-\gamma(x_P)\sqrt{\Omega^T\Omega}\right]\frac{\Omega}{\sqrt{\Omega^T\Omega}}. \quad (5.1)$$

Similarly, we then have the corresponding differential equations in the (u, v, w) space,

$$\frac{du}{d\tau} = -\frac{1}{2}u^2 + v^2 + a_{T1} + \text{sat}_{M1}\left[\alpha(x_P)v^2\right], \quad (5.2a)$$
$$\frac{dv}{d\tau} = -\frac{3}{2}uv + a_{T2} + \text{sat}_{M2}\left[\beta(x_P)uv\right], \quad (5.2b)$$
$$\frac{dw}{d\tau} = -\frac{1}{2}uw. \quad (5.2c)$$

Likewise, if $\alpha(x_P), \beta(x_P)$ and a_{T1}, a_{T2} are functions of u and v only, then the differential equations for u and v will be decoupled from the one for w. Note that $\beta(x_P)$ must be chosen such that $\exists u, v$ at which point $\frac{dv}{dt} < 0$. Therefore, the lower bound of $\beta(x_P)$ may need to be larger than 2 to ensure the existence of capture area for missile with limited acceleration.

To determine the target's policy to escape, let's do the following analysis. Assume that $\|a_M\|$ is unlimited. After some algebraic manipulations, the differential equations for ρ are given as follows

$$\frac{d^2\rho}{dt^2} = \frac{d}{dt}\left(\frac{x_{P4}}{x_{P3}}\right) = [\alpha(x_P)+1]\frac{\Omega^T\Omega}{x_{P3}} + a_{T1}, \quad (5.3a)$$
$$\frac{d^3\rho}{dt^3} = [\alpha(x_P)+1]\left\{[1+2(\beta(x_P)-2)]\frac{x_{P4}}{x_{P3}}\Omega^T\Omega + 2a_{T2}\sqrt{\Omega^T\Omega}\right\} + \frac{d}{dt}a_{T1} + \frac{\Omega^T\Omega}{x_{P3}}\frac{d}{dt}\alpha(x_P) \quad (5.3b)$$

Apparently, when target tries to escape, it uses the maximum available power to maximize $\ddot{\rho}$ and $\dddot{\rho}$, hence, we may assume that during the intercepting process

$$a_{T1} = a_{T1max}, \text{ and } a_{T2} = \text{sgn}(\alpha(x_P)+1)a_{T2max}.$$

Henceforth, $a_{T2} = -a_{T2max}$ when $\alpha(x_P) < -1$. This is contrary to the point of view of maximizing $\frac{du}{d\tau}, \frac{dv}{d\tau}$ only. Although the differential equations (5.2) describing the relative dynamics of target and missile becomes with limited maneuverability are involved, however, we still may express the capture area on the (u, v) plane graphically. Similarly, we consider the following two cases:

5.1 Target uses maximum available power

Assume target utilizes available maximum power to maximize $\frac{du}{d\tau}$ and $\frac{dv}{d\tau}$ (or $\ddot{\rho}, \dddot{\rho}$), therefore,

$$a_{T1} = a_{T1max}, \quad a_{T2} = a_{T2max} \text{ and } a_{T\Omega} = 0.$$

Figure 4 gives the mushroom type phase portraits on the (u, v) plane of the differential equations (5.2) when $\alpha = 1, \beta(x_P) = 6, a_{M1max} = a_{M2max} = 20g_0$, and target is utilizing maximum available power $a_{T1max} = a_{T2max} = 8g_0$. The capture region is bounded by the following three curves:

1. the two stable manifolds of the saddle equilibrium point $(-12.8138, 1.3596)$,
2. one particular trajectory starting from $(-15.9282, 4.9221)$ and approaching to $(-\infty, 0)$.

One interesting observation is that all the trajectories inside the capture region eventually approach to the one of the unstable manifold of the saddle point $(-12.8138, 1.3596)$. Note that in this case $\beta(x_P) = 3$ is not enough to provide capture region.

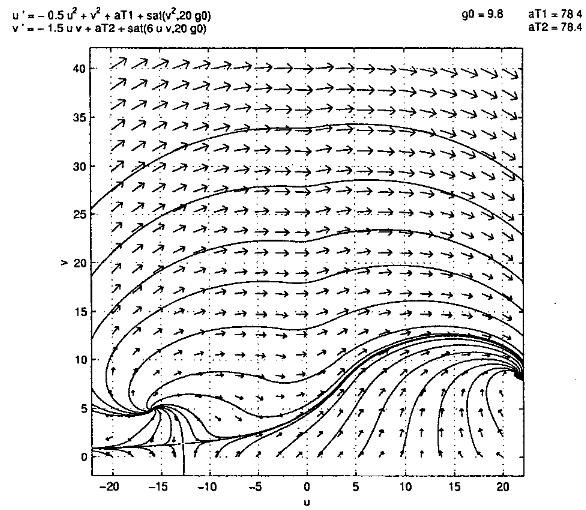

Figure 4: The phase portrait for $\alpha = 1, \beta = 6, a_{T1max} = a_{T2max} = 8g_0$, $a_{M1max} = a_{M2max} = 20g_0$, with a_T is not measurable.

5.2 Intelligent target

$$a_{T1} = \text{sat}_{T1}\left[\lambda_T\frac{\Omega^T\Omega}{x_{P3}}\right] = \text{sat}_{T1}\left[\lambda_T v^2\right],$$
$$a_{T2} = \text{sat}_{T2}\left[-\lambda_T\frac{x_{P4}}{x_{P3}}\sqrt{\Omega^T\Omega}\right] = \text{sat}_{T2}\left[-\lambda_T uv\right],$$
$$a_{T\Omega} = 0.$$

Figure 5 shows the mushroom type phase portraits on the (u, v) plane of the differential equations (5.2) when $\alpha = 1, \beta(x_P) = 6, a_{M1max} = a_{M2max} = 20g_0$, and the intelligent target is performing an escape with $\lambda_T = 5, a_{T1max} = a_{T2max} = 8g_0$. The capture region is bounded by the following four curves:

1. the stable manifold of the saddle equilibrium point $(-8.0739, 2.1578)$ connecting the unsta-

ble equilibrium point $(-15.9282, 4.9221)$ and $(-8.0739, 2.1578)$,

2. the stable manifold of the saddle equilibrium point $(-8.0739, 2.1578)$ connecting $(-8.0739, 2.1578)$ and $(0, 0)$,

3. the unstable manifold $u < 0, v = 0$,

4. one particular trajectory starting from $(-15.9282, 4.9221)$ and approaching to $(-\infty, 0)$.

Similarly, all the trajectories inside the capture region eventually approach to the one of the unstable manifold of the saddle point $(-8.0739, 2.1578)$. Again, in this case $\beta(x_P) = 3$ is not enough to provide capture region. Comparing the above two cases indicates that when target utilize maximum available power to escape has the smaller capture region.

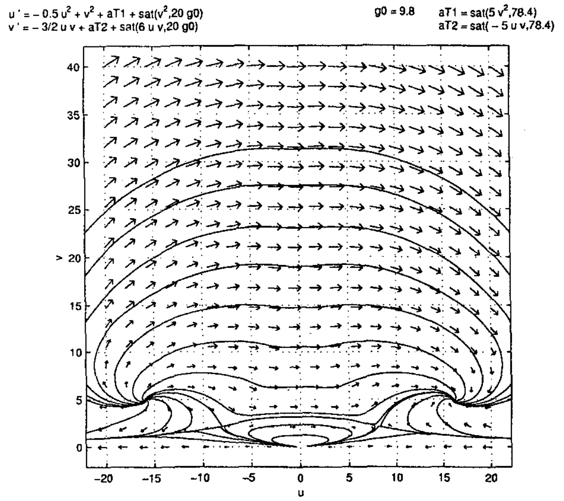

Figure 5: The phase portrait for $\alpha = 1, \beta = 6, \lambda_T = 5, a_{T1max} = a_{T2max} = 8g_0$, $a_{M1max} = a_{M2max} = 20g_0$, and an intelligent target with a_T is not measurable.

6 Conclusion

The relative dynamics between target and missile is formulated in an orthogonal but not orthonormal Modified polar coordinate. With the cost of two redundant states, the expression of the differential equations are simple and no trigonometric functions are involved. Then the author introduces three modified polar variables. It turns out that the three corresponding differential equations are all we need to analyze the general 3D TPN guidance law. In addition, only two of the MPVs, namely u and v are required to analyze the intercept of target for the given definition. Owing to this, for the first time on literature, we would be able to show the capture region of TPN guidance law for both target and missile with limited maneuverability. For the unlimited maneuverability missile and target, we derived the bound of final time, t_f, and the analytical expression of capture region. By virtue of the independent saturation assumption of target and missile's acceleration, we still are able to identify the capture region graphically on the (u, v) plane. Finally, the following conclusions can be drawn to increase the capture region from the missile's perspective: 1. Select the navigation variable $\beta(x_P)$ as large as possible , choose the navigation variable $\alpha(x_P)$ as small (or negative) as possible. 2. Increase the saturation level of missile's endurable acceleration a_M. 3. Provide estimate of target's acceleration a_T.

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