Output Feedback Control of Fuzzy Descriptor Systems with Interval Time-Varying Delay

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***Abstract*—This paper proposes output feedback control for fuzzy descriptor systems with interval time-varying delay. First, singular nonlinear dynamic systems with interval time-varying delay are taken into consideration. Then using a Takagi-Sugeno (T-S) fuzzy model, we design a fuzzy representation of the original nonlinear system. This fuzzy representation consists of local linear descriptor systems. To achieve the control objective, a fuzzy controller and observer is designed in a systematic manner. The stability analysis of the overall closed-loop fuzzy system leads to formulation of linear matrix inequalities. Using the observer and controller gains by solving LMIs, we carry out numerical simulations which verify theoretical statements.**

I. Introduction

In the past decade fuzzy control has been proved to be very fruitful in many applications. Using the T-S fuzzy model [1] representation of nonlinear systems into local linear fuzzy models has lead to vast amounts of research. For example fuzzy control [2], [3]; fuzzy model based chaotic control and synchronization [4], [5]; robust fuzzy control and observer based approaches [6], [7], [8], [9], [10], [11]. Many of the mentioned works approach the design of controllers and observers in an systematic manner. The stability analysis of the closed-loop system leads to formulation of linear matrix inequalities (LMIs) [12]. Then the controller and observer gains are found once the feasible LMIs are solved. The process of solving LMIs can be done numerically by powerful packaged software toolboxes (e.g., MATLAB LMI Toolbox) [13].

Descriptor systems have a tighter representation for a wider class of systems in comparison to traditional state-space representation. Recently this concept has been extended to T-S fuzzy model systems [14]. Note that using traditional T-S fuzzy modeling for Lagrangian mechanical systems, we will need a fuzzy model representation for the inverse of the inertia matrix. This matrix inverse will drastically increase the rule numbers. On the other hand, if the fuzzy descriptor system is used, the number fuzzy rules will be decreased. This rule reduction is an important issue for LMI-based control synthesis since larger number of LMI rules may leads to infeasible problems.

In this paper, we extend the good properties of fuzzy descriptor systems and fuzzy observers into the design of output feedback control for fuzzy descriptor systems. In addition to immeasurable states, we consider interval time-varying delays. The controller and observer design leads to formulating LMIs. Then a two-stage process is utilized in place of simultaneously solving controller and observer parameters which is a complex problem.

The rest of the paper is organized as follows. In Sec. II, the fuzzy descriptor system representation of a singular nonlinear dynamic system with interval time-varying delay is introduced. In Sec. III, we carry out the stability analysis of the open-loop fuzzy descriptor system where the intrinsic stability criterion is given. In Sec. IV, numerical simulations on the control design is carried out. Finally some conclusions are made in Sec. V.

II. Preliminaries and Problem Formulation

We consider a singular nonlinear system

$$\begin{aligned} M\left(x\left(t\right)\right)\dot{x}\left(t\right) &= f_1\left(x\left(t\right)\right)+f_2\left(x\left(t-h(t)\right)\right) \\ &\quad +g\left(x\left(t\right)\right)u\left(t\right)\left(t\right) \\ y\left(t\right) &= h\left(x\right) \end{aligned} \tag{1}$$

where $x\left(t\right) = \left[x_1\left(t\right)\ x_2\left(t\right)\ \cdots\ x_n\left(t\right)\right]^T \in R^n$ is the state vector; $x\left(t-h(t)\right) = \left[x_1\left(t-h(t)\right)\ x_2\left(t-h(t)\right)\ \cdots\ x_p\left(t-h(t)\right)\right]^T \in R^p$ is the state time delay vector; $u\left(t\right) = \left[u_1\left(t\right)\ u_2\left(t\right)\ \cdots\ u_m\left(t\right)\right]^T \in R^m$ is the control input; $M\left(x\left(t\right)\right)$, $f_1\left(x\left(t\right)\right)$, $f_2\left(x\left(t-h(t)\right)\right)$, $g\left(x\left(t\right)\right)$, $h\left(x\left(t\right)\right)$ are smooth functions with $f\left(0\right)=0$; and $y\left(t\right) \in R^q$ is the output. The T-S fuzzy representation is:

$$\begin{aligned} &\text{Plant Rule } i : \\ &\text{IF } z_1\left(t\right) \text{ is } F_{i1} \text{ and } \cdots \text{ and } z_g\left(t\right) \text{ is } F_{gi} \\ &\text{THEN } E_k\dot{x}\left(t\right) = A_i x\left(t\right)+A_{hi}x(t-h(t))+B_i u\left(t\right) \\ &\qquad y\left(t\right) = C_i x\left(t\right), \\ &\qquad x(t) = \varphi(t), t\in[-\tau_M,0]. \end{aligned}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $u(t)$, $y(t) \in R^m$ are the control input and output, respectively; A, A_{hi}, B and C are constant matrices with appropriate dimensions; $\varphi(t)$ is a continuously differentiable initial function of $t \in [-\tau_M, 0]$; and $h(t)$ denotes the time-varying delay and τ_M is the upper of $h(t)$ which satisfies one of the following assumptions:

***Assumption 1*:** The time delay $h(t)$ is a continuous function satisfying $0 \le h(t) \le \tau_M,\ \dot{h}(t) \le \beta < 1$, where τ_M and β are both constants.

***Assumption 2*:** The time delay $h(t)$ is a differentiable function satisfying $0 \le \tau_m \le h(t) \le \tau_M,\ \dot{h}(t) \le \beta$, where τ_m, τ_M and β are positive constants.

The length of allowable delay time will be denoted by δ, i.e., $\delta = \tau_M - \tau_m$. Our objective is to determine the maximum allowable delay time while the system stability is kept. The inferred output

$$\begin{aligned} \sum_{k=1}^{r_e} \mu_k(z(t)) E_k \dot{x}(t) &= \sum_{i=1}^{r} v_i(z(t)) \left(A_i x(t)\right. \\ &\quad \left. + A_{hi} x(t-h(t)) + B_i u(t)\right) \\ y(t) &= \sum_{i=1}^{r} v_i(z(t)) C_i x(t) \end{aligned} \tag{2}$$

for all $i = 1,2,\ldots,r;\ k = 1,2,\ldots,r_e$. Define $x^* = [x^T(t)\ \dot{x}^T(t)]^T$, we rewrite the fuzzy descriptor system (2)

$$\begin{aligned} E^* \dot{x}^*(t) &= \sum_{i=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) \mu_k(z(t)) \left(A_{ik}^* x^*(t)\right. \\ &\quad \left. + A_{hi}^* x^*(t-h(t)) + B_i^* u(t)\right), \\ y(t) &= \sum_{i=1}^{r} v_i(z(t)) C_i^* x^*(t) \end{aligned} \tag{3}$$

for all $i = 1,2,\ldots,r;\ k = 1,2,\ldots,r_e$, where

$$\begin{aligned} E^* &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},\ A_{ik}^* = \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}, \\ A_{hi}^* &= \begin{bmatrix} 0 & 0 \\ A_{hi} & 0 \end{bmatrix},\ B_i^* = \begin{bmatrix} 0 \\ B_i \end{bmatrix},\ C_i^* = \begin{bmatrix} C_i & 0 \end{bmatrix}. \end{aligned}$$

We now design the controller rule as:

$$\begin{aligned} &\text{Control Rule } i: \\ &\text{IF } z_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } z_g(t) \text{ is } F_{gi} \\ &\text{THEN } u(t) = -K_{ik}^* \hat{x}^*(t) \text{ for } i = 1,\ 2,\ \ldots,\ r. \end{aligned}$$

where $K_{ik}^* = [K_{ik}\ 0],\ \ K_{ik} = [K_{ik11}\ K_{ik12}]$; and K_{ik} are controller gains to be chosen later.

We propose a modified PDC (4) to stabilize the fuzzy descriptor system (3):

$$u(t) = -\sum_{i=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) \mu_k(z(t)) K_{ik}^* x^*(t). \tag{4}$$

By substituting (4) into (3), the closed-loop fuzzy control system

$$\begin{aligned} E^* \dot{x}^*(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \\ &\quad \times \left((A_{ik}^* - B_i^* K_{jk}^*) x^*(t) + A_{hi}^* x^*(t-h(t))\right). \end{aligned} \tag{5}$$

To estimate the immeasurable states, we design the observer as:

$$\begin{aligned} &\text{Observer Rule } i: \\ &\text{IF } z_1(t) \text{ is } F_{1i} \text{ and } \cdots \text{ and } z_g(t) \text{ is } F_{gi} \\ &\text{THEN } E_k \dot{\hat{x}}(t) = A_i \hat{x}(t) + A_{hi} \hat{x}(t-h(t)) + B_i u(t) \\ &\qquad\qquad + L_i\left(y(t) - \hat{y}(t)\right) \\ &\qquad \hat{y}(t) = C_i \hat{x}(t), \end{aligned}$$

where $L_{ik}^* = [0\ L_{ik}^T]^T,\ \ L_{ik} = [L_{ik11}^T\ L_{i12}^T]^T$; and L_i is the observer gain of the i-th observer rule to be chosen later. The overall inferred output is

$$\begin{aligned} &\sum_{k=1}^{r_e} \mu_k(z(t)) E_k \dot{\hat{x}}(t) = \sum_{i=1}^{r} v_i(z(t)) \left(A_i \hat{x}(t)\right. \\ &\left. + A_{hi} \hat{x}(t-h(t)) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right. \\ &y(t) = \sum_{i=1}^{r} v_i(z(t)) C_i x(t) \end{aligned} \tag{6}$$

where $z_1(t) \sim z_g(t)$ are the premise variables which consist of the states of the system; F_{ji} $(j = 1,2,\ldots,g)$ are the fuzzy sets; r is the number of fuzzy rules; A_i and B_i are system matrices with appropriate dimensions. For simplicity, we assume that the membership functions have been normalized, i.e., $\sum_{i=1}^{r} \prod_{j=1}^{g} F_{ji}(z_j(t)) = 1$. Using singleton fuzzifier, product inferred, and weighted defuzzifier, the fuzzy system is inferred as follows:

$$\begin{aligned} &E^* \dot{\hat{x}}^*(t) = \sum_{i=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) \mu_k(z(t)) \left(A_{ik}^* \hat{x}^*(t)\right. \\ &\left. + A_{hi}^* \hat{x}^*(t-h(t)) + B_i^* u(t) + L_{ik}^*(y(t) - \hat{y}(t))\right) \\ &y(t) = \sum_{i=1}^{r} v_i(z(t)) C_i^* \hat{x}^*(t), \end{aligned} \tag{7}$$

for all $i = 1,2,\ldots,r;\ k = 1,2,\ldots,r_e$. Then the PDC fuzzy controller with immeasurable states is as follows:

$$u(t) = -\sum_{i=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) \mu_k(z(t)) K_{ik}^* \hat{x}^*(t) \tag{8}$$

where $x^*(t) = [x^{*T}(t)\ \ \dot{\hat{x}}^{*T}(t)]^T$. Combining the fuzzy controller (8) and fuzzy observer (7) and denoting $e^*(t) = x^*(t) - \hat{x}^*(t)$, we arrive with the system representations:

$$\begin{aligned} &E^* \dot{x}^*(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \\ &\times \left((A_{ik}^* - B_i^* K_{jk}^*) x^*(t) + A_{hi}^* x^*(t-h(t)) + B_i^* K_{jk}^* e^*(t)\right) \\ &E^* \dot{e}^*(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \\ &\times \left((A_{ik}^* - L_{ik}^* C_j^*) e^*(t) + A_{hi}^* e^*(t-h(t))\right). \end{aligned} \tag{9}$$

III. Stability Analysis

Under Assumption 1, the stability criterion for system (9) is as follows.

***Theorem 1*:** The fuzzy descriptor system (2) along with controller (4) forming the closed-loop system (5) is asymptotically stable, if there exist positive definite Q_1, nonsingular matrices P, matrices Z_1 and Z_3 satisfying the following LMIs,

$$Z_1^T = Z_1 > 0, \tag{10}$$

$$\begin{bmatrix} -Z_3^T - Z_3 + \tilde{Q}_{111} & \left(Z_1^T A_i^T - M_{jk}^T B_i^T + Z_3^T E_k^T + Z_1 + \tilde{Q}_{112}\right) & 0 & 0 \\ \left(A_i Z_1 - B_i M_{jk} + E_k Z_3 + Z_1^T + \tilde{Q}_{112}^T\right) & \left(-Z_1^T E_k^T - E_k Z_1 + \tilde{Q}_{122}\right) & A_{hi} Z_1 & 0 \\ 0 & Z_1^T A_{hi}^T & -(1-\beta)\tilde{Q}_{111} & -(1-\beta)\tilde{Q}_{112} \\ 0 & 0 & -(1-\beta)\tilde{Q}_{112}^T & -(1-\beta)\tilde{Q}_{122} \end{bmatrix} < 0, \ i,j = 1,2,\ldots,r. \quad k = 1,2,\ldots,r_e. \tag{11}$$

where controller gain is accordingly $K_{jk} = M_{jk} Z_1^{-1}$.

***Proof*:** We can rewrite $E^{*T}P = P^T E^* \geq 0$ as $P^{-T}E^{*T} = E^* P^{-1} \geq 0$. The above inequality is

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-T} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix}^{-1} \geq 0.$$

Therefore, we obtain

$$\begin{bmatrix} Z_1^T & -Z_3^T \\ 0 & Z_1^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 \\ 0 & 0 \end{bmatrix} \geq 0.$$

where $Z_1 = S_1^{-1}$ and $Z_3 = S_1^{-1} S_3 S_1^{-1}$. Note that the following relation holds:

$$\begin{bmatrix} S_1 & 0 \\ S_3 & S_1 \end{bmatrix} \begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Let

$$Q_1 = \begin{bmatrix} Q_{111} & Q_{112} \\ Q_{112}^T & Q_{122} \end{bmatrix}.$$

Here, we consider the Lyapunov-Krasovskii functional candidate $V_1(x^*(t)) = x^{*T}(t) E^{*T} P x^*(t) + \int_{t-h(t)}^{t} x^{*T}(s) Q_1 x^*(s) ds$. The time derivative $\dot{V}_1(x^*(t)) = \dot{x}^{*T}(t) E^{*T} P x^*(t) + x^{*T}(t) E^{*T} P \dot{x}^*(t) + x^{*T}(t) Q_1 x^*(t) - x^{*T}(t-h(t))(1-\dot{h}(t)) Q_1 x^*(t-h(t))$. Taking the time derivative of $V(x^*(t))$ along (9), we have $\dot{V}_1(x^*(t)) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \{x^{*T}(t)(G_{ijk}^T P + P^T G_{ijk}) x^*(t) + x^{*T}(t) P^T A_{hi}^* x^*(t-h(t)) + x^{*T}(t-h(t)) A_{hi}^{*T} P x^*(t) + x^{*T}(t) Q_1 x^*(t) - x^{*T}(t-h(t))(1-\beta) Q_1 x^*(t-h(t))\}$ where $G_{ijk} = A_{ik}^* - B_i^* K_{jk}^*$. Therefore $\dot{V}(x^*(t)) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \xi(t)^T \Lambda \xi(t)$, where $\xi(t)^T = [x^{*T}(t) \ x^{*T}(t-h(t))]$ and

$$\Lambda = \begin{bmatrix} G_{ijk}^T P + P^T G_{ijk} + Q_1 & P^T A_{hi}^* \\ A_{hi}^{*T} P & -(1-\beta) Q_1 \end{bmatrix} < 0.$$

Therefore, when $E^{*T}P = P^T E^* \geq 0$, $\Lambda < 0$, the stability and the closed-loop system (9) is proven. We multiply the inequality $\Lambda < 0$ by the matrix $diag[P^{-T}, \ P^{-T}]$ and its transpose on the left and right, respectively. Then by setting

$$P^{-T} = \begin{bmatrix} Z_1^T & -Z_3^T \\ 0 & Z_1^T \end{bmatrix}$$

where $Z_1 > 0$, and by Schur complement, the inequalities $\Lambda < 0$ are equivalent to (10), where $M_{jk} = K_{jk} Z_1$. This completes the proof of the theorem. □

***Theorem 2*:** The fuzzy descriptor system (2) along with controller (8) and observer (7) forming the closed-loop system (9) is asymptotically stable, if there exist positive definite Q_1 and Q_2, nonsingular matrices P and R, matrices Z_1, Z_3, R_1, R_3, M_{jk} and H_{ik}, and scalars ρ, $\varepsilon_n > 0$, $n = 1,2,3$ satisfying the following LMIs: $Z_1^T = Z_1 > 0$,

$$\begin{bmatrix} -Z_3^T - Z_3 + \tilde{Q}_{111} & \left(Z_1^T A_i^T - M_{ik}^T B_i^T + Z_3^T E_k^T + Z_1 + \tilde{Q}_{112}\right) & 0 & 0 \\ \left(A_i Z_1 - B_i M_{ik} + E_k Z_3 + Z_1^T + \tilde{Q}_{112}^T\right) & \left(-Z_1^T E_k^T - E_k Z_1 + \tilde{Q}_{122} + \varepsilon_1 B_i B_i^T\right) & A_{hi} Z_1 & 0 \\ 0 & Z_1^T A_{hi}^T & -(1-\beta)\tilde{Q}_{111} & -(1-\beta)\tilde{Q}_{112} \\ 0 & 0 & -(1-\beta)\tilde{Q}_{112}^T & -(1-\beta)\tilde{Q}_{122} \end{bmatrix} \tag{12}$$

$$< 0, \ i = 1,2,\ldots,r. \quad k = 1,2,\ldots,r_e. \tag{13}$$

$$\begin{bmatrix} \phi_{11} & \phi_{12} & 0 & 0 \\ \phi_{12}^T & \phi_{22} & (A_{hi} + A_{hj}) Z_1 & 0 \\ 0 & Z_1^T (A_{hi}^T + A_{hj}^T) & -2(1-\beta)\tilde{Q}_{111} & -2(1-\beta)\tilde{Q}_{112} \\ 0 & 0 & -2(1-\beta)\tilde{Q}_{112}^T & -2(1-\beta)\tilde{Q}_{122} \end{bmatrix} \tag{14}$$

$$< 0, \ i < j.$$

$$R_1^T = R_1 > 0, \tag{15}$$

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \rho R_1^T A_{hi} & 0 & K_{ik11}^T \\ \lambda_{12}^T & \lambda_{22} & R_1^T A_{hi} & 0 & K_{ik12}^T \\ \rho A_{hi}^T R_1 & A_{hi}^T R_1 & -(1-\beta)\tilde{Q}_{211} & -(1-\beta)\tilde{Q}_{212} & 0 \\ 0 & 0 & -(1-\beta)\tilde{Q}_{212}^T & -(1-\beta)\tilde{Q}_{222} & 0 \\ K_{ik11} & K_{ik12} & 0 & 0 & -\varepsilon_1 I \end{bmatrix} < 0, \tag{16}$$

$$i = 1, 2, \dots, r. \quad k = 1, 2, \dots, r_e.$$

$$\begin{bmatrix} \tilde{\lambda}_{11} & \tilde{\lambda}_{12} & \rho R_1^T (A_{hi} + A_{hj}) & 0 & K_{jk11}^T & K_{ik11}^T \\ \tilde{\lambda}_{12}^T & \tilde{\lambda}_{22} & R_1^T (A_{hi} + A_{hj}) & 0 & K_{jk12}^T & K_{ik12}^T \\ \rho(A_{hi}^T + A_{hj}^T) R_1 & (A_{hi}^T + A_{hj}^T) R_1 & -2(1-\beta)\tilde{Q}_{211} & -2(1-\beta)\tilde{Q}_{212} & 0 & 0 \\ 0 & 0 & -2(1-\beta)\tilde{Q}_{212}^T & -2(1-\beta)\tilde{Q}_{222} & 0 & 0 \\ K_{jk11} & K_{jk12} & 0 & 0 & -\varepsilon_2 I & 0 \\ K_{ik11} & K_{ik12} & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0, \tag{17}$$

$$i < j.$$

where $\phi_{11} = -2Z_3^T - 2Z_3 + 2\tilde{Q}_{111}$, $\phi_{12} = Z_1^T A_i^T - M_{jk}^T B_i^T + Z_1^T A_j^T - M_{ik}^T B_j^T + 2Z_3^T E_k^T + 2Z_1 + 2\tilde{Q}_{112}$, $\phi_{22} = -2E_k Z_1 - 2Z_1^T E_k^T + 2\tilde{Q}_{122} + \varepsilon_2 B_i B_i^T + \varepsilon_3 B_j B_j^T$, $\lambda_{11} = \rho A_i^T R_1 - \rho C_i^T H_{ik}^T + \rho R_1^T A_i - \rho H_{ik} C_i + \tilde{Q}_{211}$, $\lambda_{12} = A_i^T R_1 - C_i^T H_{ik}^T + R_1^T - \rho R_1^T E_k + \tilde{Q}_{212}$, $\lambda_{22} = -E_k^T R_1 - R_1^T E_k + \tilde{Q}_{222}$, $\tilde{\lambda}_{11} = \rho A_i^T R_1 - \rho C_j^T H_{ik}^T + \rho A_j^T R_1 - \rho C_i^T H_{jk}^T + \rho R_1^T A_i - \rho H_{ik} C_j + \rho R_1^T A_j - \rho H_{jk} C_i + 2\tilde{Q}_{211}$, $\tilde{\lambda}_{12} = A_i^T R_1 - C_j^T H_{ik}^T + A_j^T R_1 - C_i^T L_{jk}^T + R_1^T - \rho R_1^T E_k + 2\tilde{Q}_{212}$, $\tilde{\lambda}_{22} = -E_k^T R_1 - R_1^T E_k + 2\tilde{Q}_{222}$, where controller and observer gains are accordingly $K_{jk} = M_{jk} Z_1^{-1}$ and $L_{ik} = R_1^{-1} H_{ik}$.

***Proof*:** $E^{*T} R = R^T E^* \geq 0$, can be rewritten as $R^{-T} E^{*T} = E^* R^{-1} \geq 0$. The above inequality is

$$\begin{bmatrix} W_1 & 0 \\ W_3 & W_1 \end{bmatrix}^{-T} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} W_1 & 0 \\ W_3 & W_1 \end{bmatrix}^{-1} \geq 0.$$

Therefore, we obtain

$$\begin{bmatrix} R_1^T & -R_3^T \\ 0 & R_1^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R_1 & 0 \\ -R_3 & R_1 \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix} \geq 0.$$

where $R_1 = W_1^{-1}$ and $R_3 = W_1^{-1} W_3 W_1^{-1}$. Note that the following relation holds:

$$\begin{bmatrix} W_1 & 0 \\ W_3 & W_1 \end{bmatrix} \begin{bmatrix} R_1 & 0 \\ -R_3 & R_1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

Let

$$Q_2 = \begin{bmatrix} Q_{211} & Q_{212} \\ Q_{212}^T & Q_{222} \end{bmatrix}, \ \psi^T(t) = [x^{*T}(t) \ \ e^{*T}(t)],$$

and Q_1 is the same as those defined in Theorem 1. We consider the Lyapunov-Krasovskii functional candidate $V(\psi(t)) = \sum_{i=1}^{2} V_i(\psi(t))$ where $V_1(x^*(t)) = x^{*T}(t) E^{*T} P x^*(t) + \int_{t-h(t)}^{t} x^{*T}(s) Q_1 x^*(s) ds)$, $V_2(e^*(t)) = e^{*T}(t) E^{*T} R e^*(t) + \int_{t-h(t)}^{t} e^{*T}(s) Q_2 e^*(s) ds)$. From the time derivative $\dot{V}_1(x^*(t)) = \dot{x}^{*T}(t) E^{*T} P x^*(t) + x^{*T}(t) E^{*T} P \dot{x}^*(t) + x^{*T}(t) Q_1 x^*(t) - x^{*T}(t - h(t))(1 - \dot{h}(t)) Q_1 x^*(t - h(t))$ an upper bound of time-derivative $\dot{V}_2(e^*(t)) = \dot{e}^{*T}(t) E^{*T} R e^*(t) + e^{*T}(t) E^{*T} R \dot{e}^*(t) + e^{*T}(t) Q_2 e^*(t) - e^{*T}(t - h(t))(1 - \dot{h}(t)) Q_2 e^*(t - h(t))$. Taking the time derivative of $V(\psi(t))$ along (9), we have $\dot{V}(\psi(t)) = \sum_{i=1}^{2} \dot{V}_i(\psi(t))$. Then $\dot{V}_1(x^*(t)) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \{x^{*T}(t)(G_{ijk}^T P + P^T G_{ijk}) x^*(t) + x^{*T}(t) P^T A_{hi}^* x^*(t - h(t)) + x^{*T}(t - h(t)) A_{hi}^{*T} P x^*(t) + e^{*T}(t)(B_i^* K_{jk}^*)^T P x^*(t) + x^{*T}(t) P^T (B_i^* K_{jk}^*) e^*(t) + x^{*T}(t) Q_1 x^*(t) - x^{*T}(t - h(t))(1 - \beta) Q_1 x^*(t - h(t))\}$ which further leads to

$$\begin{aligned} & \dot{V}_1(x^*(t)) \\ \leq \ & \sum_{i=1}^{r} \sum_{k=1}^{r_e} v_i^2(z(t)) \mu_k(z(t)) \\ & \times \big[x^{*T}(t)(G_{iik}^T P + P^T G_{iik}) x^*(t) \\ & + x^{*T}(t) P^T A_{hi}^* x^*(t - h(t)) \\ & + x^{*T}(t - h(t)) A_{hi}^{*T} P x^*(t) + e^{*T}(t)(B_i^* K_{ik}^*)^T P x^*(t) \\ & + x^{*T}(t) P^T (B_i^* K_{ik}^*) e^*(t) + x^{*T}(t) Q_1 x^*(t) \\ & - x^{*T}(t - h(t))(1 - \beta) Q_1 x^*(t - h(t))\big] \\ & + 2 \sum_{i=1}^{r} \sum_{i<j} \sum_{k=1}^{r_e} v_i(z(t)) v_j(z(t)) \mu_k(z(t)) \\ & \times \Bigg[x^{*T}(t)\Big(\frac{(G_{ijk} + G_{jik})}{2}^T P \\ & + \frac{(G_{ijk} + G_{jik})}{2} P^T\Big) x^*(t) \\ & + x^{*T}(t) \frac{(A_{hi}^* + A_{hj}^*)}{2} P^T x^*(t - h(t)) \\ & + x^{*T}(t - h(t)) \frac{(A_{hi}^* + A_{hj}^*)}{2}^T P x^*(t) \\ & + e^{*T}(t) \frac{(B_i^* K_{ik}^* + B_j^* K_{jk}^*)}{2}^T P x^*(t) \\ & + x^{*T}(t) \frac{(B_i^* K_{ik}^* + B_j^* K_{jk}^*)}{2} P^T e^*(t) + x^{*T}(t) Q_1 x^*(t) \\ & - x^{*T}(t - h(t))(1 - \beta) Q_1 x^*(t - h(t))\Bigg] \end{aligned}$$

According to inequality $2x^T y \leq \varepsilon x^T x + \varepsilon^{-1} y^T y$, where $\varepsilon > 0$, it is obtained that $e^{*^T}(t)(B_i^* K_{ik}^*)^T P x^*(t) + x^{*^T}(t) P (B_i^* K_{ik}^*) e^*(t) \leq \varepsilon_1 x^{*^T}(t) P B_i^* B_i^{*^T} P x^*(t) +$

$\varepsilon_1^{-1} e^{*^T}(t) K_{ik}^{*^T} K_{ik}^* e^*(t)$ and

$$\begin{aligned} & e^{*^T}(t)\frac{(B_i^* K_{jk}^* + B_j^* K_{ik}^*)^T}{2} P x^*(t) \\ & + x^{*^T}(t) P \frac{(B_i^* K_{jk}^* + B_j^* K_{ik}^*)}{2} e^*(t) \\ \leq \; & \varepsilon_2 x^{*^T}(t) \frac{B_i^* B_i^{*T}}{2} P x^*(t) + \varepsilon_2^{-1} e^{*^T}(t) \frac{K_{jk}^{*T} K_{jk}^*}{2} e^*(t) \\ & + \varepsilon_3 x^{*^T}(t) \frac{B_j^* B_j^{*T}}{2} P x^*(t) + \varepsilon_3^{-1} e^{*^T}(t) \frac{K_{ik}^{*T} K_{ik}^*}{2} e^*(t). \end{aligned}$$

We therefore have $\dot{V}_2(x^*(t)) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\{((A_i^* - L_{ik}^* C_j^*)e^*(t) + A_{hi}^* e^*(t-h(t)))^T R e^*(t) + e^{*^T}(t) R^T ((A_i^* - L_{iK}^* C_j^*)e^*(t) + A_{hi}^* e^*(t-h(t))) + e^{*T}(t) Q_2 e^*(t) - e^{*T}(t-h(t))(1-\beta) Q_2 e^*(t-h(t))\}$which further arrives to

$$\begin{aligned} & \dot{V}_2(x^*(t)) \\ \leq \; & \sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i^2(z(t))\mu_k(z(t)) \\ & \times \big[e^{*T}(t)(W_{iik}^T R + R^T W_{iik}) e^*(t) \\ & + e^{*T}(t) R^T A_{hi}^* e^*(t-h(t)) \\ & + e^{*T}(t-h(t)) A_{hi}^{*T} R e^*(t) + e^{*T}(t) Q_2 e^*(t) \\ & - e^{*T}(t-h(t))(1-\beta) Q_2 e^*(t-h(t)) \big] \\ & + 2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t)) \\ & \times \Bigg[x^{*T}(t)(\frac{(W_{ijk}+W_{jik})}{2}^T R \\ & + \frac{(W_{ijk}+W_{jik})}{2} R^T) x^*(t) \\ & + e^{*T}(t) R^T \frac{(A_{hi}^* + A_{hj}^*)}{2} e^*(t-h(t)) \\ & + e^{*T}(t-h(t)) \frac{(A_{hi}^* + A_{hj}^*)}{2}^T R e^*(t) + e^{*T}(t) Q_2 e^*(t) \\ & - e^{*T}(t-h(t))(1-\beta) Q_2 e^*(t-h(t)) \Bigg]. \end{aligned}$$

where $G_{ijk} = A_{ik}^* - B_i^* K_{jk}^*$, $W_{ijk} = A_i^* - L_{ik}^* C_j^*$. Therefore $\dot{V}(\psi^T(t)) \leq \sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i^2(z(t))\mu_k(z(t))\xi(t)^T \Lambda_1 \xi(t) + 2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t))\xi(t)^T \Lambda_2 \xi(t) + \sum_{i=1}^{r}\sum_{k=1}^{r_e} v_i^2(z(t))\mu_k(z(t))\zeta^T(t)\Lambda_3\zeta(t) + 2\sum_{i=1}^{r}\sum_{i<j}\sum_{k=1}^{r_e} v_i(z(t))v_j(z(t))\mu_k(z(t))\zeta^T(t)\Lambda_4\zeta(t)$, where $\xi(t)^T = [x^{*T}(t) \;\; x^{*T}(t-h(t))]$, $\zeta^T(t) = [e^{*T}(t) \; e^{*T}(t-h(t))]$ and

$$\Lambda_1 = \begin{bmatrix} G_{iik}^T P + P^T G_{iik} + Q_1 + \varepsilon_1 P^T B_i^* B_i^{*^T} P & P^T A_{hi}^* \\ A_{hi}^{*^T} P & -(1-\beta) Q_1 \end{bmatrix} < 0,$$

$$\Lambda_2 = \begin{bmatrix} \big((G_{ijk}+G_{jik})^T P + P^T (G_{ijk}+G_{jik}) + 2Q_1 + P^T(\varepsilon_2 B_i^* B_i^{*^T} + \varepsilon_3 B_j^* B_j^{*^T}) P\big) & P^T (A_{hi}^* + A_{hj}^*) \\ (A_{hi}^{*^T} + A_{hj}^{*^T}) P & -2(1-\beta) Q_1 \end{bmatrix} < 0,$$

$$\Lambda_3 = \begin{bmatrix} W_{iik}^T R + R^T W_{iik} + Q_2 + \varepsilon_1 K_{ik}^{*^T} K_{ik} & R^T A_{hi}^* \\ A_{hi}^{*^T} R & -(1-\beta) Q_2 \end{bmatrix} < 0,$$

$$\Lambda_4 = \begin{bmatrix} \big((W_{ijk}+W_{jik})^T R + R^T (W_{ijk}+W_{jik}) + 2Q_2 + \varepsilon_1 K_{jk}^{*^T} K_{jk}\big) & R^T (A_{hi}^* + A_{hj}^*) \\ (A_{hi}^{*^T} + A_{hj}^{*^T}) R & -2(1-\beta) Q_2 \end{bmatrix} < 0.$$

Therefore, when $E^{*T} P = P^T E^* \geq 0$, $E^{*T} R = R^T E^* \geq 0$, $\Lambda_1 < 0$, $\Lambda_2 < 0$, $\Lambda_3 < 0$, $\Lambda_4 < 0$ the stability and the closed-loop system (9) is proven. We multiply the inequality $\Lambda_1 < 0$ and $\Lambda_2 < 0$ by the matrix $diag[P^{-T}, \; P^{-T}]$ and its transpose on the left and right, respectively. Then by setting

$$P^{-T} = \begin{bmatrix} Z_1^T & -Z_3^T \\ 0 & Z_1^T \end{bmatrix}$$

where $Z_1 > 0$, and by Schur complement, the inequalities $\Lambda_1 < 0$ and $\Lambda_2 < 0$ are equivalent to (12)-(14) , which $M_{jk} = K_{jk} Z_1$ (or $M_{ik} = K_{ik} Z_1$), the substituting $K_{jk} = M_{jk} Z_1^{-1}$ (or $K_{ik} = M_{ik} Z_1^{-1}$), ε_1, ε_2, ε_3 which were obtained from the feasible solutions of (15)-(17), into the inequality $\Lambda_3 < 0$, $\Lambda_4 < 0$ and letting $H_{ik} = R_1^T L_{ik}$, $R_3 = \rho R_1$,

$$R = \begin{bmatrix} R_1 & 0 \\ -R_3 & R_1 \end{bmatrix}$$

where $R_1 > 0$. We could get $\Lambda_3 < 0$ and $\Lambda_4 < 0$, which are equivalent to (15)-(17) by the Schur complement. This completes the proof of the theorem. □

Under Assumption 2, the stability criterion for system (9) is as follows.

***Proposition 1*:** Given scalars τ_m, τ_M and β, system (9) is asymptotically stable for any time delay $h(t)$ satisfying (**??**) if there exist real matrices $P = P^T > 0$, $R = R^T > 0$, $Q_i = Q_i^T > 0$, $i = 1,2,3,4,5,6$, $Z_j = Z_j^T > 0$, $j = 1,2,3,4$, and any appropriately dimensioned matrices N, M, S, T, H, J, U, V such that

$$\Pi + \Omega < 0 \tag{18}$$

$$\tilde{\Pi} + \tilde{\Omega} < 0 \tag{19}$$

where

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} & \pi_{15} \\ & \pi_{22} & \pi_{23} & \pi_{24} & \pi_{25} \\ & * & \pi_{33} & \pi_{34} & \pi_{35} \\ & * & * & \pi_{44} & \pi_{45} \\ & * & * & * & \pi_{55} \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} & \chi_{14} & \chi_{15} \\ & \chi_{22} & \chi_{23} & \chi_{24} & \chi_{25} \\ & * & \chi_{33} & \chi_{34} & \chi_{35} \\ & * & * & \chi_{44} & \chi_{45} \\ & * & * & * & 0 \end{bmatrix},$$

$$\tilde{\Pi} = \begin{bmatrix} \tilde{\pi}_{11} & \tilde{\pi}_{12} & \tilde{\pi}_{13} & \tilde{\pi}_{14} & \tilde{\pi}_{15} \\ & \tilde{\pi}_{22} & \tilde{\pi}_{23} & \tilde{\pi}_{24} & \tilde{\pi}_{25} \\ & * & \tilde{\pi}_{33} & \tilde{\pi}_{34} & \tilde{\pi}_{35} \\ & * & * & \tilde{\pi}_{44} & \tilde{\pi}_{45} \\ & * & * & * & \tilde{\pi}_{55} \end{bmatrix}$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\chi}_{11} & \tilde{\chi}_{12} & \tilde{\chi}_{13} & \tilde{\chi}_{14} & \tilde{\chi}_{15} \\ & \tilde{\chi}_{22} & \tilde{\chi}_{23} & \tilde{\chi}_{24} & \tilde{\chi}_{25} \\ & * & \tilde{\chi}_{33} & \tilde{\chi}_{34} & \tilde{\chi}_{35} \\ & * & * & \tilde{\chi}_{44} & \tilde{\chi}_{45} \\ & * & * & * & 0 \end{bmatrix}$$

where

$$\begin{aligned}
\pi_{11} &= T_1(A^*_{ik} - B^*_i K^*_{jk}) + (A^*_{ik} - B^*_i K^*_{jk})^T T_1^T \\
&\quad + \varepsilon_1 T_1 B^*_i B^{*T}_i T_1^T + Q_1 + Q_2 + Q_3, \\
\pi_{12} &= (A^*_{ik} - B^*_i K^*_{jk})^T T_2^T + T_1 A^*_{hi}, \\
\pi_{13} &= (A^*_{ik} - B^*_i K^*_{jk})^T T_3^T,\ \pi_{14} = (A^*_{ik} - B^*_i K^*_{jk})^T T_4^T, \\
\pi_{15} &= (A^*_{ik} - B^*_i K^*_{jk})^T T_5^T + T_1 E^*, \\
\pi_{22} &= -(1-\beta)Q_2 + T_2 A^*_{hi} + A^{*T}_{hi} T_2^T + \varepsilon_2 T_2 B^*_i B^{*T}_i T_2, \\
\pi_{23} &= A^*_{hi} T_3^T,\ \pi_{24} = A^*_{hi} T_4^T,\ \pi_{25} = A^*_{hi} T_5^T + T_2 E^*, \\
\pi_{33} &= -Q_1 + \varepsilon_3 T_3 B^*_i B^{*T}_i T_3^T, \\
\pi_{34} &= 0,\ \pi_{35} = T_3 E^*,\ \pi_{44} = -Q_3 + \varepsilon_4 T_4 B^*_i B^{*T}_i T_4^T, \\
\pi_{45} &= +T_4 E^*, \\
\pi_{55} &= \tau_M E^{*T} Z_1 E^* + \delta E^{*T} Z_2 E^* \\
&\quad + \varepsilon_5 T_5 B^*_i B^{*T}_i T_5^T + T_5 E^* + E^{*T} T_5^T,
\end{aligned}$$

$$\begin{aligned}
\chi_{11} &= N_1 E^* + E^{*T} N_1^T, \\
\chi_{12} &= E^{*T} N_2^T - M_1 E^* + S_1 E^*, \\
\chi_{13} &= E^{*T} N_3^T - N_1 E^* + M_1 E^*, \\
\chi_{14} &= E^{*T} N_4^T - S_1 E^*,\ \chi_{15} = E^{*T} N_5^T, \\
\chi_{22} &= -M_2 E^* - E^{*T} M_2^T + S_2 E^* + E^{*T} S_2^T, \\
\chi_{23} &= -N_2 E^* + M_2 E^* - E^{*T} M_3^T + E^{*T} S_3^T, \\
\chi_{24} &= -E^{*T} M_4^T + E^{*T} S_4^T - S_2 E^*, \\
\chi_{25} &= -E^{*T} M_5^T + E^* S_5^T, \\
\chi_{33} &= -N_3 E^* + M_3 E^* - E^{*T} M_3^T + E^{*T} S_3^T, \\
\chi_{34} &= -E^{*T} N_4^T + E^{*T} M_4^T - S_3 E^*, \\
\chi_{35} &= -E^{*T} N_5^T + E^{*T} M_5^T, \\
\chi_{44} &= -S_4 E^* + E^{*T} S_4^T,\ \chi_{45} = -E^{*T} S_5^T,
\end{aligned}$$

$$\begin{aligned}
\tilde{\pi}_{11} &= Q_4 + Q_5 + Q_6 + V_1(A^*_{ik} - L^*_{ik} C^*_j) \\
&\quad + (A^*_{ik} - L^*_{ik} C^*_j)^T V_1^T, \\
\tilde{\pi}_{12} &= (A^*_{ik} - L^*_{ik} C^*_j)^T V_2^T + V_1 A^*_{hi}, \\
\tilde{\pi}_{13} &= (A^*_{ik} - L^*_{ik} C^*_j)^T V_3^T,\ \tilde{\pi}_{14} = (A^*_{ik} - L^*_{ik} C^*_j)^T V_4^T, \\
\tilde{\pi}_{15} &= E^{*T} R + (A^*_{ik} - L^*_{ik} C^*_j)^T V_5^T - V_1 E^* \\
\tilde{\pi}_{22} &= -(1-\beta)Q_5 + V_2 A^*_{hi} + A^{*T}_{hi} V_2^T, \\
\tilde{\pi}_{23} &= A^{*T}_{hi} V_3^T,\ \tilde{\pi}_{24} = A^{*T}_{hi} V_4^T, \\
\tilde{\pi}_{25} &= A^{*T}_{hi} V_5^T - V_2 E^*,\ \tilde{\pi}_{33} = -Q_4,\ \tilde{\pi}_{34} = 0, \\
\tilde{\pi}_{35} &= -V_3 E^*,\ \tilde{\pi}_{44} = -Q_6,\ \tilde{\pi}_{45} = -V_4 E^*, \\
\tilde{\pi}_{55} &= E^{*T} Z_3 E^* + \delta E^{*T} Z_4 E^* - V_5 E^* - E^{*T} V_5^T,
\end{aligned}$$

$$\begin{aligned}
\tilde{\chi}_{11} &= H_1 E^* + E^{*T} H_1^T + \varepsilon_1^{-1} K^{*T}_{jk} K^*_{jk} + \varepsilon_2^{-1} K^{*T}_{jk} K^*_{jk} \\
&\quad + \varepsilon_3^{-1} K^{*T}_{jk} K^*_{jk} + \varepsilon_4^{-1} K^{*T}_{jk} K^*_{jk} + \varepsilon_5^{-1} K^{*T}_{jk} K^*_{jk}, \\
\tilde{\chi}_{12} &= E^{*T} H_2^T - J_1 E^* + U_1 E^*, \\
\tilde{\chi}_{13} &= E^{*T} H_3^T - H_1 E^* + J_1 E^*, \\
\tilde{\chi}_{14} &= E^{*T} H_4^T - U_1 E^*,\ \tilde{\chi}_{15} = E^{*T} H_5^T, \\
\tilde{\chi}_{22} &= -J_2 E^* - E^{*T} J_2^T + U_2 E^* + E^{*T} U_2^T, \\
\tilde{\chi}_{23} &= -H_2 E^* + J_2 E^* - E^{*T} J_3^T + E^{*T} U_3^T, \\
\tilde{\chi}_{24} &= -E^{*T} J_4^T + E^{*T} U_4^T - U_2 E^*, \\
\tilde{\chi}_{25} &= -E^{*T} J_5^T + E^{*T} U_5^T, \\
\tilde{\chi}_{33} &= -H_3 E^* - E^{*T} H_3^T - J_3 E^* + E^{*T} J_3^T, \\
\tilde{\chi}_{34} &= -E^{*T} H_4^T + E^{*T} J_4^T - U_3 E^*, \\
\tilde{\chi}_{35} &= -E^{*T} H_5^T + E^{*T} J_5^T, \\
\tilde{\chi}_{44} &= -U_4 E^* - E^{*T} U_4^T,\ \tilde{\chi}_{45} = -E^{*T} U_5^T.
\end{aligned}$$

***Proof*:** We consider the Lyapunov-Krasovskii functional candidate $V(\psi(t)) = \sum_{i=1}^{4} V_i(\psi(t))$, where $V_1(x^*(t)) = x^{*T}(t)E^{*T}Px^*(t) + \int_{t-\tau_m}^{t} x^{*T}(s)Q_1x^*(s)ds + \int_{t-h(t)}^{t} x^{*T}(s)Q_2x^*(s)ds + \int_{t-\tau_M}^{t} x^{*T}(s)Q_3x^*(s)ds$, $V_2(x^*(t)) = \int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{x}^{*T}(s)E^{*T}Z_1E^*\dot{x}^*(s)dsd\theta + \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} \dot{x}^{*T}(s)E^{*T}Z_2E^*\dot{x}^*(s)dsd\theta$, $V_3(e^*(t)) = e^{*T}(t)E^{*T}Re^*(t) + \int_{t-\tau_m}^{t} e^{*T}(s)Q_4e^*(s)ds + \int_{t-h(t)}^{t} e^{*T}(s)Q_5e^*(s)ds + \int_{t-\tau_M}^{t} e^{*T}(s)Q_6e^*(s)ds$, $V_4(e^*(t)) = \int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{e}^{*T}(s)E^{*T}Z_3E^*\dot{e}^*(s)dsd\theta + \int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t} \dot{e}^{*T}(s)E^{*T}Z_4E^*\dot{e}^*(s)dsd\theta$ From the time derivative $\dot{V}_1(x^*(t)) = \dot{x}^{*T}(t)E^{*T}Px^*(t) + x^{*T}(t)E^{*T}P\dot{x}^*(t) + x^{*T}(t)Q_1x^*(t) - x^{*T}(t-\tau_m)Q_1x^*(t-\tau_m) + x^{*T}(t)Q_2x^*(t) - x^{*T}(t-h(t))(1-\dot{h}(t))Q_2x^*(t-h(t)) + x^{*T}(t)Q_3x^*(t) - x^{*T}(t-\tau_M)(1-\dot{h}(t))Q_3x^*(t-\tau_M)$, $\dot{V}_2(x^*(t)) = \dot{x}^{*T}(s)\tau_M E^{*T}Z_1E^*\dot{x}^*(s) + \int_{t-\tau_M}^{t} \dot{x}^{*T}(s)E^{*T}Z_1E^*\dot{x}^*(s)ds + \dot{x}^{*T}(s)\delta E^{*T}Z_2E^*\dot{x}^*(s) + \int_{t-\tau_M}^{t-\tau_m} \dot{x}^{*T}(s)E^{*T}Z_2E^*\dot{x}^*(s)ds$. With the lower and upper bound of time-derivative of $V(\psi(t))$ can be obtained as $\dot{V}_3(e^*(t)) = \dot{e}^{*T}(t)E^{*T}Re^*(t) + e^{*T}(t)E^{*T}R\dot{e}^*(t) + e^{*T}(t)Q_4e^*(t) + e^{*T}(t-\tau_m)Q_4e^*(t-\tau_m) + e^{*T}(t)Q_5e^*(t) - e^{*T}(t-h(t))(1-\dot{h}(t))Q_5e^*(t-h(t)) + e^{*T}(t)Q_6e^*(t) + e^{*T}(t-\tau_M)Q_6e^*(t-\tau_M)$, $\dot{V}_4(e^*(t)) = \dot{e}^{*T}(s)\tau_M E^{*T}Z_3E^*\dot{e}^*(s) + \int_{t-\tau_M}^{t} \dot{e}^{*T}(s)E^{*T}Z_3E^*\dot{e}^*(s)ds +$

$\dot{e}^{*T}(s)\delta E^{*T}Z_4E^*\dot{e}^*(s) + \int_{t-\tau_M}^{t-\tau_m}\dot{e}^{*T}(s)E^{*T}Z_4E^*\dot{e}^*(s)ds$. Then we have the following Zero Equalities I: $2\zeta_1^T(t)N\{E^*x(t) - E^*x(t-\tau_m) - \int_{t-\tau_m}^{t}E^*\dot{x}^*(s)ds\} = 0$, $2\zeta_1^T(t)M\{E^*x(t-\tau_m) - E^*x(t-h(t)) - \int_{t-h(t)}^{t-\tau_m}E^*\dot{x}^*(s)ds\} = 0$, $2\zeta_1^T(t)S\{E^*x(t-h(t)) - E^*x(t-\tau_M) - \int_{t-\tau_M}^{t-h(t)}E^*\dot{x}^*(s)ds\} = 0$, $2\zeta_1^T(t)T\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}v_i(z(t))v_j(z(t))\mu_k(z(t))\{(A_{ik}^* - B_i^*K_{jk}^*)x^*(t) + A_{hi}^*x^*(t-h(t)) + B_i^*K_{jk}^*e^*(t)\} - (E^*\dot{x}^*(t)) = 0$ and Zero Equalities II: $2\zeta_2^T(t)H\{E^*e(t) - E^*e(t-\tau_m) - \int_{t-\tau_m}^{t}E^*\dot{e}^*(s)ds\} = 0$, $2\zeta_2^T(t)J\{E^*e(t-\tau_m) - E^*e(t-h(t)) - \int_{t-h(t)}^{t-\tau_m}E^*\dot{e}^*(s)ds\} = 0$, $2\zeta_2^T(t)U\{E^*e(t-h(t)) - E^*e(t-\tau_M) - \int_{t-\tau_M}^{t-h(t)}E^*\dot{e}^*(s)ds\} = 0$, $2\zeta_2^T(t)V\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r_e}v_i(z(t))v_j(z(t))\mu_k(z(t))\{(A_{ik}^* - L_{ik}^*C_j^*)e^*(t) + A_{hi}^*e^*(t-h(t)) - E^*\dot{e}^*(t)\} = 0$, where $\xi_1(t)^T = [x^{*T}(t)\ \ x^{*T}(t-h(t))\ \ x^{*T}(t-\tau_m)\ \ x^{*T}(t-\tau_M)\ \ \dot{x}^{*T}(t)]$, $\zeta_2^T(t) = [e^{*T}(t)\ \ e^{*T}(t-h(t))\ \ e^{*T}(t-\tau_m)\ \ e^{*T}(t-\tau_M)\ \ \dot{e}^{*T}(t)]$. Taking the time derivative of $V(\psi(t))$ along (9), we have $\dot{V}(\psi(t)) = \sum_{i=1}^{4}\dot{V}_i(\psi(t))$. Hence $V(\psi(t))$ is negative as long as the inequalities (18)-(19) hold, which implies that system (9) is asymptotically stable. This completes the proof. □

IV. Numerical Simulations

We carry out numerical simulations on the following example to verify the theoretical derivations. Consider a nonlinear time-delay system

$$\begin{aligned}(1 + a\cos\theta(t))\ddot{\theta}(t) &= -b\dot{\theta}^3(t) + c\theta(t) - 0.4\theta(t-h(t))\\ &\quad +0.6\dot{\theta}(t-h(t)) + du(t)\end{aligned}$$

where the range of $|\dot{\theta}(t)| < \gamma$. This can be expressed exactly by the following fuzzy descriptor form $\sum_{k=1}^{2}\mu_k(z(t))E_k\dot{x}(t) = \sum_{i=1}^{2}v_i(z(t))\{(A_ix(t) + A_{hi}x(t-h(t)) + B_iu(t)\}$ where $x^*(t) = [x_1(t)\ x_2(t)]^T = [\theta(t)\ \dot{\theta}(t)]$,

$$E_1 = \begin{bmatrix}1 & 0\\ 0 & 1+a\end{bmatrix},\ E_2 = \begin{bmatrix}1 & 0\\ 0 & 1-a\end{bmatrix},$$
$$A_1 = \begin{bmatrix}0 & 1\\ c & -b\phi_1^2\end{bmatrix},\ A_2 = \begin{bmatrix}0 & 1\\ c & -b\phi_2^2\end{bmatrix},$$
$$A_{h1} = A_{h2} = \begin{bmatrix}0 & 0\\ -0.4 & 0.6\end{bmatrix},\ B_1 = B_2 = \begin{bmatrix}0\\ d\end{bmatrix},$$

We let $a = 0.2$, $b = 1$, $c = -1$, $d = 10$, $\phi_1 = 4$, $\phi_2 = 0$, $h(t) = 0.8(1 + \sin 0.5t)$ and $\beta = 0.4$. The membership functions $\mu_1(x_1(t)) = \frac{1+\cos x_1(t)}{2}$, $\mu_2(x_1(t)) = \frac{1-\cos x_1(t)}{2}$, $v_1(x_2(t)) = \frac{x_2^2(t)}{2}$, $v_2(x_2(t)) = 1 - \frac{x_2^2(t)}{2}$. According to LMIs (10)-(11) , we can obtain control gains K_{jk} separately, where $K_{11} = [0.6560\ \ 3.9564]$, $K_{12} = [0.4110\ \ 2.4066]$, $K_{21} = [0.6560\ \ 3.9564]$, $K_{22} = [0.4110\ \ 2.4066]$.

Figure 1 shows the perfect convergence result under the overall control law $u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{r_e}v_i(z(t))\mu_k(z(t))K_{ik}^*x^*(t)$ with initial condition $x(0) = [0.8\ \ -0.4]^T$.

Considering output feedback case with immeasurable states, the observer descriptor form $\sum_{k=1}^{2}\mu_k(z(t))E_k\dot{\hat{x}}(t) = \sum_{i=1}^{2}v_i(z(t))\{(A_i\hat{x}(t) + A_{hi}\hat{x}(t-h(t)) + B_iu(t) + L_i(y(t) - \hat{y}(t)\}$ and $y(t) = \sum_{i=1}^{2}v_i(z(t))C_ix(t)$ where $x^*(t)$ and E_1, E_2, A_1, A_2, A_{h1}, A_{h2}, B_1, B_2 are same as controller only example with output matrices $C_1 = C_2 = [0.1\ \ 1]$. We let $a = 0.2$, $b = 1$, $c = -1$, $d = 10$, $\phi_1 = 4$, $\phi_2 = 0$, $\varepsilon_1 = 8$, $\varepsilon_2 = 1.8$, $\varepsilon_3 = 3$, $\rho = 0.8$, $h(t) = 0.5$ and $\beta = 0$ The observer membership functions are defined as $\mu_1(\hat{x}_1(t)) = \frac{1+\cos\hat{x}_1(t)}{2}$, $\mu_2(\hat{x}_1(t)) = \frac{1-\cos\hat{x}_1(t)}{2}$, $v_1(\hat{x}_2(t)) = \frac{\hat{x}_2^2(t)}{2}$, $v_2(\hat{x}_2(t)) = 1 - \frac{\hat{x}_2^2(t)}{2}$. According to LMIs (12)-(17) , we can obtain control gains K_{jk} and observer gains L_{ik} separately where $K_{11} = [0.1680\ \ -1.1992]$, $K_{12} = [0.1012\ \ -1.2856]$, $K_{21} = [0.1680\ \ 0.4008]$, $K_{22} = [0.1012\ \ 0.3144]$ $L_{11} = [0.9655\ \ -14.5572]^T$, $L_{12} = [0.9952\ \ -14.8545]^T$, $L_{21} = [0.9146\ \ 1.3916]^T$, $L_{22} = [0.8889\ \ 1.1348]^T$.

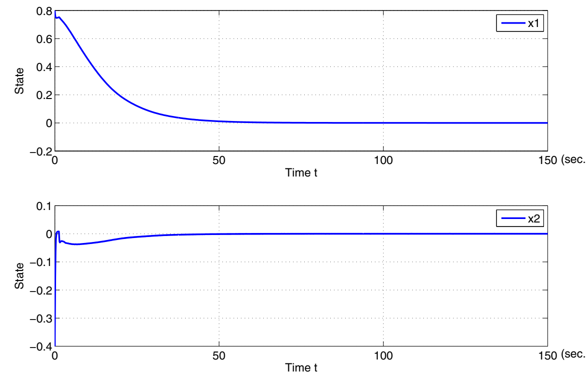

Fig. 1. State trajectories of system.

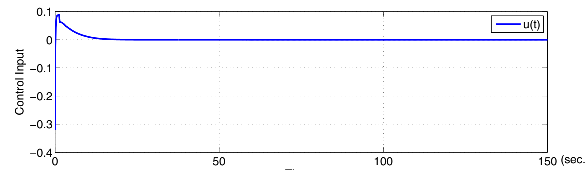

Fig. 2. Controller performance with the fuzzy control approach.

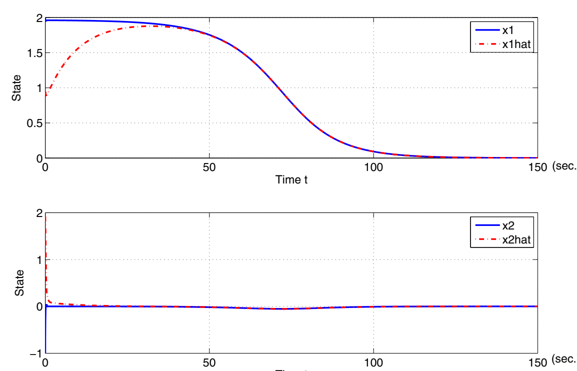

Fig. 3. State trajectories of system.

Figure 3 shows the perfect convergence result under the overall control law $u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{r_e}v_i(z(t))\mu_k(z(t))K_{ik}^*\hat{x}^*(t)$ with initial condition $x(0) = [2\ \ -1]^T$ and $\hat{x}(0) = [1\ 2]^T$.

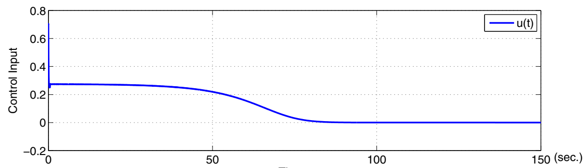

Fig. 4. Controller performance with the fuzzy control approach.

V. CONCLUSIONS

This paper studies a class of fuzzy interval time-delay descriptor systems. Sufficient conditions for the stability and stabilization problems are obtained by using appropriate analysis methods for descriptor systems. The present results are in terms of LMIs. and can be viewed as extensions of some existing developments.

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