ELECTROMAGNETIC IMAGING FOR A CONDUCTING CYLINDER BURIED IN THE THREE LAYERS STRUCTURE

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Abstract-The paper presents a computational approach to the imaging of a conducting cylinder buried in the three layers structure. A conducting cylinder of unknown shape, which is buried in the second layer scatters the incident wave from the first layer and the scattered field is recorded in the first and third layers. Based on the boundary condition and the measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is optimized by the genetic algorithm. Numerical results demonstrated that, even when the initial guess is far away from the exact one, good reconstruction has been obtained.

I. Introduction

The image problem of conducting objects has been a subject of considerable importance in noninvasive measurement, medical imaging, and biological application. In the past 20 years, many rigorous methods have been developed to solve the exact equation. However, inverse problem of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient search approach such as the Newton-Kantorovitch method [1], the Levenberg-Marguart algorithm [2] and the successive-overrelaxation method [3]. This method is highly dependent on the initial guess and tends to get trapped in a local extreme. In contrast, the second approach is based on the genetic algorithm [4]. It usually converges to the global extreme of the problem, no matter what the initial estimate is.

II. Theoretical Formulation

In this paper, a conducting cylinder buried in a two-dimensional three layers structure is presented. Let $(\varepsilon_i, \sigma_i)$, $i = 1,2,3$, denote the permittivity and conductivity in each layer. A conducting cylinder is buried in the second layer as shown in Fig.1.The metallic cylinder with cross section described in polar coordinates in x-y plane by the equation $\rho = F(\theta)$ is illuminated by an incident plane wave whose electric field vector is parallel to the z axis (i.e., transverse magnetic, or TM, polarization). We assume that time dependence of the field is harmonic with the factor $\exp(j\omega t)$. The E_1

denote the incident field form region 1 with incident angle θ_1, as shown in Fig. 1. Then the total electric field in the absence of the conducting object, E, in each region is given by

$$E = \begin{cases} E_1 = E_1^+ e^{+jk_1 \cos\theta_1 y} e^{-jk_1 \sin\theta_1 x} \bar{z} + E_1^- e^{-jk_1 \cos\theta_1 y} e^{-jk_1 \sin\theta_1 x} \bar{z} & , y \geq a \\ E_2 = E_2^+ e^{+jk_2 \cos\theta_2 y} e^{-jk_2 \sin\theta_2 x} \bar{z} + E_2^- e^{-jk_2 \cos\theta_2 y} e^{-jk_2 \sin\theta_2 x} \bar{z} & , a \geq y \geq -a \\ E_3 = E_3^+ e^{+jk_3 \cos\theta_3 y} e^{-jk_3 \sin\theta_3 x} \bar{z} & , y \leq -a \end{cases} \tag{1}$$

At an arbitrary point (x, y) in Cartesian coordinates or (r,θ) in polar coordinates outside the scatterer the scattered field, $\bar{E}_s$, can be expressed

$$E_s(\bar{r}) = -\int_0^{2\pi} G(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' \tag{2}$$

$$J(\theta) = -j\omega\mu_0 \sqrt{F^2(\theta) + F'^2(\theta)} J_s(\theta)$$

with

$$G(x, y; x', y') = \begin{cases} G_1(x, y; x', y'), & y > a \\ G_2(x, y; x', y'), & a > y > -a \\ G_3(x, y; x', y'), & y < -a \end{cases}$$

and $J_s(\theta)$ is the induced surface current density, which is proportional to the normal derivative of the electric field on the conductor surface. Note that G_1, G_2 and G_3 denote the Green's function for the line source in region 2. The boundary condition on the surface of the scatter on states that the total tangential electrical field must be zero and yield an integral equation for $J(\theta)$:

$$E_2(\bar{r}) = -\int_0^{2\pi} G_2(\bar{r}, F(\theta'), \theta') J(\theta') d\theta' \tag{3}$$

For the direct scattering problem, the scattered field E_s is calculated by assuming that the shapes are known. This can be achieved by first solving J in (3) and calculating E_s in (2). For the inverse problem, we assume that the shape $F(\theta)$ function can be expanded as:

$$F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \tag{4}$$

where B_n and C_n are real coefficient to be determined, and N+1 is the number of unknowns for shape function. In the inversion procedure, the genetic algorithm is used to minimize the following cost function:

$$CF = \{\frac{1}{M_t}\sum_{m=1}^{M_t}\left|E_s^{\exp}(\bar{r}_m) - E_s^{cal}(\bar{r}_m)\right|^2 / \left|E_s^{\exp}(\bar{r}_m)\right|^2\}^{1/2} \tag{5}$$

where M_t is the total number of measured points. $E_s^{\exp}(\bar{r})$ and $E_s^{cal}(\bar{r})$ are the measured scattered field and the calculated scattered field respectively. The basic GA for which a flowchart is shown in Fig. 3.

III. Numerical Results

Let us consider a perfectly conducting cylinder buried in region 2, as shown in Fig. 1. The permittivity in each region is characterized by $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = 2.56\varepsilon_0$ and $\varepsilon_3 = \varepsilon_0$ respectively. The frequency of the incident wave is chosen to be 1 GHz and the incident angles are 45° and 135°. The width of the second layer 0.2 m. In the first example, the shape function is chosen to be $F(\theta) = 0.05 + 0.01\cos 3\theta - 0.01\sin 3\theta$ m .The reconstructed shape function for the best population member is plotted in Fig. 3(a) with the error shown in Fig. 3(b). It is seen that the reconstructed result is quite good. In the second example, the shape function is chosen to be $F(\theta) = 0.05 + 0.01\cos 2\theta - 0.01\sin 2\theta$ m. Good results are obtained in Fig. 4(a) and Fig. 4(b).

IV. Conclusions

We have presented a study of applying the genetic algorithm to reconstruct the shapes of a conducting cylinder buried in a three layer structure. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization problem. By using the genetic algorithm, the shape of the object can be reconstructed from the scattered fields. Numerical results are presented and good reconstruction is obtained.

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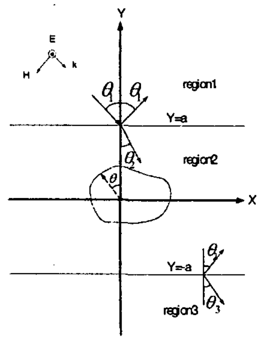

Fig.1. Geometry of problem in (x,y)-plane

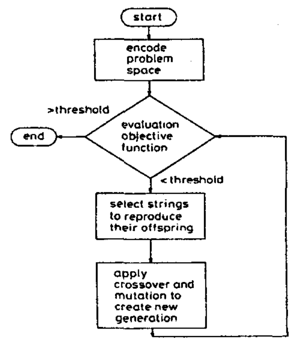

Fig.2. The flowchart of GA

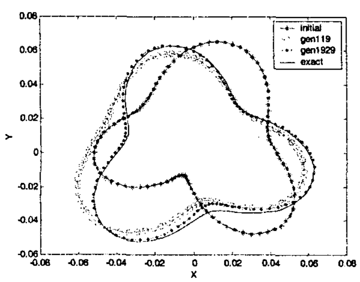

Fig.3 (a) shape function. The solid curve represents the exact shape, and the others represent the best shape function of each generation

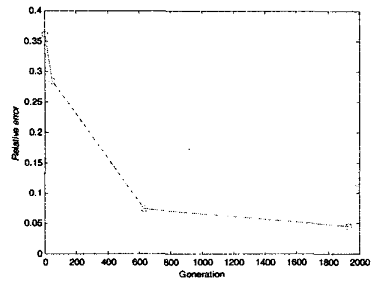

Fig.3(b) shape function error in each generation

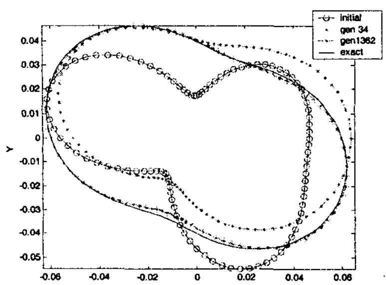

Fig.4 (a) shape function. The solid curve represents the exact shape, and the others represent the best shape function of each generation

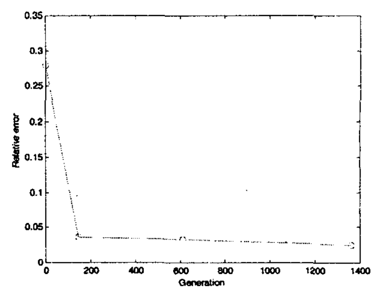

Fig.4(b) shape function error in each generation