IMAGE RECONSTRUCTION OF A TWO-DIMENSIONAL DIELECTRIC OBJECT BY TE WAVE ILLUMINATION

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Introduction

In recent years, Most microwave inverse scattering algorithms developed are for TM wave illuminations in which the vectorial problem can be simplified to a scalar one [1] . On the other hand, much fewer works have been reported on the more complicated TE case [2] In this paper, the inverse problem of a homogeneous dielectric cylinder with unknown cross-section shape and dielectric constant by TE wave illumination is investigated. By applying the genetic algorithm and the moment method [3], the inverse problem can be solved as an optimized problem. Good reconstruction is obtained as multi-incident waves are applied to get the measured data.

Theoretical Formulation

Consider a homogeneous dielectric cylinder located in free space as shown in Fig.1. The cross section of the dielectric cylinder is assumed to be described in polar coordinates by $\rho = F(\theta)$. The permittivity and permeability of free space and the dielectric object are denoted by (ε_0, μ_0) and (ε_2, μ_2), respectively.

The dielectric object is illuminated by an incident plane wave whose electric field vector is perpendicular to the Z axis (i.e., TE polarization). We assume that time dependence of the field is harmonic with the factor $e^{j\omega t}$. Then, the incident electric and magnetic fields can be given by

$$\vec{E}_i(x, y) = (\cos\varphi\hat{x} + \sin\varphi\hat{y})e^{-jk(x\sin\phi + y\cos\phi)} ,$$

$$k_0^2 = \omega^2\varepsilon_0\mu_0, \quad \vec{H}_i(x, y) = \hat{z}\sqrt{\frac{\varepsilon_0}{\mu_0}}e^{-jk(x\sin\phi + y\cos\phi)}$$

where ϕ is the incident angle and k_0 is the free-space wave number.

Since the tangential components of $\bar{E}$ and $\bar{H}$ fields should be continuous across the surface of the dielectric object, we can derive two integral equations as follows:

$$\hat{n}\times\bar{E}^i(\theta) = -\hat{n}\times\int_0^{2\pi}[j\omega(\mu_2 G_2 + \mu_0 G_0)\bar{J}(\theta') - \bar{M}(\theta')\times\nabla'(G_2 + G_0)]d\theta' - \rho_e(\theta')\nabla'(\frac{G_2}{\varepsilon_2} + \frac{G_0}{\varepsilon_0})]d\theta' \quad (1)$$

$$\hat{n}\times\bar{H}^i(\theta) = -\hat{n}\times\int_0^{2\pi}[j\omega(\varepsilon_2 G_2 + \varepsilon_0 G_0)\bar{M}(\theta) - \bar{J}(\theta)\times\nabla'(G_2 + G_0)]d\theta' \quad (2)$$

with $r_0(\theta,\theta') = |\bar{r} - \bar{r}'| = \sqrt{F^2(\theta) + F^2(\theta') - 2F(\theta)F(\theta')\cos(\theta - \theta')}$, $\bar{J}(\theta) = \sqrt{F^2(\theta) + F'^2(\theta)}\bar{J}_s(\theta)$

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$$\vec{M}(\theta)=\sqrt{F^2(\theta)+F'^2(\theta)}\vec{M}_s(\theta),\ \rho_e(\theta)=\sqrt{F^2(\theta)+F'^2(\theta)}\rho_{es}(\theta),\ \rho_{es}=\frac{j}{\omega}\nabla\bullet\vec{J}_s$$

where $\vec{J}_s(\theta)$ and $\vec{M}_s(\theta)$ are the equivalent surface electric and magnetic current densities, $\hat{n}$ is the outward unit normal on the object surface, and $G_0(k_0r_0)$, $G_2(k_2r_0)$ are the Green's functions in free-space and in a homogeneous space with relative dielectric constant $\varepsilon_r=\varepsilon_2/\varepsilon_0$ respectively.

For TE case, the magnetic field has only one component along the Z axis such that the scattered electric field $\vec{H}^s$ at the point (x,y) outside the scatter can be expressed by

$$\vec{H}^s(x,y)=\int_0^{2\pi}[j\omega\varepsilon_0G_0(k_0r_0)\vec{M}(\theta')+\vec{J}(\theta')\times\nabla'G_0(k_0r_0)]d\theta' \tag{3}$$

In order to solve the direct problem for a given $F(\theta)$ and ε_2 (let $\mu_2=\mu_0$), the moment method is applied. By employing the point-matching technique, the above integral equations (1) and (2) can be transformed into matrix form as

$$\begin{cases}[E^{inc}]_{N_d\times1}=[C_1]_{N_d\times N_d}[J]_{N_d\times1}+[C_2]_{N_d\times N_d}[M]_{N_d\times1}\\ [H^{inc}]_{N_d\times1}=[C_3]_{N_d\times N_d}[J]_{N_d\times1}+[C_4]_{N_d\times N_d}[M]_{N_d\times1}\end{cases} \tag{4}$$

and the scattered electric field $\vec{H}^s$ in Eqn.(3) can be transformed as

$$\vec{H}^s(x,y)=-\left\{\sum_{n=1}^{N}\int_{\Delta\ell}\left[\frac{\omega\varepsilon_0H_0^{(2)}(k_0\xi)}{4}\beta_n+j\frac{k_0H_1^{(2)}(k_0\xi)}{\sqrt{F^2(\theta')+F'^2(\theta')}\xi}\alpha_n\right]d\theta'\right\}\hat{z} \tag{5}$$

with $\xi=\sqrt{(x-F(\theta')\cos\theta')^2+(y-F(\theta')\sin\theta')^2}$

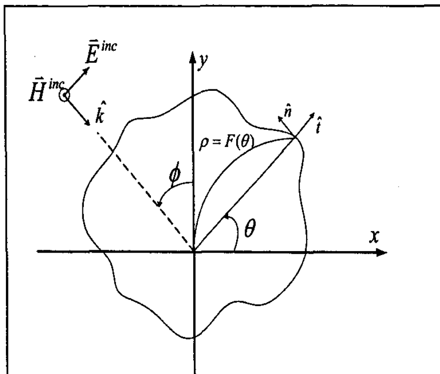

Figure 1 Geometry of problem in (x,y) plane.

For the direct scattering problem, the scattered field $\vec{H}^s$ is calculated by assuming that the shape functions are known. This can be achieved by first solving $\vec{J}(\theta)$ and $\vec{M}(\theta)$ using (1) and (2) and then the scattered field outside the scatter can be calculated from Eqn. (5). It serves as the measured data of the inverse problem for the purpose of numerical simulation.

For the inverse problem, we assume the approximate center of the scatter, which in fact can be any point inside the scatter, is known. Then the shape $F(\theta)$ function can be expanded as: $F(\theta)=\sum_{n=0}^{N}B_n\cos(n\theta)+\sum_{n=1}^{N}C_n\sin(n\theta)$,

where B_n and C_n are real number to be determined, and 2N+1 is the number of unknown coefficients for shape function. In the inversion procedure, the genetic algorithm is used to minimize the following cost function: $CF=\{\frac{1}{M_t}\sum_{m=1}^{M_t}\left|E^{s,\exp}(\bar{r}_m)-E^{s,cal}(\bar{r}_m)\right|^2/\left|E^{s,\exp}(\bar{r}_m)\right|^2\}^{1/2}$, where M_t is the total number of the measurement. $E^{s,\exp}(\bar{r})$ and $E^{s,cal}(\bar{r})$ are the measured scattered field and the calculated scattered field respectively.

Numerical Results

In this section, we report some numerical results of using the scheme described in Section II. The permittivity of the dielectric object ε_2 is assumed in the following examples. The frequency of the incident wave is chosen for 3 GHz. To reconstruct the shape and the permittivity, the dielectric objects in the following examples are illuminated by plane waves of unit amplitude from three directions ($\phi = 0°, 120°, 240°$), and the measurement points ($M_t = 8$) are equally separated on a circle of radius $r_m = 0.5m$. Note that the simulated result using only one incident wave is much worse than that using two incident waves.

In this example, the shape function is chosen to be F(θ)=0.04+0.015*sin(3θ) m , and the relative permittivity of the object is $\varepsilon_r = 2.3$. The reconstructed shape function of the best population member is plotted in Fig.2 (a). The r.m.s. error (DF) of the reconstructed shape $F^{cal}(\theta)$ and the relative error (DIPE) of ε_r^{cal} with respect to the exact values versus generation are shown in Fig. 2(b). Here, DF and DIPE are defined as

$$DF = \{\frac{1}{N'}\sum_{i=1}^{N'}[F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i)\}^{1/2}, \quad DIPE = \frac{\left|\varepsilon^{cal} - \varepsilon\right|}{\varepsilon}$$

The r.m.s. error DF is about 0.18%and DIPE= 1.14% n final.

Conclusions

We have presented a study of applying the genetic algorithm to reconstruct the shapes and relative permittivity of a homogeneous dielectric cylinder by TE wave. Based on the equivalence principle, boundary condition and measured scattered fields, we have derived a set of nonlinear surface integral equations and reformulated the imaging problem into an optimized problem. Numerical results have been carried out and good reconstruction has been obtained.

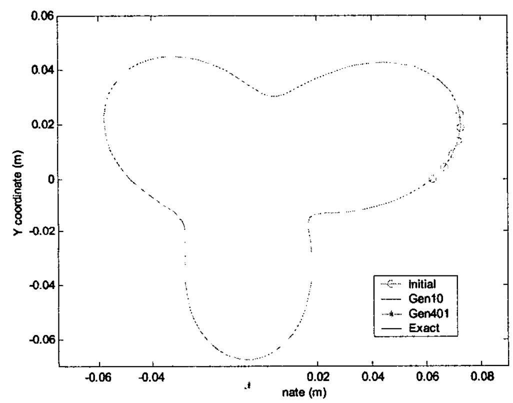

Figure 2 (a). The solid curve represents the exact shape, while the star curves are calculated shape in iteration process.

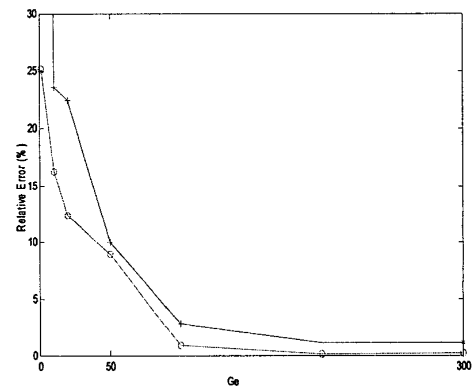

Figure 2 (b) Shape-function error and permittivity error in each generation.

References

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