# Inverse Scattering of a Two-Dimensional Periodic Conductor

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*Abstract* -The image reconstruction of a two -dimensional periodic conductor by the genetic algorithm is investigated. A periodic conducting cylinder of unknown periodic length and shape scatters the incident wave in free space and the scattered field is recorded outside. Based on the boundary condition and measured scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. The genetic algorithm is then employed to find out the global extreme solution of the cost function. As a result, the shape and the periodic length of the conductor can be obtained. Numerical results are given to demonstrate that even in the presence of noise, good reconstruction has been obtained.

## I. INTRODUCTION

The development of inverse scattering techniques for imaging the shape of conducting objects is very important. Due to its noninvasive nature, inverse scattering can be applied to remote sensing, medical imaging and nondestructive testing. Two main categories of approaches have been developed. The first is an approximate approach, which makes use of the physical optics approximation [1]-[2]. This method is comparatively straightforward to apply and usually computationally efficient. But the resulted resolution is rough and only partial knowledge of the scatterer can be retrieved. The second approach is to solve the exact equations rigorously by numerical methods based on optimization techniques [3]-[4].

Many papers use the gradient search methods, such as Newton-Kantorovitch method and the successive -overrelaxation method, to find the solution. However, this method is highly dependent on the initial guess and trends to get trapped in a local extreme. Recently a relative new optimization approach, the genetic algorithm, has been applied to the inverse problem [5]-[6]. Compared to gradient search optimization techniques, the genetic algorithm is less prone to convergence to a local minimum.

In this paper, the electromagnetic imaging of a periodic perfectly conducting cylinder is investigated. The genetic algorithm is used to recover the periodic length and the shape of a scatterer, by using the scattered field. In Section II, the relevant theory and formulations is presented. The general principle of genetic algorithms and the way we applied them to the imaging problem are also described. Numerical simulation is presented to demonstrate the proposed algorithms in Section III. Finally, conclusions are given in Section IV.

## II. THEORETICAL FORMULATION

A periodic two-dimensional metallic cylinder is situated in a background medium with a permittivity $\varepsilon_o$ and a permeability $\mu_o$ as shown in Fig. 1. The array is periodic in the x-direction with a periodic length $d$ and uniform in the z-direction. The cross section of the metallic cylinder is assumed to be described in polar coordinates in xy plane by the equation $\rho=F(\theta)$. A plane wave whose electric field vector is parallel to the z-axis (i.e., transverse magnetic, or TM, polarization) is incident upon the periodic cylinder. Let $\bar{E}_i$ denote the incident wave with incident angel $\phi$, as shown in Fig. 1. The scattered field, $\bar{E}_s = E_s\hat{z}$ can be expressed by

$$E_s(x,y)=\int_0^{2\pi} G_i(x,y,x',y')J(\theta')d\theta' \qquad (1)$$

with

$$J(\theta) = -j\omega\mu_0\sqrt{F^2(\theta)+F'^2(\theta)}\,J_s(\theta) \qquad (2)$$

$$k^2 = \omega^2\mu_0\varepsilon_0$$

Here $G_i(x,y,x',y')$ is the two–dimensional periodic Green's function [7], and $J_s(\theta)$ is the induced surface current density which is proportional to the normal derivative of electric field on the conductor surface. The boundary condition at the surface of the scatterer states that the total tangential electric field must be zero and this yield an integral equation for $J(\theta)$:

$$E_i\,(F(\theta),\theta) = -\int_0^{2\pi} G_i(x,y,x',y')J(\theta')d\theta' \qquad (3)$$

For the direct scattering problem, the scattered field $E_s$ is calculated by assuming that the shape of the object and the periodic length $d$ is known. This can be achieved by first solving $J(\theta)$ and calculating $E_s$ in (1). For numerical calculation of the direct problem, the contour is first divided into sufficient small segments so that the induced surface current can be considered constant over each segment. Then the moment method is used to solve (1) with pulse basis function for expanding and Dirac delta function for testing.

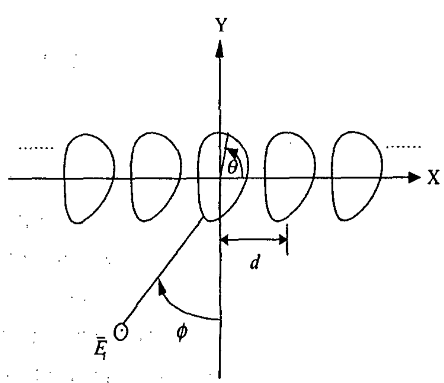


Fig. 1. Geometry of a periodic conducting cylinder with a periodic spacing $d$ along the x-direction.

Let us consider the following inverse problem: given the scattered field $E_s$ measured outside the scatterer, determine the shape $F(\theta)$ and the periodic length $d$ of the object. Assume the approximate center of the scatterer, which in fact can be any point inside the scatter, is known. Then the shape $F(\theta)$ function can be expanded as

$$F(\theta) = \sum_{n=0}^{N/2} B_n\cos(n\theta) + \sum_{n=1}^{N/2} C_n\sin(n\theta)$$

where $B_n$ and $C_n$ are real coefficient to be determined, and N+1 is the number of unknowns for shape function. In the inversion procedure, the genetic algorithm is used to minimize the following cost function:

$$CF = \left\{\frac{1}{M}\sum_{m=1}^{M}\frac{\left|E_s^{exp}(\bar{r}_m)-E_s^{cal}(\bar{r}_m)\right|^2}{\left|E_s^{exp}(\bar{r}_m)\right|^2} + \alpha\left|F'(\theta)\right|^2\right\}^{1/2} \qquad (4)$$

where M is the total number of measurement points. $E_s^{exp}(\bar{r}_m)$ and $E_s^{cal}(\bar{r}_m)$ are the measured scattered field and calculated scattered field respectively. Note that the regularization term $\alpha|F'(\theta)|^2$ was added in (4). Please refer the references [6] for detail.

Genetic algorithms are the global numerical optimization methods based on genetic recombination and evolution in nature [8]. They use the iterative optimization procedures that start with a randomly selected population of potential solutions, and then gradually evolve toward a better solution through the application of the genetic operators: reproduction, crossover and mutation operators. As soon as the cost function $(CF)$ changes by <1% in two successive generations, the algorithm will be terminated and a solution is then obtained.

## III. Numerical Results

Let us consider a perfectly conducting cylinder array with the periodic length $d$ in free space and a plane wave of unit amplitude is incident upon the object, as shown in Fig. 1. The frequency of the incident wave is chosen to be 3 GHz; i.e., the wavelength $\lambda$ is 0.1 m. In the examples, the size of the scatterer is about one third the wavelength, so the frequency is in the resonance range.

To reconstruct the shape and periodic length of the cylinder, the object is illuminated by two incident waves with incident angles $\phi$ =45° and 135°, and the measurement points are taken on two lines with $Y = R'$ meter and $Y = -R'$

meters from x=-0.045 to 0.045 meters. Each line has nine measurement points. In our cases, $R'$ is chosen much larger than $2D'^2/\lambda$, where $D'$ is the largest dimension of the scatterer. Note that for each incident angle eighteen measurement points at equal spacing are used, and there are totally 36 measurement points in each simulation. The number of unknowns is set to 10 (i.e., N+2=10), to save computing time. The population size is chosen as 250 (i.e., $M$=250). The search range for unknown coefficient of the shape function is chosen to be from 0 to 0.1. The search range for unknown periodic length is chosen from 0.05 to 0.1. The extreme value of the coefficient of the shape function and periodic length can be determined by the prior knowledge of the objects. The crossover probability $p_c$ and mutation probability $p_m$ are set to be 0.8 and 0.04 respectively. The value of $\alpha$ is chosen to be 0.001.

In the example, the shape function is chosen to be $F(\theta)=(0.03+0.006\cos\theta+0.003\cos 2\theta+0.003\sin\theta)\,\mathrm{m}$ with periodic length $d$=0.09 m. The reconstructed shape function for the best population member (chromosome) is plotted in Fig. 2(a) with the error shown in Fig. 2(b), while the error for the reconstructed periodic length is also given in Fig. 2(b). Here *DR* and *PD*, which are called shape function and periodic length discrepancies respectively, are defined as

$$DR=\{\frac{1}{N'}\sum_{i=1}^{N'}[F^{cal}(\theta_i)-F(\theta_i)]^2/F^2(\theta_i)\}^{1/2}$$

$$PD=\frac{\left|d^{cal}-d\right|}{d}$$

where $N'$ is set to 100. Quantities *DR* and *PD* provide measures of how well $F^{cal}(\theta)$ approximates $F(\theta)$ and $d^{cal}$ approximates $d$ respectively. From Fig. 2, it is clear that the reconstruction of the shape function and periodic length is quite good. In addition, we also see that the reconstruction of shape function does not change rapidly toward the exact value until *PD* is small enough. This can be explained by the fact that the periodic length makes a stronger contribution to the scattered field than the shape function does.

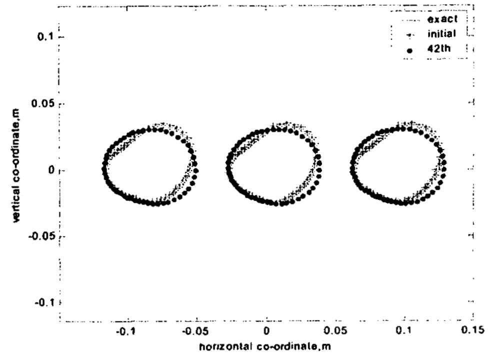


Fig. 2(a) Shape function for example 1. The solid curve, star curve and dot curve represent the exact shape, initial shape and reconstructed shape, respectively.

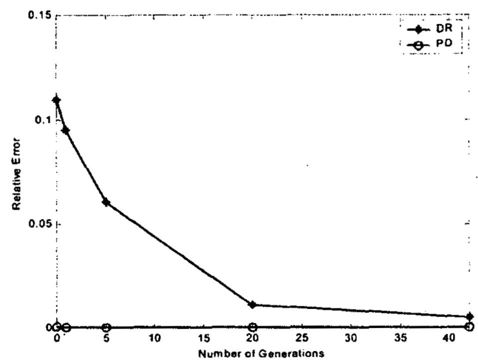


Fig. 2(b) Periodic length error and shape function error in each generation.

## IV. CONCLUSIONS

We have presented a study of applying the genetic algorithm to reconstruct periodic length and the shape of a periodic conducting cylinder through knowledge of scattered field. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization problem. By using the genetic algorithm, the periodic length and shape of the object can be reconstructed. Even when the initial guess is far away from exact, the genetic algorithm converges to a global extreme of the cost function. According to our experience, the main difficulties in applying the genetic algorithm to this problem are how to choose the parameters, such as the population size $(M)$, bit length of the string $(L)$, crossover probability $(p_c)$, and mutation probability $(p_m)$. Different parameter sets will affect the speed of convergence as well as the computing time required. From the numerical simulation, it is concluded that a population size from 100 to 300, a string length from 8 to 16 bits, and $p_c$ and $p_m$ in the ranges of $0.7 < p_c < 0.9$ and $0.0005 < p_m < 0.05$ are suitable for imaging problems of this type.

## V. REFERENCES


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