Image Reconstruction for the Partially Immersed Conductor by Dynamic Differential Evolution

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***Abstract* - The application of one technique for the reconstruction of shape reconstruction of a metallic cylinder from scattered field measurements is studied in this paper. Considering that the microwave imaging is recast as a nonlinear optimization problem, a cost functional is defined by the norm of a difference between the measured scattered electric field and that calculated for an estimated the shape of metallic cylinder. Thus, the shape of metallic cylinder can be obtained by minimizing the cost function. In order to solve this inverse scattering problem, dynamic differential evolution (DDE) is employed. The technique has been tested in the case of simulated measurement contaminated by additive white Gaussian noise. Numerical results demonstrate that, even when the initial guess is far away from the exact one, good reconstruction can be obtained.**

keywords - inverse scattering problem, partially immersed conductor, dynamic differential evolution, transverse electric wave.

I. INTRODUCTION

The solution of an electromagnetic inverse scattering problem has been an area of research for many years because of its wide variety of applications which include, for example, geological investigations of the earth structure, nondestructive evaluation and characterization of different food grains, imaging of human organs in biomedical microwave tomography, ground-penetrating radar, and possible detection of diseases in trees and forests.

It is well known that the main difficulties of inverse problems are the nonlinearity, ill-posedness and non-uniqueness [1]. Generally speaking, two kinds of approaches have been developed to solve the problem. The first is based on gradient searching schemes [2]. In these methods, the initial guesses are very important and they tend to get trapped in a local extreme. The second method is a global searching scheme, such as genetic algorithm. They tend to converge to the global extreme of the problem, no matter what the initial guesses are [3].

In microwave imaging, one tries to reconstruct the shape of an unknown object from the scattering microwave data measured outside. Dealing with a transverse electric (TE) illumination requires a more complex mathematical description, since the arising vectorial integral equations present derivatives of both the background Green's function and the field [4]. The TE polarization case is useful because it provides additional information in the scenario under test. Moreover, as pointed out in [5], the influence of the ill-posedness for TE inverse scattering is lower than that for transverse magnetic (TM) since the Green's function is more singular in the former case.

In this paper, the inverse scattering problem of the partially immersed perfectly conducting cylinder by TE wave illumination is investigated. We use the dynamic differential evolution to recover the shape of a partially immersed perfectly conducting cylinder. In section II, the theoretical formulation for the inverse scattering is derived. The numerical result for the object of different shape is presented in section III. Section IV gives the conclusions.

II. THEORETICAL FORMULATION

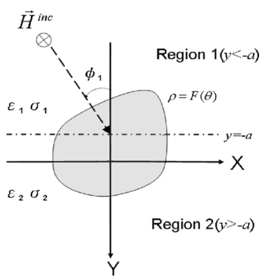

Figure 1. Geometry of the problem in (x,y) plane

A. Direct Problem

Let us consider a perfectly conducting cylinder which is partially immersed in a lossy homogeneous half-space, as shown in Fig. 1. Media in regions 1 and 2 are characterized by permittivities and conductivities (ε_1,σ_1) and (ε_2,σ_2) respectively. A perfectly conducting cylinder is illuminated by a TE plane wave. The cylinder is of an infinite extent in the z direction, and its cross-section is described in polar coordinates in the x, y plane by the equation $\rho = F(\theta)$. We assume that the time dependence of the field is harmonic with the factor $e^{j\omega t}$. Let $\vec{H}^{inc}$ denote the incident field form region 1 with incident angle ϕ_1 as follow:

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$$\vec{H}^{inc} = e^{-jk_1(y\cos\phi_1 + x\sin\phi_1)}\hat{z} \tag{1}$$

Owing to the interface between regions 1 and 2, the incident plane wave generates two waves that would exist in the absence of the conducting object. Thus, the unperturbed field is given by

$$\vec{H}^i = \begin{cases} [e^{-jk_1(y\cos\phi_1 + x\sin\phi_1)} + H_1 e^{-jk_1(-y\cos\phi_1 + x\sin\phi_1)}]\hat{z} & ,y \le -a \\ H_2 e^{-jk_2(y\cos\phi_2 + x\sin\phi_2)}\hat{z} & ,y > -a \end{cases} \tag{2}$$

where

$$H_1 = (\frac{Z_1 - Z_2}{Z_1 + Z_2})e^{2jk_1 a\cos\phi_1}$$

$$H_2 = \frac{2Z_1 e^{jk_1 a\cos\phi_1} e^{-jk_2 a\cos\phi_2}}{Z_1 + Z_2}$$

$$\phi_2 = \sin^{-1}\frac{k_1}{k_2}\sin\phi_1$$

$$Z_1 = \eta_1\cos\phi_1 \;,\; Z_2 = \eta_2\cos\phi_2 \;,\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \;,\; \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Since the cylinder is partially immersed, the equivalent current exists both in the upper half space and the lower half space [6].

For a perfectly conducting scatterer, the total tangential electric field at the surface of the scatterer is equal to zero.

$$\hat{n}\times(\frac{1}{j\omega\varepsilon}\nabla\times\vec{H}^{tot}) = 0 \tag{3}$$

with $\vec{H}^{tot} = \vec{H}^i + \vec{H}^s$

where $\hat{n}$ is the outward unit vector normal to the surface of the scatterer and $\vec{H}^s$ is the scattered fielded.

For the direct scattering problem, the scattered field H^s is calculated by assuming that the shape is known. For the inverse problem, assume the approximate center of scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function $F(\theta)$ can be expanded as:

$$F(\theta) = \sum_{n=0}^{N/2} B_n\cos(n\theta) + \sum_{n=1}^{N/2} C_n\sin(n\theta) \tag{4}$$

where B_n and C_n are real coefficients to be determined, and N+1 is the number of unknowns for the shape function. In the inversion procedure, the dynamic differential evolution is used to minimize the following cost function [7]:

$$CF = \{\frac{1}{M_t}\sum_{m=1}^{M_t}\left|H^s_{exp}(\vec{r}_m) - H^s_{cal}(\vec{r}_m)\right|^2 / \left|H^s_{exp}(\vec{r}_m)\right|^2\}^{1/2} \tag{5}$$

where M_t is the total number of measurement points. $H^s_{exp}(\vec{r}_m)$ and $H^s_{cal}(\vec{r}_m)$ are the measured and calculated scattered fields, respectively.

B. Dynamic Differential Evolution

DDE algorithm starts with an initial population of potential solutions that is composed by a group of randomly generated individuals which represents the dielectric constant, center position and the geometrical radiuses of the cylinders. Each individual in DDE algorithm is a *D*-dimensional vector consisting of *D* optimization parameters. After initialization, DDE algorithm performs the genetic evolution until the termination criterion is met. DDE algorithm relies on the genetic operations to evolve generation by generation. DDE algorithm goes through six procedures as follows:

- Initialize a starting population: DDE algorithm is initialized with a population that is composed by a group of randomly generated candidate individuals. Individuals in DDE algorithm represent a set of *D*-dimensional vectors in the parameter space for the problem, $\{x_i :\; i = 1,2,\cdots,Np\}$, where *D* is the number of parameters to be optimized and *Np* is the population size.
- Evaluate the population using cost function: After initialization, DDE algorithm evaluates the cost function (5) for each individual in the population.
- Perform mutation operation to generate trial vectors: The mutation operation of DDE algorithm is performed by arithmetical combination of individual. For each parameter vector x^g_j of the parent generation, a trial vector v^{g+1}_j is generated according to following equation:

$$(v^{g+1}_j)_i = (x^g_j)_i + F\cdot[(x^g_{best})_i - (x^g_j)_i] + \lambda\cdot[(x^g_{r1})_i - (x^g_{r2})_i] \tag{6}$$

Where $F > 0$ and $\lambda > 0$ both are scaling factors that control the amplification of the differential variation $(X^g_{best} - X^g_j)$ and $(x^g_{r1}) - (x^g_{r2})$. The in*dices i, r_1 and r_2 of* individuals are randomly chosen. The subscript *g* stands for the generation index of the parent generation. The *best* refer to the optimal individual in the parent population.

- Perform crossover operation with probability of crossover *Cr* to deliver crossover vectors: The crossover operation in DDE algorithm is performed to increase the diversity of the parameter vectors. The crossover operation in DDE algorithm allows to deliver the crossover vector u^{g+1}_j by mixing component of the current vector x^g_j and the trial vector v^{g+1}_j. It can be expressed as :

$$u^{g+1}_j = \begin{cases} (v^{g+1}_j)_i, & \zeta < Cr \\ (x^g_j)_i, & \zeta \ge Cr \end{cases} \tag{7}$$

where *Cr* is the probability of crossover, ζ is the random number generated uniformly between 0 and 1.

- Perform selection operation to produce offspring: Selection operation is conducted by comparing the current vector x^g_j with the crossover vector u^g_j. The vector with smaller cost function value is selected as a member of the next generation. Explicitly, the selection operation for the minimization problem is given by:

$$x_i^{g+1} = \begin{cases} u_i^g, & if \quad CF(u_i^g) < CF(x_i^g) \\ x_i^g, & otherwise \end{cases} \qquad (8)$$

DDE algorithm is carried out in a dynamic way: each parent individual would be replaced by its offspring if the offspring has obtained a better cost function value than its parent. Thus, DDE algorithm can respond the progress of population status immediately and yield faster convergence speed than the typical DE [8]. Based on the convergent characteristic of DDE algorithm, we are able to reduce the numbers of cost function evaluation and find the solution efficiently.

Stop the process and obtain the best individual if the termination criterion is satisfied, else go to step II.

Based on the convergent characteristic of DDE algorithm, we are able to reduce the numbers of cost function evaluation, reconstruct the microwave image efficiently.

III. NUMERICAL RESULTS

We illustrate the performance of the proposed inversion algorithm and its sensitivity to random noise in the scattered field. Let us consider a perfectly conducting cylinder buried in a lossless half-space ($\sigma_1 = \sigma_2 = 0$). The permittivity in each region is characterized by $\varepsilon_1 = \varepsilon_0$ and $\varepsilon_2 = 2.56\varepsilon_0$ respectively. The frequency of the incident wave is chosen to be 1GHz with incident angles ϕ_1 equal to -45°, 0° and 45°, respectively. The wavelength λ_0 is 0.3m. For each incident wave, 8 measurements are made at the points equally separated on a semi-circle with the radius of 3m in region 1. There are 24 measurement points in each simulation. We set the generation to be 500, c_1 and c_2 to be 1.3 and 2.8 respectively. The number of unknowns is set to be 7 (i.e., N+1=7), N_p set to be 30. The search range for the unknown coefficient of the shape function is chosen to be from 0 to 0.05, B_0 is chosen to be 0.07 to 0.15. Our purpose is to reconstruct the shape of the object by using the scattered field at different incident angles.

In this example, the shape function is chosen to be $F(\theta) = (0.1 + 0.04\cos 2\theta)$ m. The reconstructed shape function for the best population member is plotted in Fig. 2 with the shape error shown in Fig. 3. The reconstructed shape error is 5.6%. It is clear that the reconstructed result is good.

To investigate the effects of noise, we add to each complex scattered field a quantity b+cj, where b and c are independent random numbers having a uniform distribution over 0 to the noise level times the R.M.S value of the scattered field. Normalized standard deviations of 10^{-4}, 10^{-3}, 10^{-2} and 10^{-1}, respectively, are used in the simulations. Fig. 4 shows the reconstructed results under the condition that the scattered H fields to mimic the measurement data are contaminated by the noise. Here DF, which is called shape function discrepancies, is defined as

$$DF = \{\frac{1}{N'}\sum_{i=1}^{N'}[F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i)\}^{1/2} \qquad (9)$$

It is noted that the effect of noise is negligible for normalized standard deviations below 10^{-2}.

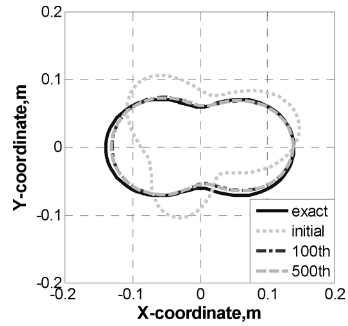

Figure 2. The reconstructed shape of the cylinder a different generations

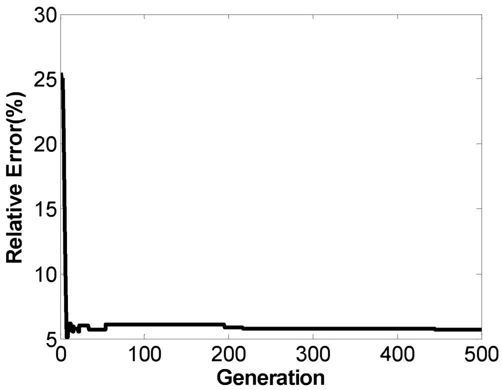

Figure 3. Shape function error versus generation

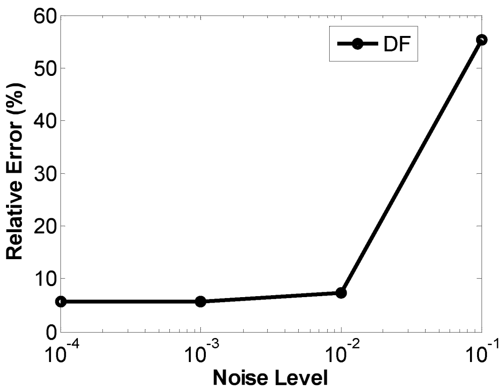

Figure 4. Shape function error as a function of noise levels

IV. CONCLUSION

We have presented a study of applying the dynamic differential evolution to reconstruct the shapes of a partially immersed conducting cylinder illuminated by TE waves. Numerical example has been given, and a good agreement between the exact and the reconstructed profiles has been achieved. The effect of noise on the overall reconstruction has been observed that the proposed method is able to provide good shape reconstruction.

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