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Planning Life Tests for Burr XII Distributed Products Under Progressive Group-censoring with Cost Considerations

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ABSTRACT

In this paper, a progressive type I group-censoring life test for Burr XII distribution is considered. We use the maximum likelihood method to obtain the point estimators of the Burr XII parameters. The approximate confidence intervals for the parameters of Burr XII distribution are also obtained. We use modified algorithm proposed by Kus *et al.* [20] to decide the number of test units, number of inspections, and length of inspection interval under a restricted budget of experiment such that the determinant of the asymptotic variances-covariance of estimators of parameters is minimum. A numerical example is presented to illustrate the proposed approach. The sensitivity analysis is also investigated.

Key words: D-Optimality, Grouped data, Interval censoring, Nonlinear mixed integer programming, Maximum likelihood method, Progressive censoring.

1. INTRODUCTION

In reliability analysis, censored sampling arises in a life test whenever the experimenter does not observe the lifetimes of all test units. The most common censoring schemes are type I censoring and type II censoring. They are used to reduce the test time and cost reduction. Censoring schemes usually give the exact failure times of some test units. However, in some situations, it is often impossible to observe the testing process continuously, even with censoring. The test units might be able to be inspected intermittently. That is, we can only record whether a test unit fails in an interval instead of measuring failure time exactly. Data of this type are called grouped data. In the literature, group-censored data have been studied by many researchers such as Cheng and Chen [16], Chen and Mi [15], Aggarwala [1], Qian and Correa [24], Xiang and Tse [28], Yang and Tse [29], Wu *et al.* [27] and Lu and Tsai [21] .

One of the most popular group censoring scheme is the progressive type I group-censoring. To conduct a progressive type I group-censored life test more efficiently, one has to address the problem of determining the number of test units, the number of inspections, and the length of the inspection intervals. In practice, the budget of a life experiment is limited. The size of the budget always affects the decisions of the number of test units, the

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number of inspections, and the length of inspection intervals and, hence, affects the precision of estimation. In this study, we will obtain the optimal settings of a progressive type I group-censored life test under the constraint that the total experimental cost does not exceed a pre-determined budget. In the literature, some researchers took cost considerations into account when reliability plans were designed. Some of them are Lui *et al*. [22], Tse *et al*. [25], Kuş *et al*. [19, 20], Akdoğan *et al*. [4], Kuş and Akdoğan [18], Akdoğan and Kuş [3] and Akdoğan [2] .

In this study, we assume that the lifetimes have a Burr XII distribution, which was constructed by Burr [11]. The Burr XII distribution has a non-monotone hazard function and with decreasing failure rates at larger times. This is similar to the log-normal distribution. Therefore, the Burr XII distribution can be considered as an alternative to the log-normal. Zimmer *et al.* [30] pointed out that the Burr type XII distribution has two advantages. They are: (1) the Burr cumulative distribution function and reliability function can be written in closed form; thus, it simplifies the computation of the percentiles and the likelihood function for censored data; (2) the Burr type XII distribution has algebraic tails which are effective for modeling failures that occur with lesser frequency than with corresponding models based on exponential tails. Hence, the Burr type XII distribution gives the reliability analyst another model for representing failure time data.

The main purpose of this study focuses on the designing problem of a progressive type I group-censored life test. We integrate the decision variables and the cost of the experiment to construct a mathematical model, and use the method of nonlinear mixed integer programming to obtain the optimal plans. The rest of the paper is organized as follows. In Section 2, we use the maximum likelihood method to obtain the point and interval estimators of the model parameters. In Section 3, an algorithm for obtaining the optimal plans is provided. In Section 4, a numerical example is discussed. In Section 5, results of the sensitivity analysis are presented. Finally, we conclude the paper in Section 6.

2. MODEL DESCRIPTION AND PARAMETER ESTIMATION

Let the lifetime of a particular unit have a Burr XII distribution with probability density function

$f(x) = \lambda\beta x^{\beta-1}\left(1+x^{\beta}\right)^{-(\lambda+1)}, \quad x>0,$ where λ and β are both shape parameters. The corresponding cumulative distribution function and hazard function are given respectively by

$$F(x) = 1-\left(1+x^{\beta}\right)^{-\lambda}, \quad x>0 \qquad (1)$$
$$h(x) = \lambda\beta x^{\beta-1}\left(1+x^{\beta}\right)^{-1}, \quad x>0.$$

Burr XII distribution can be used for modelling to describe biological, clinical or other experimental data, see for instance Burr [11] and Burr and Cislak [12]. It has also been applied in areas of quality control, reliability studies, duration and failure time modeling, see for example Papadopoulos [23] and more recently AL-Hussaini and Jaheen [5,6], Gupta et al. [17], and Ali Mousa and Jaheen [7-10].

Let us consider the following *k*-stage progressive type I group-censoring: n units are simultaneously placed on a life test, and run until time τ_1, at which point the number of failed units n_1 is counted and r_1 surviving units are removed from the test; starting from time τ_1, the $n-n_1-r_1$ non-removed surviving units are run until time τ_2, at which point the number of failures n_2 is counted and r_2 surviving units are removed from the test, and so on. At time τ_k, the number of failed units n_k is counted and the remaining $r_k = n-\sum_{i=1}^{k} n_i - \sum_{j=1}^{k-1} r_j$ surviving units are all removed, thereby terminating the test. This scheme may be depicted pictorially in Figure 1. In practice, one may assume that the lengths of inspection intervals are all equal; that is, $\tau_i - \tau_{i-1} = \tau$, $i = 1, 2, \ldots, k$. This assumption is convenient for practitioners. Thus, we will consider equal length of inspection interval throughout this paper.

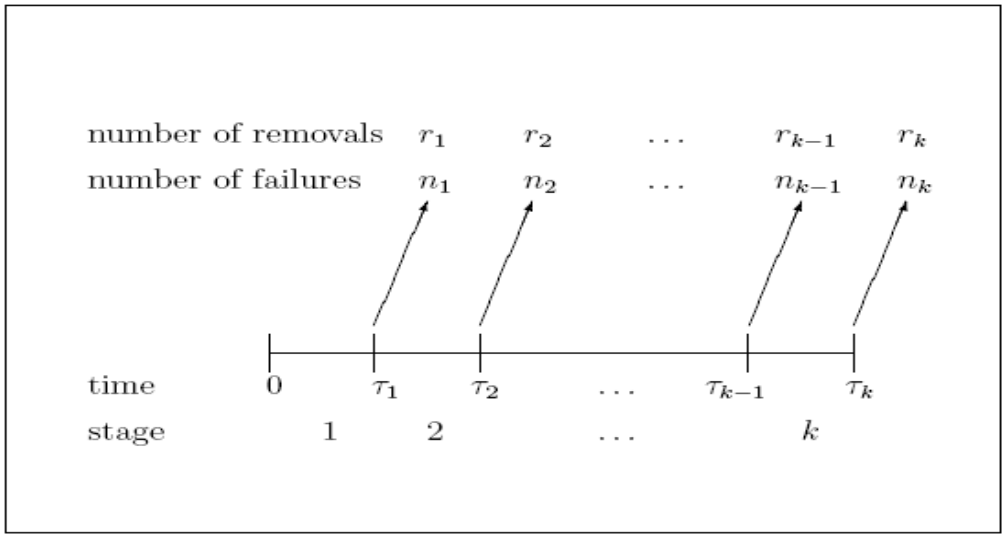

Figure 1. k -stage progressive type I group-censoring scheme

Note that the number of failed units n_i, and the number of removed units r_i are random variables. Generally, the values of $r_1, r_2, \ldots, r_k$ can be computed by the pre-determined percentages of the remaining live units $p_1, p_2, \ldots, p_k$ (with $p_k = 1$). That is, $r_i = (m_i - n_i)\, p_i$, where $m_1 = n$ and $m_i = n - \sum_{j=1}^{i-1} n_j - \sum_{j=1}^{i-1} r_j$, $i = 2,3,\ldots k$, are the number of non-removed surviving units at the beginning of the i -th stage. Under a progressive type I group-censoring scheme, we have the fact that

$$N_i \mid n_{i-1}, \ldots, n_1, r_{i-1}, \ldots, r_1 \sim Binomial(m_i, q_i),$$

where N_i is the random variable of observed value n_i and

$$q_i = \left(F(\tau_i) - F(\tau_{i-1})\right) / \left(1 - F(\tau_{i-1})\right) = 1 - \left[\left(1 + (i\tau)^{\beta}\right) / \left(1 + ((i-1)\tau)^{\beta}\right)\right]^{-\lambda}$$

is the probability that a unit survives at time τ_{i-1} and will fail before time τ_i, for $i = 1, 2, \ldots, k$, $\tau_0 = 0$, and the function $F(\cdot)$ is defined in Equation (1).

Given observations $(n_1, n_2, \ldots, n_k)$ and $(r_1, r_2, \ldots, r_k)$, the log-likelihood function can be written as

$$\log(L(\beta, \lambda)) \propto \sum_{i=1}^{k} n_i \log(q_i) + (m_i - n_i) \log(1 - q_i)$$

The maximum likelihood estimators (MLEs) of β and λ can be obtained by solving

$$\frac{\partial \log\left(L(\beta, \lambda)\right)}{\partial \beta} = \sum_{i=1}^{k} \frac{n_i}{q_i} \frac{\partial q_i}{\partial \beta} - \frac{(m_i - n_i)}{(1 - q_i)} \frac{\partial q_i}{\partial \beta}$$

and

$$\frac{\partial \log\left(L(\beta, \lambda)\right)}{\partial \lambda} = \sum_{i=1}^{k} \frac{n_i}{q_i} \frac{\partial q_i}{\partial \lambda} - \frac{(m_i - n_i)}{(1 - q_i)} \frac{\partial q_i}{\partial \lambda}.$$

Because the log-likelihood equations can not be solve analytically, one may employ Newton–Raphson method for finding the MLEs numerically.

Under some mild regularity conditions (see *e.g.*, Casella and Berger [13, p.516]), the property of asymptotic normality of the MLEs can be easily constructed. First, the second derivatives of the log-likelihood function are:

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\beta^2}=\sum_{i=1}^{k}\frac{n_i}{q_i^2}\left(\frac{\partial^2 q_i}{\partial\beta^2}q_i-\left(\frac{\partial q_i}{\partial\beta}\right)^2\right)-\frac{\left(m_i-n_i\right)}{\left(1-q_i\right)^2}\left(\frac{\partial^2 q_i}{\partial\beta^2}\left(1-q_i\right)+\left(\frac{\partial q_i}{\partial\beta}\right)^2\right)$$

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\lambda^2}=\sum_{i=1}^{k}\frac{n_i}{q_i^2}\left(\frac{\partial^2 q_i}{\partial\lambda^2}q_i-\left(\frac{\partial q_i}{\partial\lambda}\right)^2\right)-\frac{\left(m_i-n_i\right)}{\left(1-q_i\right)^2}\left(\frac{\partial^2 q_i}{\partial\lambda^2}\left(1-q_i\right)+\left(\frac{\partial q_i}{\partial\lambda}\right)^2\right)$$

and

$$\frac{\partial^2 \log\left(L(\beta,\lambda)\right)}{\partial\beta\partial\lambda}=\sum_{i=1}^{k}\left\{-\frac{n_i}{q_i^2}\frac{\partial q_i}{\partial\lambda}\frac{\partial q_i}{\partial\beta}+\frac{\partial^2 q_i}{\partial\beta\partial\lambda}\frac{n_i}{q_i}-\frac{\left(m_i-n_i\right)}{\left(1-q_i\right)^2}\frac{\partial q_i}{\partial\lambda}\frac{\partial q_i}{\partial\beta}-\frac{\partial^2 q_i}{\partial\beta\partial\lambda}\frac{\left(m_i-n_i\right)}{\left(1-q_i\right)}\right\},$$

where

$$\frac{\partial q_i}{\partial\lambda}=-\left(1-q_i\right)\frac{\log\left(1-q_i\right)}{\lambda}$$

$$\frac{\partial q_i}{\partial\beta}=\lambda\left(1-q_i\right)\left[\frac{\left((i\tau)^{\beta}\right)\log(i\tau)}{1+(i\tau)^{\beta}}-\frac{\left((i-1)\tau\right)^{\beta}\log\left((i-1)\tau\right)}{1+\left((i-1)\tau\right)^{\beta}}\right]$$

$$\frac{\partial^2 q_i}{\partial\lambda^2}=\left(\frac{\partial q_i}{\partial\lambda}\right)\frac{\log\left(1-q_i\right)}{\lambda}$$

$$\frac{\partial^2 q_i}{\partial\beta^2}=\lambda\left(1-q_i\right)\left[\frac{\left((i\tau)^{\beta}\right)\log(i\tau)\log(i\tau)}{\left(1+(i\tau)^{\beta}\right)^2}-\frac{\left((i-1)\tau\right)^{\beta}\log\left((i-1)\tau\right)\log\left((i-1)\tau\right)}{\left(1+\left((i-1)\tau\right)^{\beta}\right)^2}\right]-\left(\frac{\partial q_i}{\partial\beta}\right)^2\frac{1}{1-q_i}$$

and

$$\frac{\partial q_i}{\partial\beta\partial\lambda}=\frac{\partial q_i}{\partial\beta}\frac{1}{\lambda}-\frac{\partial q_i}{\partial\beta}\frac{\partial q_i}{\partial\lambda}\frac{1}{1-q_i}.$$

Thus, the Fisher information matrix is then obtained by taking the negative of the expectations of the second derivatives. That is,

$$\mathbf{I}(\beta,\lambda)=\begin{bmatrix}\sum_{i=1}^{k}E(M_i)\frac{\left(\partial q_i/\partial\beta\right)^2}{q_i\left(1-q_i\right)} & \sum_{i=1}^{k}E(M_i)\frac{\left(\partial q_i/\partial\beta\right)\left(\partial q_i/\partial\lambda\right)}{q_i\left(1-q_i\right)}\\ \sum_{i=1}^{k}E(M_i)\frac{\left(\partial q_i/\partial\beta\right)\left(\partial q_i/\partial\lambda\right)}{q_i\left(1-q_i\right)} & \sum_{i=1}^{k}E(M_i)\frac{\left(\partial q_i/\partial\lambda\right)^2}{q_i\left(1-q_i\right)}\end{bmatrix},$$

where M_i is the random variable of observed value m_i with $E(M_i) = n\prod_{j=1}^{i-1}(1-q_j)(1-p_j),\ i=1,2,\ldots,k.$ For a large sample size n, the MLEs $(\hat{\beta},\hat{\lambda})'$ have an approximate bivariate normal distribution with mean vector $(\beta,\lambda)'$ and variance-covariance matrix $\mathbf{I}^{-1}(\beta,\lambda)$. Thus, the approximate confidence intervals or confidence region for β and λ can be easily established.

3. OPTIMAL PLANS

To obtain a precise estimation of life parameters, frequently asked questions include `How many units does the experimenter need to test?', `How long does the experimenter need to run the life test?' or `How many times does the experimenter need to inspect the units in the life test?' Simply put, more test units, more test time, and more number of inspections will generate more information, which improves the precision of estimates. However, in practice, the budget of an experiment is limited. Therefore, the problem of obtaining a precise estimation of life parameters under a restricted cost of experiment is an important issue for the reliability analyst.

There are a lot of decision variables that affect the cost of experiment. The most important three decision variables are: (1) the number of test units, (2) the number of inspections, and (3) the length of inspection interval. Let n denote the number of units on test, k the number of inspections, and τ the length of the inspection interval. The cost of experiment consists of the following four parts.

(1) Installation cost: This is the cost of installing all test units at the beginning of the life experiment, say C_a. It does not depend on the number of test units.

(2) Sample cost: This is the cost of test units. Let C_s be the cost of a test unit. Then, the total sample cost is nC_s.

(3) Inspection cost: This cost includes the cost of using inspection equipment and material. It depends on the number of inspections. Let C_i denote the cost of one inspection. Then, the total inspection cost is kC_i.

(4) Operation cost: This includes salaries of operators, utilities, and depreciation of test equipment, etc. It is proportional to the testing time. Let C_o be the operation cost in the time interval between two inspections. Then, the total operation cost is $k\tau C_o$.

Therefore, the total cost of experiment is:

$$C_T = C_a + nC_s + kC_i + k\tau C_o.$$

Note that the asymptotic variance-covariance matrix $\mathbf{I}^{-1}(\beta,\lambda)$ of the MLEs $\hat{\beta}$ and $\hat{\lambda}$ is a function of n, k and τ. For a specific plan (n,k,τ), we can compute the asymptotic variance-covariance matrix of the MLEs. We want to determine the optimal plan (n,k,τ) under cost considerations. Since the parameter (β,λ) is two-dimensional, some possible optimality criteria include A-optimality, D-optimality, and E-optimality. The A-optimality, D-optimality, and E-optimality are to minimize the trace, the determinant, and the largest eigenvalue of the asymptotic variance-covariance matrix, respectively. Since D-optimality provides an overall measure of variability of the estimates, but the A-optimality and E-optimality do not implement all available information on the parameters, we only investigate D-optimality in this paper.

Let $G(n,k,\tau)$ be the determinant of the asymptotic variance-covariance matrix of the MLEs. Then, the optimal design problem can be formulated as follows:

$$\begin{aligned} \min \quad & G(n,k,\tau) \\ subject\ to \quad & C_a + nC_s + kC_i + k\tau C_o \le C_r \\ & k,n \in \square,\ k \ge 1\ and\ \tau > 0, \end{aligned}$$

where $\square$ is the set of positive integers and C_r is the budget of the experiment. Since the decision variables n and k are integer, the decision variable τ is real, and the objective function and constraint are both nonlinear functions of n, k, and τ, the nonlinear mixed integer programming can be used to find the optimal solution.

Kuş *et al*. [20] modified the algorithm proposed by Wu *et al*. [27] for finding the optimal solution as follows:

Algorithm

Step 1. Set the values of cost parameters C_a, C_s, C_i, C_o and C_r and give the values parameters (β, λ).

Step 2. Calculate the upper bound of the number of test units. Under the constraint of total experimental cost, the upper bound is

$$\tilde{n} = \left[\frac{C_r - C_a - C_i}{C_s} \right],$$

where $[x]$ is greastest integer that is less than or equal to x. Set $n = 2$.

Step 3. Compute the upper bound of the number of inspections for a given n. Using the constraint of total experimental cost and a given value of n, compute the upper bound

$$\tilde{k}_n = \left[\frac{C_r - C_a - nC_s}{C_i} \right].$$

Step 4. Compute the upper bound of the length of inspection interval. Using the constraint of total experimental cost, for all $k \in \mathrm{N}, \quad 1 \le k \le \tilde{k}_n$, compute the upper bound of the length of inspection interval $\tilde{\tau}_{kn} = \frac{C_r - C_a - nC_s - kC_i}{kC_o}$, and obtain $\tau' = \arg\min_{\tau} = G(n, k, \tau)$.

Step 5. Calculate the corresponding value of objective function $G(n, k, \tilde{\tau}_{kn})$ and $G(n, k, \tau')$.

Step 6. If $\tau' < \tilde{\tau}_{kn}$, set $\tau_{kn} = \tau'$, else $\tau_{kn} = \tilde{\tau}_{kn}$.

Step 7. Let function $\varphi(n) = G(n, k_n, \tau_{kn}) = \min_{1 \le k \le \tilde{k}_n} G(n, k, \tau_{kn})$.

Step 8. Set $n = n + 1$. If $n \le \tilde{n}$ go to Step 3 else go to Step 9.

Step 9. Compute the optimal value of objective function $\varphi(n^*) = \min_{2 \le n \le \tilde{n}} \varphi(n) = G(n^*, k^*, \tau^*) = \min_{2 \le n \le \tilde{n}} G(n, k_n, \tau_{kn})$.

Step 10. The optimal design (n^*, k^*, τ^*) is obtained.

4. NUMERICAL EXAMPLE

We apply the proposed methods to a numerical example. Assume that $n = 60$, $k = 6$, $\tau = 0.2500$, $\beta = 3$, $\lambda = 2$, $(p_1, p_2, p_3, p_4, p_5) = (0.05, 0.05, 0.05, 0.05, 1)$. We use an algorithm described in Aggarwala [1] to generate the progressively type I group-censored sample which are presented in Table 1.

Table 1.Progressively group-censored sample

i	1	2	3	4	5	6
n_i	1	8	11	20	8	5
r_i	2	2	1	0	0	2
p_i	0.05	0.05	0.05	0.05	0.05	1

The MLEs of λ and β are obtained as $\hat{\lambda} = 1.7765$ and $\hat{\beta} = 3.6538$, respectively. We use these estimates in the design of our new experiment. Assume that the percentages of removals are $(p_1 = p_2 = \cdots = p_{k-1} = 0.05)$ and $p_k = 1$. Suppose further that the values of cost parameters are as

follows: $C_a = 3, C_s = 75, C_i = 5.25, C_o = 6$ and $C_r = 5000$. Thus, the optimal design problem is:

$$\begin{aligned} &\min && G(n,k,\tau) \\ &subject\ to && 3 + 75n + 5.25k + 6k\tau \le 5000 \end{aligned}$$

According to the D-optimality criterion, one can obtain the optimal number of test units, number of inspections, and length of inspection interval are, respectively,

$$n^* = 65, \quad k^* = 17 \quad \text{and} \quad \tau^* = 0.2569.$$

5. SENSITIVITY ANALYSIS

The sensitivity study of the optimal solution to change in the values of the different parameters is an important issue to the planning of life test. These parameters consist of two parts: (1) the parameter of lifetime distribution, *i.e.*, β and λ; and (2) the parameters of experimental cost, *i.e.*, C_a, C_s, C_i, C_o and C_r. We now investigate the influence of the parameter of lifetime distribution and the parameters of experimental cost on the optimal solution, respectively.

In practice, the values of distribution parameters β and λ are usually unknown. We have to use prior information or data from a pilot study to get their estimates. Thus, one needs to investigate the effect of changing values of estimated parameters on the optimal solution. From Section 4, the MLEs of β and λ are obtained as $\hat{\lambda} = 1.7765$ and $\hat{\beta} = 3.6538$, respectively. The 95% approximate confidence interval for β is $(2.8201, 4.4875)$, and the 95% approximate confidence interval for and λ is $(1.2742, 2.2789)$. We choose different values of β and λ in their 95% approximate confidence intervals for sensitivity analysis. Let $(C_a, C_s, C_i, C_o, C_r)$ = $(3, 75, 5.25, 6, 5000)$ which are the values of cost parameters used in Section 4. We further assume that the pre-specified percentages of removals in each stage are all equal. That is, $p_1 = p_2 = \cdots = p_{k-1} = p$ and $p_k = 1$. The optimal solutions of n, k and τ for various values of β and λ are presented in Table 2. It shows that n is not sensitive to the changes in these values of parameters. The length of the inspection interval τ and the termination time of experiment $k\tau$ are decreasing functions of β and λ. The values of D-optimality is also increasing functions of β and λ.

Table 2. Optimal values of n, k and τ for fixed $C_a = 3, C_s = 75, C_i = 5.25, C_o = 6$ and $C_r = 5000$ and with $p = 0.05$

λ	β	n	k	τ	$k\tau$	D-optimality
1.2742	2.8201	65	17	0.3098	5.2666	0.0032
	3.6538	65	17	0.2905	4.9385	0.0056
	4.4875	65	17	0.2647	4.4999	0.0087
1.7765	2.8201	65	17	0.271	4.6070	0.0054
	3.6538	65	17	0.2569	4.3673	0.0093
	4.4875	66	7	0.244	1.7080	0.0146
2.2789	2.8201	65	18	0.2442	4.3956	0.0081
	3.6538	66	7	0.244	1.7080	0.0140
	4.4875	66	7	0.2358	1.6506	0.0222

Changes in cost parameters of the experiment can affect the determination of the optimal design. Let us consider the value of distribution parameters $\lambda = 3.6778$ and $\beta = 1.6048$. Using these values of the distribution parameters, the sensitivity of each of the decision variables n, k and τ to changes in the cost parameters of the experiment is examined. Tables 3 shows that a higher value of C_r causes a higher value of n; the number of test units is insensitive to changes in C_i, C_a and C_o; and a larger value of C_s results in a smaller value of n. In addition, the length of the inspection interval τ and the number of inspections k are sensitive to all cost parameters.

Table 3. D-Optimal values of n, k and τ for different costs values under $\lambda = 3.6778$ and $\beta = 1.6048$ with $p = 0.05$.

C_r	C_s	C_i	C_o	C_a	n	k	τ	Variance
1000	75	5.25	6	3	12	14	0.2572	0.2741
2000					26	6	0.3070	0.0605
3000					39	10	0.2614	0.026
4000					52	14	0.2572	0.0146
5000					65	17	0.2569	0.0093
6000					79	10	0.2614	0.0063
7000					92	14	0.2572	0.0047
5000	45	5.25	6	3	109	13	0.2576	0.0033
	55				89	15	0.2570	0.005
	65				76	8	0.2731	0.0069
	75				65	17	0.2569	0.0093
	85				58	9	0.2655	0.0118
	95				52	8	0.2731	0.0147
5000	75	4.50	6	3	66	7	0.2863	0.0092
		4.75			66	7	0.2863	0.0092
		5.00			66	7	0.2857	0.0092
		5.25			65	17	0.2569	0.0093
		5.50			65	17	0.2569	0.0093
		5.75			65	16	0.2569	0.0093
		6.00			65	16	0.2569	0.0093
5000	75	5.25	5	3	66	7	0.2569	0.0092
			6		65	17	0.2569	0.0093
			7		65	17	0.2569	0.0093
			8		65	16	0.2569	0.0093
			9		65	16	0.2569	0.0093
			10		65	15	0.2570	0.0093
			11		65	15	0.2570	0.0093
5000	75	5.25	6	1	66	7	0.2863	0.0092
				2	66	7	0.2679	0.0092

3	65	17	0.2569	0.0093
4	65	17	0.2569	0.0093
5	65	17	0.2569	0.0093
6	65	17	0.2569	0.0093
7	65	17	0.2569	0.0093

6. CONCLUSION

The decision problem of obtaining appropriate number of test units, number of inspections, and length of inspection interval under limited budget of life experiment is an important issue for practitioners. Wu *et al*. [27] use an algorithm based on nonlinear mixed integer programming to set tup the optimal design for progressive type I group censoring. Kuş et al. [20] modified their algorithm and this algorithm is used in this paper. By using this algorithm, we can obtain the optimal values of decision variables based on D-Optimality. Finally, the proposed approach can lead to better designs for conducting progressive type I group censoring life tests. It provides an efficient use of one's resources and to achieve the precision that one can expect to have with such a design. This approach is intuitive and can be useful to engineers.

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