The *k*-Barrier Coverage Mechanism in Wireless Visual Sensor Networks

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***Abstract*—Wireless Visual Sensor Networks (WVSNs) consist of a set of camera sensor nodes each of which equips with a camera and is capable of communicating with the other camera sensors within a specific distance range. As an extension of wireless sensor networks (WSNs), the WVSNs can provide richer information such as image and picture during executing targets monitoring and tracking tasks. Since the sensing area of each camera sensor is fan-shaped, existing barrier-coverage algorithms developed for WSNs cannot be applied to the WVSNs. This paper is considering to address the *k*-barrier coverage problems in WVSNs and to propose a barrier-coverage approach aiming at finding a maximal number of distinct defense curves with each of which consists of as few camera sensors as possible but still guarantees *k*-barrier coverage. Compared with the related work, experimental study reveals that the proposed *k*-barrier coverage mechanism constructs more defense curves than the *k*-barrier coverage and the number of camera sensors participating in each defense curve is smaller.**

Keywords- k-barrier coverage; visual sensor networks; wireless sensor networks

I. INTRODUCTION

The *k*-barrier coverage which definition is that a belt region with a sensor network deployed over it is said to be *k*-barrier coverage if and only if all crossing paths through the belt are *k*-covered, which means the crossing paths intersecting with the sensing range of at least *k* distinct sensors, by the sensor network [5]. In literature, the *k*-barrier coverage problem has been widely discussed in the wireless sensor networks (WSNs) in the past few years. As an extension of WSNs, the wireless visual sensor networks (WVSNs) consists of visual sensor nodes. Compared with the traditional sensor node in the WSNs, each visual sensor node in WVSNs equips with a camera which provides rich information. Different from traditional sensor, the sensing range of visual sensor node is a Field of View (FoV) in camera's lens, which can be viewed as a fan-shaped. Hence the existing barrier-coverage algorithms [1][2][5] developed for WSNs cannot be applied to the WVSNs.

In the past few years, barrier coverage problem in WVSNs has attracted much attention. Since the images of camera nodes will be further processed in the sink node, most of the existing approaches aim to reduce the number of active camera sensors for reducing the computing loads at the sink node. Ma et al. [3] proposed a deployment mechanism which maintains connectivity between cameras sensors. However, the authors did not consider how to construct a defense curves from beginning to the end. Zhang et al. [4] introduced the concepts of strong and weak camera barrier. The study initially transformed the barrier coverage problem into an integer linear programming formulation. Then, it proposed a barrier coverage mechanism where each cluster header constructs the defense curve by combining each fragment of defense curves. Nevertheless, the research did not take into consideration the *k*-coverage problem for $k \geq 2$. Furthermore, the proposed approach did not find out the defense curves as many as possible. Shih et al. [6] constructed the defense curves for barrier coverage according to the geographical relations of neighboring sensors. However, the discovered defense curves only support 1-coverd barrier which constrains the monitoring quality. In addition, there is only one defense curve constructed by the proposed approach. Camera sensors that participate in the defense curve should always work. A barrier coverage mechanism that intends to schedule different sets of camera sensors working in turns should construct more than one defense curves. Furthermore, the number of camera sensors that construct the defense curve can be further reduced.

This paper aims to develop a decentralized algorithm to cope with the *k*-barrier coverage problem. Initially, the network region will be partitioned into grids, aiming to simplify the *k*-barrier coverage problem. Then a *Basic Algorithm* (*BA*) is proposed aiming to construct a number of defense curves each is composed of minimum number of visual sensor nodes but supports *k*-barrier coverage. Based on the proposed *BA*, the *Branch Algorithm* (*BRA*) is further proposed for constructing more defense curve with the capability of *k*-barrier coverage. As a result, visual sensor nodes belonging to the constructed defense curves can be active in turn to achieve the load balance purpose. The remainder of this paper is organized as follows.

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Section 2 introduces the network environment and assumption. Section 3 gives the detailed description on how to select visual sensor nodes to form *k*-barrier coverage in an arbitrary deployed WVSN. Section 4 presents the simulation results while Section 5 concludes this paper.

II. NETWORK ENVIRONMENT AND ASSUMPTION

In this paper, the monitoring region of the WVSN is considered to a rectangle region *R*. The size of region *R* is $W \times L$, where *W* and *L* are the width and length of the region. The notations L_N, L_S, L_E, L_W denote the north, south, east and west boundaries of *R*. There are *n* visual sensor nodes $U=\{v_1,\ldots,v_w\}$ randomly deployed in the WVSN. Each visual sensor node v_x has a unique *ID* and is aware of its own location and the boundary coordinates of *R*. Furthermore, each v_x collects the *IDs* and location information of its neighboring visual sensor nodes through the exchange of the beacon with one hop neighbors. The movement trajectory that starts from L_S to L_N and crosses the width of *R* is called a valid crossing path in the WVSN. The monitoring region *R* is with *k*-barrier coverage if any valid crossing path in *R* is detected by at least *k* visual sensor nodes. Let DB^k be a defense barrier with degree *k*. The DB^k is composed of *k* disjoint defense barriers DB^1s that support *k*-barrier coverage. The following introduces the proposed *k-BCC* algorithm for constructing DB^k.

III. THE PROPOSED *K*-BARRIER COVERAGE CONSTRUCTION ALGORITHM (*K-BCC*)

The proposed *k-BCC* algorithm can be divided into two phases. In the Initialization phase, the network is partitioned into a number of equal-sized grids. The second phase, called *Barrier Construction* (*BC*) *phase*, aims to construct a defense barrier DB^k.

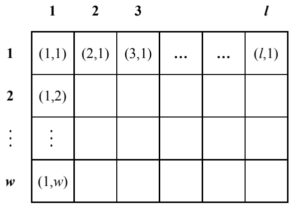

Figure 1: Grid-based network.

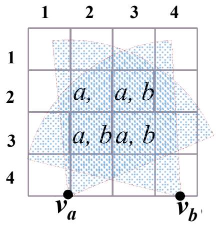

Figure. 2: *Fully covered grids of* v_a *and* v_b.

A. Initialization Phase

Initially, the network is partitioned into $w \times l$ equal-sized grids as shown in Fig. 1. Each grid is assigned with a coordinates (*m*, *n*). The rules for assigning coordinates are described below. The most left-up grid is initially assigned with coordinates (1, 1). As shown in Fig. 1, the *x*-coordinate and *y*-coordinate are increased by one if the location of a grid shifts one position toward right and down directions, respectively. Notation $g_{m,n}$ represents the grid with coordinates (*m*, *n*). Each visual sensor node in this phase will firstly identify the coordinates of the located grid. Since the sensing range can covers more than one grid, a grid might be commonly covered by several visual sensor nodes. Another important task of each visual sensor node in the *Initialization* phase is to evaluate the coverage degree of the grid it covers. The following defines *Fully Cover Grids of* v_x.

Definition: *Fully Covered Set of* v_x; G_x

A grid $g_{m,n}$ is a fully covered grid of v_x if the grid $g_{m,n}$ is fully covered by v_x. The *fully covered set of* v_x, denoted by G_x. consists of all grids that are fully covered by v_x. □

In Fig. 2, the symbols marked in each grid represent the *IDs* of the visual sensor nodes whose sensing ranges can fully cover that grid. As shown in Fig. 2, grids $g_{2,2}$, $g_{2,3}$, $g_{3,2}$, and $g_{3,3}$ are fully covered by v_a while grids $g_{2,3}$, $g_{3,2}$, and $g_{3,3}$ are fully covered by v_b. Therefore, we have $G_a=\{g_{2,2}, g_{2,3}, g_{3,2}, g_{3,3}\}$ and $G_b=\{g_{2,3}, g_{3,2}, g_{3,3}\}$. Since a field-of-view (FoV) angle and an orientation vector of camera lens are known by each visual sensor node, each v_x can evaluate its own G_x. Besides, each sensor v_x can evaluate the coverage degree of each grid $g_{m,n} \in G_x$ based on the neighboring information of v_x.

Herein, a grid $g_{m,n}$ is said to be *p*-covered if the number of coverage is *p* that *is* covered by *p* visual sensors. The partition of the network region into a number of equal-sized grids can simplify the *k*-barrier coverage problem.

B. Barrier Construction (BC) phase

The *Barrier Construction* (*BC*) *phase* aims to construct a DB^k based on the grid-based network depicted in the *Initialization Phase*. In conceptual level, the DB^k is composed of a sequence of interconnected *segments*. The *BC* phase will construct the DB^k *segment* by *segment* from the boundary L_W to the boundary L_E of the monitoring region *R*. Each *segment* is composed of several connected grids. These grids should satisfy the *k*-covered requirement which is contributed by a set of visual sensors, called *best working set*. Let the *starting grid* and *ending grid* of a *segment* be the leftmost and rightmost grids of the *segment*, respectively. To connect the neighboring *segments*, the *starting grid* of the successive *segment* should neighbor to the *ending grid* of the previous *segment*. Therefore, a DB^k can be constructed by a sequence of interconnected *segments*. For accomplish a DB^k, in each *segment*, a visual sensor v_x are selected to be the *Decision Maker* (*DM*) for executing the operations proposed in *BC* phase. Consider the grid $g_{m,n}$. The visual sensor nodes that can fully cover $g_{m,n}$ are called *DM* candidates of $g_{m,n}$. The *DM* of the $g_{m,n}$ is the *DM* candidate that has the largest number of neighbors. Note that, the *DM* of a $g_{m,n}$ is not necessarily located in the $g_{m,n}$ because that the sensing range of v_x might fully cover several grids. In the *BC* phase, two approaches, including *Basic Approach* (*BA*), *Basic* and *Branch Approach* (*BRA*), are proposed to implement the *BC* phase.

1) *Basic Approach(BA)*

The following proposes a *Basic Approach* (*BA*) for constructing a DB^k. Recall that, the DB^k is constructed in a manner of *segment* by *segment*. Let a grid $g_{m,n}$ that is currently considered by *BA* approach be called *current grid*, denoted by $g_{m,n}^{current}$. Let the *DM* of $g_{m,n}^{current}$ be called $DM_{m,n}^{current}$ (or $DM^{current}$ in short). The *BA* approach considers a grid $g_{m,n}^{current}$ at a time and tries to find a set of *k* visual sensors, which is also referred to as the *best working set* $\hat{q}_{m,n}^{k}$, to monitor the grid $g_{m,n}^{current}$ and the other grids covered by all visual sensors in the same $\hat{q}_{m,n}^{k}$. Let there be *w candidate working sets* of $g_{m,n}^{current}$, which are denoted by the notations $q_{m,n}^{k,j}$ where $1 \le j \le w$. Let $Q_{m,n}^{k,w} = \{q_{m,n}^{k,j} \mid 1 \le j \le w\}$ denote the set of *w candidate working sets*. The $DM_{m,n}^{current}$ will be responsible for selecting the *best working set* $\hat{q}_{m,n}^{k}$ from *set* $Q_{m,n}^{k,w}$. For selecting the $\hat{q}_{m,n}^{k}$, a process in *BA* called *Best Working Set Construction Process* is described below.

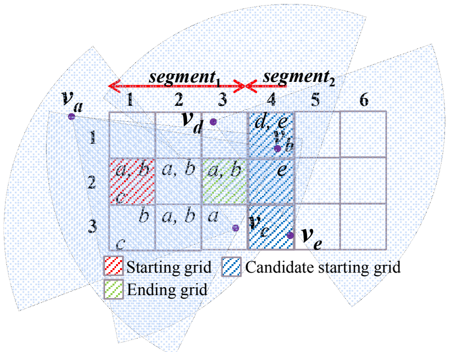

Figure 3: An example of constructing DB^2 by *BA*

a) <u>*Best Working Set Construction Process*</u> :

The following illustrates how $DM_{m,n}^{current}$ selects the *best working set* $\hat{q}_{m,n}^{k}$. The policy for determining the $\hat{q}_{m,n}^{k}$ is based on the contribution of each *candidate working set* $q_{m,n}^{k,j}$. The evaluation of contribution is described below. A grid $g_{\alpha,\beta}$ is an *extended grid* of $q_{m,n}^{k,j}$ if both grids $g_{\alpha,\beta}$ and $g_{m,n}^{current}$ are fully covered by all visual sensors in $q_{m,n}^{k,j}$. Let notation $g_{\alpha,\beta}^{k,j,m,n}$ denote an *extended grid* $g_{\alpha,\beta}$ of *candidate working set* $q_{m,n}^{k,j}$. Note that the $g_{m,n}^{current}$ is not an *extended grid* which belong to any $q_{m,n}^{k,j}$. The contribution is evaluated by relative position between $g_{m,n}^{current}$ and $g_{\alpha,\beta}^{k,j,m,n}$. The *weighted distance d* between relative positions is evaluated by Equ. (1).

$$d\left(g_{m,n}^{current}, g_{\alpha,\beta}^{k,j,m,n}\right) = (n-\beta) + (\alpha - m)\cdot Max(W,L) \qquad (1)$$

where *W* and *L* are the width and length of region *R*, respectively. The grid $g_{\alpha,\beta}^{k,j,m,n}$ closer boundary L_E has a larger *weighted distance d* and thus is considered has a larger contribution. This is because that the *weighted distance* is the length of barrier coverage contributed by the working set $q_{m,n}^{k,j}$. A larger *d* contributed by the *candidate working set* $q_{m,n}^{k,j}$ also means the defense barrier requires fewer visual sensor nodes. In Equ. (1), the multiplication of $Max(W,L)$ and $(\alpha - m)$ is to emphasize the distance increasing in the direction of *x*-axis since a straight barrier is the most appreciated in constructing the barrier. Let *farthest extended grid* $\hat{g}_{\alpha,\beta}^{k,j,m,n}$ be the *extended grid* $g_{\alpha,\beta}^{k,j,m,n}$ that is with highest *d*.

Applying Equ. (1), the $\hat{g}_{\alpha,\beta}^{k,j,m,n}$ located in upper right corner of region *R* will have the largest contribution than the other contribution of *extended grids* in $E_{m,n}^{k,j}$. Therefore, the *candidate working set* $q_{m,n}^{k,j}$ that can fully cover the grid $\hat{g}_{\alpha,\beta}^{k,j,m,n}$ will have the largest contribution and will be selected as the $\hat{q}_{m,n}^{k}$ by $DM_{m,n}^{current}$. Then the grids covered by all visual sensors in $\hat{q}_{m,n}^{k}$ will become *defense grids*. These selected *defense grids* can be treated as a constructed *segment* of DB^k.

Recall that the DB^k is constructed *segment* by *segment*. The *current grid* $g_{m,n}^{current}$ and the *farthest extended grid* $\hat{g}_{\alpha,\beta}^{k,j,m,n}$ will be the *starting grid* and *ending grid* of the constructed *segment*, respectively. The next step of *BA* approach is to construct the next *segment* which connects to the previous one. To accomplish this, the $DM_{m,n}^{current}$ will select a proper *starting grid* of the next *segment*. Let $g_{m',n'}^{next}$ be the *starting grid* of the next *segment*. Then the *best working set construction process* and the $g_{m',n'}^{next}$ *selection process* (discussed later), will be recursively performed to construct the next *segment*. As a result, the DB^k can be constructed by recursively executing the above-mentioned procedure until a constructed *segment* reaches the boundary L_E.

b) <u>*Start Grid Selection Process*</u> :

The policy for $DM_{m,n}^{current}$ to select the $g_{m',n'}^{next}$ is described below. Let the constructed *segment* be S_i and the next *segment* be denoted by S_{i+1}. The *starting grid* of *segment* S_{i+1} should be able to connect to the *ending grid* of *segment* S_i. The grids whose coordinate are $\hat{g}_{\alpha+1,\beta+v}^{k,j,m,n}$, where $-1 \le v \le 1$ are called the *candidate starting grids* of *segment* S_{i+1}. To fully support the *k*-barrier coverage, the selected *starting grid* $g_{m',n'}^{next}$ should be *k*-covered. The $DM_{m,n}^{current}$ will arbitrary select the *starting grid* $g_{m',n'}^{next}$ from the *candidate starting grids* of *segment* S_{i+1} where $g_{m',n'}^{next}$ is *k*-covered.

Figure 3 depicts an example of constructing DB^2. In Fig. 3, the grid $g_{1,1}$ is the *starting grid* where has *w*=3 *candidate working sets* $q_{1,2}^{2,1} = \{s_a, s_b\}$, $q_{1,2}^{2,2} = \{s_a, s_c\}$ and $q_{1,2}^{2,3} = \{s_b, s_c\}$. Besides, the *extended grids* of $q_{1,2}^{2,1}$, $q_{1,2}^{2,2}$ and $q_{1,2}^{2,3}$ are $\{g_{2,2}^{2,1,1,2}, g_{2,3}^{2,1,1,2}, g_{3,2}^{2,1,1,2}\}$, $\{\phi\}$ and $\{g_{1,3}^{2,3,1,2}\}$, respectively. The *farthest extended grid* of $q_{1,2}^{2,1}$, $q_{1,2}^{2,2}$ and $q_{1,2}^{2,3}$ are $g_{3,2}^{2,1,1,2}$, ϕ and $g_{1,3}^{2,3,1,2}$, respectively. By applying formula (1), the contribution of $q_{1,2}^{2,1}$, $q_{1,2}^{2,2}$ and $q_{1,2}^{2,3}$ are 12, $-\infty$ and 1, respectively. The *contributions* of these grids are evaluated based upon the following calculations.

$$q_{1,2}^{2,1} : d\left(g_{1,2}, \hat{g}_{3,2}^{2,1,1,2}\right) = (2-2) + (3-1) \cdot \max(3,6) = 12$$

$$q_{1,2}^{2,2} : d\left(g_{1,2}, \phi\right) = -\infty$$

$$q_{1,2}^{2,3} : d\left(g_{1,2}, \hat{g}_{1,3}^{2,1,1,2}\right) = (3-2) + (1-1) \cdot \max(3,6) = 1$$

As the result, the $q_{1,2}^{2,1} = \{s_a, s_b\}$ will be selected as the *best working set* by $DM_{1,2}^{current}$ for being in charge of monitoring the *segment*$_1$ between $g_{1,2}$ and $g_{3,2}$. Then the $DM_{1,2}^{current}$ will select the $g_{4,1}$ which is satisfied 2-covered to be the next start grid of segment. The recursive operation will keep running the above-mention procedure until a grid that reach the boundary L_E is selected and the DB^2 will be constructed by *BA*.

The major advantage of *BA* is simple and easy to be implemented. However, only those grids that are at least *k*-covered will be invited to join the DB^k. The next subsection proposes a *Branch Approach* aiming to fully utilize the grids that do not satisfy the *k*-covered requirement.

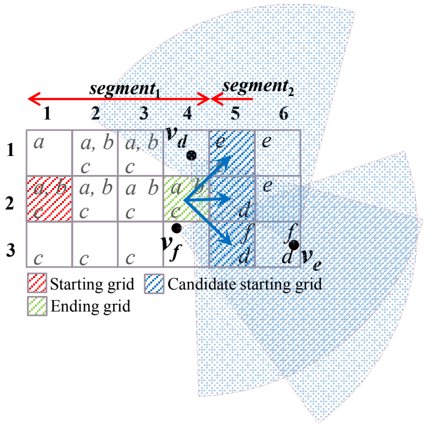

Figure 4: An example of the proposed *BRA*

2) *Branch Approach (BRA)*

The main idea of *BRA* is to give more opportunities for the neighboring grids to join the DB^k even though each of the neighboring grids contains less than *k*-covered. Since the random deployment might cause an imbalanced distribution of visual sensor nodes, the coverage degree in each grid might be different. Though some grids contain less than *k* coverage, they can contribute their potential coverage for constructing a DB^k. In constructing a DB^k, the *BA* did not consider those grids whose coverage degree is less than *k*.

Recall that, a curve with DB^k can have *k* branches DB_1^1, DB_2^1, …, DB_k^1 from some grids. Let *weak grid* represents the grid whose coverage degree is less than *k*. A grid with at least *k*-coverage is called *qualified grid*. The *BRA* can construct more defense barriers with the DB^k than *BA* by inviting *weak grids* to join the DB^k when all neighboring grids of $DM_{current}$ are *weak grids*. That is, the *BRA* gives more opportunities for the *weak grids* to join the DB^k and hence balances the workload of visual sensor nodes. Similar to *BA*, the *BRA* selects a *qualified grid* from the leftmost column to the rightmost column. As soon as a *DM* fails to find a *qualified grid* from its neighboring grids, it tries to select some *weak grids* $g_{m+1,n-1}$, $g_{m+1,n}$ and $g_{m+1,n+1}$ that satisfy Condition (2). Herein, we assume that the *DM* of $g_{m,n}$ cannot find any grid satisfying *k*-coverage requirement.

$$\left|C_{m+1,n-1} \cup C_{m+1,n} \cup C_{m+1,n+1}\right| \geq k \tag{2}$$

where the notation $C_{m+1,n-1}$, $C_{m+1,n}$ and $C_{m+1,n+1}$ denotes the numbers of different *ID*s in $g_{m+1,n-1}$, $g_{m+1,n}$ and $g_{m+1,n+1}$, respectively. The total number of these grids should greater than or equal to *k*-covered requirement. That is to say that these grids can be gathered as a set for achieving the *k*-covered requirement in column (*m*+1). However, The *BRA* aims to find a set of neighboring grids so that their coverage can cooperatively contribute *k*-coverage. In *BRA*, the *DM*s of the selected grids will execute the *BRA* until the boundary L_E is reached.

Figure 4 gives an example for constructing a DB^3 by applying *BRA*. Herein, We assume that the *segment*$_1$ between $g_{1,2}$ and $g_{4,2}$ is already covered by *best working set* $\{v_a,v_b,v_c\}$. Then the *DM* of *segment*$_1$ $DM_{1,2}^{current}$ will select the next starting grid from candidate starting grids. However, it cannot find any feasible neighboring grid that satisfies 3-covered requirement. Different from the *BA*, when the *BRA* fails to find a *qualified grid* from its neighboring girds, it considers the *weak grids* for constructing as more as possible DB^k. As shown in Fig. 4, the $DM_{1,2}^{current}$ initials a branch procedure trying to invite the *weak grids* $g_{5,1}$, $g_{5,2}$ and $g_{5,3}$ to satisfy the 3-coverage requirement. After $DM_{1,2}^{current}$ checks the total number of visual sensor nodes in *weak grids* $g_{5,1}$, $g_{5,2}$ and $g_{5,3}$, it sends a branch message which contains its own location and the required coverage degree for each selected *weak grid*. In this example, the *DM* of $g_{5,1}$, $g_{5,2}$ and $g_{5,3}$ will be notified that the required coverage degrees are 1. Upon obtaining the construction authority, the *DMs* of $g_{5,1}$, $g_{5,2}$ and $g_{5,3}$ individually apply the same *BRA* approach to construct the DB_1^1 , DB_2^1 and DB_3^1, respectively. By applying the *BRA*, a branched DB^2 can be successfully constructed.

The *BRA* approach not only improves the defense strength of barrier coverage but also balances the workload of visual sensor nodes.

Table 1. Simulation Parameters

Monitor Area	$400m \times 400m$
Number of *VSNs*	300, 400, 500, 600
Grid Size	1m, 2m, 3m
Sensing Radius	20 m
Comm. Radius	40 m
FoV	$\pi/3$, $\pi/2$, $2\pi/3$
Deployment	Random

IV. SIMULATION

This section studies the performance of the proposed *BA* and *BRA* approaches against the Maximum Disjoint Paths (*MDP*) mechanism which is a centralized algorithm proposed in [7]. The *MDP* finds *k* starting and terminal points at the left and right boundaries, respectively. For each combinational pair of starting and end points, *MDP* constructs a shortest path that is disjoint with the other paths constructed previously. The number of combination pairs will be k^2. Then the *MDP* selects the constructed *k* disjoint paths as the solution of DB^K. Since the *MDP* is a centralized algorithm with considering a large number of possible solutions, it will be treated as the optimal solution. To investigate how far the proposed *BA* and *BRA* approaches closed to the optimal solution in terms of number of participated visual sensors, the proposed three approached will be compared with the *MDP*. The simulation parameters used in our simulation is shown in Table 1.

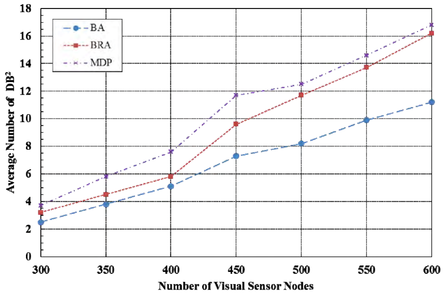

Figure 5: The average numbers of DB^2 constructed by applying *BA*, *BRA* and *MDP*.

Figure 5 investigates the average numbers of DB^2 constructed by applying *BA*, *BRA* and *MDP* by varying the number of deployed visual sensor nodes ranging from 300 to 600. The *BRA* adopts branch policy to further invite the weak grids participating in the defense barrier. Therefore, the *BRA* outperforms *BA* and approaches to the optimal performance produced by *MDP*.

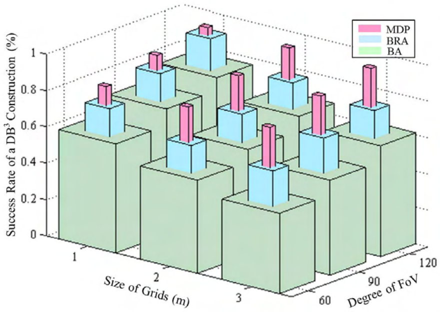

Figure 6: The success rate of constructing a DB^3 by applying *BA*, *BRA* and *MDP*

The size of grid and degree of FoV will impact the success rate of DB^k construction. Fig. 6 shows the success rate of a DB^3 construction by applying the proposed *BA*, *BRA* and the existing *MDP*. The success rate of constructing a DB^3 is decreased with the size of grid and is decreased with the degree of FoV. The major reason is that the smaller size of grids will increase the number of grids fully covered by each visual sensor node. As a result, there are more candidate working sets can be selected in each grid. Besides, larger degree of FoV can fully cover more grids and hence results in more candidate working sets can be considered. These candidate working sets can be further utilized by $DM^{current}$ to construct DB^k easier when the number of visual sensors is limited.

V. CONCLUSION

Barrier Coverage is an important issue in defense and intruder detection applications. This issue especially important and has a big challenge when the *k*-barrier coverage would be constructed by the visual sensors which improve the monitoring quality by providing the image information. This paper presents a *k*-barrier coverage algorithm with two barrier-construction policies, called *BA* and *BRA*. Initially the network region is partitioned into grids to simplify the investigated problem. Then a decentralized *BA* mechanism is proposed to cope with the *k*-barrier coverage problem. In addition to *BA*, the *BRA* mechanism that adopts branch police is proposed to further improve the performance of *BA*. Simulation study reveals that the proposed *BRA* outperforms *BA* and likely approaches to the optimal performance of constructing a DB^k in case of $k \geq 2$.

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